BIRNBAUM'S MEASURE OF COMPONENT IMPORTANCE FOR NON-COHERENT SYSTEMS

S. Beeson and J. D. Andrews; Department of Mathematical Sciences, Loughborough University; Loughborough, Leicestershire, UK

Keywords: Importance Measures, Non-coherent, Fault Trees, Structure Function

SUMMARY

Importance analysis of non-coherent systems is limited and generally inaccurate since all measures of importance that have been developed are strictly for coherent analysis. This paper considers the probabilistic measure of component importance developed by Birnbaum in 1969. An extension of this measure is proposed which enables non-coherent importance analysis. As a result of the proposed extension the expected number of system failures in a given interval for non-coherent systems can be calculated more efficiently.

1. NOTATION

$\phi(\underline{x})$	The Structure Function, which defines the system state in terms of the states of the system components.
$G_i(\underline{q})$	Birnbaum's measure of component reliability Importance
$G_i^*(\underline{q})$	Jackson's proposed extension of Birnbaum's measure
$G_i^R(\underline{q})$	Component repair criticality. The probability that the system is in a working state such that the repair of component ***i*** would cause system failure.
$G_i^F(\underline{q})$	Component failure criticality. The probability that the system is in a working state such that the failure of component ***i*** would cause system failure.
$Q_{SYS}(t)$	System Unavailability Function, which is the probability that the system is in a failed state at time ***t***.
$Q_{SYS}(1_i, \underline{q})$	The probability that the system fails with component ***i*** failed.
$Q_{SYS}(0_i, \underline{q})$	The probability that the system fails with component ***i*** working
q_i	Unreliability of component ***i***
p_i	Reliability of component ***i***
α	Indicator variable taking the value of either 0 or 1.
n	Total number of system components.
n_p	Total number of prime implicants
n_e	Total number of elements in a selected prime implicant
n_{crit}	Total number of critical system states
$w_{SYS}(t)$	System unconditional failure intensity
$W_{SYS}(0,t)$	Expected number of system failures in the interval $[0,t)$.
$w_i(t)$	Unconditional failure intensity of component ***i***.
$v_i(t)$	Unconditional repair intensity of component ***i***.
θ_i	Occurrence of prime implicant set ***i*** in the interval $[t, t+dt)$.

$B \equiv \bigcup_{i=1}^{n} \varepsilon_i$ Where, ε_i is the event that prime implicant ***i*** exists at time ***t***.

2. INTRODUCTION

Safety systems are designed to protect against hazardous events; if failure occurs on a potentially hazardous system the consequences can be disastrous. Many examples are possible one such is the recent crash of the Concord aeroplane in Paris, July 2000. This left all 113 passengers and crewmembers dead. Such disasters make clear the need to minimise the likelihood of system failure. Today reliability assessment plays a critical role in analysing and improving system safety.

Fault Tree Analysis [1,2] is a well known and widely used deductive technique developed by Watson in the early 1960's to enable reliability assessment of a wide variety of systems. A fault tree diagram expresses the causes of a particular system failure mode (top event) in terms of component failure modes that are connected by logical operators called gates.

The three fundamental gate types used in the fault tree are the AND gate, the OR gate and the NOT gate. Generally the use of the NOT gate is discouraged since a fault tree is non-coherent if the NOT gate is used or directly implied. In a non-coherent system component failed states and component working states can contribute to system failure. This may be considered philosophically to be a poor analysis as intuitively it is a bad design that has components working correctly contributing to system failure. From a practical viewpoint, NOT logic also increases the complexity of analysis and rarely provides additional information about the system. If only the AND gate and the OR gate types are used in the fault tree and all basic events represent failures then it is coherent and only component failures can contribute to system failure.

A fault tree is non-coherent if its structure function $\phi(\underline{x})$ does not comply with the definition of coherency given by the properties of relevance and monotonicity given below, see [3, 4]:

- Every component i is relevant.
  i.e.

$$\phi(1_i, \underline{x}) \neq \phi(0_i, \underline{x})$$

  For some $\underline{x}$

- Its structure function is monotonically increasing

$$\phi(1_i, \underline{x}) \geq \phi(0_i, \underline{x}) \quad \forall i$$

Where

$$\phi(1_i, \underline{x}) = \phi(x_1, ..., x_{i-1}, 1, x_{i+1}, ..., x_n)$$
$$\phi(0_i, \underline{x}) = \phi(x_1, ...x_{i-1}, 0, x_{i+1}, ..., x_n)$$

The first condition ensures that each component contributes to the system state. The second, an increasing[1] structure function, ensures that the system state deteriorates (at least does not improve) with increasing number of component failures. Component failures cannot improve the system state.

Although the use of NOT logic is often discouraged, Andrews [5] demonstrated that in the case of multi-tasking systems NOT logic is essential if successful and meaningful analysis is to be performed. This is also true for event tree analysis [6, 7, 8]. Hence it will be essential to consider NOT logic for such systems and be able to analyse resulting non-coherent fault trees efficiently and accurately.

Fault Tree Analysis can be split into two stages. The first is qualitative analysis, which identifies the minimal cut sets or for non-coherent fault trees the prime implicant sets. The second is quantitative analysis, which involves calculating the system unavailability and the system unreliability; it can also involve analysis of component and minimal cut set (prime implicant) importance. Importance analysis and Birnbaum's measure of component reliability importance in particular is the focus of this paper.

3. BIRNBAUM'S COMPONENT IMPORTANCE

When assessing a system its performance is dependent on that of its components. Some components will play a more significant role in causing or contributing to system failure than others. The concept of importance measures is to numerically rank the contribution of each component or basic event to reflect the susceptibility of the system to the occurrence of this event.

In 1969 Birnbaum [9] introduced the concept of importance and developed a probabilistic measure of component reliability importance. This measure is denoted by $G_i\left(\underline{q}\right)$ and defined as the ***probability that component i is critical to system failure***, i.e. when ***i*** fails it causes the system to pass from a working to a failed state. Birnbaum's measure is also referred to as the criticality function and is expressed as:

$$G_i\left(\underline{q}\right) = Q_{SYS}\left(1_i, \underline{q}\right) - Q_{SYS}\left(0_i, \underline{q}\right) \quad (1)$$

Where $Q_{SYS}\left(1_i, \underline{q}\right)$ is the probability that the system fails with component ***i*** failed and $Q_{SYS}\left(0_i, \underline{q}\right)$ is the probability that the system fails with component ***i*** working and $\underline{q}$ denotes the vector of component unreliabilities for the remaining components.

Although this field has received a great deal of attention over the last 30 years, the majority of measures that have been developed, have been developed specifically for the analysis of coherent systems, and therefore have ranked component failures. Importance analysis of non-coherent systems is extremely limited; it is generally inaccurate and

[1] Non-decreasing.

misleading because importance is approximated using the measures developed for the analysis of coherent systems.

In 1983 Jackson [10] considered the extension of some of the most commonly used measures of importance to enable analysis of non-coherent systems. Jackson began by developing an extension of Birnbaum's measure and then used this extension to extend a number of others measures based on Birnbaum's measure. Jackson's proposed extension of Birnbaum's measure is given below.

$$G_i^*(\underline{q}) = \left| Q_{SYS}(1_i, \underline{q}) - Q_{SYS}(0_i, \underline{q}) \right| \tag{2}$$

It is unclear exactly how this measure should be interpreted. Jackson considered a simple system introduced by Henley and Inagaki [11]. The system has three prime implicant sets, $\{X1, X2\}, \{X1, X3\}, \{X2, \overline{X3}\}$ the system unavailability function, the component unreliabilities assigned by Jackson and the results Jackson obtained are given in table 1 (where the component reliability $p_i = 1 - q_i$)

$$Q_{SYS}(t) = q_{X1}q_{X2} + q_{X1}q_{X3} + q_{X2}p_{X3} - q_{X1}q_{X2}q_{X3} - q_{X1}q_{X2}p_{X3}$$

$$q_{X1} = 9.90099 \times 10^{-3}, q_{X2} = 3.84615 \times 10^{-2}, q_{X3} = 1.52534 \times 10^{-1}$$

Event	Jackson’s Results	Ranking
$X1$	5.665×10^{-3}	4
$X2$	8.105×10^{-3}	3
$X3$	9.575×10^{-3}	2
$\overline{X3}$	3.839×10^{-2}	1

Table 1: The results obtained by Jackson

An alternative way of considering Birnbaum's measure for this same example is to consider component criticality by an exhaustive tabular approach. Consider a system with ***n*** components: the system state can then be expressed in terms of the component state. It is possible to determine whether a component is critical to system failure given the states of the remaining $n-1$ components. There are possible 2^{n-1} states of the other $n-1$ components. By identifying the critical situations for component ***i*** and summing their probability of occurrence it is possible to calculate the probability that component ***i*** is critical to system failure.

Thus for Jackson's example Table 2 identifies the critical states for each of the 3 components. Table 3 records the sum of the critical situations for each event and the final column records the probability that each event is critical to system failure.

State of X1	State of X2	Is X3 critical	State of X1	State of X3	Is X2 Critical	State of X2	State of X3	Is X1 critical
W	W	No	W	W	Yes (F)	W	W	No
W	F	Yes (R)	W	F	No	W	F	Yes (F)
F	W	Yes (F)	F	W	Yes (F)	F	W	No
F	F	No	F	F	No	F	F	Yes (F)

Table 2: Possible and critical states for the events

Event	Sum of Critical Situations	Expected Result	Ranking
X1	$p_{X2}q_{X3} + q_{X2}q_{X3}$	0.152534	2
X2	$p_{X1}p_{X3} + q_{X1}p_{X3}$	0.84747	1
X3	$q_{X1}p_{X2}$	0.00952	4
$\overline{X}3$	$p_{X1}q_{X2}$	0.03808	3

Table 3: Expected Results

Comparing Jackson's results in Table 1 to those given in Table 3 it is clear that not only does Jackson's extension calculate component criticality incorrectly but that it also ranks the components incorrectly. Hence it can be concluded that Jackson's extension is not conceptually equivalent.

4. EXTENSION OF BIRNBAUM'S MEASURE OF COMPONENT IMPORTANCE FOR NON-COHERENT ANALYSIS

Birnbaum's measure of component reliability importance (importance defined as the probability that component ***i*** is critical to system failure) is the fundamental probabilistic measure of. Many other measures of importance are extensions of this measure. Birnbaum developed this measure for the analysis of coherent systems only. It is calculated from the system unavailability function, $Q_{sys}(t)$, which is obtained using the exclusion-inclusion principle and Boolean reduction laws. $G_i(\underline{q})$ can be evaluated from equation (1) above which, since $Q_{SYS}(t)$ is linear in each q_i can be expressed as:

$$G_i(\underline{q}) = \frac{\partial Q_{sys}(t)}{\partial q_i(t)} \tag{3}$$

Since for coherent systems Birnbaum's measure is central to so many other measures of importance its extension to enable analysis of non-coherent systems needs to provide a consistent foundation to extend these measures for non-coherent analysis.

Birnbaum's measure calculates the probability that component ***i*** is critical to system failure. When dealing with a coherent system, system failure can only be caused by component failure. Hence a component in a coherent system can only be **failure critical**.

However, when dealing with a non-coherent system, system failure can be caused not only by component failure referred to as event ***i***, but also by component repair referred to as event $\bar{i}$, thus a component in a non-coherent system can be **failure critical** or **repair critical**. These two criticalities must be considered separately since component ***i*** can exist in only one state at any time

The probability required is the probability that component ***i*** is critical to system failure, which can be expressed as the probability that component ***i*** is repair critical $G_i^R\left(\underline{q}\right)$or the probability that component ***i*** is failure critical $G_i^F\left(\underline{q}\right)$

$$G_i(q) = G_i^R\left(\underline{q}\right) + G_i^F\left(\underline{q}\right) \tag{4}$$

By denoting the component failure probability by q_i and component repair probability by p_i, Henley and Inagaki's [11] calculation procedure can be used to calculate an expression for the system unavailability function of a non-coherent system.

$$Q_{SYS}(t) = \sum_{i=1}^{n_p} \Pr(\varepsilon_i) - \sum_{i=1}^{j-1} \sum_{j=i+1}^{n_p} \Pr\left(\varepsilon_i \cap \varepsilon_j\right) + ... + (-1)^{n_p - 1} \Pr\left(\varepsilon_1 \cap \varepsilon_2 \cap .... \varepsilon_{n_p}\right) \tag{5}$$

ε_i is prime implicant set ***i*** $i = 1,...,n_p$ with $\alpha = 1$ if event ***i*** is a member of ε_i or $\alpha = 0$ if event $\bar{i}$ is a member of ε_i.

$$\Pr\left(\varepsilon_i\right) = \prod_{j=1}^{n_e} q^{\alpha j}(t) \text{ and } q^{\alpha j}(t) = \begin{cases} q_j, \ if\ \alpha = 1 \\ p_j, \ if\ \alpha = 0 \end{cases}$$

Note: $p_i q_i = 0$, since any component can exist in only one state at any given time.

Component ***i*** is failure critical if the system is working but will fail if component ***i*** fails. Thus the probability that component ***i*** is failure critical is the probability that the system is in a working state such that the failure of component ***i*** causes at least one prime implicant set containing event ***i*** to occur. This probability is calculated by obtaining the probability that at least one prime implicant set containing event ***i*** exists at time ***t*** and then dividing this probability by the unreliability of component ***i***.

$$G_i^F\left(\underline{q}\right) = \frac{\Pr\left[\bigcup_{\substack{j=1 \\ i \in \varepsilon_j}}^{n_p} \varepsilon_j\right]}{q_i} \tag{6}$$

Similarly the probability that component ***i*** is repair critical is the probability that the system is in a working state such that the repair of component ***i*** causes at least one prime implicant set containing event $\bar{i}$ to occur. This is calculated by obtaining the probability that at least one prime implicant set containing event $\bar{i}$ exists at time ***t*** and then dividing this probability by the reliability of component ***i***.

$$G_i^R(\underline{q}) = \frac{\Pr\left[\bigcup_{\substack{j=1 \\ \bar{i}\in\varepsilon_j}}^{n_p} \varepsilon_j\right]}{p_i} \tag{7}$$

The top event can only exist at time ***t*** if at least one prime implicant set exists at time ***t***. Hence, the repair and failure criticality can be calculated separately by differentiating the system unavailability function, $Q_{SYS}(t)$ with respect to p_i and q_i respectively.

$$G_i^F(\underline{q}) = \frac{\partial Q_{SYS}(t)}{\partial q_i} \tag{8}$$

$$G_i^R(\underline{q}) = \frac{\partial Q_{SYS}(t)}{\partial p_i} \tag{9}$$

EXAMPLE

Given the following Boolean expression for the top event, $T = ab + ac + b\bar{c}$

Let component reliability be denoted by p_i and component unreliability be denoted by q_i for $i = a, b, c$. Then using the proposed method it is possible to calculate the repair and the failure importance of any component.

From equation (5)

$$Q_{SYS}(t) = q_a q_b + q_a q_c + q_b p_c - q_a q_b q_c - q_a q_b p_c \tag{10}$$

The failure importance for component ***c*** can be calculated from equation (8)

$$G_c^F(\underline{q}) = q_a - q_a q_b = q_a p_b \tag{11}$$

Similarly the repair importance for component ***c*** can be calculated from equation (9)

$$G_c^R(\underline{q}) = q_b - q_a q_b = q_b p_a \tag{12}$$

Hence from equation (4)

$$\boxed{G_c(\underline{q}) = p_a q_b + q_a p_b}$$

The result obtained can be checked by employing the tabular approach introduced earlier. There are 4 situations for which component ***c*** could be FAILURE or REPAIR critical to the system failure according to the states of components ***a*** and ***b***. Table 4 outlines the four situations and the final column records whether component ***c*** is critical to system failure.

State of component ***a***	State of component ***b***	Is component ***c*** critical
W	W	No
W	F	Yes (REPAIR)
F	W	Yes (FAILURE)
F	F	No

Table 4: Criticality assessment for component ***c***

From this table it is clear that component ***c*** is critical for 2 of the 4 situations hence Birnbaum's measure for component ***c*** is calculated as follows.

$$\boxed{G_c(\underline{q}) = \sum_{k=1}^{n_{crit}} \Pr(\text{Critical Situation k}) = p_a q_b + q_a p_b}$$

The result obtained using this tabular approach is the same as the result obtained using the proposed equation. The proposed extension calculates the probability that component ***i*** is critical to system failure. Having calculated the component repair and failure criticality, components need to be ranked and the results analysed; this will be considered in section 6.

5. EXPECTED NUMBER OF SYSTEM FAILURES

The expression for calculating the expected number of system failures, $W_{SYS}(0,t)$ in an interval $[0,t]$, when analysis is coherent, can be given in terms of Birnbaum's measure of component reliability importance.

$$W_{SYS}(0,t) = \int_0^t \sum_{i=1}^{n} G_i(\underline{q}) w_i(u) du \qquad (13)$$

Where $w_i(t)$ denotes the component unconditional failure intensity, and n denotes the total number of system components.

This identity can be extended to non-coherent systems as follows.

$$W_{SYS}(0,t) = \int_0^t \left[\sum_{i=1}^{n} G_i^F(\underline{q}) w_i(u) + \sum_{i=1}^{n} G_i^R(\underline{q}) v_i(u) \right] du \tag{14}$$

Where $v_i(t)$ denotes the component unconditional repair intensity.

The first term on the right hand side of equation (14) calculates the number of occurrences of system failure due to the failure of component ***i*** in a given interval. The second term calculates the number of occurrences of system failure in the given interval due to the repair of component ***i***.

If the proposed extension of Birnbaum's importance measure has the desired properties then equation (14) will hold. This section will test the proposed extension by considering a basic example and comparing the results obtained for $W_{SYS}(0,t)$ using a method developed by Inagaki and Henley [11] and using the above expression in equation (14).

The Henley and Inagaki procedure works directly with the Boolean expression for the top event obtained from qualitative analysis.

$$W_{SYS}(0,t) = \int_0^t w_{SYS}(u) du \tag{15}$$

$w_{SYS}(t)dt$ denotes the system unconditional failure intensity which is defined as the probability that the top event occurs in the interval $[t, t+dt)$, i.e. $w_{SYS}(t)$ is the probability that the top event occurs at ***t*** per unit time. The top event occurs in the interval $[t, t+dt)$ if and only if none of the prime implicants sets exist at time ***t*** and at least one prime implicant set occurs in the interval $[t, t+dt)$. The unconditional failure intensity is expressed in equation (16).

$$w_{SYS}(t)dt = \Pr\left\{ \bigcup_{i=1}^{n} \theta_i \right\} - \Pr\left\{ B \bigcup_{i=1}^{n} \theta_i \right\} \tag{16}$$

Where θ_i is the occurrence of prime implicant set ***i*** in the interval $[t, t+dt)$ and $B \equiv \bigcup_{i=1}^{n} \varepsilon_i$ where ε_i is the event that prime implicant ***i*** exists at time ***t***.

The first term represents the probability that one or more prime implicant sets occur in the interval $[t, t+dt)$. The second term is a correction term that calculates the probability that one or more prime implicant sets occur in the interval $[t, t+dt)$ but do not fail the system because it is already failed as one or more prime implicant sets already exist.

The unconditional failure intensity will be calculated for the example considered earlier with the Boolean expression for the top event.

$$T = ab + ac + b\bar{c}$$

The two terms of equation (16) are calculated separately. The first term on the right hand side of equation (16) can be expressed using the inclusion-exclusion expansion to give:

$$\Pr\left\{\bigcup_{i=1}^{3}\theta_i\right\} = \sum_{i=1}^{3}\Pr\{\theta_i\} - \sum_{i=1}^{2}\sum_{j=i+1}^{3}\Pr\{\theta_i\theta_j\} + \Pr\{\theta_1\theta_2\theta_3\}$$

$$\begin{aligned}
&= \Pr\{\theta_1\} + \Pr\{\theta_2\} + \Pr\{\theta_3\} - \Pr\{\theta_1\theta_2\} - \Pr\{\theta_1\theta_3\} \\
&\quad - \Pr\{\theta_2\theta_3\} + \Pr\{\theta_1\theta_2\theta_3\} \\
&= \left\{\begin{matrix} w_a q_b + w_b q_a + w_a q_c + w_c q_a + w_b p_c + v_c q_b \\ -\left(q_b q_c w_a + q_a p_c w_b\right) \end{matrix}\right\} dt \\
&= \left\{w_a\left(q_b + q_c - q_b q_c\right) + w_b\left(q_a + p_c - q_a p_c\right) + w_c q_a + v_c q_b\right\} dt
\end{aligned}$$

Similarly expanding the second term on the right hand side of equation (16) gives:

$$\Pr\left\{B\bigcup_{i=1}^{3}\theta_i\right\} = \sum_{i=1}^{3}\Pr\{\theta_i B\} - \sum_{i=1}^{2}\sum_{j=i+1}^{3}\Pr\{\theta_i\theta_j B\} + \Pr\{\theta_1\theta_2\theta_3 B\} \qquad (17)$$

Each term is also expanded about B, so for example expansion of the first term gives:

$$\begin{aligned}
\Pr\{\theta_1 B\} &= \Pr\{\theta_1\varepsilon_1\} + \Pr\{\theta_1\varepsilon_2\} + \Pr\{\theta_1\varepsilon_3\} - \Pr\{\theta_1\varepsilon_1\varepsilon_2\} \\
&\quad - \Pr\{\theta_1\varepsilon_1\varepsilon_3\} - \Pr\{\theta_1\varepsilon_2\varepsilon_3\} + \Pr\{\theta_1\varepsilon_1\varepsilon_2\varepsilon_3\}
\end{aligned} \qquad (18)$$

Considering other terms in the same way gives:

$$\Pr\{\theta_1 B\} = w_b q_a q_c + w_a q_b p_c$$

$$\Pr\{\theta_2 B\} = w_c q_a q_b$$

$$\Pr\{\theta_3 B\} = v_c q_a q_b$$

$$\sum_{i=1}^{j-1}\sum_{j=i+1}^{3}\Pr\{\theta_i\theta_j B\} = 0$$

$$\Pr\{\theta_1\theta_2\theta_3 B\} = 0$$

Hence the expected number of system failure is given below.

$$W_{SYS}(0,t)=\int_0^t w_{sys}(u)du$$

Where $\int_0^t w_{sys}(u)du=\int_0^t\begin{bmatrix} w_a\left(q_b+q_c-q_bq_c-q_bp_c\right)+ \\ w_b\left(q_a+p_c-q_ap_c-q_aq_c\right)+ \\ w_c\left(q_a-q_aq_b\right)+v_c\left(q_b-q_aq_b\right)\end{bmatrix}du$

Which simplifies to:

$$W_{SYS}(0,t)=\int_0^t\left[w_aq_c+w_bp_c+w_c\left(q_ap_b\right)+v_cq_bp_a\right]du$$

Now using the extended expression given in equation (14) to calculate the expected number of system failures.

The system unavailability function for this example is given in equation (10).

Hence

$G_a^F\left(\underline{q}\right)=q_b+q_c-q_bq_c-q_bp_c=q_c$

$G_a^R\left(\underline{q}\right)=0$

$G_b^F\left(\underline{q}\right)=q_a+p_c-q_aq_c-q_ap_c=p_c$

$G_b^R\left(\underline{q}\right)=0$

$G_c^F\left(\underline{q}\right)=q_a-q_aq_b=q_ap_b$

$G_c^R\left(\underline{q}\right)=q_b-q_aq_b=q_bp_a$

From equation (14) the expected number of system failures is:

$$\boxed{W(0,t)=\int_0^t w_{sys}(u)du=\int_0^t\left[w_aq_c+w_bp_c+w_cq_ap_b+v_cq_bp_a\right]du}$$

Notice the result obtained for the expected number of system failures is the same for both calculation procedures demonstrating that, firstly, the identity in (13) can be extended as shown in equation (14) for non-coherent analysis and, secondly, that the proposed measure calculates the desired probability.

6. GAS DETECTION SYSTEM EXAMPLE

Consider the simplified gas detection system in figure 3. This is a multitasking system introduced by Andrews [5]. This system has two sensors, D1 and D2, which detect leakage in a confined space. The detectors send signals along individual cables to the computer logic control unit, LU. If the LU receives a signal that there is a gas leak from any sensor three functions must be performed:

- Process shut down: de-energise relay R1
- Inform the operator of the leak by a lamp and siren Labelled L
- Remove the power supply to affected areas: de-energise relay R2

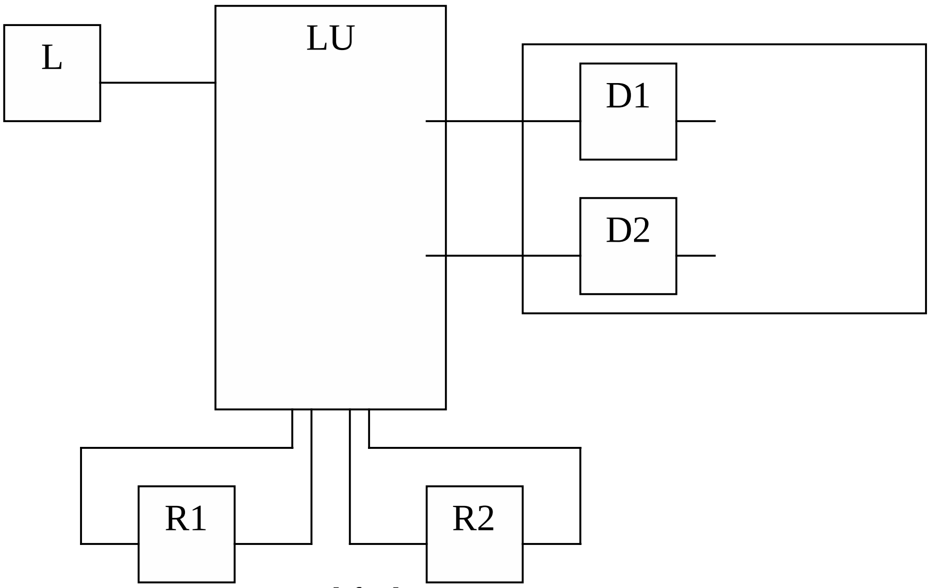

Figure 3: Simplified Gas Detection System

Andrews considered one particular failure scenario whereby, although the operator is informed of the gas release, both process shut down and power supply isolation fail. It was demonstrated that NOT logic was essential if successful analysis was to be performed. The fault tree obtained for this particular mode of failure has two prime implicant sets, $\left\{\overline{L}, \overline{LU}, R1, R2, \overline{D1}\right\}, \left\{\overline{L}, \overline{LU}, R1, R2, \overline{D2}\right\}$. In order to illustrate the method used to analyse component importance reliability values have been assigned to each component. Let, $q_L = 0.01$, $q_{LU} = 0.04$, $q_{R1} = q_{R2} = 0.06$, $q_{D1} = q_{D2} = 0.02$. The system unavailability function is:

$$Q_{SYS}(t) = p_L p_{LU} q_{R1} q_{R2} p_{D1} + p_L p_{LU} q_{R1} q_{R2} p_{D2} - p_L p_{LU} q_{R1} q_{R2} p_{D1} p_{D2}$$

From equations (8) and (9)

$$G_L^F(\underline{q}) = 0$$

$$G_L^R(\underline{q}) = P_{LU} q_{R1} q_{R2} (p_{D1} + p_{D2} - p_{D1} p_{D2}) = 0.00346$$

$$G_{LU}^F(\underline{q}) = 0$$

$$G_{LU}^R(\underline{q}) = p_L q_{R1} q_{R2} (p_{D1} + p_{D2} - p_{D1} p_{D2}) = 0.00358$$

$$G_{R1}^F(\underline{q}) = G_{R2}^F(\underline{q}) = p_L p_{LU} q_{R2} (p_{D1} + p_{D2} - p_{D1} p_{D2}) = 0.057$$

$$G_{R1}^R(\underline{q}) = G_{R2}^R(\underline{q}) = 0$$

$$G_{D1}^F(\underline{q}) = G_{D2}^F(\underline{q}) = 0$$

$$G_{D1}^R(\underline{q}) = G_{D2}^R(\underline{q}) = p_L p_{LU} q_{R1} q_{R2} (1 - p_{D2}) = 0.00007$$

Tables 5-7 record the results and the ranking obtained for the total criticality, the failure criticality and the repair criticality.

Event	Total Criticality	Ranking
L	0.00346	3
LU	0.00358	2
R1	0.057	1
R2	0.057	1
D2	0.00007	4
D1	0.00007	4

Table 5: Results and ranking for total criticality

Event	Failure Criticality	Ranking
L	0	N/A
LU	0	N/A
R1	0.057	1
R2	0.057	1
D1	0	N/A
D2	0	N/A

Table 6: Results and ranking for Failure Criticality

Event	Repair Criticality	Ranking
L	0.00346	2
LU	0.00358	1
R1	0	N/A
R2	0	N/A
D1	0.00007	3
D2	0.00007	3

Table 7: Results and Ranking for Repair Criticality

Table 5 records the total criticality of each component and the ranking obtained. From this table it is clear that system is most likely to be in a critical state for components R2 and R1. The importance of components LU and L are close numerically ranked 2nd and 3rd respectively. The failure and repair criticality of each component is given in, tables 6 and 7 respectively.

Components R1 and R2 are ranked highest and can only be failure critical. From this ranking it can be concluded that the system is most likely to be in a working but critical state for components R1 and R2. Should system performance be inadequate two steps can be taken to increase system reliability.

- Firstly, the likelihood of this critical state occurring for either R1 or R2 can be reduced. In general this can be achieved by increasing the reliability of any components whose failure is necessary for component ***i*** to be failure critical.
- Secondly, the reliability of components R1 and R2 can be increased to reduce the likelihood of either causing system failure.

Components LU and L were ranked 2nd and 3rd highest but both components can only be repair critical. Thus if the system is in such a state that components LU and L are repair critical, it is vital that they are not repaired to a working state until the system state changes and they are not repair critical. It is not appropriate to reduce the reliability of components that can be repair critical, instead,

- The probability of existence of the necessary and sufficient conditions for the component to be repair critical needs to be minimised.
- The repair of a component which can be repair critical needs to be carried out at an appropriate time, i.e. when it is not repair critical (other component failures repaired first).

6. CONCLUSIONS

This paper has introduced an extension of Birnbaum's measure of importance to enable non-coherent importance analysis. This extension calculates the probability that component ***i*** is critical to system failure by considering two types of criticality, failure and repair criticality.

This extension enables more efficient calculation of the system unconditional failure intensity and thus the expected number of system failures. An expression for calculating the expected number of system failures which is expressed in terms of Birnbaum's measure of importance can be extended to apply to non-coherent systems using the proposed extension to Birnbaum's measure.

Birnbaum's measure of component importance is central to many other measures of importance; hence its extension should make the derivation of other measures possible.

REFERENCES:

[1] Henley & Kumamoto, "**Reliability Engineering and Risk Assessment**", Englewood Cliffs, 1981

[2] J. D. Andrews & T. R. Moss, "**Reliability and Risk Assessment**", Longman Scientific and Technical, UK, 1993

[3] R. E. Barlow & F. Proschan, “**Mathematical Theory of Reliability**”, Wiley, 1965

[4] A. Bendell, & J. Ansell, “The incoherency of Multistate Coherent Systems”, **Reliability Engineering**, vol. 8, 1984, pp165-178

[5] J. D. Andrews, "To not or not to not", **Proceedings of the International System Safety Conference**, Fort Worth, Sept. 2000, pp 267-275

[6] I. A. Papazoglou, "Mathematical Foundations of Event Trees", **Reliability Engineering and System Safety**, Vol. 61, 1998, pp168-183

[7] S. Dunnett & J. D. Andrews, "Improving Accuracy in Event Tree Analysis", **Foresight and Precaution**, Cottam, Harvey, Pape & Tait (eds), **Proceedings of ESREL 2000, SARS and SRA-EUROPE annual Conference**, 15-17 May 2000

[8] J. D. Andrews & S. J. Dunnett, “Event Tree Analysis Using Binary Decision Diagrams”, **IEEE Transactions on Reliability**, vol 49 No.2, June 2000, pp230-239

[9] Z.W.Birnbaum, "On the importance of different components in a multi-component system", **Multivariate Analysis 11**, P.R.Krishnaiah, ed.,Academic Press, 1969

[10] P. S. Jackson, "One the s-importance of elements and prime implicants of non-coherent systems", **IEEE Transactions on Reliability**, vol. r-32, No.1, APR 1983

[11] Henley and Inagaki, Probabilistic evaluation of prime implicants and top events for non-coherent systems", **IEEE Transactions on Reliability**, vol. r-29, No.5, DEC 1980