Block-Circulant Low-Density Parity-Check Codes for Optical Communication Systems

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***Abstract*—A novel family of structured low-density parity-check (LDPC) codes with block-circulant parity-check matrices that consist of permutation blocks is proposed. The codes from this family are based on new combinatorial objects termed cycle-invariant difference sets, and they have low storage requirements, fast encoding algorithms, and girth of at least six. Most importantly, they tend to outperform many other known structured LDPCs of comparable rate and length.**

***Index Terms*—Cycle-invariant difference sets, forward error correction, low-density parity-check codes, optical communications.**

I. Introduction

The role of forward error correction (FEC) in maintaining acceptable transmission quality is becoming increasingly important as the capacity of dense wavelength-division multiplexing (DWDM) long-haul transmission systems grows. FEC can improve the performance of optical communications systems without introducing major changes to optical materials and devices and without major paradigm shifts in hardware such as converting to all-optical networks. This characteristic is especially important as the physical properties of optical networks are being pushed to their limits. The FEC proposed in this paper can readily be adapted to follow any shift in system implementation.

In state-of-the-art optical communication systems, two FEC standards are currently in use, both based on Bose–Chaudhuri–Hocquenghen (BCH) codes [1]. For example, "in-band" FEC used in SONET/SDH framing is a 3-[4359, 4320] shortened BCH code (ITU-T G.707 standard). "Outband" FEC coding is accomplished by using interleaved Reed–Solomon (RS) [255, 238] codes, as specified in the digital wrapper of the ITU G.709 standard. The FEC used in submarine systems is based on a Reed–Solomon code with parameters [255, 239], as specified by the ITU-T G.975 standard [2]. For optical communication systems with bit rates exceeding 10 Gb/s, more powerful FEC codes with large coding gains and high code rates are desirable. Two such schemes have been proposed recently: Ait Sab *et al.* [3] proposed a concatenated scheme with two RS codes, and Pyndiah [4] and Ait Sab *et al.* [5] proposed block turbo codes.

Recently, the authors showed in [6]–[8] that the error-correcting capability of turbo codes can be surpassed by low-density parity-check (LDPC) codes; at the same time, the hardware complexity of LDPC codes tends to be significantly smaller than that corresponding to turbo codes [9]. LDPC codes are usually designed in a pseudorandom fashion [10], but this leads to encoders/decoders with complexity exceeding practical limits imposed on high-speed optical communication systems. A more attractive approach that we propose in this paper is to design block-circulant LDPC codes with permutation blocks based on a new class of combinatorial objects; these structures allow for small storage requirements, as well as for performing encoding and decoding via fast and simple circuits. Such high-speed FEC architectures are of crucial importance in optical communications, and there has been a great deal of activity in this area. For example, Agere Systems (see Azadet *et al.* [11]) reported an optical networking interface device with four parallel RS codecs, each operating at 2.5 Gb/s. Additionally, Agere Systems [12], [13] and Flarion [14] researchers developed codecs employing LDPC designs that seem to be a combination of structured and random-like components.

The main result of this paper is a novel class of LDPC codes based on a combinatorial structure referred to as an *m-fold cycle-invariant difference set* (CIDS). The parity-check matrices of the proposed codes are block-circulants with permutation blocks. Both the minimum distance and the girth of the corresponding bipartite graph [8] are at least six. By carefully designing the cycle-invariant difference set, and by appropriately permuting the elements in some block-rows, code graphs with $(q-1)(q^m-1)$ (q an odd prime) variable nodes can be made to have girth at least $2m+2$. As the authors demonstrated by extensive computer simulations in [8], codes with high girth tend to perform very well, since a larger number of message-passing iterations can be performed before extrinsic information of nodes in the graph becomes correlated. Due to the block-circulant structure of the parity-check matrix, encoding can be performed very efficiently. Additionally, the min-sum decoding algorithm employed for soft-decision decoding of these codes requires a number of binary logic operations at least one order of magnitude smaller than that corresponding to hard-decision decoding of RS or turbo codes [4], [5] with comparable performance. For more details on min-sum algorithm complexity, the reader is referred to [17, Appendix B] and [20]–[22].

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The performance of the codes proposed in this paper is assessed in a very realistic simulation environment, outlined in Section III, that includes impairments originating from amplified spontaneous emission (ASE) noise, pulse distortions due to fiber nonlinearities, chromatic dispersion, crosstalk effects, and intersymbol interference. For more details on the transmission system model, the reader is referred to our previous work [17], [18]. Since the coding gain of these LDPC codes is much higher than the gain of RS or turbo codes, they are excellent candidates for use in long-haul optical transmission systems.

This paper is organized as follows. In Section II our novel family of LDPC codes is described. In Section III the system model used for obtaining the simulation results is briefly described. The bit error rate (BER) results are given in Section IV, and Section V contains the concluding remarks.

II. CIDS Based LDPC Codes

The codes proposed in this paper are based on the theory of generalized difference sets, more precisely on $m{+}1$-fold CIDSs, introduced by the authors in [15].

A *general* (v,k,λ) *difference set* over an additive Abelian group V of order v is a set $S=\{s_1,\ldots,s_k\}$ of distinct elements from V, such that each nonzero element s from V can be represented as $s=s_i-s_j$ in at most λ ways. Codes based on difference sets with $\lambda>1$ have short cycles, and therefore we will restrict our attention to the case $\lambda=1$. The constraint $\lambda=1$ implies that every element of the group can be represented at most once as the difference of two distinct elements from the difference set. For the sake of simplicity, we will henceforth call general difference sets by *difference sets.*

Let Z_{N} denote the additive group of integers modulo N and let the elements of a general difference set S over Z_N be arranged in a given order. Furthermore, let C^i denote the cyclic shift operator that shifts a sequence cyclically i positions to the right. If for $i=1,\ldots,m$, the ordered sets $\Omega_i = C^iS-S \bmod N$ (where subtraction is performed component-wise) are themselves difference sets, we say that S is an $(m{+}1)$-fold CIDS.

For example, consider Z_7 and the ordered difference set $\mathrm{S}=\{1,2,4\}$. For this set, $\Omega_1=\{4,1,2\}-\{1,2,4\}=\{3,6,5\} \bmod 7$ is itself a difference set. Since $\Omega_2=\{1,2,4\}=S$, S is in fact a twofold CIDS over Z_7. On the other hand, the difference set {0,1,3,9} over Z_{13} gives rise to three different sets, namely, $\Omega_1=\{9,12,11,17\}$, $\Omega_2=\{3,8,10,5\}$, and $\Omega_3=\{1,2,6,4\}$, none of which is a difference set. Therefore, {0,1,3,9} is a trivial (one-fold) CIDS over Z_{13}.

The parity-check matrix of the proposed regular LDPC codes can now be described as follows.

Let H be an $(m\times N)\times(s\times N)$ block-circulant matrix with permutation blocks determined as different powers of the basic permutation matrix P of order N. In other words, let

$$H=\begin{bmatrix} P^{i_1} & P^{i_2} & P^{i_3} & \cdots & P^{i_s} \\ P^{i_s} & P^{i_1} & P^{i_2} & \cdots & P^{i_{s-1}} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ P^{i_{s-m+2}} & P^{i_{s-m+3}} & P^{i_{s-m+4}} & \cdots & P^{i_{s-m+1}} \end{bmatrix} \quad (1)$$

where $i_1,i_2,\ldots,i_s$ are nonnegative integers and where P is an $N\times N$ basic circulant permutation matrix

$$P=\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 \\ 1 & 0 & 0 & \cdots & 0 \end{bmatrix}.$$

The row and column weights of the code described by (1) are s and m, respectively, and the girth of the corresponding Tanner graph is at most equal to six. The last claim can be proved by simply observing the following three facts: first, two permutation matrices on the diagonals of the first two block-rows are the same. For example, the blocks at position (1, 1) and (2, 2) of the parity-check matrix are both P_1^i. The last two permutation blocks within the first two rows are P_{s}^i and $P_{\mathrm{s}-1}^i$, respectively, and they additionally appear as blocks at position (3, 1) and (3, 2), closing up a cycle of length six. In order to ensure that codes constructed in this way have no cycles of length four in their corresponding bipartite graphs, and that they have good minimum distance, the exponents of the basic permutation matrix must be chosen with care. For example, it can be shown that if $i_l=l(l-1)/2, 1\le l\le s, 2\le m<l$, and $N\ge s(s+1)/2+(s-m)(m-2)$, then the LDPC code specified by H of the form given by (1) has girth equal to six and minimum distance of at least four.

In general, the code generated by (1) is an *intersection of quasi-circulant (quasicyclic) codes* of order s. A quasi-circulant code of order s is a code with the property that any cyclic shift of a codeword in s positions represents another codeword. The intersection of a class of codes $\boldsymbol{C}$ represents the set of codewords contained in all codes belonging to $\boldsymbol{C}$. Quasi-circulant codes are well known in classical coding theory literature, and they can be efficiently encoded in linear time.

We will give two examples illustrating possible encoding strategies for codes specified by (1).

Example 1: The generator matrix of a code from the block-circulant class can sometimes be obtained directly from the parity check matrix by solving a simple set of linear equations. One such illustrative example for H and G with parameters $N=10, m=2$, and $s=4$, is shown as

$$H=\begin{bmatrix} I & P & P^3 & P^6 \\ P^6 & I & P & P^3 \end{bmatrix}$$
$$G=\begin{bmatrix} Q^6 & Q^4 & Q^3 & Q^9 \\ Q^9 & Q^6 & Q^4 & Q^3 \end{bmatrix}.$$

Here, I denotes the identity matrix and $Q=P^T$. It can be observed that he generator matrix G is also a block-circulant matrix with permutation blocks. This implies that encoding is very straightforward to accomplish.

Example 2: Let $m=3, s>4$, and for the sake of simplicity write $P^{i_t}=P_t, 1\le t\le s$. Additionally, let a codeword $\boldsymbol{x}$ be parsed into s subwords of length N each, i.e.,

$$\boldsymbol{x}=[\boldsymbol{x}_1,\boldsymbol{x}_2,\ldots,\boldsymbol{x}_s].$$

Then encoding can be performed by setting $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_{s-3}$ to the desired information sequence, and by rewriting $H\boldsymbol{x}^{\mathrm{T}} = \mathbf{0}$ as

$$\begin{bmatrix} P_{s-2} & P_{s-1} & P_s \\ P_{s-3} & P_{s-2} & P_{s-1} \\ P_{s-4} & P_{s-3} & P_{s-2} \end{bmatrix} \begin{bmatrix} x_{s-2}^T \\ x_{s-1}^T \\ x_s^T \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \tag{2}$$

where

$$v_1 = P_1 x_1^T + \cdots + P_{s-3} x_{s-3}^T,$$
$$v_2 = P_s x_1^T + \cdots + P_{s-4} x_{s-3}^T,$$
$$v_3 = P_{s-1} x_1^T + \cdots + P_{s-5} x_{s-3}^T.$$

Observe that the matrix on the left-hand side of (2) is *block-Toeplitz* with *circulant* (permutation) blocks. Inverses of such matrices can be computed very efficiently.

If, on the other hand, the set of exponents $\{i_1, i_2, \ldots, i_k\}$ is an $(m+1)$-fold CIDS, then both the minimum distance and the girth of the corresponding code is at least six. Furthermore, the construction for CIDS sets to be described shortly also allows for constructing codes with girths eight, ten, or values exceeding ten, although for this purpose the permutation blocks of the matrix (1) have to be permuted in a certain manner. We will illustrate these ideas in Example 5.

Let θ be a primitive element of the finite field GF(p^4) (where p is an odd prime). Then the set S of integers specified by

$$S = \{a : 0 \leq a < p^4 - 1, \theta^a + \theta \in \text{GF}(p)\} \tag{3}$$

is a p-fold CIDS mod p^4-1 [15]. The length of a CIDS code described by this set is $p(p^4-1)$, which can be very large even for small prime values of p. If one uses the following construction instead:

$$\tilde{S} = \{a : o \leq a < p^2 - 1, \omega^a + \omega \in \text{GF}(p)\} \tag{4}$$

where ω a primitive element GF(p^2), the resulting code length is $p(p^2-1)$, but the set $\tilde{S}$ is not a CIDS. By erasing a carefully chosen subset of elements from $\tilde{S}$ and, if necessary, permuting the order of the elements, one can obtain CIDS set of cardinality usually larger than described by (3).

We will refer to the constructions described above as the *Bose construction,* due to the fact that Bose was the first researcher to use finite field methods to construct *ordinary* difference sets.

Example 3: Let $q = 17$ and let ω be a primitive element of the field GF(17^2) with minimal polynomial $x^2 + x + 3$. Then, the set

$$\{2, 17, 27, 39, 47, 58, 79, 85, 102, 136, 145\,149\,150\,152\,178\,231\,266\}$$

is a difference set modulo $17^2 - 1 = 288$.

For $q = 19$ and ω, a primitive element of the field GF(19^2) with minimal polynomial x^2+x+2, the difference set obtained from the construction given by (4) is

$$\{2, 3, 5, 13, 19, 46, 67, 151, 169, 176, 181, 218, 237, 252, 275, 284, 288, 310, 334\}.$$

For $q = 7$ and θ, a primitive element of GF(7^4) with minimal polynomial $x^4 + x^2 + 3x + 5$, the CIDS set described by (3) contains the following elements:

$$\{431, 561, 1201, 1312, 1406, 1579, 1883\}.$$

The advantage of using the p^2-1 instead of the p^4-1 method is that a larger family of codes can be constructed. The disadvantage lies in the fact that the construction over a field of smaller order usually produces more cycles of short length in the code graph [15].

Example 4: The following sets, obtained by deleting a proper subset of elements from $\tilde{S}$ [constructed according to (4) and Example 3] and subsequently rearranging it, are CIDSs:

$$\{5, 13, 19, 46, 2, 3, 151, 169, 218\}\ (3\text{–fold CIDS})$$
$$\{5, 13, 19, 3, 46, 2, 151, 169, 218\}\ (3\text{–fold CIDS})$$
$$\{5, 275, 310, 13, 19, 284, 3, 2, 151, 176, 46, 218\}\ (3\text{–fold CIDS}).$$

Example 5: A code with girth eight can be constructed by choosing the exponents of the permutation matrices to be rearranged subsets of CIDS sets. For example, a code of rate $\mathrm{R} = 0.75$, length 4320, and girth of eight can be constructed by using 12 block-columns (i.e., 12 permutation matrices within a block-row) and three block-rows (i.e., three permutation matrices within a block-column) with exponents $\{j_1, j_2, \ldots, j_{12}\}$

$$\textbf{first block}: 5, 275, 310, 13, 19, 284, 3, 2, 151, 176, 46, 218;$$
$$\textbf{second block}: 344, 5, 275, 310, 13, 19, 284, 3, 2, 151, 176, 46;$$
$$\textbf{third block}: 151, 334, 275, 13, 284, 2, 176, 218, 5, 310, 19, 3.$$

Based on Tanner's bound for the minimum distance of a code, we have the equation shown at the bottom of the page, where g denotes the girth of the code graph and c denotes the column weight of the parity-check matrix, it follows that the code with girth eight also has minimum distance of at least six. Unfortunately, the encoding problem for this construction becomes significantly more complex because of the fact that the matrix to be inverted during this process is no longer block-Toeplitz.

$$d \geq \begin{cases} 1 + \frac{c}{c-2}((c-1)^{\lfloor (g-2)/4 \rfloor} - 1), & g/2 = 2f + 1 \\ 1 + \frac{c}{c-2}((c-1)^{\lfloor (g-2)/4 \rfloor} - 1) + (c-1)^{\lfloor (g-2)/4 \rfloor}, & g/2 = 2f \end{cases}$$

Some of the sets described above will be used to evaluate the performance of structured LDPC based on (1) and its modifications over optical communication systems. The last element of the first set of Example 3 will be omitted in order to design a code length of 4608, rather than 4896.

Remark: Since the parity check matrix of the codes investigated in this paper consists of permutation matrices, it is fairly straightforward to determine the number of cycles of a given length in the Tanner graph of the code. By referring to the results in [23], one can, for example, see that for the GF}(19^2) construction without subset removal, there are 59 block-cycles of length six in the code. This results in exactly $59 \times 19 = 1121$ six-cycles. By slightly modifying the construction of the parity check matrix as given by (1) into

$$H = \begin{bmatrix} P^{i_1} & P^{i_2} & \cdots & \cdots & P^{i_{s-1}} \\ P^{i_s} & P^{i_1} & \cdots & \cdots & P^{i_{s-2}} \\ P^{i_{s+2(s-2)}} & P^{i_s} & P^{i_2} & \cdots & P^{i_{s+2(s-3)}} \end{bmatrix}$$

it is possible to obtain a code-graph with girth six, but with a significantly smaller number of cycles; for the aforementioned example, the number of cycles becomes only $7 \times 19 = 133$.

For the code construction utilizing the modified difference sets constructed over GF(q^2), the number of cycles of length six also tends to be small. For the third set in Example 4, the number of six-cycles is only $13 \times 19 = 247$.

III. System Model Description

A wavelength-division multiplexing (WDM) system is considered.The continuous-wave laser signals at different wavelengths are modulated by using independently encoded electrical streams and a Mach–Zehnder (MZ) modulator, WDM multiplexed and transmitted over the same fiber. The utilized carrier-suppressed return-to-zero (CSRZ) modulator is composed of a laser diode, two MZ intensity modulators (the first serving as modulator, the second as non-RZ to RZ converter), a pseudorandom bit sequence generator, and an LDPC encoder. The optical signal at the receiver side is split into separate channels by using an optical demultiplexer. Erbium-doped fiber amplifiers (EDFAs) and dispersion compensating fibers (DCF) are deployed periodically to compensate the loss and accumulated dispersion of the standard single-mode fiber (SMF). The direct detection receiver observed is composed of a WDM demultiplexer, a PIN photodiode, an electrical filter, and a sampler followed by a decoder. An EDFA is used as a preamplifier.

The propagation of a signal through the transmission media is modeled by the nonlinear Schrödinger equation [16] solved using the split-step Fourier method, as described in [16].

For more details on the transmission system model implemented, the reader is referred to our previous paper [17], and corresponding derivations may be found in [18].

In optical communication systems, it is customary to use the Q-factor [19] as a figure of merit, rather than the signal-to-noise ratio, and in this paper we will follow this convention.

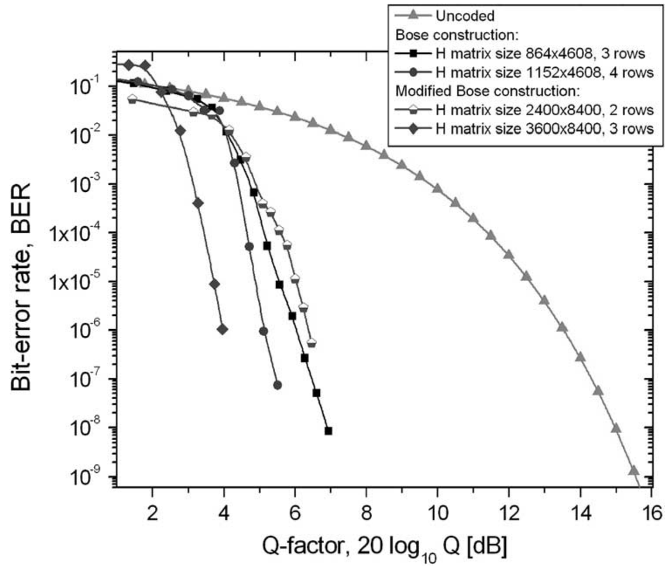

Fig. 1. BER performance of CIDS-based LDPC codes at 40 Gb/s with six iterations of the min-sum algorithm.

IV. Performance of CIDS-Based LDPC Codes

Figs. 1 and 2 plot the BER obtained from Monte Carlo simulations with six iterations for CIDS-based LDPC [4608, 3744] codes with

a) code rate $R = 0.8125$ (redundancy of 23%) employing three block-rows in (1);
b) CIDS-based LDPC [4608, 3456] with code rate 0.75 (redundancy of 33%) with four block-rows;
c) modified CIDS-based codes of length 4320 and 3960 and girth six and eight, respectively.

A WDM system with 40-Gb/s bit rate per channel and a channel spacing of 100 GHz is considered. It is assumed that the observed channel is located at 1552.524 nm (193.1 THz) and that there exists a nonnegligible interaction with six neighboring channels. The influence of optical and electrical filters is taken into account as well. The transmission system considered has a dispersion map composed of an SMF section (of length 80 km), followed by an EDFA to compensate for the fiber losses in the SMF section, and a DCF section to compensate for both GVD and second-order GVD, as well as another EDFA to compensate for the fiber losses in the DCF section. Eight SMF-DCF sections are considered, and the Q-factor is additionally decreased by noise loading.

Although of smaller redundancy, the LDPC [4608, 3744] performs comparably or better than the turbo block code BCH [128, 113, 6]^2 [5] of redundancy 28%. The LDPC [4608, 3456] of redundancy 33% has about a 1-dB larger coding gain then the described turbo block code. The expected coding gain of the LDPC [4608, 3456] code at BER$= 10^{-9}$ is about 10 dB. When a code obtained from the modified Bose construction is employed, the coding gain larger exceeds 11 dB at 10^{-9}, which is the best result reported so far. Notice that the complexity of the min-sum algorithm is significantly lower than the complexity of the turbo-decoding algorithm, since the only operations performed are addition and calculating the minimum. Since the

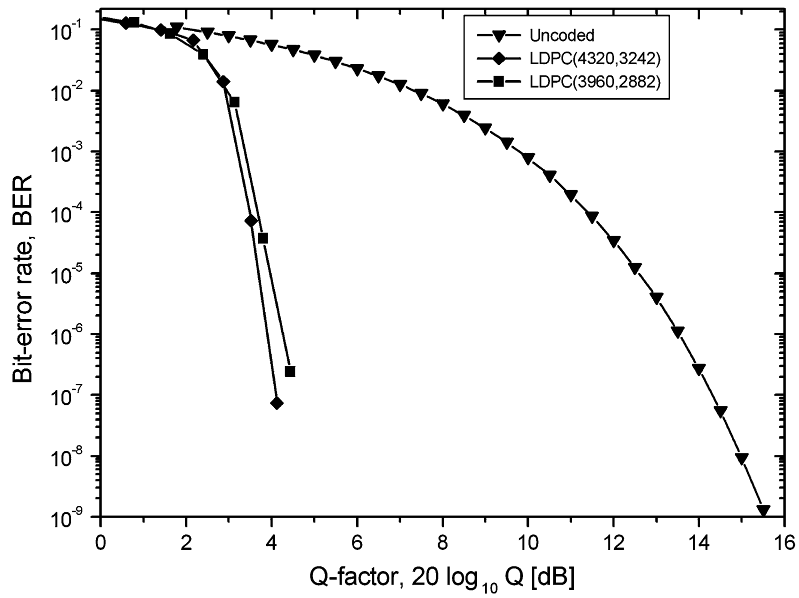

Fig. 2. BER performance of CIDS-based LDPC codes at 40 Gb/s with six iterations of the min-sum algorithm.

code rate influence is included in Q-factor calculation, the reported coding gain is equivalent to the net effective coding gain of an additive white Gaussian noise channel.

V. Conclusion

We proposed a novel class of FEC regular LDPC codes suitable for a long-haul optical transmission system based on cycle-invariant difference sets. The codes have a simple construction, fast-encoding and low-complexity decoding algorithms, high girth, large minimum distance, and excellent BER performance. Since the simulation results show that these block-circulant LPDC codes perform very well in the presence of ASE noise, fiber nonlinearities, chromatic dispersion, and intersymbol interference, and significantly outperform previously proposed FEC schemes utilizing turbo codes, we are confident in proposing their use for high-speed long-haul optical transmission.

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