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Blind identification and allocation of multivariate disturbances

Matthijs Boerlage, Bram de Jager, and Maarten Steinbuch

Abstract—A second order statistics based blind identification technique is used to recover the physical sources from disturbances acting on a multivariable system. The results are then used to find the physical location of disturbance sources in an active vibration isolation platform. Furthermore, implications for multivariable controller design are discussed.

I. INTRODUCTION

In multivariable systems, the same disturbance, due to a single physical cause (source), can enter in more than one controlled variable. A typical example can be found in motion control, where multiple degrees of freedom of a controlled plant suffer from the same disturbance sources, e.g., pump, floor and machine vibrations. The sources cannot be measured directly, only an unknown mixture of these sources is observed at a certain place in the feedback loop. In order to study the physical nature of the disturbances, one has to recover both the sources *and* the mixture of these sources.

In this work, we show that this is equivalent to solving a *blind* identification problem. Blind identification problems appear in information theory, direction of arrival problems and array processing, see [3], [5] for a survey. Blind identification methods often rely on higher order statistics of the observed signals. An example of this is the independent component analysis technique used in [15]. Using higher order statistics implies that Gaussian sources cannot be retrieved. Also, as estimates of higher order statistics have high variance, long data sets are required. The novelty of our contribution is that we show that for time colored sources, only a set of second order statistics are required to solve the blind identification problem. Hence, the method from [1] can be used to solve the blind identification problem within some indeterminacies.

As opposed to other disturbance modeling techniques, e.g. [14], we are able to find a *structured* disturbance model. Herein, the contribution of each source can be studied individually. Also, the direction of each source is identified which is crucial for multivariable control design, as suggested in [9], [11, p. 85]. Furthermore, we show that with some additional assumptions, the physical location of sources can be recovered. This offers the possibility to trace down the sources and reduce their influence through mechanical redesign. The theory is demonstrated on a non trivial 6×6 MIMO active vibration platform.

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In the next section, we discuss how multivariate disturbances can be recovered from closed loop measurements. From thereon, we introduce the blind identification method used in this work. This is then applied to the active vibration isolation platform. Herein, disturbances are allocated using the proposed methodology. Finally the implications on multivariable feedback control design are discussed.

II. MULTIVARIATE DISTURBANCES

We consider a plant G with n inputs and n outputs which is controlled by a feedback controller K in the architecture depicted in Fig. 1. The disturbance vector d enters the

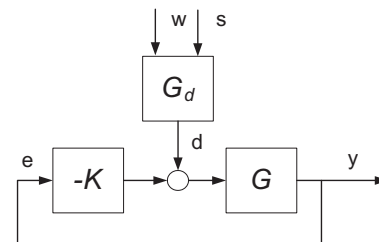


Fig. 1. Feedback control structure. G is the plant, K the controller, G_d the disturbance model.

loop at the input of the plant. In this paper, we assume that the plant is invertible and known within negligible uncertainties. Hence, disturbances at the output of the plant can be considered at the input of the plant (and vice versa).

The error e equals

$$e = S_o G d \quad (1)$$

where $S_o = (I + GK)^{-1}$ is the output sensitivity function. From an initial experiment, a batch of observations of the error can be obtained, $e(t) \in \mathbb{R}^n$ for $t = 0, \dots, T_s N$, where T_s denotes the sample time and $N + 1$ is the number of samples. With $e(t)$ given, one can reconstruct $d(t)$ by using the inverse of the output sensitivity, $d(t) = G^{-1}(q) S_o^{-1}(q) e(t)$, with q the differential operator, $qu(t) = \frac{\partial u(t)}{\partial t}$. The disturbance at each channel $d_i, i = 1, \dots, n$ results from a mixture of sources that are to be identified $s_j, j = 1, \dots, m$, where $m \leq n$ and sources that are not identified $w_l, l = 1, \dots, n$. The challenge is to find 1) the number of physical disturbances (m), 2) the physical disturbance sources ($s(t)$), and 3) the matrix that mixes the sources (the relevant part of G_d).

III. BLIND IDENTIFICATION

As we do not know the mixing matrix G_d and have no knowledge about the sources $s(t)$, we face a *blind* identification problem. Key in this analysis, is the use of the τ -lagged covariance matrix, defined for a multivariate signal $x(t)$,

$$R_x(\tau) = E\{x(t)x(t-\tau)^T\} \quad (2)$$

where $E\{\cdot\}$ is the statistical expectancy. The components of $x(t)$ are *uncorrelated* if $R_x(0)$ is diagonal.

In order to solve the blind identification problem, several assumptions must be made. Here, we make use of the following assumptions,

- A 1: The sources are mutually statistical independent
- A 2: Each source is a different time colored phenomena.
- A 3: The mixing process G_d is constant in time
- A 4: The sources and noise are uncorrelated

The first assumption A1 states that the sources are statistical independent physical phenomena. Two variables are called *independent* when knowing the value of one, does not provide any information about the other. In that case, their joint probability density function equals the product of the marginal probability density functions, [5]. Sources that are independent are also uncorrelated, but not vice versa. The second assumption A2 assumes that the sources are time colored. Then, the lagged covariance $R_s(\tau)$, is positive semi-definite for non-zero τ . The sources are different so that the lagged covariance is different for at least one value of τ . Assumption A3 states that G_d is a static transfer function, at least at the frequencies of interest. This is justified when sources have a narrow band spectrum and locations of the sources do not change in time. For ease of notation, we assume the sources are zero mean.

We divide the sources in two classes, namely the sources $s(t)$ which are to be identified, and the *noise* signals $w(t)$ which are not identified directly. Depending on the nature of $w(t)$, great simplifications in the blind identification problem can be realized. We assume that the noise and the sources are uncorrelated, A4. The disturbance model becomes;

$$d(t) = \underbrace{\begin{bmatrix} G_s & | & G_w \end{bmatrix}}_{G_d} \begin{bmatrix} s(t) \\ w(t) \end{bmatrix} \quad (3)$$

The blind identification procedure consists of two steps; Step A) principal component analysis and scaling, Step B) independent component analysis.

A. Principal component analysis

The objective of principal component analysis (PCA) is to find the minimal number of uncorrelated components $z(t)$ in the observed disturbances $d(t)$. In addition, the uncorrelated components are scaled to unit covariance. This procedure is

also known as *whitening*, [1]. Hence the objective is to find a possibly non-square matrix W so that,

$$z(t) = Wd(t) \quad (4)$$

with, $R_z(0) = I$. Using (3), the covariance of the disturbance equals,

$$R_d(0) = G_s R_s(0) G_s^T + G_w R_w(0) G_w^T \quad (5)$$

The issue is that the structure at the right hand side of this equation is to be determined while only $R_d(0)$ is known. The unknown singular value decompositions of the source and noise parts, is defined as,

$$G_s R_s(0) G_s^T = U_s \Sigma_s U_s^T, \quad G_w R_w(0) G_w^T = U_w \Sigma_w U_w^T, \quad (6)$$

where $R_s(\tau), R_w(\tau)$ are symmetric. Next, the singular value decomposition of the covariance of $d(t)$ is studied, so that,

$$\begin{aligned} R_d(0) &= U_d \Sigma_d U_d^T \\ &= U_s \Sigma_s U_s^T + U_w \Sigma_w U_w^T \\ &= \begin{bmatrix} U_{ds} & | & U_{dw} \end{bmatrix} \begin{bmatrix} \Sigma_{ds} & | & 0 \\ \hline 0 & | & \Sigma_{dw} \end{bmatrix} \begin{bmatrix} U_{ds}^T \\ \hline U_{dw}^T \end{bmatrix} \end{aligned} \quad (7)$$

The i^{th} singular value of $R_d(0)$, σ_{di} , namely the i^{th} diagonal element of Σ_d , equals the square of the variance of the i^{th} principal component in decreasing order. The best rank m approximation of $R_d(0)$ is achieved by considering the m dimensional subspace related to the first, hence largest, m principal components. The costs of this dimension reduction is small when $\frac{\sigma(\Sigma_{ds}) - \bar{\sigma}(\Sigma_{dw})}{\bar{\sigma}(\Sigma_{ds})}$ is large. When the sources dominate the noise signals, one finds the subspace of the sources as, $G_s R_s(0) G_s^T \approx U_{ds} \Sigma_{ds} U_{ds}^T$.

In the case that the noise space outside the source space can be approximated as $\Sigma_{dw} \approx \rho^2 I_{n-m}$. And one can assume, e.g., on physical bases, that $\Sigma_{ds} = \Sigma_s + \rho^2 I_m$, one can determine the variance of the spatially white noise space as $\rho^2 \approx \frac{1}{n-m} \text{tr}(\Sigma_{dw})$. Hence, an unbiased estimate of the source variances can be obtained using, $G_s R_s(0) G_s^T \approx U_{ds} (\Sigma_{ds} - \rho^2 I_m) U_{ds}^T$. This strategy is justified in the special case that $G_w = I_n$ and $R_n(\tau) = \rho^2 I_n \delta_{t,\tau}$. This special structure is commonly assumed in array processing applications, [12]. Herein, all sensors in the array are assumed to suffer from sensor noise with the same covariance. In control applications, these assumptions may be justified when all channels (e.g. all sensors) have the same noise variance. This is a crude assumption in most applications.

When none of the above arguments hold, and no clear decay of the singular values of $R_d(0)$ is visible, it is questionable if reduction of the dimension of the disturbance signal space is justified. If the dimension is reduced, performance of the blind identification procedure may decrease significantly, as shown in [8]. Blind identification procedures that make no use of a post-processing step as principal component analysis, such as [7], can then be considered.

In the following, we assume that we can use $G_s R_s(0) G_s^T \approx U_{ds} \Sigma_{ds} U_{ds}^T$, so that the signal dimension can be reduced to m . Next, the issue is to find the whitening matrix $W \in \mathbb{R}^{m \times n}$ so that,

$$\begin{aligned} R_z(0) &= W R_d(0) W^T \\ &= W U_{ds} \Sigma_{ds} U_{ds}^T W^T = I \end{aligned} \quad (8)$$

when we assume, without loss of generality, that $R_s(0) = I$, we find that,

$$W = \Sigma_{ds}^{-\frac{1}{2}} U_{ds}^T \quad (9)$$

Now, the directions of the dominant disturbances are contained in U_{ds} . The most dominant disturbance lies in the direction of the first column of U_{ds} . Note that when $m < n$, all signals in the subspace orthogonal to W , i.e., signals in the image of U_{dw} , are not considered in future steps of the blind identification procedure.

The covariance matrix $R_z(0)$ does not change when $z(t)$ is transformed with any unitary matrix U . Due to this freedom, the uncorrelated components $z(t)$ can still result from a mixture of the sources with any unknown unitary matrix U ,

$$z(t) = U s(t). \quad (10)$$

Even though the components are uncorrelated, their behavior can be much different from the behavior of the physical sources $s(t)$. The next step in the blind identification procedure is to reduce this freedom by using a stronger statistical condition, namely *statistical independence*.

B. Independent component analysis

In this step, the uncorrelated components $z(t)$ are transformed to components that are mutually statistical independent. As independence is a stronger statistical condition, the residual freedom in the blind identification problem reduces. As the sources are time colored signals, their τ lagged covariance, $R_s(\tau)$ is non-zero. The case is studied where the noise signals are small and fast compared to the source signals. Hence $R_s(\tau) \gg R_n(\tau), \tau > 0$, so that,

$$R_d(\tau) = G_s R_s(\tau) G_s^T, \quad \tau > 0. \quad (11)$$

In [12] it is argued, that when the lagged covariance matrix is diagonal for multiple lags, the components are independent. Starting from the whitened components $z(t) \in \mathbb{R}^m, R_z(0) = 0, R_z(\tau) \succeq 0, \tau > 0$, the objective is to find a unitary matrix U so that,

$$R_s(\tau) = U^T R_z(\tau) U \quad (12)$$

is diagonal for a set of $\tau_k > 0, \tau_k = \{\tau_1, \dots, \tau_{N_k}\}$.

This is a unitary simultaneous diagonalization problem that can be solved within two indeterminacies; namely sign and permutation of the columns of U . We express these indeterminacies with the matrix P which is the product of a permutation matrix and a phase matrix. The solution $VP = U$, with unknown P , is the approximated eigenstructure of $R_z(\tau_k)$ for $\tau_k = \tau_1, \dots, \tau_{N_k}$. A solution

exists if at least one covariance matrix $R_s(\tau_k)$ has distinct diagonal values. As long as this covariance matrix is in the set for $\tau_k = \tau_1, \dots, \tau_{N_k}$, the sources can be separated. Hence, increasing N_k , will improve signal separation, especially for more broadband disturbances. The unitary simultaneous diagonalization can be formulated as an optimization problem. Here, we use the *joint approximate diagonalization of eigenmatrices* (JADE) solver from [4] to find V .

Knowing both W and V , the physical sources can be recovered up to the indeterminacies P as,

$$\hat{s}(t) = P s(t) = V^T W d(t). \quad (13)$$

We define $\hat{G}_s = W^\dagger V$. The indeterminacies P imply arbitrary ordering and arbitrary sign of the recovered sources. The arbitrary permutation, implies that the ordering of the recovered sources is not fixed. In the principal component analysis, it was assumed that $R_s(0) = I$, which implies that all scalings of the sources are contained in \hat{G}_s . Alternatively, one may choose to scale the columns of \hat{G}_s to unity, as we show in Section IV. This is just a matter of convention and does not play any role in further use of this disturbance model.

IV. EXPERIMENTAL SETUP

An industrial active vibration isolation platform is studied, see Fig. 2. The platform consist of an active mounted table

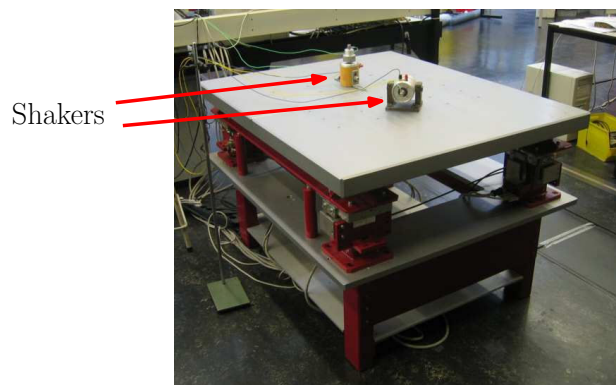


Fig. 2. Active vibration isolation platform. Shakers mounted at the table surface generate disturbances.

where actuators apply forces and moments at the center of gravity (COG). Geophones (sensors) measure the velocity of the COG. As the table behaves rigid in the domain of interest, the transfer function matrix of the plant is diagonal. Hence, all six cartesian degrees of freedom can be controlled independently.

Two disturbances are added synthetically to the system, by means of two shakers placed at the surface of the table. Both the location and the time behavior of the disturbances are considered to be unknown. For validation purposes, the acceleration of the shakers is measured. The errors are

measured, so that using (1), the disturbances $d(t)$ can be recovered, see Fig. 3. It is visible that due to the location of the shakers, the shakers excite all controlled axes. In the

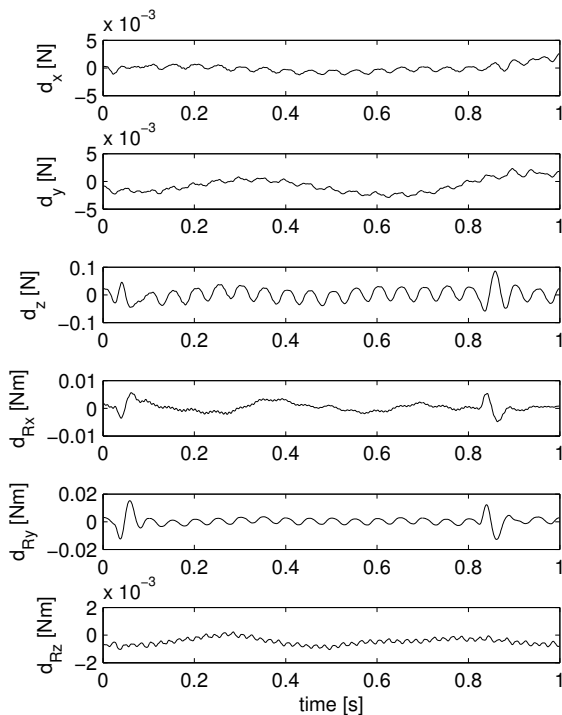


Fig. 3. Reconstructed input disturbance of the active vibration isolation platform.

principal component analysis, we find the singular values of the covariance matrix $R_d(0)$, Fig. 4. We take in account three principal components ($m = 3$). There is no clear distinction between the third and fourth singular value of $R_d(0)$. The influence of other sources in the m dimensional signal subspace of $R_d(0)$ can therefore be significant. We choose to do this, to illustrate the power of the blind identification procedure. In Fig. 5 the three components are shown, as $z(t) = Wd(t)$, $W \in \mathbb{R}^{3 \times 6}$. Next, using independent component analysis the uncorrelated components $z(t)$ are transformed to independent components, Fig. 6. Herein, we used $N_k = 50$ lagged covariance matrices in the simultaneous diagonalization problem, (12). We decompose the recovered matrix \hat{G}_s in the directions of, \overline{G}_s , and the input gains Γ , so that $\hat{G}_s = \overline{G}_s \Gamma$

$$\overline{G}_s = \begin{bmatrix} -0.013 & 0.003 & 0.209 \\ -0.010 & -0.007 & 0.582 \\ -0.997 & 0.970 & 0.781 \\ -0.008 & -0.093 & -0.078 \\ -0.081 & -0.224 & 0.026 \\ -0.001 & 0.002 & 0.020 \end{bmatrix}$$

$$\Gamma = \begin{bmatrix} 0.020 & 0 & 0 \\ 0 & 0.013 & 0 \\ 0 & 0 & 0.002 \end{bmatrix} \quad (14)$$

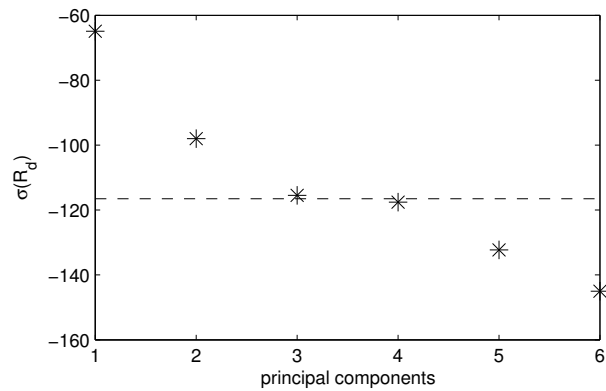


Fig. 4. Singular values of $R_d(0)$, equals the squared variance per principal component.

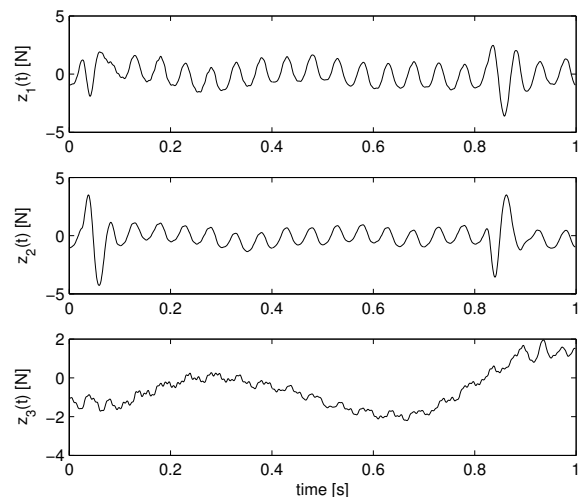


Fig. 5. Three principal components z recovered from the observed disturbances.

The j^{th} column of \overline{G}_s is the direction of the j^{th} independent component. It is visible that the first and second independent component act mostly in z direction. Also, both two sources cause a rotation about the y -axis. The third source is a disturbance in both z and y direction, this is due to the motions of the floor in our laboratory for which the table is poorly isolated. In Γ , it is visible that the first and second source are much larger than the third source.

For validation, we compare the first two independent components with the measurement of the acceleration sensors of the shakers, Fig. 7. We see that, within a scale, sign and permutation indeterminacy, the wave forms match closely, so it is justified to conclude that the independent components describe the behavior of the sources. It is clear that the principal components, Fig. 5, do not possess this property. Hence, for this application, the practical significance of requiring statistical independence is demonstrated.

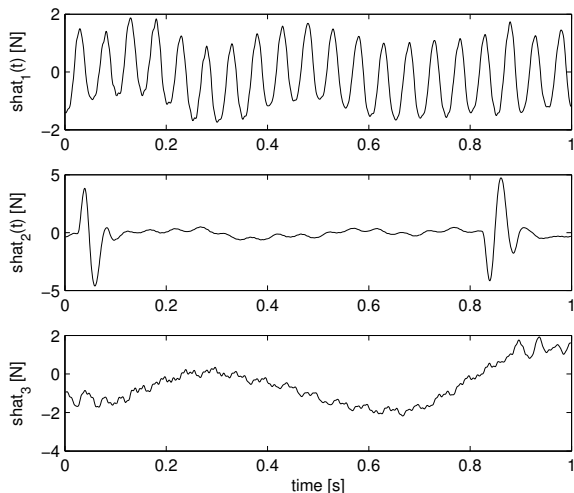


Fig. 6. Three independent components \hat{s} recovered from the observed disturbances.

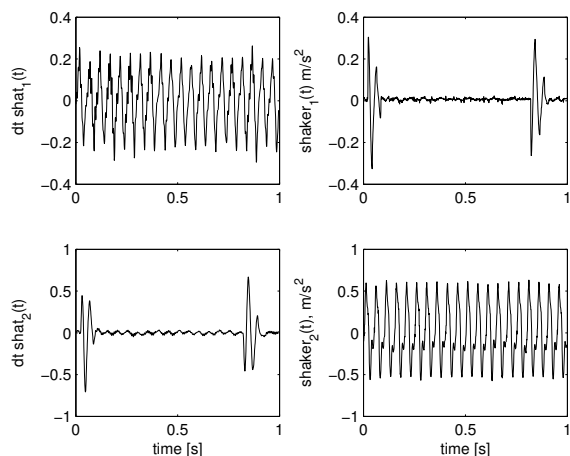


Fig. 7. Derivative of $\hat{s}_1(t)$, $\hat{s}_2(t)$ and the acceleration of the shakers.

V. ALLOCATION

Finding the location of disturbance sources is of great value in system design. When the physical source of a disturbance is recovered, measures to reduce the influence of that source on machine performance can be taken either through mechanical or control (re)design. From the blind identification procedure, the contribution of each j^{th} disturbance source to the disturbance is identified, see (18).

As the disturbances are studied at the input of the plant, d_{sj} is expressed in terms of cartesian forces and moments acting on the center of gravity of the table, so that for the j^{th} disturbance source holds that,

$$d_{sj} = [d_x^j, d_y^j, d_z^j, d_{Rx}^j, d_{Ry}^j, d_{Rz}^j]^T \quad (15)$$

We assume that the disturbance source acts point wise and the table is rigid. Allocation of the source then boils down

to finding the vector from the center of gravity to the point where the disturbance source acts, see Fig. 8. As we have

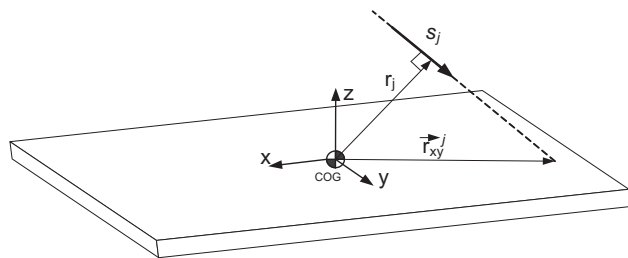


Fig. 8. Multi body model to find the location of the j^{th} independent component from the disturbance forces acting on the center of gravity.

$\vec{M}_{COG} = \vec{r} \times \vec{F}_{COG}$, see [10], it follows that

$$\begin{bmatrix} d_{Rx}^j \\ d_{Ry}^j \\ d_{Rz}^j \end{bmatrix} = \begin{bmatrix} 0 & d_z^j & -d_y^j \\ -d_z^j & 0 & d_x^j \\ d_y^j & -d_x^j & 0 \end{bmatrix} \begin{bmatrix} r_x^j \\ r_y^j \\ r_z^j \end{bmatrix} \quad (16)$$

Note that the matrix at the right hand side of this equation has rank 2. Hence, it is only possible to find the shortest distance r_j to a line on which the j^{th} source is located. When we assume that the source is located at the surface of the table, the vector \vec{r}_{xy}^j from the center of gravity to the source location can be uniquely determined, Fig. 8. In (16), the ratio between the elements in d_{sj} (15) is important, it suffices to take in account the direction of d_{sj} , which equals the j^{th} column of \vec{G}_s (14). Hence, we recovered the vector \vec{r}_{xy}^j from the center of gravity to each source, and the direction of the source (equals direction of $[d_x^j, d_y^j, d_z^j]^T$ at COG).

The first two sources, recovered in Section IV, are used to demonstrate this allocation procedure. For ten different measurements, the estimated locations are depicted in Fig. 9. The actual location of the shakers is marked with the diamonds. The estimation accuracy improves when the number of lagged covariance matrices (N_k) is increased.

VI. IMPLICATIONS FOR CONTROL DESIGN

The insights after blind identification offer new opportunities for multivariable control design. Here we discuss possibilities for using the knowledge from blind identification for physical interpretation and redesign of a multivariable feedback controller.

A. Changes in the disturbances

After blind identification, one can study the contribution of each source in the disturbance $d(t)$. Hence, the benefit of eliminating a particular source in order to improve machine performance can be studied. Here, we measure the performance with a norm on $e(t)$. The disturbances can be factored as contributions from the independent components $d_s(t)$ and contributions from the noise space $d_w(t)$,

$$d(t) = d_s(t) + d_w(t), \quad (17)$$

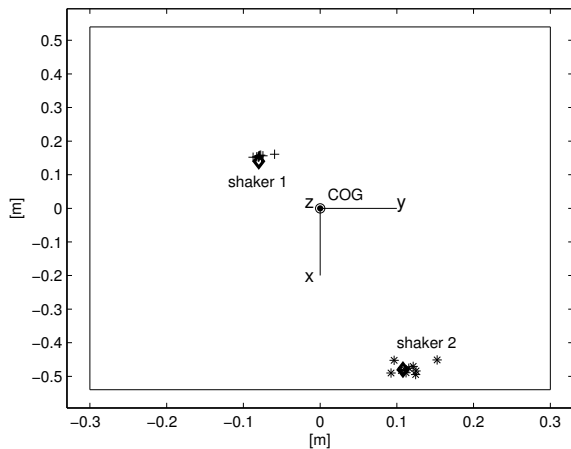


Fig. 9. Top view of table surface. Estimated location of the sources for 10 experiments, \hat{s}_1 (+), \hat{s}_2 (*) and actual location of the shakers \diamond .

with $d_s(t) = \hat{G}_s \hat{s}(t)$. The contribution of each j^{th} independent component $\hat{s}_j(t)$ to the disturbance equals,

$$d_{sj}(t) = g_{dj} \hat{s}_j(t). \quad (18)$$

Herein g_{dj} equals the j^{th} column of \hat{G}_s , so that $d_s(t) = \sum_{j=1}^m d_{sj}(t)$. Using (1), the error as a result of each component $\hat{s}_j(t)$ is then,

$$e_{sj}(t) = S_o(p)G(p)d_{sj}(t) \quad (19)$$

The benefit of eliminating the j^{th} source can be obtained from calculating $\|e(t) - e_{sj}(t)\|_p$, for any p -norm.

B. Redesign of feedback controller

Given the structured disturbance model, one has the ability to redesign the feedback controller to reject the disturbances related to the individual sources. Defining $v_j(t) = G(p)d_{sj}(t)$, and using (19), we have that

$$e_{sj}(t) = S_o(p)v_j(t). \quad (20)$$

As $S_o(p)$ is a transfer function *matrix*, the size of $e_{sj}(t)$ depends on the gain at the input direction of $S_o(p)$ corresponding to the direction of $v_j(t)$. When the direction of $v_j(t)$ is fixed, one may consider shaping the sensitivity function so that attenuation is high in that direction while attenuation in orthogonal directions is decreased. Hence, design freedom is exploit that has no scalar analogue, [9]. An \mathcal{H}_∞ controller design that demonstrates this is discussed in [2].

VII. CONCLUSIONS

It is demonstrated that the spatial diversity and time evolution of disturbances can be used to identify both the disturbance sources and the way they are mixed in a multivariable controlled system. Component wise analysis offers great insight in the physical nature of disturbances and facilitates allocation of sources and improved multivariable controller design choices. The benefit of using directional

information of disturbances for improved weighting filter selection in \mathcal{H}_∞ control design, is discussed in our recent paper, [2].

More advanced blind identification techniques, that have less stringent assumptions on the disturbance model are currently under investigation. In [13] more general blind identification techniques are discussed. Also, the case of underdetermined mixtures ($m > n$), in [6], is subject to future research.

REFERENCES

- [1] A. Belouchrani, K. Abed-Meraim, J. Cardoso, and E. Moulines, "A blind source separation technique using second-order statistics," *IEEE Transactions on Signal Processing*, vol. 45, no. 2, pp. 123–145, 1997.
- [2] M. Boerlage and B. de Jager, "Multivariable control design for fixed direction disturbances," in *Proceedings American Control Conference, Accepted*, 2007.
- [3] J. Cardoso, "Blind signal separation: statistical principles," *Proceedings of the IEEE*, vol. 9, no. 10, pp. 2009–2025, 1998.
- [4] J. Cardoso and A. Soulmiac, "Jacobi-angles for simultaneous diagonalization," *SIAM J. Mat. Anal. Appl.*, vol. 17, no. 1, pp. 161–164, 1996.
- [5] P. Comon, "Independent component analysis, a new concept?" *Signal Processing*, vol. 36, no. 3, pp. 287–314, 1994.
- [6] L. De Lathauwer and J. Castaing, "Second-order blind identification of underdetermined mixtures," in *6th Int. Conference on Independent Component Analysis and Blind Signal Separation (ICA 2006)*, 2006, pp. 40–47.
- [7] L. De Lathauwer, B. De Moor, and J. Vandewalle, "Dimensionality reduction in higher-order-only ICA," in *Proceedings of the IEEE Signal Processing Workshop on Higher-Order Statistics*, 1997, pp. 316–320.
- [8] L. De Lathauwer, B. D. Moor, and J. Vandewalle, "A prewhitening-induced bound on the identification error in independent component analysis," *IEEE Transactions on Circuits and Systems I*, vol. 52, no. 3, pp. 546–554, 2005.
- [9] J. Freudenberg and D. Looze, *Frequency Domain Properties of Scalar and Multivariable Feedback Systems, Lecture Notes in Control and Information Sciences*, M. Thoma and A. Wyner, Eds. Springer-Verlag, 1988.
- [10] R. Huston, *Multibody Dynamics*. Butterworth-Heinemann, London, 1990.
- [11] J. Maciejowski, *Multivariable feedback design*. Addison-Wesley, 1989.
- [12] L. Tong, R.-w. Liu, V. Soon, and Y.-F. Huang, "Indeterminacy and identifiability of blind identification," in *IEEE Transactions on Circuits and Systems*, vol. 38, 1991.
- [13] K. Tugnait, J., "Identification and Deconvolution of Multichannel Linear Non-Gaussian Processes Using Higher Order Statistics and Inverse Filter Criteria," *IEEE Transaction on Signal Processing*, vol. 45, no. 3, pp. 658–672, 1997.
- [14] P. Van Overschee, B. De Moor, W. DeHandschutter, and J. Swevers, "A subspace algorithm for the identification of discrete time frequency domain power spectra," *Automatica*, vol. 33, no. 2, pp. 2147–2157, 1997.
- [15] C. Xia, H. J., and N. Thornhill, "Isolating multiple sources of plant-wide oscillations via independent component analysis," *IEEE Control Engineering Practice*, vol. 13, no. 8, pp. 1027–1035, 2005.