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BOND AND STOCK RETURNS IN A SIMPLE EXCHANGE MODEL

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ABSTRACT

In this paper I analyze a simple "representative agent" exchange model of general equilibrium, and derive closed form solutions for returns on stocks and real and nominal bonds.

The model restricts the representative agent's utility function to be time-separable with isoelastic period utility, and the endowment to be conditionally lognormal. These assumptions allow me to examine a general stationary stochastic process for the log of the endowment. Money and nominal prices are modelled by means of a Clower constraint.

Risk premia on stocks and real and nominal discount bonds are simple functions of the coefficient of relative risk aversion, the variance of the innovation to the log endowment, and the weights in the moving average representation of the log endowment. One-period holding premia on real bonds may be positive or negative, but the limit as maturity increases is positive. When the money supply is deterministic, stocks and nominal bonds are perfect substitutes. Their expected returns to maturity are higher than those on real bonds of equal maturity, but need not be higher over other holding periods. Nominal interest rates vary positively with prices (the "Gibson paradox") if the coefficient of relative risk aversion is greater than one.

In the last section of the paper I consider random shocks to the agent's utility function. These shocks may generate risk premia even when the agent is risk-neutral.

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In this paper I analyze a simple "representative agent" exchange model of general equilibrium, and derive some new propositions about the determination of returns and risk premia in the term structure and the stock market.

The model presented here derives closed form solutions for asset prices from first-order conditions of the representative agent's intertemporal optimization problem. Net supplies of all assets are zero; therefore there are no income effects of changes in asset prices, and the results of the model arise from substitution effects alone.[1] The driving variable in the model is the representative agent's nonstorable endowment.

In these respects the model is similar to those of Lucas [1978] and LeRoy [1982]. However Lucas does not derive closed forms. LeRoy obtains closed form solutions for a world in which the representative agent's endowment follows a 2-state Markov process, whereas I allow the log of the endowment to follow any stationary stochastic process about a trend. In LeRoy's model, if the state is currently bad it cannot get worse; this is a crucial restriction which I relax by considering general stationary processes. I also consider the effect of "taste shocks" in the utility function.

The cost of greater generality in the stochastic process for the endowment is that I must impose restrictions on the form of the representative agent's utility function and the distribution of endowment and taste shocks. I assume that the log of the endowment, $\log \omega_t$,

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[1] Stiglitz [1970] studied an equilibrium model of asset pricing allowing for both income and substitution effects. In general he was unable to sign the combined result of these two effects.

follows a stationary stochastic process about a trend which for simplicity I assume to be linear. By Wold's Decomposition Theorem, we can write

$$(1)\quad \log \omega_t = gt + \sum_{k=0}^{\infty} \xi_k e_{t-k}$$

where $\sum_{k=0}^{\infty} \xi_k^2 < \infty$ for stationarity, and $\xi_0 = 1$.

The trend growth rate of the endowment is g. The innovation in the log endowment, e_t, is i.i.d. normal with mean zero and standard deviation s_e; thus ω_t is lognormal. I assume that at time t the representative agent possesses no information about ω_{t+i}, $i > 0$, superior to that contained in equation (1). That is, the univariate innovation e_t is the true innovation in the agent's best forecasting equation for ω_t.

I assume that the representative agent's utility function is time-separable with isoelastic period utility discounted by a factor β. The coefficient of relative risk aversion is α. Ignoring taste shocks for the moment, the representative agent solves the problem

$$(2)\quad \text{Max } E_t \sum_{k=0}^{\infty} \beta^k u(c_{t+k}) = E_t \sum_{k=0}^{\infty} \beta^k c_{t+k}^{1-\alpha}/(1-\alpha)$$

where in equilibrium $c_{t+k} = \omega_{t+k}$, all k, because there is no production or storage in the model.

Of course, this is a trivial maximization problem since the agent has no production or even exchange opportunities and simply consumes his endowment. Nevertheless, it can be used to price various assets, since we can ask "at what interest rates is the price-taking agent content to consume his endowment?"

I begin by discussing the determination of real prices of real bonds, then the determination of stock prices. I introduce money, and therefore nominal prices, using a Clower constraint of the type discussed by Lucas [1982], which generates a unit velocity of money. If the money supply is deterministic, I show that stocks and nominal bonds are perfect substitutes. I discuss the relationship between the nominal price level and nominal interest rates. Finally, I modify the basic model by introducing taste shocks into the agent's utility function.

Cox, Ingersoll and Ross [1981] have stressed that conclusions about asset returns in theoretical models are sensitive to the exact definition and holding period considered. Accordingly they argue for continuous time modelling of instantaneous returns. The present paper offers an alternative modelling strategy, in which closed form solutions for discrete time returns may be obtained and compared for any holding period.

1. A Term Structure of Real Bonds

The first-order condition for an i-period real discount bond, which costs P_{it} units of the consumption good today and returns 1 unit of the good in period t+i, is simply

$$(3) \quad E_t[\beta^i c_{t+i}^{-\alpha}] = P_{it} c_t^{-\alpha}$$

$$\rightarrow \quad E_t[\beta^i (c_{t+i}/c_t)^{-\alpha}] = P_{it} = (1+R_{it})^{-i}$$

where R_{it} is the net return per period (or yield) on the i-period bond. The term in square brackets is conditionally lognormal, and thus one can apply the formula for the expected value of a lognormal random variable to obtain

$$(4) \quad i.\log(1+R_{it}) = i.\log(1/\beta) + \alpha E_t[\log c_{t+i} - \log c_t] - (\alpha^2/2)\mathrm{Var}_t[\log c_{t+i} - \log c_t]$$

$$= i.\log(1/\beta) + i\alpha g + \alpha[\sum_{k=0}^{\infty} (\xi_{k+i}-\xi_k)\, e_{t-k}] - (\alpha^2/2)[\sum_{k=0}^{i-1} \xi_k^2]\, s_e^2$$

The first part of this equation is equivalent to expressions in Mankiw [1981] and Hansen and Singleton [1983]; the second part follows from the representation of consumption in equation (1).

For given i, the real interest rate on a real bond is inversely related to the discount factor β and therefore positively related to the rate of time preference. The interest rate rises with the expected increase in log consumption from time t to time t+i; this expected increase has trend and stochastic components.

The real interest rate on a real bond falls as the variance of the endowment shock increases.[2] This can be explained as follows: Miller [1976] has shown that a sufficient condition for saving to increase with labor income uncertainty, in a multi-period model with a known return to saving, is that $u'(c)$ is positive and convex. The isoelastic utility function satisfies this condition. But the equilibrium interest rate is just that rate at which the agent is content to save exactly zero; therefore it falls with endowment uncertainty.

Equivalently, note that the expected value of a convex function of a random argument increases with the variance of the argument: therefore the left hand side of equation (3) increases with the variance of consumption, driving asset prices up and interest rates down.

Risk premia are most conveniently defined in this lognormal model to be the log of the ratio of expected gross returns on alternative investment strategies. These "log ratio" risk premia are constant through time.

Defined this way, Campbell and Shiller's [1984] holding period premium on an i-period bond held for j periods, over a j-period bond, is

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[2] This is a comparative static statement. The variance of the endowment was assumed to be constant through time in the derivation of (4).

$$\phi_{ijt} = i.\log(1+R_{it}) + \log E_t[(1+R_{i-j,t+j})^{-(i-j)}] - j.\log(1+R_{jt})$$

$$= i.\log(1+R_{it}) - (i-j)E_t \log(1+R_{i-j,t+j}) - j.\log(1+R_{jt})$$

$$+ (1/2)\mathrm{Var}_t[\log(1+R_{i-j,t+j})]$$

Once we know ϕ_{ijt}, it is trivial to obtain the conventional "difference" risk premium as

$$(1+R_{jt})^{j}(\exp[\phi_{ijt}]-1)$$

The difference risk premium varies in proportion with the shorter j-period rate, and has the same sign as ϕ_{ijt} when the j-period rate is positive.

Now the conditional variance of the (i-j)-period rate, j periods ahead, is just

$$\mathrm{Var}_t[E_{t+j}\log c_{t+i} - \log c_{t+j}]$$

$$= \mathrm{Var}_t[\sum_{k=0}^{\infty} (\xi_{i-j+k}-\xi_k)\, e_{t+j-k}]$$

$$= [\sum_{k=0}^{j-1} (\xi_{i-j+k}-\xi_k)^2]\, s_e^2$$

Then straightforward but tedious calculation shows that

$$(5)\quad \phi_{ijt} = \alpha^2 \Big[\sum_{k=0}^{j-1} \xi_k(\xi_k - \xi_{i-j+k})\Big] s_e^2$$

We are particularly interested in the special case of a unit holding period. (5) implies that

$$(6)\quad \phi_{i1t} = \alpha^2 [1-\xi_{i-1}] s_e^2$$

The 1-period holding premium for an i-period bond is positive whenever $\xi_{i-1}<1$. The intuition behind this result is simple. When $\xi_{i-1}<1$, a positive endowment shock between t and t+1 raises the t+1 endowment more than it raises the expected endowment at t+i when the bond matures. Thus a positive endowment shock lowers the yield on the bond and gives a capital gain to bondholders. Capital gains on such bonds are positively correlated with the ratio of consumption tomorrow to consumption today, and negatively correlated with the corresponding ratio of marginal utilities. These bonds must therefore have a higher expected yield.

In general there will be some i for which $\xi_i>1$, and thus some negative term premia.[3] But the assumption that the endowment is stationary restricts the proportion of negative 1-period holding premia. Since

$\sum_{i=0}^{\infty} \xi_i^2 < \infty$, we must have $\lim_{n\to\infty} \sum_{i=0}^{n} \xi_i^2/n = 0$ and $\lim_{i\to\infty} \xi_i = 0$.

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[3] These cases were ruled out by LeRoy's 2-state model.

The first limit implies that the proportion of maturities i for which the square of ξ_i exceeds unity, in a sample of the first n maturities, must go to zero as $n \to \infty$. A fortiori the proportion of negative 1-period holding premia must go to zero. The second limit implies that 1-period holding premia approach

$\alpha^2 s_e^2$ as $i \to \infty$.

Two simple examples of stochastic processes for the log endowment may help to clarify the implications of the model. When the log endowment follows an AR(1) with parameter a such that $-1<a<1$, then

$$\xi_i = a^i$$

and 1-period holding premia are always positive. They increase monotonically with maturity when $0<a<1$. When the log endowment follows an AR(2) with parameters a_1 and a_2 and roots κ_1 and κ_2, then

$$\xi_i = \sum_{k=0}^{i} \kappa_1^k \kappa_2^{i-k}.$$

If the roots are both real and positive, ξ_i may have a "hump shape" (Blanchard [1981]), and there will be some negative 1-period holding premia at the short end of the term structure.

Figure 1 is a graphical illustration of the determination of one-period holding premia. It displays a typical impulse response function, ξ_i as a function of i, and the regions of negative and positive holding premia. The slope of a line between the points $(0,\xi_0)$ and $(i-1,\xi_{i-1})$ determines the response of the yield on an i-period bond to a unit positive innovation in the endowment. When this slope is positive, the holding premium is negative, and vice versa.

The expected excess return on an i-period bond held to maturity, over a sequence of one-period bonds, is what Campbell and Shiller [1984] call the <u>rolling premium</u>. For the purposes of this model, it is written as

$$\phi'_{i1t} = i.\log(1+R_{it}) - \log E_t[(1+R_{1t})\ldots(1+R_{1,t+i-1})]$$

$$= i.\log(1+R_{it}) - E_t\left[\sum_{k=0}^{i-1}\log(1+R_{i,t+k})\right]$$

$$- (1/2)\mathrm{Var}_t\left[\sum_{k=0}^{i-1}\log(1+R_{i,t+k})\right]$$

As in the case of the holding premium, it is trivial to obtain the conventional "difference" rolling premium as

$$E_t[(1+R_{1t})\ldots(1+R_{1,t+i-1})](\exp[\phi'_{i1t}]-1)$$

which has the same sign as ϕ'_{i1t} so long as the expected return on the rollover strategy is positive.

It turns out that

$$(7)\quad \phi'_{i1t} = \alpha^2\left[\sum_{k=0}^{i-1}\xi_k(1-\xi_k)\right]s_e^2$$

To understand the intuition of this result, consider the case where i=2. Then

(8) $\phi'_{21t} = \alpha^2 [\xi_1(1-\xi_1)]s_e^2$

ϕ'_{21t} can be negative if $(1-\xi_1)$ is negative, a case we have already discussed, or if ξ_1 is negative. To understand the latter condition, note that the return on a 2-period rollover strategy is particularly high when short rates are higher than expected in period t+1. With $\xi_1<1$, this occurs when there is a negative endowment shock in period t+1. If ξ_1 is negative, the endowment will on average rebound to a higher level in period t+2 than the level that was expected in period t; thus returns on the rollover strategy are positively correlated with the ratio of period t+2 consumption to period t consumption, and must be higher on average than the returns on a "safe" strategy of holding a 2-period bond for 2 periods. In other words, when ξ_1 is negative the 2-period bond has a negative risk premium. If ξ_1 is positive, however, a negative endowment shock at t+1 will tend to be followed by a lower endowment at t+2 than was originally expected, and the above conclusions are reversed.

Stationarity alone does not generate a presumption that rolling premia are positive. Under the strong condition that $0 \leq \xi_i \leq 1$, all i, rolling premia are positive for all i (as are all holding premia).

Finally I consider whether in this model the spread between the i-period rate and the j-period rate, j<i, is positively related to the level of risk premia. Campbell and Shiller [1984] argue that this should be true in general. From equation (4),

$$(9) \quad \log(1+R_{it}) - \log(1+R_{jt}) =$$

$$(\alpha/i)[\sum_{k=0}^{\infty} (\xi_{i+k}-\xi_k)\, e_{t-k}]$$

$$- (\alpha/j)[\sum_{k=0}^{\infty} (\xi_{j+k}-\xi_k)\, e_{t-k}]$$

$$+ \{(\alpha^2/j)[\sum_{k=0}^{j-1} \xi_k^2] - (\alpha^2/i)[\sum_{k=0}^{i-1} \xi_k^2]\}$$

The first two terms relate the spread to expected future interest rates. The last term relates the spread to the variance of the endowment. This variance is multiplied by the difference between the average of the first j terms in the series of squared ξ's, and the average of the first i terms. This difference can in general be positive or negative, but once again stationarity imposes the restriction that as i approaches ∞ with fixed j, the difference becomes positive. Thus in the limit the spread is positively related to the variance of the endowment and thus the level of risk premia.

2. Stocks and Nominal Bonds

It is simple to price stock in this model. A "stock" is simply a claim to a share of the random endowment at some period in the future.[4] Define P^*_{it} as the real time t price of a claim to the whole endowment at time t+i, $\omega_{t+i} = c_{t+i}$. Then

$$(10) \quad E_t[\beta^i c_{t+i}^{1-\alpha}] = P^*_{it} c_t^{-\alpha}$$

and we can use this first order condition to solve for P^*_{it}. The real log return on the stock over j periods, j<i, is $\log P^*_{i-j,t+j} - \log P^*_{it}$, and the real log return over i periods is $\log c_{t+i} - \log P^*_{it} = i.\log(1+R^*_{it})$. By contrast with real bonds, the i-period real return on an i-period stock is random, since the final payoff is random.

Solving for the real price and i-period log expected return of an i-period stock, we find

$$(11) \quad \log P^*_{it} = i.\log(\beta) + E_t \log c_{t+i} - \alpha[E_t \log c_{t+i} - \log c_t] + (\alpha^2/2)\, Var_t \log c_{t+i}$$

$$(12) \quad \log E_t[(1+R^*_{it})^i] = i.\log(1+R_{it}) + \alpha[\sum_{k=0}^{i-1} \xi_k^2] s_e^2$$

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[4] I do not explicitly consider a more realistic "consol-like" stock, a claim to the whole stream of future endowments. The price of such a stock is the sum over i of the prices P* in equation (10). Unfortunately, the formula for this price is messy, since the model generates simple solutions for log rather than natural prices.

Over i periods, the real return on an i-period stock is always expected to be higher than the real return on an i-period real bond. This result is unsurprising since the payoff and therefore the return on the stock are perfectly correlated with consumption at time t+i.

In equation (4), we saw that an increase in the variance of the endowment innovation lowered expected real bond returns. It follows from equation (12) that it lowers expected real stock returns only if $\alpha < \alpha^2/2$, that is if $\alpha > 2$. For these high values of α, the fall in the real bond return outweighs the increase in the stock risk premium.

Although an i-period stock is always expected to yield more than an i-period real bond over i periods, this result does not carry over to other holding periods or real bonds of other maturities. The expression for the j-period holding premium on an i-period stock, over a j-period real bond, is

$$(13)\ \phi^*_{ijt} = \log E_t P^*_{i-j,t+j} - \log P^*_{it} - j.(1+R_{jt})$$

$$= \alpha^2 \left[\sum_{k=0}^{j-1} \xi_k^2 \right] + \alpha(1-\alpha) \left[\sum_{k=0}^{j-1} \xi_k \xi_{i-j+k} \right]$$

$$= \phi_{ijt} + \alpha\left[\sum_{k=0}^{j-1} \xi_k \xi_{i-j+k} \right] s_e^2$$

The j-period holding premium on an i-period stock is the sum of the j-period holding premium on an i-period bond, and a term resulting from the dividend uncertainty on stock. Neither term is unambiguously positive or negative in general. This illustrates the basic point that, for assets with a single payoff, payoff uncertainty translates directly into uncertainty about returns only when the holding period equals the maturity of an asset. Over other holding periods, an asset whose payoff is positively correlated with consumption may have a return which is negatively correlated with consumption and thus a negative holding premium.

I now discuss the introduction of money and nominal prices into the model. There is one major problem with this extension. The possibility of transferring resources from one period to another by means of money, at a zero nominal interest rate, constrains the nominal interest rate to be non-negative. In some models it is possible to assume that this constraint is never binding (Lucas [1982]); unfortunately the lognormal distributions of the present model are inconsistent with this assumption.

If the constraint binds periodically, the solution for prices and nominal interest rates becomes intractable. Accordingly I introduce "money" but ignore its role as a store of value. This could be justified either by postulating some confiscatory tax on end-of-period money balances, or as an approximation to the exact solution of the model when the parameters are such that the constraint binds only very rarely.

Following Lucas [1982], I assume that the representative agent faces a Clower constraint of the form $P_t c_t \leq M_t$, and that this always holds with equality. In order to focus on the nominal effects of endowment shocks, I further assume that the supply of money follows a deterministic trend: $\log M_t = g_M t$. Then

$$(14) \quad \log P_t = g_M t - \log c_t$$

so the log price level moves inversely with the log endowment.

Now consider the pricing of nominal bonds. The log real payoff on an i-period nominal bond is $\log (1/P_{t+i}) = \log c_{t+i} - g_M(t+i)$. But this payoff is perfectly correlated with consumption at time t+i, so with a deterministic money supply an i-period nominal bond is equivalent to an i-period stock.[5] The propositions stated above for expected real returns on stocks carry over directly to nominal bonds.

However once we have a nominal price of goods, we may be interested in nominal prices and expected returns of nominal bonds. The log nominal price of an i-period nominal bond is just $\log P^*_{it} + \log P_t$, and the known log nominal return is the negative of this. But

$$(15) \quad -\log P^*_{it} - \log P_t = i.\log (1/\beta) + i.g_M$$

$$- (1-\alpha)[E_t \log c_{t+i} - \log c_t]$$

$$-(\alpha^2/2)\mathrm{Var}_t \log c_{t+i}$$

[5] Lucas [1982] also notes this (p.348).

There are two important features of this equation. First, for any parameter values there is a finite probability that the right hand side of (15) is negative: this is the problem with the monetary extension of the model mentioned above. However, the probability may be very small with a high rate of time preference, a high rate of trend inflation, and a low variance of the endowment.

Secondly, the equation characterizes the covariance of detrended nominal interest rates and the price level. We find

$$(16)\quad \mathrm{Cov}[-\log P^*_{it} - \log P_t,\ \log P_t]$$

$$= (1-\alpha)\left[\sum_{k=0}^{\infty} (\xi_{k+i} - \xi_k)\xi_k\right] s_e^2$$

The term in square brackets is the difference between the i'th autocovariance and the variance of the detrended endowment. By the Cauchy-Schwartz inequality, this must be negative, so the covariance of prices and nominal interest rates is positive when $\alpha > 1$ and negative when $\alpha < 1$. This result is independent of i.

A positive covariance seems counter-intuitive at first: when prices are unusually high, on average they are expected to fall so one might expect nominal interest rates to be unusually low. However a positive covariance has been found in much historical data, and is often referred to as the "Gibson paradox" (Sargent [1973], Shiller and Siegel [1977]).

The reason why this model may generate a "Gibson paradox" is as follows. When the endowment is unusually low, the price level is unusually high and is expected to fall. This expected deflation lowers the log nominal interest rate one for one. However, the log real interest rate is also unusually high when the endowment is low: it is increased by a factor of α. The real interest rate effect outweighs the inflation effect when $\alpha > 1$.

3. Taste Shocks

In the traditional literature on the term structure, it was often asserted that investor preferences for consumption at a particular date would lower yields on bonds due to mature at that date, and furthermore would cause such bonds to have negative risk premia.[6]

The model of the previous sections can be adapted to study this question by adding multiplicative shocks λ_t to the utility function. Then (2) becomes

$$\text{(17)} \quad \text{Max} \sum_{i=0}^{\infty} \beta^i u(c_{t+i}) = \sum_{i=0}^{\infty} \beta^i \lambda_{t+i} c_{t+i}^{1-\alpha}/(1-\alpha)$$

The first-order condition (3) becomes

$$\text{(18)} \quad E_t[\beta^i (\lambda_{t+i}/\lambda_t)(c_{t+i}/c_t)^{-\alpha}] = P_{it} = (1+R_{it})^{-i}$$

Note that when $\alpha = 1$, that is when the agent has a log utility function, taste shocks enter the first order condition (18) in exactly the same way as endowment shocks. The effect of taste shocks should not be confused with the effect of a non-geometric discount function. The latter would cause expected changes through time in the relative valuation of consumption at two dates, and thus would generate a time inconsistency problem. Taste shocks, however, are indexed by time t+i rather than by distance from the present time i, and so do not lead to time inconsistency.

[6] See for example Modigliani and Sutch [1966].

When the taste shocks are deterministic, that is when λ_{t+i} is known at time t, for all i, then we find

$$(19) \quad i.\log(1+R_{it}) = i.\log(1/\beta) + \alpha E_t[\log c_{t+i} - \log c_t] - [\log \lambda_{t+i} - \log \lambda_t] - (\alpha^2/2)\mathrm{Var}_t[\log c_{t+i} - \log c_t]$$

Clearly it is true that a positive shock to the marginal utility of consumption at time t+j, λ_{t+j}, lowers the yield on an i-period bond. However since the shock is deterministic it also lowers the expected return on all other investment strategies maturing at t+j, and therefore does not generate negative risk premia.[7]

If taste shocks are to generate risk premia in this model, they must be random and therefore contribute a conditional variance term to the formula for the interest rate. Suppose that the log of the taste parameter λ_t follows a stationary stochastic process, in a manner analogous to the process for the endowment. We write

$$(20) \quad \log \lambda_t = \sum_{k=0}^{\infty} \psi_k u_{t-k}$$

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[7] Since difference premia vary with expected returns, they will be affected by deterministic taste shocks. The rolling difference premium on a j-period bond will fall with a positive taste shock j periods ahead. However the holding difference premium on a longer i-period bond over a j-period bond will also fall, so this pattern of premia is not the one predicted by the traditional literature.

As before, we assume that u_t is i.i.d. normal with mean zero and standard deviation s_u. For simplicity, assume that u_t and e_t are independent. Then

$$\begin{aligned}(21)\quad i.\log(1+R_{it}) = {} & i.\log(1/\beta) + \alpha E_t[\log c_{t+i} - \log c_t] \\ & - E_t[\log \lambda_{t+i} - \log \lambda_t] \\ & - (\alpha^2/2)\mathrm{Var}_t[\log c_{t+i} - \log c_t] \\ & - (1/2)\ \mathrm{Var}_t[\log \lambda_{t+i} - \log \lambda_t]\end{aligned}$$

where $\mathrm{Var}_t[\log \lambda_{t+i} - \log \lambda_t] = [\sum_{k=0}^{i-1} \psi_k^2]\, s_u^2$.

Note that with taste shocks, interest rates may vary randomly through time even when the representative agent is risk-neutral (α=0).

The random taste shocks add new terms to the formulae for risk premia developed in the first section. We find

$$\begin{aligned}(22)\quad \phi_{ijt} = {} & \alpha^2 [\sum_{k=0}^{j-1} \xi_k(\xi_k - \xi_{i-j+k})]s_e^2 \\ & + [\sum_{k=0}^{j-1} \psi_k(\psi_k - \psi_{i-j+k})]s_u^2\end{aligned}$$

$$\begin{aligned}(23)\quad \phi_{i1t} = {} & \alpha^2 [1-\xi_{i-1}]s_e^2 \\ & + [1-\psi_{i-1}]s_u^2\end{aligned}$$

$$(24)\quad \phi'_{i1t} = \alpha^2 \left[\sum_{k=0}^{i-1} \xi_k(1-\xi_k)\right] s_e^2 + \left[\sum_{k=0}^{i-1} \psi_k(1-\psi_k)\right] s_u^2$$

However the risk premia on stocks and nominal bonds over real bonds are not increased by taste shocks, because stock and nominal bond payoffs are unaffected by these shocks.

As before, it is most instructive to focus the discussion on the 1-period holding premia. Consider the taste-shock components of these premia. By contrast with the endowment-shock components, they do not vanish as the coefficient of relative risk-aversion α goes to zero. Random preferences may generate risk premia even when agents are risk-neutral, which provides a counter-example to the traditional view that risk premia are zero under risk-neutrality.[8]

The taste shock components of 1-period holding premia may in general be positive or negative, but stationarity of taste shocks generates a presumption that they are positive. Thus the analysis of both endowment shocks and taste shocks lends some support to Hicks' [1939] proposition that risk premia on long bonds are positive.

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[8] Cox, Ingersoll and Ross [1981] provide an alternative counter-example in which interest rates are random because of shocks to the marginal productivity of capital. They also discuss deterministic preferences for consumption at one particular date, but do not consider random taste shocks.

4. Conclusion

In this paper I have presented a simple exchange model and discussed its implications for asset pricing. The model restricts the form of the representative agent's utility function and the distribution of shocks in a way which enables the derivation of closed-form solutions for asset prices and returns. In a significant generalization of previous work, the representative agent's endowment is modelled as a general stationary stochastic process rather than as a univariate Markov process.

The model sheds light on four major issues:

1) It supports the view of Hicks [1939] that risk premia on long bonds should generally be positive. The risk premium on a bond of any particular maturity i may be negative, but the limit as i approaches ∞ must be positive if the agent has positive relative risk aversion and interest rates are random.

2) The model does not support the contention of Modigliani and Sutch [1966] that investor preferences for consumption at a particular date lower risk premia on bonds maturing at that date. Random taste shocks do generate risk premia, however, and as above there is a presumption that these premia are positive for long bonds. This effect is independent of the agent's degree of relative risk aversion.

3) The model shows that stocks are not necessarily expected to yield more than real bonds except when both assets have the same maturity date and are held to maturity. In general, an asset with greater payoff uncertainty need not have greater uncertainty of return over some short holding period.

4) The model suggests a possible explanation for the "Gibson paradox", the positive correlation of prices and nominal interest rates noted in much historical data. If the supply of money is deterministic, prices move inversely with the endowment and expected inflation moves inversely with the real interest rate. If the coefficient of relative risk aversion is greater than one, high real interest rates raise nominal interest rates when the price level is high.

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FIGURE 1

DETERMINATION OF 1-PERIOD HOLDING PREMIA ON i-PERIOD BONDS