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BODY FORCE EQUIVALENTS FOR STRESS-DROP SEISMIC SOURCES

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ABSTRACT

The equivalent body forces for a stress-drop seismic source are found. When the isotropic stress drop and one of the three principal stress drops are zero, then the equivalent body forces are the same double couple without moment which would result from a shear dislocation. In general however, all six stress-drop components must be specified as independent functions of time.

THE STRESS-DROP SOURCE

Gilbert (1970) introduced the concept of a seismic moment tensor, which he defined as the volume integral of the stress drop. Gilbert's "stress drop" does not consider the change in stress resulting from the elastic waves radiated by the source. Thus his stress drop, $\tau(\mathbf{x}, t)$ serves only to define the kinematic body forces

$$f_i = -\tau_{ij,j} \tag{1}$$

which in turn generate the dynamic displacement field through the equation of motion

$$\sigma_{ij,j} - \rho \ddot{u}_i = -f_i. \tag{2}$$

On the other hand, several investigators, e.g., Richards (1973), have considered the problem of finding the dynamic dislocation on a fault plane which is compatible with a prescribed stress drop (total change in the stress field at each point on the fault, rather than Gilbert's definition). The dislocation field then is used with the representation theorem to calculate displacements everywhere in the medium. In this paper, we consider Gilbert's source and hence "stress drop" refers to his definition.

The displacements from an arbitrary body-force distribution are given by the representation theorem of de Hoop (1958) and Burridge and Knopoff (1964),

$$u_i(\mathbf{x}, t) = \int_{V_s} G_{ij}[f_j] dV(\xi) \tag{3}$$

where V_s is a volume completely enclosing the region of nonzero body forces, and G_{ij} is the Green's tensor operator.

For the stress-drop source (3) becomes

$$u_i(\mathbf{x}, t) = \int_{V_s} -G_{ij}[\tau_{jk,k}(\xi, t)] dV(\xi). \tag{4}$$

But by Green's theorem (4) becomes

$$u_i(\mathbf{x}, t) = \int_{A_s} -G_{ij}[\tau_{jk} n_k] dA(\xi) + \int_{V_s} \frac{\partial}{\partial x_k} G_{ij}[\tau_{jk}] dV(\xi)\,. \tag{5}$$

Because $\tau_{ij} = 0$ everywhere outside the source region and A_s is a surface completely enclosing the source, the surface integral vanishes, thus

$$u_i(\mathbf{x}, t) = \int_{V_s} G_{ij,k}[\tau_{jk}(\xi, t)] dV(\xi) \tag{6}$$

Point Stress-Drop Sources

Gilbert (1970) introduced the concept of using a moment tensor (volume integral of stress drop) to represent a point source. Gilbert (1973) gives the moment tensor elements for an isotropic source, a shear dislocation and a compensated linear vector dipole. McGarr (1976) uses the moment tensor representation to study earthquakes resulting from volume changes. Randall (1971) showed that seismic moment of a "generalized dislocation" is a tensor.

In general a moment tensor representation has degrees of freedom (the six stress-drop elements or alternatively the three principal axes and the three principal stress drops) and thus six independent time functions. One cannot find "the" source function for a seismic event without first having determined from observations that all six time functions are identical. In this section we will consider the simplified case of a point stress-drop source with fixed principal axes and only two independent principal stress time functions—one for the isotropic dilatational part and one for the deviatoric part.

When represented in the principal axes coordinate system the stress-drop tensor is

$$\tau = \begin{pmatrix} \tau_{11} & 0 & 0 \\ 0 & \tau_{22} & 0 \\ 0 & 0 & \tau_{33} \end{pmatrix} \delta(\mathbf{x}-\mathbf{x}_0)$$

$$= \left\{ \Delta \mathbf{I} g(t) + \begin{pmatrix} D_{11} & 0 & 0 \\ 0 & D_{22} & 0 \\ 0 & 0 & D_{33} \end{pmatrix} h(t) \right\} \delta(\mathbf{x}-\mathbf{x}_0) \tag{7a}$$

where

$$\Delta = \lim_{t\to\infty} \frac{1}{3}(\tau_{11}+\tau_{22}+\tau_{33}) \tag{7b}$$

and

$$\mathbf{D} = \lim_{t\to\infty} \begin{pmatrix} \tau_{11}-\Delta & 0 & 0 \\ 0 & \tau_{22}-\Delta & 0 \\ 0 & 0 & \tau_{33}-\Delta \end{pmatrix} \tag{7c}$$

Let $\mathbf{u}^I(\mathbf{x}, t)$ be displacement from the isotropic stress drop and $\mathbf{u}^D(\mathbf{x}, t)$ be displacement from the deviatoric stress drop. To make the notation less cumbersome, let it be implicit for the remainder of this section that G_{ij} is operating on a point source at $\boldsymbol{\xi} = \mathbf{x}_0$. Then from (6) (remembering that $G_{ij,k}$ is an operator)

$$u_i^I(\mathbf{x}, t) = \Delta(G_{i1,1}+G_{i2,2}+G_{i3,3})[g(t)] \tag{8a}$$

$$u_i^D(\mathbf{x}, t) = (D_{11}G_{i1,1}+D_{22}G_{i2,2}+D_{33}G_{i3,3})[h(t)] \tag{8b}$$

Displacements from a double-couple point source can always be represented as

$$u_i(\mathbf{x}, t) = M(G_{i1,2}+G_{i2,1})[h(t)] \tag{9}$$

after the coordinates have been rotated so that the x_1 and x_2 axes are parallel to the double couple. Burridge and Knopoff (1964) showed that after a 45° rotation of the x_1 and x_2 axes the system in (9) became

$$u_i(\mathbf{x}, t) = M(G_{i1,1}-G_{i2,2})[h(t)] \tag{10}$$

They referred to the $G_{i1,1}$ type force dipoles as doublets.

In the moment rate tensor formulation, the double couple in (10) is a pure deviatoric source with eigenvalues (principal deviatoric stress drops) of M and $-M$. Similarly the compensated linear vector dipole (CLVD) model proposed by Knopoff and Randall (1970) is equivalent to a purely deviatoric stress drop with eigenvalues M, M and $-2M$. (Note that a CLVD is just the sum of two double couples.)

The most general equivalent force representation of a point deviatoric stress-drop source is a mechanism we will name the "triple doublet" which consists of three mutually perpendicular force doublets with no net dilatation. (The double couple, CLVD and triple doublet all are shown in Figure 1.)

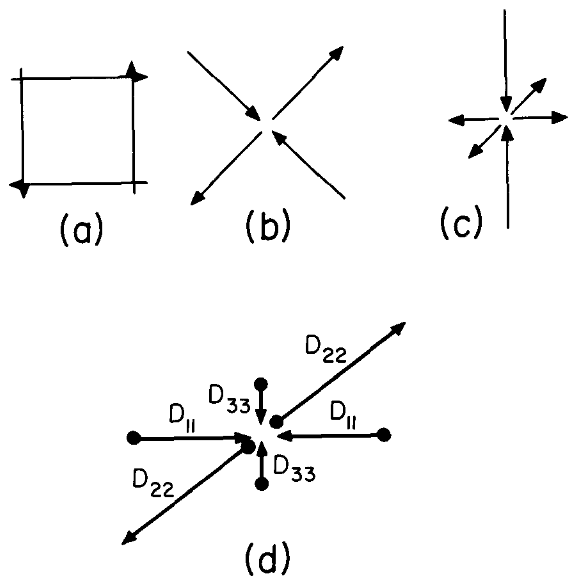

FIG. 1. Double couple, CLVD and triple doublet force systems: (a) and (b) The same double couple represented two different ways. (c) The compensated linear vector dipole (CLVD). (d) The "triple doublet," consisting of three mutually perpendicular force doublets with $D_{11}+D_{22}+D_{33} = 0$.

The "triple doublet" model can easily be extended to include volume change sources by removing the requirement that $D_{11}+D_{22}+D_{33} = 0$. It may be preferable though to separate the source into an isotropic dilatational part and a deviatoric part, because the two parts need not have the same time-history functions.

In a spatially finite source the principal axes may vary from point to point. In general it is impossible to define a set of principal axes for the entire source volume. Thus all six terms of the integrand in (6) must be considered.

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