Blind-Channel Identification for MIMO Single-Carrier Zero-Padding Block-Transmission Systems

Yi-Sheng Chen and Ching-An Lin

Abstract—**We propose a blind identification method for multiple-input multiple-output (MIMO) single-carrier zero-padding block-transmission systems. The method uses periodic precoding on the source signal before transmission. The estimation of the channel impulse response matrix consists of two steps: 1) obtain the channel product matrix by solving a lower-triangular linear system; 2) obtain the channel impulse response matrix by computing the positive eigenvalues and eigenvectors of a Hermitian matrix formed from the channel product matrix. The method is applicable to MIMO channels with more transmitters or more receivers. A sufficient condition for identifiability is simply that the channel impulse response matrix is full column rank. The design of the precoding sequence which minimizes the noise effect in covariance matrix estimation is proposed and the effect of the optimal precoding sequence on channel equalization is discussed. Simulations are used to demonstrate the performance of the method.**

Index Terms—**Blind identification, block transmission, multiple-input multiple-output (MIMO) channel, periodic precoding, zero padding (ZP).**

I. Introduction

MULTIPLE-INPUT multiple-output (MIMO) communication systems employing multiple transmit and receive antennas have received much attention due to the potential improvement in data transmission rate and link reliability they can offer. However, to exploit the potential advantage of MIMO systems, accurate channel state information is required. Channel can be identified or estimated using training signal which requires additional bandwidth. As a means to eschew the need of training signal and the associated bandwidth requirement, blind identification of MIMO channels has been the focus of much research. Many blind identification algorithms have been developed for various transmission systems (see [33]–[35] for a detailed review), including single-carrier (SC) block-transmission systems.

SC block-transmission systems can be generally classified into three kinds: the first with cyclic prefix (CP) insertion (SC-CP systems) [1]–[7], the second with zero padding (SC-ZP systems) [8]–[13], and the third with nonZP (SC-NZP) [36]–[38]. All these schemes are used to remove interblock interference (IBI) [8], [12]–[14], [36]. Discussions on these three kinds of systems can be found in [1]–[13], [36]–[38]. In this paper, we consider blind identification of the MIMO SC-ZP block-transmission system.

In the literature, to the best of our knowledge, there are two methods for blind identification of MIMO SC-ZP block-transmission systems. The subspace method [12] is suitable for white channel noise, while the eigen-decomposition method [13] can be used in the case when the channel noise is temporally and spatially colored.

Blind-channel identification using periodic precoding/modulation, originally proposed in [21], has become an active research area, because it imposes no restriction on the locations of channel zeros. Many different methods in this class have been proposed for SISO/MIMO series transmission systems [20]–[26]. Wu and Lee [7] is the first to apply periodic precoding for blind identification of SISO SC-CP block-transmission systems. Their method exploits the circulant structure for computational advantage in solving the channel product matrix. It is shown that channel impulse response can be identified up to a phase ambiguity by an eigenvalue-eigenvector decomposition.

We propose a method for blind identification of MIMO SC-ZP block-transmission systems with periodic precoding. The estimation of the channel impulse response matrix consists of two steps: 1) obtain the channel product matrix by solving a lower-triangular linear system; and 2) obtain the channel impulse response matrix by computing the positive eigenvalues and eigenvectors of a Hermitian matrix formed from the channel product matrix. The method is applicable to MIMO channels with more transmitters or more receivers. A sufficient condition for identifiability is simply that the channel impulse response matrix is full column rank. The design of the precoding sequence which minimizes the noise effect in covariance matrix estimation is proposed and the effect of the optimal precoding sequence on channel equalization is discussed. Simulations are used to demonstrate the performance of the method and to compare it with a subspace method [12]. Compared with the subspace method [12], the identifiability condition of the proposed method is more relaxed than the irreducible condition required in [12]. As a result, the proposed method is suitable for more practical scenarios.

This paper is organized as follows. Section II is the system model and problem statement. In Section III, we describe the identification method, discuss the design of precoding

Manuscript received January 8, 2007; revised June 14, 2007. First published February 2, 2008; last published July 10, 2008 (projected). This work was supported by the National Science Council under Grant NSC-93-2213-E009-041. This paper was recommended by Associate Editor P. Regalia.

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Digital Object Identifier 10.1109/TCSI.2008.916418

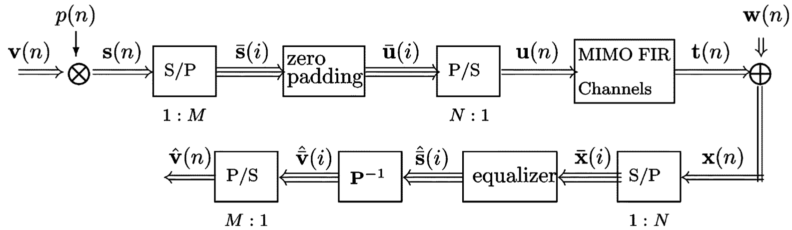

Fig. 1. MIMO SC-ZP block-transmission baseband model with periodic precoding.

sequences, and propose an identification algorithm. The effect of the precoding scheme on channel equalization is discussed in Section IV. Simulation results are given in Section V. Section VI concludes this paper.

Notations used in this paper are quite standard: Bold uppercase is used for matrices, and bold lowercase is used for vectors. $\mathbf{A}^T$ and $\mathbf{A}^*$ denote the transpose and conjugate transpose of $\mathbf{A}$, respectively. $\mathbf{I}_M$ denotes the identity matrix of dimension $M \times M$. $\mathbf{A}(r_1 : r_2, c_1 : c_2)$ is the submatrix formed from the r_1th row to the r_2th row and from the c_1th column to the c_2th column of $\mathbf{A}$. The symbols $\mathbb{R}$ and $\mathbb{C}$ stand for the set of real numbers and the set of complex numbers, respectively.

II. System Model and Problem Statement

Consider the K-input J-output discrete-time SC-ZP block-transmission baseband model shown in Fig. 1. At the transmitting end, the K-input signal vector $\mathbf{v}(n) = [v_1(n)v_2(n)\cdots v_K(n)]^T \in \mathbb{C}^K$ is first multiplied by a positive P-periodic sequence, $p(n) \in \mathbb{R}$, to obtain $\mathbf{s}(n) = p(n)\mathbf{v}(n)$, where $\mathbf{s}(n) \in \mathbb{C}^K$ is similarly defined as $\mathbf{v}(n)$ and $p(n+P) = p(n)$, $\forall n$. Then $\mathbf{s}(n)$ is passed through a serial-to-parallel block whose output is $\bar{\mathbf{s}}(i) = [\mathbf{s}(iM)^T\mathbf{s}(iM+1)^T\cdots\mathbf{s}(iM+M-1)^T]^T \in \mathbb{C}^{KM}$. Then $\bar{\mathbf{s}}(i)$ is zero padded as

$$\begin{aligned}\bar{\mathbf{u}}(i) &= [\ \underbrace{\bar{\mathbf{s}}(i)^T}_{M\text{ entries}}\ \underbrace{\mathbf{0}\cdots\mathbf{0}}_{P\text{ entries}}]^T\\ &= [\underbrace{\mathbf{u}(iN)^T\cdots\mathbf{u}(iN+M-1)^T}_{M\text{ entries}}\ \underbrace{\mathbf{0}\cdots\mathbf{0}}_{P\text{ entries}}]^T \in \mathbb{C}^{KN}\end{aligned} \tag{2.1}$$

where $N = M + P$. Finally, $\bar{\mathbf{u}}(i)$ is converted to $\mathbf{u}(n)$ via a parallel-to-serial block and transmitted through the MIMO FIR channel. At the receiving end, the J-output received signal vector $\mathbf{x}(n) = [x_1(n)x_2(n)\cdots x_J(n)]^T \in \mathbb{C}^J$ is $\mathbf{x}(n) = \mathbf{t}(n) + \mathbf{w}(n) = \sum_{m=0}^{L}\mathbf{H}(m)\mathbf{u}(n-m) + \mathbf{w}(n)$, where $\mathbf{t}(n)$ is the signal component at the output and $\mathbf{w}(n)$ is the channel noise seen at the receivers. Here $\mathbf{t}(n)$ and $\mathbf{w}(n)$ are similarly defined as $\mathbf{x}(n)$. $\mathbf{H}(m) \in \mathbb{C}^{J\times K}$ is the channel coefficient matrix whose jkth element $h_{jk}(m)$, $m = 0, 1, \ldots, L_{jk}$, is the impulse response from the kth transmitter to the jth receiver, and $L = \max_{j,k}\{L_{jk}\}$ is the order of the MIMO channel. We assume that $\mathbf{H}(L) \neq \mathbf{0}_{J\times K}$. Group the sequence of $\mathbf{x}(n)$ as $\bar{\mathbf{x}}(i) = [\mathbf{x}(iN)^T\mathbf{x}(iN+1)^T\cdots\mathbf{x}(iN+N-1)^T]^T \in \mathbb{C}^{JN}$ and define $\bar{\mathbf{w}}(i) \in \mathbb{C}^{JN}$ similarly as $\bar{\mathbf{x}}(i)$, we have

$$\bar{\mathbf{x}}(i) = \mathbf{H_0}\bar{\mathbf{u}}(i) + \mathbf{H_1}\bar{\mathbf{u}}(i-1) + \bar{\mathbf{w}}(i) \tag{2.2}$$

where $\mathbf{H_0}$ is a $JN \times KN$ block lower-triangular Toeplitz matrix with the first block column being $[\mathbf{H}(0)^T\mathbf{H}(1)^T\ \cdots\mathbf{H}(L)^T\mathbf{0}\cdots\mathbf{0}]^T \in \mathbb{C}^{JN\times K}$, and $\mathbf{H_1}$ is a $JN \times KN$ block upper-triangular Toeplitz matrix with the first block row being $[\mathbf{0}\cdots\mathbf{0}\mathbf{H}(L)\mathbf{H}(L-1)\cdots\mathbf{H}(1)] \in \mathbb{C}^{J\times KN}$.

The problem we study in this paper is blind identification of the MIMO channel coefficient matrices $\mathbf{H}(m)$, $0 \le m \le L$ using second-order statistics of the received data. We assume that the receivers are synchronized with the transmitters. In addition, the following assumptions are made throughout the paper.

(i) An upper bound $\hat{L}$ of the channel order L is known, $P = \hat{L} + 1$, and $M > P$ is a multiple of P.

(ii) The source signal $\mathbf{v}(n)$ is a zero mean white sequence with $E[\mathbf{v}(m)\mathbf{v}(n)^*] = \delta(m-n)\mathbf{I}_K$, where $\delta(\cdot)$ is the Kronecker delta function. The noise $\mathbf{w}(n)$ is white zero mean with $E[\mathbf{w}(m)\mathbf{w}(n)^*] = \delta(m-n)\sigma_w^2\mathbf{I}_J$. In addition, the source signal is uncorrelated with the noise $\mathbf{w}(n)$, i.e., $E[\mathbf{v}(m)\mathbf{w}(n)^*] = \mathbf{0}_{K\times J}, \forall m, n$.

(iii) The channel impulse response matrix $\mathbf{H} = [\mathbf{H}(0)^T\mathbf{H}(1)^T\cdots\mathbf{H}(L)^T]^T$ is full column rank, i.e., rank $(\mathbf{H}) = K$.

III. Blind-Channel Identification

In this section, we derive the proposed method under assumptions (**i**), (**ii**), and (**iii**). We propose an optimal design of the precoding sequence, which takes into account the noise effect in the estimation of covariance matrix of the received data, so as to increase the accuracy in the computation of the channel product matrix $\mathbf{HH}^*$ and thus reduce the channel estimation error. With the proposed optimal precoding sequence, the computation of $\mathbf{HH}^*$ becomes particularly simple. Taking eigen-decomposition of $\mathbf{HH}^*$, we obtain the channel impulse response matrix $\mathbf{H}$ up to a unitary matrix ambiguity. The proposed method can be used in the case of more transmitters $(K > J)$ or more receivers $(J \ge K)$.

A. Identification Method

We first derive the proposed method for the case where noise is absent, the channel order L is known, and there are more receivers, i.e., $J \ge K$. We assume that $P = L + 1$. We discuss the case where noise is present in Section III-B. The effect of channel order overestimation is discussed in Section III-E. The $K > J$ case is discussed in Section III-F.

Due to ZP of $\bar{\mathbf{u}}(i-1)$ and $\bar{\mathbf{u}}(i)$ [see (2.1)], we know $\mathbf{H_1}\bar{\mathbf{u}}(i-1) = \mathbf{0}$ and (2.2) can be written as $\bar{\mathbf{x}}(i) = \mathbf{H_e}\bar{\mathbf{s}}(i)$ (noiseless case), where $\mathbf{H_e} = \mathbf{H_0}(:, 1 : KM)$. Let $\mathbf{x}_f(i) = [\mathbf{x}(iN)^T\mathbf{x}(iN+1)^T\cdots\mathbf{x}(iN+L)^T]^T$ be the first $J(L+1)$ rows of $\bar{\mathbf{x}}(i)$. Then $\mathbf{x}_f(i) = \mathbf{H}_f\mathbf{s}_f(i)$, where $\mathbf{H}_f = \mathbf{H}_e(1 : J(L+1), 1 : K(L+1))$ and $\mathbf{s}_f(i) = [\mathbf{s}(iM)^T \mathbf{s}(iM+1)^T \cdots \mathbf{s}(iM+L)^T]^T$. Due to periodic precoding, we know $\mathbf{s}(iM+m) = p(m)\mathbf{v}(iM+m)$ for $m = 0, 1, \ldots, L$, and hence $\mathbf{x}_f(i) = \mathbf{H}_f\mathbf{s}_f(i)$ can be written as

$$\mathbf{x}_f(i) = \mathbf{H}_p\mathbf{v}_f(i) \tag{3.1}$$

where $\mathbf{v}_f(i)$ is similarly defined as $\mathbf{s}_f(i)$, and

$$\mathbf{H}_p = \begin{bmatrix} p(0)\mathbf{H}(0) & & \\ p(0)\mathbf{H}(1) & p(1)\mathbf{H}(0) & \\ \vdots & \vdots & \ddots \\ p(0)\mathbf{H}(L) & p(1)\mathbf{H}(L-1)\cdots & p(L)\mathbf{H}(0) \end{bmatrix}.$$

Define $\mathbf{S} \in \mathbb{R}^{J(L+1)\times J(L+1)}$ as the matrix whose first block sub-diagonal entries are all $\mathbf{I}_J$ (i.e., $\mathbf{S}(J+1 : J(L+1), 1 : JL) = \mathbf{I}_{JL}$), and all remaining entries are zero. Then $\mathbf{H}_p = [p(0)\mathbf{H}\ p(1)\mathbf{S}\mathbf{H}\cdots p(L)\mathbf{S}^L\mathbf{H}]$. Taking expectation of $\mathbf{x}_f(i)\mathbf{x}_f(i)^*$, we get the covariance matrix

$$\mathbf{R}_f = E[\mathbf{x}_f(i)\mathbf{x}_f(i)^*] = \mathbf{H}_p\mathbf{H}_p^*. \tag{3.2}$$

Since $\mathbf{H}_p = [p(0)\mathbf{H}\ \ p(1)\mathbf{S}\mathbf{H}\cdots p(L)\mathbf{S}^L\mathbf{H}]$, (3.2) can be written as

$$\mathbf{R}_f = \sum_{k=0}^{L} p(k)^2\mathbf{S}^k\mathbf{H}\mathbf{H}^*(\mathbf{S}^T)^k. \tag{3.3}$$

From [32, p.414], we know that the general matrix equation $\sum_{j=1}^{p}\mathbf{A}_j\mathbf{X}\mathbf{B}_j = \mathbf{C}$ can be equivalently expressed as a matrix-vector equation form, $\left[\sum_{j=1}^{p}\mathbf{B}_j^T\otimes\mathbf{A}_j\right]\mathrm{vec}(\mathbf{X}) = \mathrm{vec}(\mathbf{C})$, where $\mathrm{vec}(\cdot)$ is the vec-function which stacks up columns of a matrix. Hence, the matrix (3.3) can be written in the following vector form:

$$\begin{aligned}\mathrm{vec}(\mathbf{R}_f) &= \mathrm{vec}\left(\sum_{k=0}^{L} p(k)^2\mathbf{S}^k\mathbf{H}\mathbf{H}^*(\mathbf{S}^T)^k\right)\\ &= \left(\sum_{k=0}^{L} p(k)^2\mathbf{S}^k\otimes\mathbf{S}^k\right)\mathrm{vec}(\mathbf{H}\mathbf{H}^*)\\ &= \mathbf{G}\cdot\mathrm{vec}(\mathbf{H}\mathbf{H}^*).\end{aligned} \tag{3.4}$$

Here $\mathbf{G}$ is a block Toeplitz lower-triangular matrix shown as follows:

$$\begin{aligned}\mathbf{G} &= \sum_{k=0}^{L} p(k)^2\mathbf{S}^k\otimes\mathbf{S}^k\\ &= \begin{bmatrix} p(0)^2\mathbf{I}_{JF} & \mathbf{0} & \cdots & \mathbf{0}\\ p(1)^2\widehat{\mathbf{S}} & p(0)^2\mathbf{I}_{JF} & \cdots & \mathbf{0}\\ \vdots & \vdots & \ddots & \vdots\\ p(L)^2\widehat{\mathbf{S}}^L & p(L-1)^2\widehat{\mathbf{S}}^{L-1} & \cdots & p(0)^2\mathbf{I}_{JF}\end{bmatrix}\\ &\in\mathbb{R}^{F^2\times F^2},\end{aligned} \tag{3.5}$$

where $F = J(L+1)$ and $\widehat{\mathbf{S}} \in \mathbb{R}^{JF\times JF}$ is a block diagonal matrix with $\mathbf{S}$ on the diagonal blocks. Since $\mathbf{G}$ is square, the solution to (3.4) is

$$\mathrm{vec}(\mathbf{H}\mathbf{H}^*) = \mathbf{G}^{-1}\mathrm{vec}(\mathbf{R}_f) \tag{3.6}$$

provided $p(0) \neq 0$. The elements of the channel product matrix $\mathbf{H}\mathbf{H}^*$ obtained in (3.6) are then used to form a Hermitian matrix $\mathbf{Q}$ as follows:

$$\mathbf{Q} = \mathbf{H}\mathbf{H}^*. \tag{3.7}$$

Since rank $(\mathbf{H}) = K$ by assumption (**iii**), rank $(\mathbf{Q}) = K$. Since $\mathbf{Q}$ is Hermitian and positive semidefinite, $\mathbf{Q}$ has K positive eigenvalues, say, $\lambda_1, \ldots, \lambda_K$. We can expand $\mathbf{Q}$ as

$$\mathbf{Q} = \sum_{j=1}^{K}(\sqrt{\lambda_j}\mathbf{d}_j)(\sqrt{\lambda_j}\mathbf{d}_j)^* \tag{3.8}$$

where $\mathbf{d}_j$ is a unit norm eigenvector of $\mathbf{Q}$ associated with $\lambda_j > 0$. We can thus choose the channel impulse response matrix to be

$$\widehat{\mathbf{H}} = [\sqrt{\lambda_1}\mathbf{d}_1\sqrt{\lambda_2}\mathbf{d}_2\cdots\sqrt{\lambda_K}\mathbf{d}_K] \in \mathbb{C}^{J(L+1)\times K}. \tag{3.9}$$

We note that $\mathbf{H}$ can only be identified up to a unitary matrix ambiguity $\mathbf{U} \in \mathbb{C}^{K\times K}$, i.e., $\widehat{\mathbf{H}} = \mathbf{H}\mathbf{U}$, since $\widehat{\mathbf{H}}\widehat{\mathbf{H}}^* = \mathbf{H}\mathbf{H}^* = \mathbf{Q}$. The ambiguity matrix $\mathbf{U}$ is intrinsic to blind identification of multiple input systems using only second-order statistics technique [16]–[19]. The ambiguity can be resolved using a short pilot block sequence [12].

Remark: The method uses only $\mathbf{x}_f(i)$, the first $L+1$ received blocks of $\bar{\mathbf{x}}(i)$, to identify the channel product matrix $\mathbf{Q}$, since the lower-triangular structure of the sparse matrix $\mathbf{G}$ makes it easy to compute $\mathrm{vec}(\mathbf{H}\mathbf{H}^*)$, which can be seen in Section III-C. We can use more than $L+1$ block rows of $\bar{\mathbf{x}}(i)$ for identification. However, the computational load increases as more data are used. Computation of $\mathrm{vec}(\mathbf{H}\mathbf{H}^*)$ using more data is formulated in Appendix A.

B. *Optimal Design of the Precoding Sequence*

When the noise is present, the covariance matrix $\mathbf{R}_f$ contains the contribution of noise. Thus, (3.2) becomes

$$\mathbf{R}_f = E[\mathbf{x}_f(i)\mathbf{x}_f(i)^*] = \mathbf{H}_p\mathbf{H}_p^* + \sigma_w^2\mathbf{I}_F \tag{3.10}$$

where $F = J(L+1)$. In this case, (3.4) becomes

$$\mathrm{vec}(\mathbf{R}_f) = \mathbf{G}\cdot\mathrm{vec}(\mathbf{H}\mathbf{H}^*) + \sigma_w^2\mathrm{vec}(\mathbf{I}_F). \tag{3.11}$$

From (3.6), an approximate solution of $\mathrm{vec}(\mathbf{H}\mathbf{H}^*)$ is

$$\mathrm{vec}(\widehat{\mathbf{H}\mathbf{H}^*}) = \mathbf{G}^{-1}\mathrm{vec}(\mathbf{R}_f). \tag{3.12}$$

It follows from (3.12) and (3.11) that

$$\mathrm{vec}(\widehat{\mathbf{H}\mathbf{H}^*}) = \mathrm{vec}(\mathbf{H}\mathbf{H}^*) + \sigma_w^2 \mathbf{z} \tag{3.13}$$

where the vector $\mathbf{z} = \mathbf{G}^{-1}\mathrm{vec}(\mathbf{I}_F) = [z_1 z_2 \cdots z_{F^2}]^T$ in (3.13) is the solution of $\mathbf{G}\mathbf{z} = \mathrm{vec}(\mathbf{I}_F)$. Since the matrix $\mathbf{G}$ is completely determined by the precoding sequence $p(n)$, we seek to choose $p(n)$ so that $\|\mathbf{z}\|_2^2$ is minimized. To this end, we need to analyze the relations between $\mathbf{z}$ and $p(n)$. By expanding the matrix equation $\mathbf{G}\mathbf{z} = \mathrm{vec}(\mathbf{I}_F)$, we find that for $i = 1 + k(F+1)$, $k = 0, 1, \ldots, J-1$

$$\begin{cases} p(0)^2 z_i = 1 \\ \sum_{n=0}^{1} p(n)^2 z_{i+(1-n)J(F+1)} = 1 \\ \sum_{n=0}^{2} p(n)^2 z_{i+(2-n)J(F+1)} = 1 \\ \vdots \\ \sum_{n=0}^{L} p(n)^2 z_{i+(L-n)J(F+1)} = 1 \end{cases} \tag{3.14}$$

and $z_j = 0$ for all other indexes j. We write (3.14) as the following matrix equation,

$$\underbrace{\begin{bmatrix} g_0 & 0 & \cdots & 0 \\ g_1 & g_0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ g_L & g_{L-1} & \cdots & g_0 \end{bmatrix}}_{\mathbf{G_s}} \underbrace{\begin{bmatrix} m_0 \\ m_1 \\ \vdots \\ m_L \end{bmatrix}}_{\mathbf{m}} = \underbrace{\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}}_{\mathbf{y}} \tag{3.15}$$

where $\mathbf{G_s}$ is a lower-triangular Toeplitz matrix, $g_n = p(n)^2$ for $n = 0, 1, \ldots, L$, and $m_j = z_{i+jJ(F+1)}$ for $j = 0, 1, \ldots, L$, $i = 1 + k(F+1)$, $k = 0, 1, \ldots, J-1$. Hence, $\mathbf{G}\mathbf{z} = \mathrm{vec}(\mathbf{I}_F)$, the relations between $\mathbf{z}$ and $p(n)$, is reduced to (3.15), and minimization of $\|\mathbf{z}\|_2^2$ is equivalent to minimization of $\|\mathbf{m}\|_2^2$, which is a nonlinear function of $g_0, g_1, \ldots, g_L$. Then the problem is to minimize $\|\mathbf{m}\|_2^2$ by choosing $g_0, g_1, \ldots, g_L$, subject to suitable constraints. Specifically, we formulate the problem as

$$\mathrm{Minimize}_{g_0, g_1 \ldots, g_L} \|\mathbf{m}\|_2^2 \text{ subject to}$$

$$g_n \geq \tau > 0, \quad \forall 0 \leq n \leq L \tag{3.16}$$

$$\frac{1}{L+1} \sum_{n=0}^{L} g_n = 1. \tag{3.17}$$

Roughly, constraint (3.16) requires that at each instant, the power gain ($g_n = p(n)^2$) is no less than τ with $0 < \tau < 1$; constraint (3.17) normalizes the power gain of the precoding sequence of each transmitter to 1.

It is easy to show that for $L = 1$, the problem has a unique global minimizer given by $g_0 = 2 - \tau$ and $g_1 = \tau$. For general $L \geq 2$ case, the standard Kuhn-Tucker conditions of the nonlinear minimization problem do not seem to yield easily a unique analytical solution. However, the problem can be easily solved numerically (for fixed L and τ), say, using the *Matlab Optimization Toolbox*. Extensive numerically solutions, with different L, τ, and initial guesses, have indicated that a global minimizer exists and is given by

$$g_0 = L + 1 - L\tau, \quad g_1 = g_2 = \cdots = g_L = \tau. \tag{3.18}$$

In the following, we show that the solution (3.18) is also the global minimizer of an upper bound of $\|\mathbf{m}\|_2^2$. We know $\|\mathbf{m}\|_2^2 = \|\mathbf{G_s}^{-1}\mathbf{y}\|_2^2 \leq \|\mathbf{G_s}^{-1}\|_2^2 \cdot \|\mathbf{y}\|_2^2 = (L+1)\|\mathbf{G_s}^{-1}\|_2^2$, where $\|\mathbf{G_s}^{-1}\|_2$ is the 2-induced norm of $\mathbf{G_s}^{-1}$. Since $\mathbf{G_s}$ is triangular and Toeplitz, it follows from [29] that for any fixed integer $L \geq 1$

$$\begin{aligned} \left\|\mathbf{G_s}^{-1}\right\|_2^2 &\leq \frac{(\alpha+2)^{2(L+1)} + 2(L+1)(\alpha+2) - 1}{(\alpha+2)^2\beta^2} \\ &\triangleq f(\alpha, \beta) \end{aligned} \tag{3.19}$$

where $\alpha = \max_{i=1,2,\ldots,L} |g_i/g_0|$ and $\beta = |g_0|$. Hence, we know $\|\mathbf{m}\|_2^2 \leq (L+1)f(\alpha,\beta)$. Since for any $\alpha > 0$ and $\beta > 0$, $\partial f(\alpha,\beta)/\partial\alpha > 0$ (see Appendix B) and $\partial f(\alpha,\beta)/\partial\beta = -2/\beta f(\alpha,\beta) < 0$, we know for any fixed $\beta > 0$, $f(\alpha,\beta)$ is an increasing function of α, and for any fixed $\alpha > 0$, $f(\alpha,\beta)$ is a decreasing function of β. Hence, to minimize $f(\alpha,\beta)$, we should choose α as small as possible and choose β as large as possible subject to $\beta \leq L + 1 - L\tau$ and $\alpha \geq \tau/L + 1 - L\tau$. It follows that (3.18) is a global minimizer of the upper bound $(L+1)f(\alpha,\beta)$.

Since $g_n = p(n)^2$ and $p(n) > 0$, the optimal precoding sequence is

$$p(n) = \begin{cases} \sqrt{L+1-L\tau}, & n = 0 \\ \sqrt{\tau}, & 1 \leq n \leq L. \end{cases} \tag{3.20}$$

We consider next the effect of τ on channel identification. From (3.15) and [27], [28], we know $\mathbf{m} = \mathbf{G_s}^{-1}\mathbf{y}$, where $\mathbf{G_s}^{-1}$ is a lower-triangular Toeplitz matrix with $[\bar{g}_0 \bar{g}_1 \cdots \bar{g}_L]^T \in \mathbb{R}^{L+1}$ as its first column, and

$$\begin{cases} \bar{g}_0 = \frac{1}{g_0} \\ \bar{g}_l = -\frac{1}{g_0}\sum_{i=1}^{l} \bar{g}_{l-i} g_i, \quad l = 1, 2, \ldots, L. \end{cases} \tag{3.21}$$

Then

$$\|\mathbf{m}\|_2^2 = \sum_{k=0}^{L}\left(\sum_{j=0}^{k} \bar{g}_j\right)^2. \tag{3.22}$$

For the optimal solution in (3.18), the corresponding $\bar{g}_n$ in (3.21) can be expressed as follows:

$$\begin{cases} \bar{g}_0 = \frac{1}{L+1-L\tau} > 0 \\ \bar{g}_i = -\frac{\tau}{(L+1-L\tau)^2}\left(1 - \frac{\tau}{L+1-L\tau}\right)^{i-1} < 0, \quad i = 1, 2, \ldots, L. \end{cases} \tag{3.23}$$

The following proposition shows that $\|\mathbf{m}\|_2^2$ is a continuous and strictly increasing function of τ on $(0, 1)$. In other words, for $0 < \tau < 1$, $\|\mathbf{m}\|_2^2$ decreases as τ decreases, and thus as τ decreases, the noise effect in the estimation of the covariance matrix $\mathbf{R}_f$ is reduced and hence identification performance improves.

1) Proposition 3.1: With $\bar{g}_n$ given in (3.23),

$$\|\mathbf{m}\|_2^2 = \frac{1 - \left(1 - \frac{\tau}{L+1-L\tau}\right)^{2(L+1)}}{2(L+1-L\tau)\tau - \tau^2}$$

and

$$\frac{d\|\mathbf{m}\|_2^2}{d\tau} > 0, \qquad \text{for } 0 < \tau < 1.$$

Proof: See Appendix C.

C. Computation of $\mathbf{G}_0^{-1}$

With the precoding sequence $p(n)$ chosen as in (3.20), the matrix $\mathbf{G}$ in (3.5) becomes

$$\mathbf{G}_0 = \begin{bmatrix} a\mathbf{I}_{JF} & \mathbf{0} & \cdots & \mathbf{0} \\ b\widehat{\mathbf{S}} & a\mathbf{I}_{JF} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ b\widehat{\mathbf{S}}^L & b\widehat{\mathbf{S}}^{L-1} & \cdots & a\mathbf{I}_{JF} \end{bmatrix} \tag{3.24}$$

where $a = L + 1 - L\tau$, and $b = \tau$. The inverse of $\mathbf{G}_0$ can be obtained by forward substitutions as

$$\mathbf{G}_0^{-1} = \begin{bmatrix} k_0\mathbf{I}_{JF} & \mathbf{0} & \cdots & \mathbf{0} \\ k_1\widehat{\mathbf{S}} & k_0\mathbf{I}_{JF} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ k_L\widehat{\mathbf{S}}^L & k_{L-1}\widehat{\mathbf{S}}^{L-1} & \cdots & k_0\mathbf{I}_{JF} \end{bmatrix} \tag{3.25}$$

where $k_0 = 1/a$ and

$$k_i = -\frac{b}{a^2}(1 - b/a)^{i-1}$$

for $i = 1, 2, \ldots, L$. The solution $\mathrm{vec}(\widehat{\mathbf{HH}^*}) = \mathbf{G}_0^{-1}\mathrm{vec}(\mathbf{R}_f)$ in (3.12) is thus quite easy to compute once the precoding sequence is designed.

D. Identification Algorithm

We have proposed a new method for the identification of the MIMO channels for the SC-ZP block-transmission system using optimally designed periodic precoding sequences. With ZP, the computation of the channel product matrix $\mathbf{HH}^*$ becomes particularly simple, since it amounts to solving a lower-triangular linear system. The channel impulse response matrix $\mathbf{H}$ is then computed, up to a unitary matrix ambiguity, from the channel product matrix via an eigen-decomposition. We summarize the proposed method as the following algorithm.

Algorithm:

---

1) Select the optimal precoding sequence $p(n)$ given by (3.20), and form $\mathbf{G}_0^{-1}$ as in (3.25).
2) Collect the received data as $\bar{\mathbf{x}}(i)$ and pick up the first $(L + 1)$ block entries of $\bar{\mathbf{x}}(i)$ as $\mathbf{x}_f(i)$. Then estimate the covariance matrix $\mathbf{R}_f$ via the time average: $\hat{\mathbf{R}}_f = 1/S\sum_{i=1}^{S}\mathbf{x}_f(i)\mathbf{x}_f(i)^*$, where S is the number of data block.
3) Compute $\mathrm{vec}(\widehat{\mathbf{HH}^*}) = \mathbf{G}_0^{-1}\mathrm{vec}(\hat{\mathbf{R}}_f)$ to obtain the elements of $\mathbf{HH}^*$.
4) Form the matrix $\mathbf{Q}$ as in (3.7), and obtain the channel impulse response matrix (3.9) by computing the K largest eigenvalues and the associated eigenvectors of $\mathbf{Q}$.

E. Channel Order Overestimation

So far we have assumed that the channel order L is known. If only an upper bound $\hat{L} \geq L$ is available, then following the same process given in Section III-A, we obtain $\mathrm{vec}(\widehat{\mathbf{H}_{\mathbf{ov}}\mathbf{H}_{\mathbf{ov}}^*}) = \left[\sum_{k=0}^{\hat{L}}\mathbf{S}_k \otimes \mathbf{S}_k\right]^{-1}\mathrm{vec}(\mathbf{R}_f)$ where $\mathbf{H}_{\mathbf{ov}} = [\mathbf{H}^T\mathbf{0}\cdots\mathbf{0}]^T \in \mathbb{C}^{J(\hat{L}+1)\times K}$. Then we can also obtain $\mathbf{Q} = \mathbf{H}_{\mathbf{ov}}\mathbf{H}_{\mathbf{ov}}^*$. Note that the last $(\hat{L} - L)$ block columns and block rows of $\mathbf{Q}$ are zero. Hence, again, rank $(\mathbf{Q}) = K$ and $\mathbf{Q}$ has K positive eigenvalues. Each of the associated eigenvectors has the form $\hat{\mathbf{d}} = [\mathbf{d}^T 0\cdots 0]^T \in \mathbb{C}^{J(\hat{L}+1)}$ where $\mathbf{d} \in \mathbb{C}^{J(L+1)}$. Thus, we can identify the channel impulse response matrix, up to a unitary matrix ambiguity, from the K eigenvectors associated with the K positive eigenvalues of $\mathbf{Q}$.

F. More Transmitters Than Receivers

In the above discussions, we assume that there are more receivers than transmitters, i.e., $J \geq K$. If there are more transmitters, i.e., $K > J$, then either $J(L+1) \geq K$ or $K > J(L+1)$. If $J(L+1) \geq K$, then $\mathbf{H}$ is a tall matrix and assumption (**iii**) is generically satisfied. Hence, the proposed method still applies. If $K > J(L+1)$, then rank $(\mathbf{H}) < K$ and assumption (**iii**) does not hold. Hence, the proposed method is applicable to the more transmitters case, provided the additional condition $J(L+1) \geq K$ is satisfied.

IV. Channel Equalization

Once the received data $\bar{\mathbf{x}}(i) = \mathbf{H}_{\mathbf{e}}\bar{\mathbf{s}}(i) + \bar{\mathbf{w}}(i)$ is available and the channel is identified, the minimum mean-square error (MMSE) or zero forcing (ZF) equalization methods [8] can be used to recover the modulated sources $s_k(n)$. For example, with an MMSE equalizer $\mathbf{G}_{\mathbf{e}} = \mathbf{R}_{\mathbf{ss}}\mathbf{H}_{\mathbf{e}}^*\left(\sigma_w^2\mathbf{I}_{JN} + \mathbf{H}_{\mathbf{e}}\mathbf{R}_{\mathbf{ss}}\mathbf{H}_{\mathbf{e}}^*\right)^{-1}$ where $\mathbf{R}_{\mathbf{ss}} = E[\bar{\mathbf{s}}(i)\bar{\mathbf{s}}(i)^*]$, we estimate $\bar{\mathbf{s}}(i)$ by $\widehat{\bar{\mathbf{s}}}(i) = \mathbf{G}_{\mathbf{e}}\bar{\mathbf{x}}(i)$. Since the precoding scheme is applied at the transmitter, we need to multiply the estimated $\bar{\mathbf{s}}(i)$ by $\mathbf{P}^{-1}$ to obtain an estimate of $\bar{\mathbf{v}}(i)$, where $\bar{\mathbf{v}}(i)$ is similarly defined as $\bar{\mathbf{s}}(i)$, and $\mathbf{P} = \mathbf{I}_{\frac{M}{L+1}} \otimes (\mathrm{diag}[p(0), \ldots, p(L)] \otimes \mathbf{I}_K)$. In other words, the estimated $\widehat{\bar{\mathbf{v}}}(i)$ can be obtained by

$$\widehat{\bar{\mathbf{v}}}(i) = \mathbf{P}^{-1}\mathbf{G}_{\mathbf{e}}\bar{\mathbf{x}}(i). \tag{4.1}$$

From (4.1), we know the equalization performance is related to $\mathbf{P}^{-1}$ and $\mathbf{G}_{\mathbf{e}}$. Because $\mathbf{G}_{\mathbf{e}}$ is formed from the estimated channel coefficients, we expect good channel identification to bring an accurate $\mathbf{G}_{\mathbf{e}}$ and thus improves the equalization performance. Also we know using the optimal precoding sequence in (3.20), the identification performance improves as τ decreases. Hence, using a small τ brings good channel estimation and improves the accuracy of $\mathbf{G}_{\mathbf{e}}$, which is expected to improve the equalization performance. However, using a small τ would make the diagonal gain $p(k)^{-1} = 1/\sqrt{\tau}$ in $\mathbf{P}^{-1}$, $k = 1, 2, \ldots, L$, becomes large, which results in large noise amplification at the receivers and hence is more likely to cause decision error. Therefore, using a small τ would amplify the noise and the equalization performance deteriorates as τ decreases.

In summary, although decreasing τ improves the accuracy of $\mathbf{G_e}$, it would cause an increased amplification of noise, and vice versa. Hence, there is a tradeoff on the selection of τ when channel equalization is performed. In the work of [7], [20], [23], [26], this tradeoff is also observed. We will give a simulation example to demonstrate this tradeoff in Section V.

V. Simulation

In this section, we generate 100 two-input two-output random channels for each simulation (except simulation 2) with order $L = 2$ to demonstrate the performance of the proposed method. Each element in the channel impulse response matrix is generated by a zero-mean complex circular Gaussian random variable with unit variance. The length of symbol blocks is $M = 27$, which is zero padded to blocks of length $M + P = 30$. It means $P = 3(= L + 1)$ and transmission efficiency is 90%. The source symbols are independent and identically distributed (i.i.d.) Gray-coded quadrature phase-shift keying (QPSK) signals. The signal-to-noise ratio (SNR) at the output is defined as $\mathrm{SNR} = E\left[\|\mathbf{t}(n)\|_2^2\right]/E\left[\|\mathbf{w}(n)\|_2^2\right]$. The channel noise is zero mean, temporally and spatially white Gaussian. The channel normalized root-mean-square error (NRMSE) is defined as

$$\mathrm{NRMSE} = \frac{1}{\overline{\|\mathbf{H}\|}_F}\sqrt{\frac{1}{I}\sum_{i=1}^{I}\|\widehat{\mathbf{H}}^{(i)} - \mathbf{H}^{(i)}\|_F^2}$$

where $\|\cdot\|_F$ denotes the Frobenius norm. $\widehat{\mathbf{H}}^{(i)}$ is the estimate of the ith random channel $\mathbf{H}^{(i)} = [\mathbf{H}^{(i)}(0)^T\mathbf{H}^{(i)}(1)^T\cdots\mathbf{H}^{(i)}(L)^T]^T$ after removing the unitary matrix ambiguity by the least squares method [17], and I is the number of random channels. $\overline{\|\mathbf{H}\|}_F$ is the average Frobenius norm of I random channels.

Simulation 1–optimal selection of the precoding sequence

In this simulation, we use 5 precoding sequences which all satisfy (3.16) and (3.17) to illustrate the effect of the precoding sequences on the identification performance. The first sequence S_0 are chosen based on (3.20) for $\tau = 0.6$, i.e., S_0 is chosen as $\{\sqrt{1.8}\sqrt{0.6}\sqrt{0.6}\}$. The sequences S_1, S_2, S_A, and S_B are chosen as $\{\sqrt{0.6}\sqrt{1.8}\sqrt{0.6}\}$, $\{\sqrt{0.6}\sqrt{0.6}\sqrt{1.8}\}$, $\{\sqrt{0.6}\sqrt{1.0}\sqrt{1.4}\}$, and $\{111\}$ (i.e., no precoding), respectively. Fig. 2 shows that for $\mathrm{SNR} = 10$ dB, the NRMSE decreases as the number of symbol blocks increases for every precoding sequence. As expected, the optimal precoding sequence S_0 yields the smallest NRMSE.

Simulation 2–tradeoff in selecting τ

In this simulation, we use the optimal precoding sequences which satisfy (3.20) with various τ to test the effect of τ on the identification and MMSE equalization performances. The number of symbol block is 100. To get more smoother curves, we use 1000 random channels for simulation. Fig. 3 shows that the identification performs better for smaller τ. Fig. 4(a) shows that for $\tau \in [0.2, 0.8]$, the bit error rate (BER) performance deteriorates as τ decreases. Fig. 4(b) shows that for large τ, $\tau \geq 0.8$, the BER performance improves as τ decreases. Fig. 4 shows that there is a tradeoff between identification accuracy and noise amplification: a small τ means large noise amplification and an accurate channel estimate, and vice versa. For this example, it seems that $\tau = 0.8$ is a good choice for BER performance.

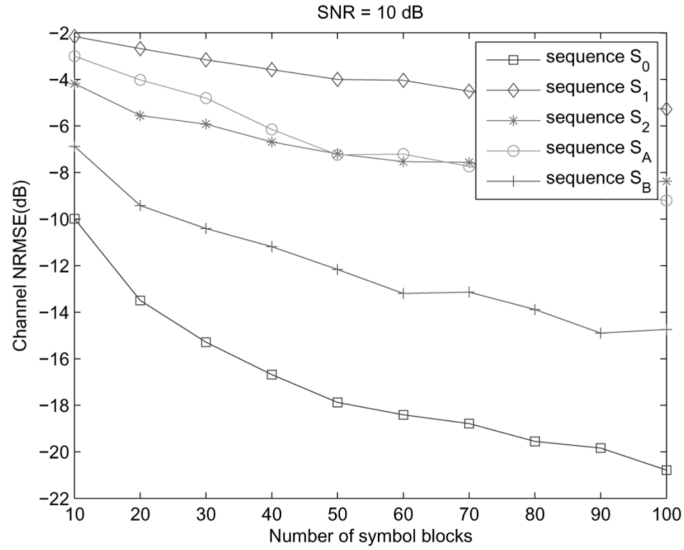

Fig. 2. Channel NRMSE for different numbers of symbol blocks.

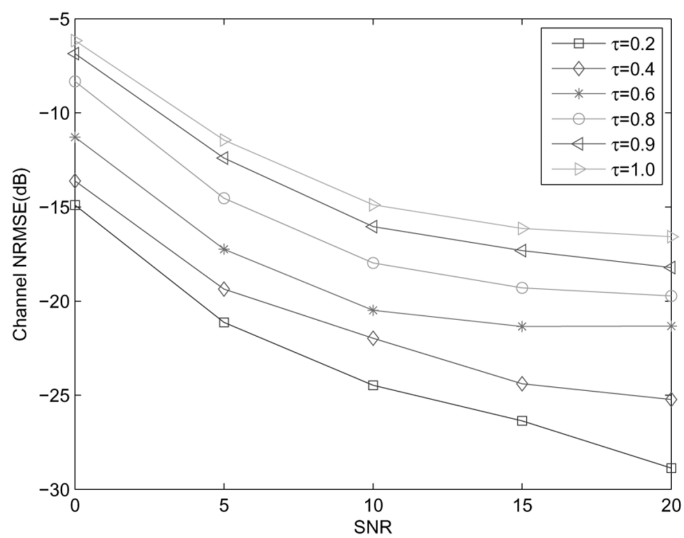

Fig. 3. Channel NRMSE at various SNR levels.

Simulation 3–channel order overestimation

In this simulation, we use precoding sequence that satisfies (3.20) with different τ and fix the number of symbol blocks at 100. For each upper bound $\hat{L}$, $0 \leq (\hat{L} - L) \leq 6$, we choose $P = \hat{L} + 1$ and $M = 9P$ for simulation such that the transmission efficiency is maintained at 90%. Fig. 5 shows the NRMSE increases with increasing channel order overestimation for each τ. We see that periodic precoding improves robustness to channel order overestimation. For example, without precoding ($\tau = 1$), the NRMSE increases about 7 dB for $(\hat{L} - L) = 3$. With precoding ($\tau = 0.8$), the corresponding increase in NRMSE is about 3 dB.

Simulation 4–comparison with the subspace method

In this simulation, we use the precoding sequence that satisfy (3.20) with $\tau = 0.7$. We compare the MMSE equalization

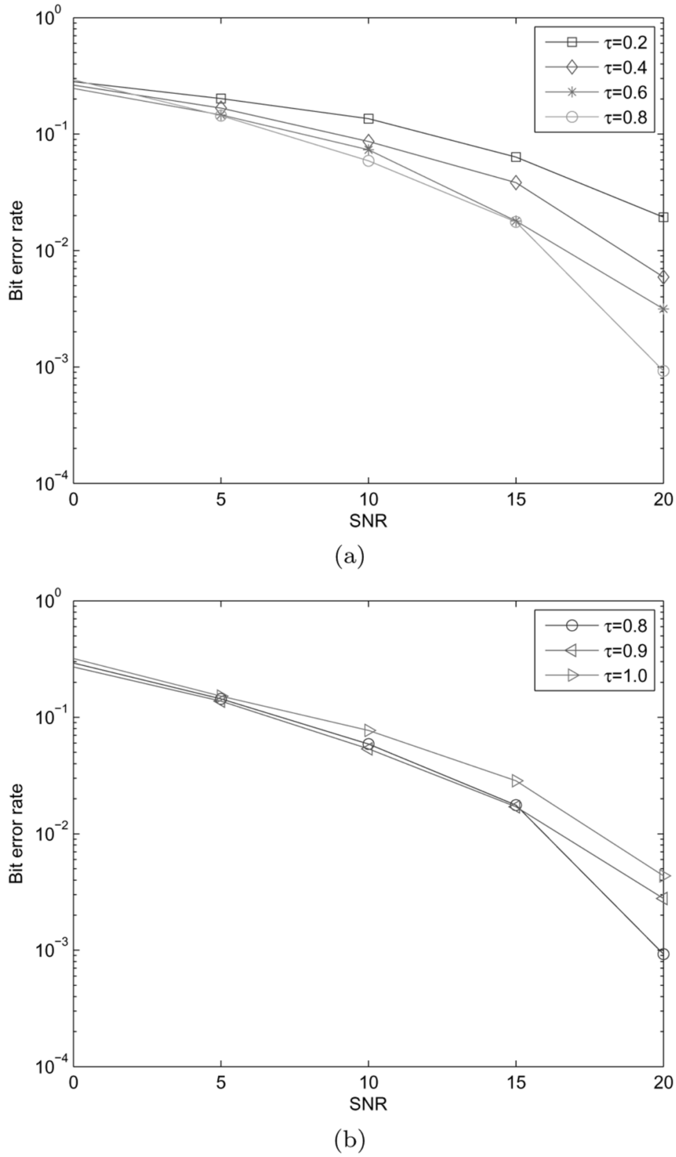

Fig. 4. BER versus output SNR.

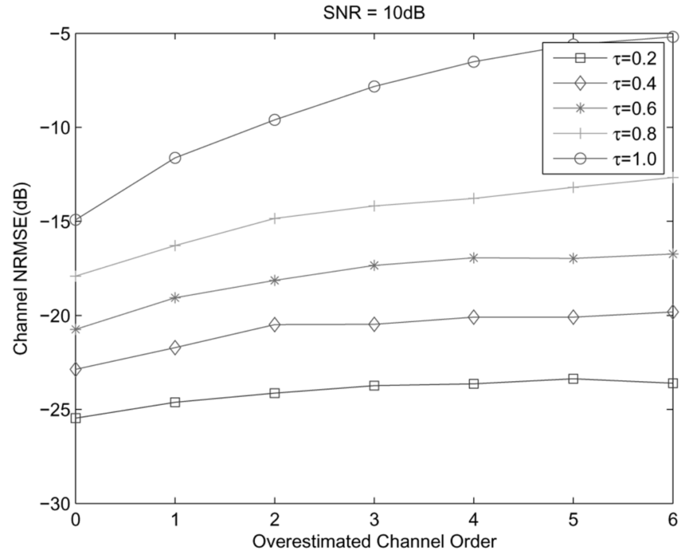

Fig. 5. Channel NRMSE versus $(\hat{L} - L)$.

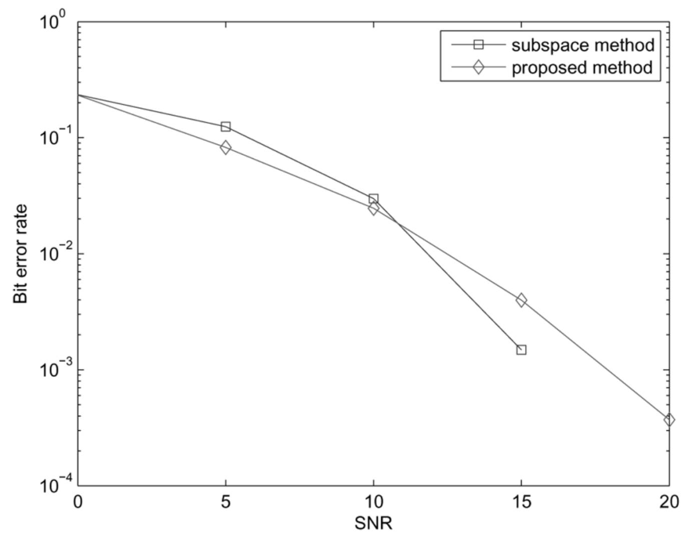

Fig. 6. Comparison with the subspace method.

performances of the proposed method and the subspace method [12] for MIMO SC-ZP systems. The number of symbol block is 100. Fig. 6 shows that the equalization performance of the proposed method is better than those of the subspace method for $\mathrm{SNR} < 10$ dB. The subspace method gives smaller BER than the proposed method for $\mathrm{SNR} > 10$ dB.

Simulation 5–identification using more recieved data

In this simulation, we use the first 3, 15, and 30 block rows of $\bar{\mathbf{x}}(i)$ to form the covariance matrices for identification. We use the precoding sequences that satisfy (3.20) with $\tau = 0.7$ and fix the number of symbol blocks at 100. Fig. 7 shows that when we use more received data, the identification performance improves. However, as we indicate at the end in Section III-A, the computational load of solving $\mathrm{vec}(\mathbf{HH}^*)$ increases as more data are used. If we define a "flop" to be a single complex multiplication or addition [30], then due to the sparse and lower-triangular structure of $\mathbf{G}^{-1}$, there requires about 4.3×10^3 flops to solve $\mathrm{vec}(\mathbf{HH}^*)$ for the first 3 block rows of $\bar{\mathbf{x}}(i)$; while for the first 15 and 30 block rows of $\bar{\mathbf{x}}(i)$, the solution of $\mathrm{vec}(\mathbf{HH}^*)$ is obtained via the least square approach, which is solved by the QR factorization [30, p.240], and the flop counts are roughly 8.4×10^5 and 3.44×10^6 flops, respectively.

VI. Conclusion

In this paper, we propose a new method for blind identification of MIMO channels for the SC-ZP block-transmission systems using periodic precoding. The identifiability condition is simply that the channel impulse response matrix is full column rank. The performance of identification algorithm depends on the choice of the precoding sequence. We propose a two-level optimal precoding scheme that minimizes the noise effect in the estimation of the covariance matrix $\mathbf{R}_f$. The effect of the optimal precoding sequence on channel equalization is also discussed.

Compared with the subspace method [12], the proposed method is shown to have better performance from low to medium SNR. Besides, the computations involved in the

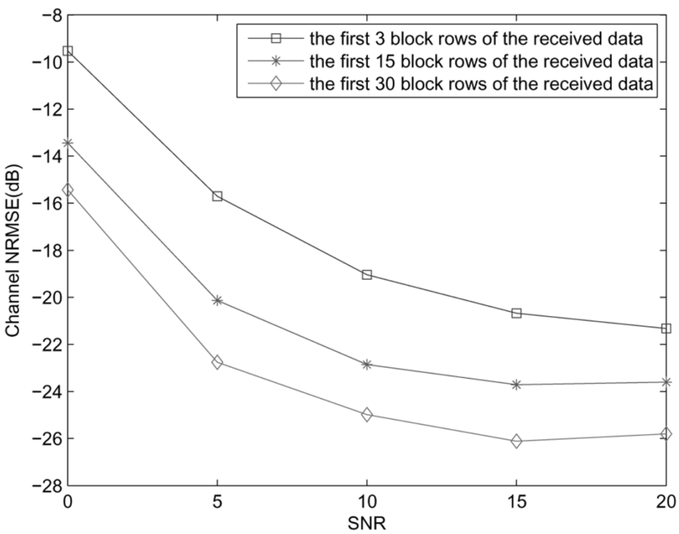

Fig. 7. Channel NRMSE for different sizes of received data.

algorithm are relatively simple: only covariance matrix estimation, a multiplication of $\mathrm{vec}(\mathbf{R}_f)$ by a lower triangular matrix to obtain $\mathrm{vec}(\widehat{\mathbf{HH}^*})$, and an eigen-decomposition of a $J(L+1) \times J(L+1)$ matrix, the main computational load; whereas, the computations of the subspace method requires a covariance matrix estimation, and two main computational loads: an eigen-decomposition of a $J(L+M) \times J(L+M)$ matrix and a singular value decomposition of a $(JN-KM)N \times J(L+1)$ matrix. Since $N = M+P$ and $M > L$, the subspace method requires substantially more computations than the proposed method.

APPENDIX

A) Computation of $\mathrm{vec}(\mathbf{HH}^*)$ *Using More Received Data:* If we use $\mathbf{x}_m(i) = [\mathbf{x}(iN)^T \mathbf{x}(iN+1)^T \cdots \mathbf{x}(iN+m)^T]^T$, the first $m+1 > L+1$ blocks of $\bar{\mathbf{x}}(i)$ for identification, then it is easy to verify that

$$\begin{aligned}\mathbf{x}_m(i) &= [p(0)\ \mathbf{E}_0\mathbf{H}\ p(1)\mathbf{E}_1\mathbf{H} \cdots p(m)\mathbf{E}_m\mathbf{H}]\mathbf{v}_m(i)\\ &= \mathbf{H}_m\mathbf{v}_m(i)\end{aligned}$$

where $\mathbf{v}_m(i)$ is similarly defined as $\mathbf{x}_m(i)$ and $\mathbf{E}_k = (\mathbf{S_E})^k\mathbf{S_I} \in \mathbb{R}^{J(m+1)\times J(L+1)}$ for $k = 0,1,\ldots,m$. Here we define $\mathbf{S_E} \in \mathbb{R}^{J(m+1)\times J(m+1)}$ as the matrix whose first block sub-diagonal entries are all $\mathbf{I}_J$ (i.e., $\mathbf{S_E}(J+1 : J(m+1), 1 : Jm) = \mathbf{I}_{Jm}$), and all remaining entries are zero; and $\mathbf{S_I} \in \mathbb{R}^{J(m+1)\times J(L+1)}$ with $\mathbf{S_I}(1 : J(L+1), 1 : J(L+1)) = \mathbf{I}_{J(L+1)}$ and all remaining entries are zero.

Taking expectation of $\mathbf{x}_m(i)\mathbf{x}_m(i)^*$, we get the covariance matrix

$$\mathbf{R}_m = \mathbf{H}_m\mathbf{H}_m^* = \sum_{k=0}^{m} p(k)^2\mathbf{E}_k\mathbf{HH}^*\mathbf{E}_k^T. \tag{A.1}$$

Write the matrix equation (A.1) in the following vector form:

$$\begin{aligned}\mathrm{vec}(\mathbf{R}_m) &= \mathrm{vec}\left(\sum_{k=0}^{m} p(k)^2\mathbf{E}_k\mathbf{HH}^*\mathbf{E}_k^T\right)\\ &= \left(\sum_{k=0}^{m} p(k)^2\mathbf{E}_k \otimes \mathbf{E}_k\right)\mathrm{vec}(\mathbf{HH}^*)\\ &= \mathbf{G_m}\cdot\mathrm{vec}(\mathbf{HH}^*).\end{aligned} \tag{A.2}$$

Since $\mathbf{G_m} = \left(\sum_{k=0}^{m} p(k)^2\mathbf{E}_k \otimes \mathbf{E}_k\right)$ is a tall matrix, the solution to (A.2) is

$$\mathrm{vec}(\mathbf{HH}^*) = \left(\mathbf{G_m}^T\mathbf{G_m}\right)^{-1}\mathbf{G_m}^T\mathrm{vec}(\mathbf{R}_m) \tag{A.3}$$

provided $\mathbf{G_m}$ is full column rank, by appropriate selection of the precoding sequence.

B) Proof of $\partial f(\alpha,\beta)/\partial\alpha > 0$*:*
Let $x = \alpha + 2$ and write $f(\alpha,\beta)$ in (3.19) as

$$f(\alpha,\beta) = \frac{x^{2(L+1)} + 2(L+1)x - 1}{\beta^2x^2}.$$

Since

$$\begin{aligned}\frac{\partial f(\alpha,\beta)}{\partial x} &= \frac{1}{\beta^2}\left[\frac{2-2x(L+1)+2Lx^{2L+2}}{x^3}\right]\\ &> \frac{1}{\beta^2}\left[\frac{2-2x(L+1)+2Lx(2L+2)}{x^3}\right]\\ &= \frac{1}{\beta^2}\left[\frac{2+2Lx-2x+4L^2x}{x^3}\right]\\ &> 0, \qquad \text{for } L \geq 1, \quad \alpha > 0\end{aligned}$$

and $\partial f(\alpha,\beta)/\partial\alpha = \partial f(\alpha,\beta)/\partial x$, we have $\partial f(\alpha,\beta)/\partial\alpha > 0$.

C) Proof of Proposition 3.1:
Let $a = L+1-L\tau$ and $b = \tau$, then according to (3.25), $\bar{g}_0 = 1/a > 0$ and $\bar{g}_i = -(b/a^2)(1-b/a)^{i-1} < 0$ for $i = 1,2,\ldots,L$, and $\sum_{j=1}^{l}\bar{g}_j = (1/a)(1-b/a)^l$. Hence, by computation, we obtain

$$\|\mathbf{m}\|_2^2 = \sum_{k=0}^{L}\left(\sum_{j=1}^{k}\bar{g}_j\right)^2 = \frac{1-(1-\frac{\tau}{L+1-L\tau})^{2(L+1)}}{2(L+1-L\tau)\tau-\tau^2}$$

and

$$\begin{aligned}&\frac{d\|\mathbf{m}\|_2{}^2}{d\tau}\\ &= \frac{\left[(1-\tau)\tau(2L+1)+\tau\right]\left[2(L+1)\left(1-\frac{\tau}{L+1-L\tau}\right)^{2L+1}\right]\cdot\left[\frac{L+1}{(L+1-L\tau)^2}\right]}{[2(L+1-L\tau)\tau-\tau^2]^2}\\ &\quad + \frac{\left[1-\left(1-\frac{\tau}{L+1-L\tau}\right)^{2(L+1)}\right]\cdot(4L\tau+2\tau)}{\left[2(L+1-L\tau)\tau-\tau^2\right]^2}\end{aligned}$$

Because $0 < 1-\tau < 1$ and $0 < \left(1-\frac{\tau}{L+1-L\tau}\right) < 1$ for $0 < \tau < 1$, $\frac{d\|\mathbf{m}\|_2^2}{d\tau}$ for $0 < \tau < 1$.

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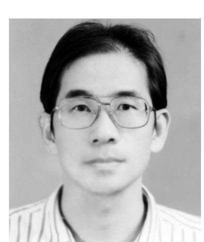

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