Topic:

Blind Identification and Equalization Based on Second-Order Statistics: A Time-Domain Approach

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IEEE Trans. On Information Theory

March, 1994

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I. Problem Formulation

A. Problem statement:

☐ Assume the channel is LTI

☐ The received baseband signal is

$$x(t) = \sum_{k=-\infty}^{\infty} s_k h(t - kT) + n(t) .$$

s_k : i.i.d source symbol, $E\{s_k s_l^*\} = d(k-l)$.

$h(t)$: FIR "composite" channel including transmit filter, channel and receiving filter.

T: symbol interval

$n(t)$: additive channel noise, zero-mean with $E\{n(t_1)n(t_2)^*\} = s^2 d(t_1 - t_2)$.

□ Assume that $n(t)$ and s_k are uncorrelated.

□ The problem of *blind channel estimation and equalization* is:

i) Estimate $h(\cdot)$ given only the received signal $x(\cdot)$.

ii) Recover the source symbols s_k when $h(\cdot)$ has been identified.

B. Why Channel Estimation ?

Traditional equalization:

1) Direct equalization without channel estimation.
2) Sending training sequences to recursively adjust the parameters of equalizer.
3) Main disadvantages: Waste of transmission time and power; slow convergence speed, etc.
4) This is overcomed by estimating channel *a priori*, and based on which we design the equalization.

C. Existing Channel Estimation Schemes:

☐ Exploit various statistics of the received signals.

1) Minimum-phase channel: Second order output statistics is sufficient to channel identification.
2) Nonminimum-phase channel: Higher order output statistics are required.

☐ A consistent estimation of high order spectrums requires large number of data samples.

☐ This means that, if the channel is nonminimum phase, we need long data records and thus long time to obtain a good channel estimation.

□ Goal of this paper:

To identify the channels, possibly nonminimum phase, using “appropriate” second-order statistics.

II. Main Results

□ A discrete-time process y_k is (*wide-sense) cyclostationary* (CS) if

$$E\left\{y_k y_l^*\right\} = E\left\{y_{k+P} y_{l+P}^*\right\}$$

for some positive integer *P*.

▣ Key ideas:

1) ***FACT1:*** If $x(t)$ is sampled at rates *P/T*, i.e., *P* times higher than symbol rate 1/*T*, the resulting sequence is CS with period *P*.

2) ***FACT2:*** Every CS process with period *P* can be represented by a *P*-dimensional vector stationary process.

□ Based on the two facts and after some manipulations, we obtain the *P*-dim. stationary model

$$\mathbf{x}(iT) = \mathbf{H}\mathbf{s}(iT) + \mathbf{n}(iT),$$

where

1) $\mathbf{x}(iT), \mathbf{n}(iT)$ are *P*-dim. vectors containing samples of $x(t), n(t)$.
2) $\mathbf{s}(iT)$ is a *d*-dim vector consisting of source symbols s_k.
3) The matrix $\mathbf{H}$ contains the channel coefficients to be identified (All formulas omitted).

- It is necessary that the matrix $\mathbf{H}$ is *full column rank*, a condition required for most existing channel identification scheme.

  This can be done if we choose $P > d$

- Thus our task is to determine $\mathbf{H}$ using the above vector representation.

▣ With some advanced matrix theory, the main result of this paper is:

Theorem 1.1: *If noise free, then* $\mathbf{H}$ *can be uniquely determined using* $\mathbf{R}_{\mathbf{x}}(0)$ *and* $\mathbf{R}_{\mathbf{x}}(1)$*, where*

$$\mathbf{R}_{\mathbf{x}}(d) := E\{\mathbf{x}(iT)\mathbf{x}((i-d)T)^*\}.$$

□ Proposed algorithm: (Details are omitted here)

The main required computations are *SVD, (singular value decomposition)* of $\mathbf{R}_{\mathbf{x}}(0)$ and $\mathbf{R}_{\mathbf{x}}(1)$.

□ Dealing with noisy case:

Estimate noise covariance matrix and subtract it from $\mathbf{R}_{\mathbf{x}}(0)$ to have a "approximate" noise-free case.

III. Conclusions

☐ Main contributions:

1) Upsampling at received signal induces cyclostationarity.
2) First time using cyclostationarity in received signal for blind channel estimation.
3) If $\mathbf{H}$ is known, the source symbols is recovered by

$$\mathbf{s}(iT) = \mathbf{H}^{+}\mathbf{x}(iT),$$

where $\mathbf{H}^{+}$ is the pseudo-inverse of $\mathbf{H}$. (equalizer is in the estimation algorithm!)

☐ Drawback:

Take $P = 2$ for example. We have 2 sets of channels. If they share common zeros (or zeros close together), the algorithm does not work well.

☐ This leads to many future research.

IV. Related Research

Key Point: Induce CS at received signal by some way other than upsampling.

☐ Transmitter-induced CS approaches:

1) With a multirate filter bank or a periodic filter at transmitter can also induce CS at received signal.
2) This avoids the numerical problem encountered in this paper, and can achieve much better performance.
3) However, we need to build a equalizer (e.g., an inverse periodic filter) at the receiver end. This is the cost.