Bond Market Exposures to
Macroeconomic and Monetary Policy Risks

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Abstract

The paper estimates a model that allows for shifts in the aggressiveness of monetary policy and time variation in the distribution of macroeconomic shocks. These model features induce variations in the cyclical properties of inflation and the riskiness of bonds. The estimation identifies inflation as procyclical from the late 1990s, when the economy shifted toward aggressive monetary policy and experienced procyclical macroeconomics shocks. Since bonds hedge stock market risks when inflation is procylical, the stock-bond return correlation turned negative in the late 1990s. The risks of encountering countercyclical inflation in the future could lead to an upward-sloping yield curve, like in the data.

1 Introduction

In the current macroeconomic environment several stylized bond market facts are different from those in the previous decades: inflation risk premium is low and possibly negative; the correlation between U.S. Treasury bond returns and stock returns, while positive in the 1980s, has turned negative in the last decade.[1,2] There is an understanding in the literature that these new facts can be reconciled in models that allow for inflation to be exogenously an either good or bad signal for future consumption growth, for example, Burkhardt and Hasseltoft (2012) and David and Veronesi (2013). When inflation carries good news to consumption growth (hence procyclical) nominal bonds are safe and provide a hedge. Since nominal bonds behave similar to real bonds in this environment, these models imply negative stock-bond return correlation, negative inflation risk premium, and a downward-sloping nominal yield curve. However, in the data, the nominal yield curve still slopes up during the same periods in which the stock-bond return correlation and inflation risk premium are negative. This recent evidence is interesting because it shows the limitations of the existing approaches and highlights the importance of understanding the sources of inflation and bond market risks and how they change over time.

This paper provides an economic mechanism underlying the inflation dynamics and bond markets by introducing three new elements in a consumption-based asset pricing model: (1) a monetary policy rule with time-varying inflation target, (2) shifts in the strength with which the Federal Reserve steers actual inflation toward the inflation target, and (3) shifts in the covariance of inflation target and real consumption growth shocks.[3] The first extension leads to "endogenous" inflation, and the nominal assets inherit the properties of monetary policy. The second and third extensions induce variations in the cyclical properties of inflation, and these lead to the risk premium and the correlation between stock and bond returns switching signs. Agents are aware of the possibility of encountering countercyclical inflation in the future due to changes in the aggressiveness of monetary policy and the distribution of macroeconomic shocks. Consequently, they demand compensations for holding nominal bonds that might be exposed to future inflation risks: risks and compensations are greater for longer-term bonds resulting in an upward-sloping nominal yield curve.

The model features three distinct economic regimes: (1) the CA regime occurs when the conditional covariance between inflation target and real consumption growth is negative (*Countercyclical*

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[1]See Fleckenstein, Longstaff, and Lustig (2015) and Chen, Engstrom, and Grishchenko (2016).

[2]Baele, Bekaert, and Inghelbrecht (2010), Campbell, Pflueger, and Viceira (2015), Campbell, Sunderam, and Viceira (2016), and David and Veronesi (2013).

[3]The model follows the long-run risk literature on the real side of the economy, and extends the previous literature to include the nominal sector and changing economic regimes.

macroeconomic shocks) and the Federal Reserve increases interest rates more than one-for-one with inflation (*Active* monetary policy); (2) the CP regime occurs when macroeconomic shocks are *Countercyclical* and the Federal Reserve raises interest rates less than one-for-one with inflation (*Passive* monetary policy); and (3) the PA regime occurs when the conditional covariance between inflation target and real consumption growth is positive (*Procyclical* macroeconomic shocks) and the monetary policy is *Active*. The model is estimated with Bayesian techniques using monthly information from asset prices (aggregate stock market and nominal Treasury yield curve) and macrovariables (consumption growth, CPI inflation) that range across the 1963-2014 period. The estimation of the model quantifies the role of monetary policy and macroeconomic shocks played in triggering changes in the inflation dynamics and, ultimately, in the bond market.

The estimation of the model delivers several important empirical findings. First, the model supports the idea that the U.S. economy was subject to *occasional* regime switches: the CP regime was prevalent until the early 80s; the economy switched to the CA regime after the appointment of Paul Volcker as Chairman of the Federal Reserve; and it switched to the PA regime in the late 90s and largely remained in that regime throughout the sample. The historical paths of the monetary policy stance are consistent with the empirical monetary literature.[4] According to the estimated transition matrix, the unconditional regime probabilities for the CA, CP, and PA regimes are 0.35, 0.33, and 0.32, respectively. The unconditional probability of staying in the active monetary policy regime, as indicated by the sum of the probability of the CA and PA regimes, is around 0.67 twice larger than that of the passive monetary policy regime, that is, the CP regime.

Second, the model accounts for significant changes in the inflation dynamics observed in the data. The estimation finds that inflation has become procyclical and less risky as the economy shifted toward an active monetary policy and experienced procyclical macroeconomic shocks. As the economy shifted from the CP regime to the CA regime and to the PA regime, the variance and persistence of inflation decreased substantially. Nevertheless, inflation risks are substantial in the model because the unconditional probability of experiencing countercyclical macroeconomic shocks, as indicated by the sum of the probability of the CA and CP regimes, is 0.68.

Third, the model finds that each regime carries distinctly different inflation risks, and uncertainty about movement across the regimes poses additional risks to bond markets. To understand the properties of regime risks, two sets of simulation exercises are conducted: one in which the regimes are fixed and the other in which regime switching is allowed in the economy. I first characterize

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[4]Active monetary policy dominates most of the sample after the early 80s. The paths for monetary policy are broadly consistent with those found in Bianchi (2012), Clarida, Gali, and Gertler (2000), Ang, Boivin, Dong, and Loo-Kung (2011), Bikbov and Chernov (2013), and Coibon and Gorodnichenko (2011).

each regime risk in a fixed-regime economy, and, subsequently, by allowing regime switching, I isolate the effect of expectations on asset prices. In a fixed-regime economy, agents dislike the CP regime since there is a large shock to inflation target that comes with low consumption growth and the Federal Reserve does not react aggressively enough to it. Inflation risks are significant in this environment. Note that the CP regime is the extreme version of the economy as described in Piazzesi and Schneider (2006), Wachter (2006), Eraker (2008), and Bansal and Shaliastovich (2013), who assume inflation is countercyclical and risky. Their intuitions carry over: the implied risk premium, stock-bond return correlation, and the slope of the yield curve are positive and largest in magnitude among all regimes. The implications of the CA regime are qualitatively similar to those of the CP regime, but implied inflation risks are smaller in magnitude. On the other hand, inflation becomes procyclical and nominal bonds are hedges and safe in the PA regime. The implied risk premium and the stock-bond return correlation are negative, and the nominal yield curve slopes downward in the PA regime.

Once regime switching is allowed, the model is able to generate an upward-sloping nominal yield curve, while maintaining negative risk premium and stock-bond return correlation in the PA regime. Agents are averse to moving into the CP regime in the future and demand compensation for holding nominal bonds exposed to inflation risks. Since longer-term bonds demand greater compensation for inflation risks, the model leads to an upward-sloping nominal yield curve. Note that this is possible because the unconditional probability of falling into the CP regime, which is a disaster regime in my model economy, is substantially high, around one-third. Consequently, an uncertainty about future changes in regimes, combined with an early resolution of uncertainty, amplifies inflation risk premium and leads to an upward-sloping nominal yield curve. How is it, then, that the model generates negative risk premium and stock-bond return correlation and, at the same time, produces an upward-sloping nominal yield curve? Under the estimated parameter configuration, I find that the risks of moving across regimes have a disproportionately larger impact on the slope of the yield curve than on the risk premium or on the stock-bond return correlation. Thus, while the risk premium and the stock-bond return correlation become much less negative in magnitude (because of the future inflation risks), only the slope of the yield curve switches sign. The key takeaway is that a regime uncertainty can go a long way in modifying equilibrium outcomes and is quantitatively very important risk factor in the bond market.

Related Literature. My work is related to a number of recent papers that study the changes in the stock-bond return correlation. Baele, Bekaert, and Inghelbrecht (2010) utilize a dynamic factor model in which stock and bond returns depend on a number of economic state variables, for example, macroeconomic, volatility, and liquidity factors. The authors attribute the changes in the

stock-bond return correlation to liquidity factors. Campbell, Sunderam, and Viceira (2016) embed time-varying stock-bond return covariance in a quadratic term-structure model and argue that the root cause is changes in nominal risks in bond markets. Different from reduced-form studies, my work builds on a consumption-based equilibrium model with monetary policy to identify the driving forces behind the changes in the stock-bond return correlation.

The works closest to my paper are those of Burkhardt and Hasseltoft (2012), Campbell, Pflueger, and Viceira (2015), and David and Veronesi (2013). Burkhardt and Hasseltoft (2012) find an inverse relation between stock-bond return correlations and correlations of growth and inflation. Burkhardt and Hasseltoft (2012) rationalize their findings in a consumption-based asset pricing model with regime switching (in the conditional dynamics of macroeconomic fundamentals) calibrated to data on fundamentals and asset returns. Campbell, Pflueger, and Viceira (2015) examine the role of monetary policy in nominal bond risks using a New Keynesian model. Using macroeconomic fundamentals and asset prices, Campbell, Pflueger, and Viceira (2015) calibrate the model separately over three different subsamples. From the simulation analysis, the authors claim that the change in monetary policy parameters is the main driver of bond risks. David and Veronesi (2013) estimate an equilibrium model of learning about inflation news and show that variations in market participants' beliefs about inflation regimes strongly affects the direction of stock-bond return correlation.

My paper is distinct from their works along several important dimensions. First, the structural changes in the economy are identified from macroeconomic fundamentals and asset prices without imposing (sometimes ad hoc) assumptions, for example, known break points, like in Burkhardt and Hasseltoft (2012) and Campbell, Pflueger, and Viceira (2015). Second, I explicitly account for the role of market participants' beliefs about regime switches in inflation and bond prices. I find strong empirical evidence in the data that the anticipation of moving across regimes is one of the key risk factors priced in the bond market. For example, ignoring the role of beliefs overstates (understates) the implications of a passive (active) monetary policy regime or countercyclical (pro-cyclical) macroeconomic shock regime because the risk properties of alternative regimes are not incorporated. Campbell, Pflueger, and Viceira (2015) do not allow for a beliefs channel to operate. Third, my model exhibits a richer structure than that of David and Veronesi (2013). By accounting for time variations in the covariance matrix of macroeconomic shocks and in monetary policy parameters, I am able to provide extensive descriptions of the bond market transmission mechanism of monetary policy and macroeconomic shocks. In this regard, my model complements the work of Burkhardt and Hasseltoft (2012), Campbell, Pflueger, and Viceira (2015), and David and Veronesi (2013).[5]

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[5]Ermolov (2015) considers stock-bond return correlation in a model with exogenous consumption and inflation

By investigating the time variation in the stance of monetary policy, my work also contributes to the monetary policy literature, for example, Bianchi (2012), Clarida, Gali, and Gertler (2000), Coibon and Gorodnichenko (2011), Davig and Doh (2014), Lubik and Schorfheide (2004), Schorfheide (2005), and Sims and Zha (2006).[6] While most of these papers study the impact of changes in monetary policy on macroeconomic aggregates, the papers of Ang, Boivin, Dong, and Loo-Kung (2011), Bikbov and Chernov (2013), Shaliastovich and Yamarthy (2015), and Ireland (2015) focus on their bond market implications (using reduced-form modeling frameworks). My work distinguishes itself from these papers, since I focus on a fully specified economic model and characterize time-varying bond market exposures to monetary policy risks.

In terms of modeling the term structure with recursive preferences, this paper is closely related to those of Gallmeyer, Hollifield, Palomino, and Zin (2007), Eraker (2008), Bansal and Shaliastovich (2013), Le and Singleton (2010), Doh (2012), Creal and Wu (2016), and Piazzesi and Schneider (2006), who work in an endowment economy setting, and van Binsbergen, Fernández-Villaverde, Koijen, and Rubio-Ramírez (2012) and Kung (2015), who study a production-based economy. While van Binsbergen, Fernández-Villaverde, Koijen, and Rubio-Ramírez (2012) and Kung (2015) allow for capital and labor supply and analyze their interaction with the yield curve, which are ignored in my analysis, they do not allow for time variation in monetary policy stance, which is key risk factor in my analysis.

The remainder of the paper is organized as follows. Section 2 discusses the new stylized facts surrounding U.S. Treasury bond markets. Section 3 introduces the model environment, describes the model solution and asset pricing implications. Section 4 discusses the data set, Bayesian inference, and empirical findings. Section 5 provides concluding remarks.

2   Empirical Evidence on Structural Changes

In this section, I empirically document changes in the cyclical properties of inflation, the Treasury yield curve, and the correlation between bond and stock returns.

A recurrent theme of macrofinance term structure models that underlies risk premiums is that inflation uncertainty makes nominal bonds risky.[7] A common approach, supported by empirical evidence, is to assume that inflation carries bad news about consumption growth. Inflation erodes

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dynamics. Ermolov's work came out after the first version of my paper.

[6]Note that I am including those that explicitly account for changes in monetary policy.

[7]Macrofinance term structure models refer to models in which the pricing kernel directly comes from a utility-maximization problem. Gürkaynak and Wright (2012) provide a nice overview of macrofinance term structure models.

the value of nominal bonds precisely at times during which consumption growth is low (or marginal utility is high). In this environment, investors demand compensation for holding nominal assets exposed to inflation risk. Since longer-term bonds require greater compensation for this inflation risk, this implies that the nominal yield curve ought to slope up, for example, Piazzesi and Schneider (2006) and Bansal and Shaliastovich (2013). Note that since large inflation shocks always come with low real growth, real stocks are also exposed to inflation risk. Therefore, the implied stock-bond return correlation is positive. This intuition hinges on the empirical correlation between inflation and consumption growth. This correlation, however, does not appear to be robust over different sub-samples.

Following Piazzesi and Schneider (2006), I assume that the vector of inflation (π_t) and consumption growth (Δc_t) has the following state-space representation

$$
\begin{aligned}
z_t &= s_{t-1} + \varepsilon_t, \quad z_t = [\pi_t, \Delta c_t]' \\
s_t &= \phi s_{t-1} + \phi K \varepsilon_t, \quad \varepsilon_t \sim N(0, \Omega).
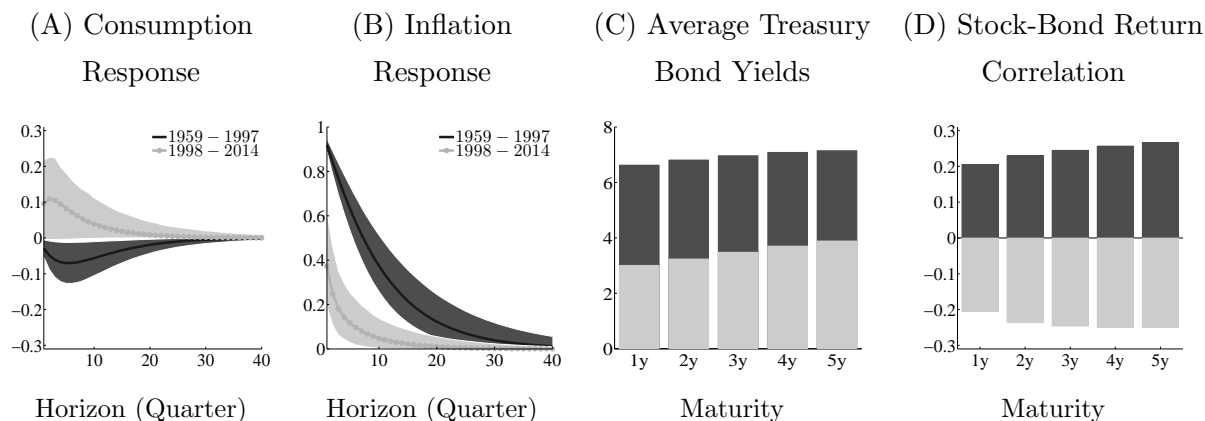\end{aligned}
\tag{1}
$$

The state vector s_t is two dimensional and contains expected inflation and expected consumption growth; ϕ is the 2×2 autoregressive matrix; and K is the 2×2 gain matrix. Using Bayesian methods, I estimate this system with quarterly consumption and inflation data that span 1959 to 2014. Details (about priors and posterior estimates) are provided in Appendix A. The estimation sample is split into two parts. One is from 1959 to 1997, and the other spans the period 1998 to 2014.[8] To understand the key properties of the estimated dynamics, I report the impulse responses of the system in the first and second panel of Figure 1. Each response represents either the change in consumption or inflation forecasts following a 1 percent inflation shock.

Several aspects of the results are noteworthy. First, the sign of consumption's reaction to an inflation shock changed from negative to positive over the last fifteen years: a 1 percent inflation surprise predicts that consumption growth will be higher by approximately 10 basis points in the next year. Inflation carries good news about consumption growth. Second, the own-shock responses for inflation decayed much faster over the last fifteen years: the impact of a 1 percent inflation surprise on itself completely dies out over the next 1-2 years. This is mainly due to a large decline in the persistence of the expected inflation process, for example, the autoregressive coefficient for inflation dropped from 0.96 to 0.41 (see Appendix A for details). Third, there is significant reduction in the variance of inflation innovations. Overall, the key aspects of the data are that the inflation dynamics have substantially changed over time and there are periods in which an inflation shock can be good news for consumption growth.[9]

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[8]The point of a structural break is formally identified via Bayesian estimation.

[9]This evidence is also documented by David and Veronesi (2013).

Figure 1: Macroeconomic Fundamentals and Treasury Yield Curve

(A) Consumption Response	(B) Inflation Response	(C) Average Treasury Bond Yields	(D) Stock-Bond Return Correlation

Notes: (A) and (B) The black (light-gray circled) line represents posterior median consumption and inflation reactions to 1 percentage point surprises in inflation from 1959-1997 (1998-2014). The black- and light-gray-shaded areas correspond to 60% credible intervals. (C) The black (light-gray) bars represents the averages of the U.S. Treasury bond yields (annualized) for maturities of 1-5 years during 1959-1997 (1998-2014). (D) The black (light-gray) bars represent the correlation between stock market returns and bond returns for a 1-month holding period for maturities of 1-5 years from 1959-1997 (1998-2014).

The third panel of Figure 1, in fact, shows that yields with longer maturities are, on average, higher than those with shorter maturities. The perspective of existing term structure models is puzzling in that during periods from 1998 to 2014, in which inflation is a carrier of good news to consumption growth, the Treasury yield curve (while shifted down significantly) still slopes upward.[10] That the correlation between bond and stock returns changed from positive to negative in those periods (the fourth panel of Figure 1) is a particularly interesting observation. The result is consistent with recent empirical studies that U.S. Treasury bonds have served as a hedge to stock market risks in the last decade.[11]

The new set of evidence is interesting not only because it shows the limitations of the existing approaches but also because it implies that the sources of risk behind the yield curve might have changed over time. There is an important reason to believe that the yield curve and inflation dynamics are sensitive to monetary policy shifts or changes in the distribution of economic shocks.[12]

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[10]As shown in Campbell (1986), positive correlation in consumption growth and inflation implies a downward-sloping nominal yield curve.

[11]See Baele, Bekaert, and Inghelbrecht (2010), Campbell, Pflueger, and Viceira (2015), Campbell, Sunderam, and Viceira (2016), and David and Veronesi (2013).

[12]See Ang, Boivin, Dong, and Loo-Kung (2011), Bikbov and Chernov (2013), and Shaliastovich and Yamarthy (2015).

Despite the extensive study on bond markets, only few papers try to investigate the origins of the bond market changes. The suggested hypotheses fall into two broad categories. The first view attributes the cause of the bond market changes to shift in the correlation between the nominal and real disturbances, for example, Campbell, Sunderam, and Viceira (2016), David and Veronesi (2013), and Ermolov (2015). The second view argues that the root cause is changes in the conduct of monetary policy (see Campbell, Pflueger, and Viceira (2015)). This paper puts forward a unified framework that enables joint assessment of the strength of these two hypotheses which in fact are not mutually exclusive. In sum, the new stylized empirical facts posit the need to look at the data from a broader perspective, which calls for a more flexible approach to the joint modeling of macroeconomic fundamentals, monetary policy, and stock and bond asset prices. I turn to this in the next section.

3   Model

I develop an asset pricing model that embeds risks through shifts in the strength with which the Federal Reserve tries to pursue its stabilization policy, as well as in the covariance matrix of nominal inflation target and real growth innovations. The real side of the model builds on the long-run risks model of Bansal and Yaron (2004) and assumes that consumption growth contains a small predictable component (i.e., long-run growth), which, in conjunction with investors' preference for an early resolution of uncertainty, determines the price of real assets. The nominal side of the model extends the model of Gallmeyer, Hollifield, Palomino, and Zin (2007) in that inflation dynamics are endogenously derived from the monetary policy rule, and the nominal assets inherit the properties of monetary policy. As a consequence of my model features, cyclical properties of inflation and bond price dynamics depend on changes in monetary policy aggressiveness and the distributions of macroeconomic shocks.

Preferences. I consider an endowment economy with a representative agent that has recursive preferences and maximizes her lifetime utility,

$$V_t = \max_{C_t} \left[(1-\delta)C_t^{\frac{1-\gamma}{\theta}} + \delta\big(\mathbb{E}_t[V_{t+1}^{1-\gamma}]\big)^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}},$$

subject to the budget constraint

$$W_{t+1} = (W_t - C_t)R_{c,t+1},$$

where W_t is the wealth of the agent, $R_{c,t+1}$ is the return on all invested wealth, γ is risk aversion, $\theta = \frac{1-\gamma}{1-1/\psi}$, and ψ is intertemporal elasticity of substitution. The log of the real stochastic discount

factor (SDF) is

$$m_{t+1} = \theta \log \delta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{c,t+1}. \tag{2}$$

Exogenous Endowment Process. Following Bansal and Yaron (2004), I decompose consumption growth, Δc_{t+1}, into a (persistent) long-run growth component, $x_{c,t}$, and a (transitory) short-run component, $\sigma_c \eta_{c,t+1}$. The persistent long-run growth component is modeled as an AR(1) process in which fluctuations are driven by a real growth innovation process, $\eta_{xc,t+1}$, which is correlated with an innovation to the inflation target, $\eta_{x\pi,t+1}$. The inflation target, $x_{\pi,t}$, is modeled by a random walk process. The covariance between the inflation target shock $\eta_{x\pi,t+1}$ and the real growth shock $\eta_{xc,t+1}$, which is captured by $\beta(S_t)\sigma_{xc}^2(S_t)$, is subject to regime changes, where S_t denotes the regime indicator variable $S_t \in \{1, ..., N\}$. The economic reasoning follows the view that there are periods in which the inflation target is above the so-called desirable rate of inflation and that any positive shock to the inflation target during those periods creates distortions and hampers long-run growth.[13] The negative values ($\beta < 0$) correspond to these periods. The periods with positive values ($\beta \geq 0$) depict periods during which the inflation target is assumed to be lower than the desirable one, and a positive shock to the inflation target removes distortions and facilitates long-run growth. Dividend streams, Δd_{t+1}, have levered exposures to $x_{c,t}$, for which magnitude is governed by the parameter ϕ. I allow $\sigma_d \eta_{d,t+1}$ to capture the idiosyncratic movements in dividend streams.

Put together, the joint dynamics for the cash flows, $G_t = [\Delta c_t, \Delta d_t]'$, are

$$
\begin{aligned}
G_{t+1} &= \mu + \varphi X_t + \Sigma \eta_{t+1}, \quad \eta_{t+1} \sim N(0, I), \\
X_{t+1} &= \Phi(S_{t+1}) X_t + \eta_{x,t+1}, \quad \eta_{x,t+1} \sim N(0, \Omega(S_{t+1})\Sigma_x(S_{t+1})\Sigma_x(S_{t+1})'\Omega(S_{t+1})'),
\end{aligned} \tag{3}
$$

where $\mu = [\mu_c, \mu_d]'$, $\eta_t = [\eta_{c,t}, \eta_{d,t}]'$, $X_t = [x_{c,t}, x_{\pi,t}, x_{i,t}]'$, $\eta_{x,t} = [\eta_{xc,t}, \eta_{x\pi,t}, \eta_{xi,t}]'$ and

$$
\varphi = \begin{bmatrix} 1 & 0 & 0 \\ \phi & 0 & 0 \end{bmatrix}, \quad \Sigma = \begin{bmatrix} \sigma_c & 0 \\ 0 & \sigma_d \end{bmatrix},
$$

$$
\Phi = \begin{bmatrix} \rho_c & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \rho_i \end{bmatrix}, \quad (\Omega\Sigma_x)(\Omega\Sigma_x)' = \begin{bmatrix} \sigma_{xc}^2 & \beta\sigma_{xc}^2 & 0 \\ \beta\sigma_{xc}^2 & \beta^2\sigma_{xc}^2 + \sigma_{x\pi}^2 & 0 \\ 0 & 0 & \sigma_{xi}^2 \end{bmatrix}.
$$

$x_{i,t}$ is monetary policy shock that follows an AR(1) process (explained later).[14]

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[13]In a New Keynesian model, the desirable rate of inflation is the rate at which prices can be changed without costs. See Aruoba and Schorfheide (2011) for a more detailed discussion.

[14]The variance-covariance matrix of $\eta_{x,t}$ is chosen to be of this particular form because the variance of the real growth shocks, $Var(\eta_{xc,t}) = \sigma_{xc}^2$, is independent from $\beta(S_t)$.

Exogenous Monetary Policy Rule. Monetary policy consists of two components: stabilization and a time-varying inflation target. I assume that the central bank makes informed decisions about inflation fluctuations at different frequencies. While the central bank attempts to steer actual inflation toward the inflation target (which itself is time varying) at low frequencies, it aims to stabilize inflation fluctuations relative to its target at high frequencies.[15] In particular, I assume that the strength with which the central bank tries to pursue its goal—a stabilization policy—changes over time along the lines explored by Clarida, Gali, and Gertler (2000). Stabilization policy is "active" ($\tau_\pi > 1$) or "passive" ($\tau_\pi \leq 1$), depending on its responsiveness to inflation fluctuations relative to the target.

In sum, monetary policy follows a regime-switching Taylor rule,

$$i_t = \tau_0(S_t) + \underbrace{\tau_c(S_t)x_{c,t}}_{\text{real growth}} + \underbrace{\tau_\pi(S_t)(\pi_t - \Gamma_0 - x_{\pi,t})}_{\text{inflation around target}} + \underbrace{x_{\pi,t}}_{\text{target}} + \underbrace{x_{i,t},}_{\text{policy shock}} \tag{4}$$

where $\tau_c(S_t)$ and $\tau_\pi(S_t)$ capture the central bank's reaction to real growth and to the variation in short-run inflation, respectively.

Several important features should be discussed. In the context of the term structure models, it is important to consider an explicit role for the inflation target since the target behaves similar to a level factor of the nominal term structure. The specification of (4) resembles specifications in which the level factor of the term structure directly enters into the monetary policy rule (see Rudebusch and Wu (2008), for example). Second, while policy rule inertia is a more plausible description of U.S. monetary policy actions (see discussions in Coibon and Gorodnichenko (2011)), it is assumed to be absent. (4) allows me to apply Davig and Leeper (2007)'s solution method and characterize inflation as exact affine functions of the "current" state variables, X_t, without any "lagged" term.[16] The solution is transparent, tractable, and verifiable—it is not a numerical black box. In summary, I ignore policy inertia to obtain a payoff in tractability and transparency.

Markov Chain. To achieve flexibility while maintaining parsimony, I assume that the model parameters evolve according to a three-state Markov chain, $S_t \in \{1, 2, 3\}$:

1. Countercyclical Macroeconomic Shocks and Active Monetary Policy (CA): $\beta < 0$, $\tau_\pi > 1$,

2. Countercyclical Macroeconomic Shocks and Passive Monetary Policy (CP): $\beta < 0$, $\tau_\pi \leq 1$,

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[15]Note that incorporating a time-varying inflation target is quite common in the monetary policy literature (see Coibon and Gorodnichenko (2011), Aruoba and Schorfheide (2011), and Davig and Doh (2014)).

[16]Rudebusch (2002) argues that, to study the term structure, it seems sensible to consider the monetary policy rule without an interest-rate-smoothing motive. Based on the term structure evidence, Rudebusch (2002) shows that monetary policy inertia is not due to the smoothing motive but is due to persistent shocks.

3. Procyclical Macroeconomic Shocks and Active Monetary Policy (PA): $\beta \geq 0$, $\tau_\pi > 1$.

I define a Markov transition probability matrix by Π, which summarizes all 3^2 transition probabilities. The labeling of the regimes is explained in detail for the asset pricing implications.

Endogenous Inflation Process and Determinacy of Equilibrium. Inflation dynamics can be endogenously determined from the monetary policy rule (4) and an asset-pricing equation, which is given below,

$$i_t \; = \; -E_t\left[m_{t+1} - \pi_{t+1}\right] - \frac{1}{2}Var_t\left[m_{t+1} - \pi_{t+1}\right]. \tag{5}$$

Substituting the asset-pricing equation (5) into the monetary policy rule (4), the system reduces to a single regime-dependent equation

$$\tau_\pi(S_t)\pi_t \; = \; E_t\left[\pi_{t+1}\right] + \Lambda_0(S_t) + \Lambda_1(S_t)X_t, \tag{6}$$

where $\Lambda_0(S_t)$ and $\Lambda_1(S_t)$ are function of the model structural parameters.

I posit regime-dependent linear solutions of the form

$$\pi_t \; = \; \Gamma_0(S_t) + \Gamma_1(S_t)X_t. \tag{7}$$

For ease of exposition, I introduce a diagonal matrix Ψ, where the ith diagonal component is $\tau_\pi(i)$. According to Proposition 2 of Davig and Leeper (2007), a unique bounded solution (determinate equilibrium) exists provided that the "long-run Taylor principle" (summarized by the two conditions) is satisfied:

1. $\tau_\pi(i) > 0$, for $i \in \{1, 2, 3\}$,

2. All the eigenvalues of $\Psi^{-1} \times \Pi$ lie inside the unit circle.

A detailed derivation is provided in Appendix B.6.

In a fixed-regime environment, the equilibrium inflation is not unique, and multiple solutions exist, including stationary sunspot equilibria when monetary policy is passive, $\tau_\pi \leq 1$. A striking feature is that with regime switching, there exists determinate equilibrium, even with passive monetary policy. Figure 2 provides admissible ranges (black-shaded regions) of monetary policy coefficients consistent with the long-run Taylor principle. According to Figure 2, monetary policy can be passive some of the time, as long as the passive regime is sufficiently short-lived (see discussion in Davig and Leeper (2007) and Foerster (2016)). Allowing for (short-lived) passive monetary policy has several important asset pricing implications that I discuss below.

Figure 2: Determinacy Regions

Notations. Before I explain the solution of the model, I introduce some notations. $r_{m,t+1}$ denotes the log real stock market return. I use $ to distinguish nominal from real values. The nominal n-maturity log zero-coupon bond price is $p_{n,t}^\$$, and the respective log bond yield is $y_{n,t}^\$ = -\frac{1}{n}p_{n,t}^\$$. $r_{n,t+1}^\$$ denotes the log return to holding a n-maturity nominal bond from t to $t+1$. $rx_{n,t+1}^\$$ is the log return to holding a n-maturity nominal bond from t to $t+1$ in excess of the log return to a one period nominal bond.

Asset Solutions and Asset Pricing Implication. The first-order condition of the agent's expected utility maximization problem yields the Euler equations

$$E_t\left[\exp\left(m_{t+1} + r_{k,t+1}\right)\right] = 1, \quad k \in \{c, m\}, \quad \text{Real Assets}, \tag{8}$$

$$p_{n,t}^\$ = \log E_t[\exp(m_{t+1} - \pi_{t+1} + p_{n-1,t+1}^\$)], \quad \text{Nominal Assets}, \tag{9}$$

where $r_{c,t+1}$ is the log return on the consumption claim and $r_{m,t+1}$ is the log market return. The solutions to (8) and (9) depend on the joint dynamics of consumption and dividend growth (3) and inflation (7). Asset prices are determined from the approximate analytical solution described by Bansal and Zhou (2002), who assume that asset prices are affine function of state X_t conditional on regime S_t. Details are provided in Appendix B.

For the sake of exposition, I set monetary policy shock to zero and reduce the state variables from three to two: real growth and inflation target.[17] The nominal n-maturity log bond price is

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[17]Since monetary policy shock is orthogonal to the real growth and inflation target shocks, its role in the asset markets is not as important as that of the previous two shocks. I am shutting monetary policy shock down for the purpose of providing intuition of the model.

an affine function of the state conditional on the current regime S_t (here I omit S_t for simplicity)

$$p_{n,t}^{\$} \;=\; C_{n,0}^{\$} + C_{n,1}^{\$} X_t, \tag{10}$$

where $C_{n,1}^{\$} = [C_{n,1,c}^{\$}, C_{n,1,\pi}^{\$}]$ and $X_t = [x_{c,t}, x_{\pi,t}]'$. The respective nominal bond yield loadings can be expressed by

$$B_{n,1,c}^{\$} \;=\; -\frac{1}{n} C_{n,1,c}^{\$} = \left(\frac{1/\psi\tau_\pi - \rho_c\tau_c}{\tau_\pi - \rho_c}\right)\frac{1}{n}\left(\frac{1-\rho_c^n}{1-\rho_c}\right), \quad B_{n,1,\pi}^{\$} = -\frac{1}{n} C_{n,1,\pi}^{\$} = 1. \tag{11}$$

Note that $B_{n,1,c}^{\$}$ decays to zero for long maturity bonds, and $B_{n,1,\pi}^{\$}$ is always one, implying that any change in inflation target induces parallel shifts in the entire yield curve. Under $\frac{1}{\psi}\min\{1, \tau_\pi/\rho_c\} > \tau_c$, the sign of bond yield loading $B_{n,1,c}^{\$}$ is positive if $\tau_\pi > 1$, that is, monetary policy is active and negative when monetary policy is passive, $\tau_\pi \le 1$.[18] When monetary policy is active (passive), bond prices rise (fall) in response to decrease in real growth and bond yields become procyclical (countercyclical).

After some tedious algebra, the sign of the one-period expected excess return of a n-maturity nominal bond (bond risk premium) is expressed as

$$\text{sign}\big(E_t(rx_{n,t+1}^{\$}) + \frac{1}{2}Var_t(rx_{n,t+1}^{\$})\big) \;\approx\; -\text{sign}\left((B_{n-1,1,c}^{\$} + \beta)\frac{(\gamma - 1/\psi)\kappa_1}{1 - \kappa_1\rho_c}\right). \tag{12}$$

The approximation is accurate for highly persistent real growth process, ρ_c, and the Campbell-Shiller log approximation constant, κ_1. Similarly, the sign of the conditional correlation between the real stock market and the n-maturity nominal bond return is characterized by

$$\text{sign}\big(Corr_t\big(r_{m,t+1}, rx_{n,t+1}^{\$}\big)\big) \;=\; -\text{sign}\big(B_{n-1,1,c}^{\$} + \beta\big). \tag{13}$$

I refer to Appendices B.7 and B.8 for the exact expression.

To build intuition into (12) and (13), I start by considering the limiting case of a fixed-regime economy. Throughout the analysis, I assume that agents have a preference for an early resolution of uncertainty ($\gamma > 1/\psi$). To facilitate intuition, I ignore inflation non-neutrality, that is, $\beta = 0$.[19] Suppose if monetary policy is active, then nominal bonds are hedges ($B_{n-1,1,c}^{\$} \ge 0$) and bond risk premium falls in response to increase in real growth and inflation target uncertainty. In this environment, nominal bonds are qualitatively similar to real bonds.[20] The implied stock-bond

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[18]The sign of $B_{n,1,c}^{\$}$ depends on the relative magnitude of τ_π and ρ_c, and I assume that ρ_c is fairly close to 1 in this analysis.

[19]Note that $\beta \ne 0$ breaks the long-run dichotomy between the nominal and real sides of the economy.

[20]It is easy to show that when agents have a preference for an early resolution of uncertainty ($\gamma > 1/\psi$), real bonds are hedges against low growth and real bond risk premiums are always negative. Because these hedging effects are stronger at longer horizons, this implies a downward-sloping real term structure.

Table 1: Asset Pricing Implications in a Fixed-Regime Economy

		CA	CP	PA
Unique Inflation Solution		Yes	No	Yes
Bond Loading	$B_{n,1,c}^{\$}$	+	−	+
Stock-Bond Return Correlation	$Corr_t\big(r_{m,t+1}, r_{n,t+1}^{\$}\big)$	+	+	−
Bond Risk Premium	$E_t(rx_{n,t+1}^{\$}) + \frac{1}{2}Var_t(rx_{n,t+1}^{\$})$	+	+	−

Notes: The regimes are labeled as (1) the CA regime, countercyclical macroeconomic shocks and active monetary policy; (2) the CP regime, countercyclical macroeconomic shocks and passive monetary policy; and (3) the PA regime, procyclical macroeconomic shocks and active monetary policy.

return correlation is negative. The same could be said for the reverse logic: the signs of bond risk premium and stock-bond return correlation flip and become positive under passive monetary policy regime.

The introduction of inflation non-neutrality, that is, $\beta \neq 0$ complicates the analysis. Suppose that monetary policy is active. As long as the covariance of inflation target and real growth shocks is small in magnitude, $\beta \geq -B_{n-1,1,c}^{\$}$, the implication on bond risk premium and stock-bond return correlation will be identical as before. In such case, the economy experiences "procyclical" macroeconomic shocks. For the sake of labeling purpose, the respective regime is PA. However, a sufficiently large "countercyclical" macroeconomic shocks, captured by $\beta < -B_{n-1,1,c}^{\$}$, can reverse the sign and generate positive bond risk premium and stock-bond return correlation. Here, the regime is CA. Suppose now that monetary policy is passive. Following large countercyclical macroeconomic shocks (which are bad news for the economy), the implied bond risk premium and stock-bond return correlation are positive and largest in magnitude across all cases. This environment, called the CP regime, is the extreme version of the economy described by Piazzesi and Schneider (2006), Wachter (2006), Eraker (2008), and Bansal and Shaliastovich (2013) who assume inflation is countercyclical and risky.

Table 1 summarizes the model's intuition. In general, the signs of relevant asset pricing moments are unambiguously determined in the CP and PA regimes, while they are not in the CA regime. They ultimately depend on the distribution of macroeconomic shocks (captured by β) and the aggressiveness of the monetary policy (captured by τ_π). The signs will be determined only for sufficiently large countercyclical macroeconomic shocks, that is, $\beta < -B_{n-1,1,c}^{\$}$, which is assumed in Table 1.

Until now, I have characterized each of the "within-regime" risks. However, it is sensible to

argue that the economic agents anticipate future regime changes and the associated regime risks are reflected in today's asset market. This is called "across-regime" risks—the extent to which the risk properties of alternative regimes are incorporated due to regime switching dynamics. With regime switching, it is difficult to understand the asset pricing implications because all regime risks are mixed together through the iterated expectation over the regimes. Determining what the driving forces behind the changes in the asset markets is entirely an empirical question. To answer this, I now turn to the estimation part of the model.

4 Empirical Results

The data set used in the empirical analysis is described in Section 4.1. Bayesian inference is discussed in Section 4.2. Section 4.3 discusses parameter restrictions of the model and identification of the regime. Section 4.4 discusses parameter estimates and regime probabilities. The model's implications for macroeconomic aggregates and asset prices are explained in Section 4.5. Finally, Section 4.6 discusses model caveats and provides robustness check on the identification of regimes.

4.1 Data

Monthly consumption data represent per capita series of real consumption expenditures on non-durables and services from the National Income and Product Accounts (NIPA) tables, which are available from the Bureau of Economic Analysis. Aggregate stock market data consist of monthly observations of returns, dividends, and prices of the CRSP value-weighted portfolio of all stocks traded on the NYSE, AMEX, and NASDAQ. Price and dividend series are constructed on a per share basis, like in Campbell and Shiller (1988b) and Hodrick (1992). Market data are converted to real data using the consumer price index (CPI) from the Bureau of Labor Statistics. Growth rates of consumption and dividends are constructed by taking the first difference of the corresponding log series. Inflation represents the log difference of the CPI. Monthly observations of U.S. Treasury bonds with maturities at one to five years are from CRSP. The time series spans the monthly data from 1963:M1 to 2014:M12. A detailed description of the data is provided in Appendix C.

4.2 Bayesian Inference

Posterior inference is implemented with a Metropolis-within-Gibbs sampler (see the previous work of Carter and Kohn (1994) and Kim and Nelson (1999)). $Y_{1:T}$ denotes the sequence of observations,

where

$$Y_t = (\Delta c_t, \pi_t, pd_t, y_{1,t}, y_{2,t}, y_{3,t}, y_{4,t}, y_{5,t}).$$

Moreover, let $S_{1:T}$ be the sequence of hidden states, and let $\Theta = (\Theta_1, \Theta_2)$, where

$$
\begin{aligned}
\Theta_1 &= (\delta, \gamma, \psi), \\
\Theta_2 &= (\mu_c, \mu_d, \rho_c, \rho_i, \phi, \sigma_c, \sigma_d, \sigma_{xc}, \sigma_{x\pi}, \sigma_{xi}, \beta(-), \beta(+), \tau_0(P), \tau_0(A), \tau_c(P), \tau_c(A), \tau_\pi(P), \tau_\pi(A)), \\
\Phi &= (\{\Pi_{ij}\}_{i,j=\{1,2,3\}}).
\end{aligned}
$$

Metropolis-within-Gibbs algorithm involves iteratively sampling from three conditional posterior distributions. Details are provided in Appendix D.

4.3 Identification and Parameter Restriction

As discussed before, the identification of the regime is achieved by segmenting the economy into the following three cases:

1. Countercyclical Macroeconomic Shocks and Active Monetary Policy (CA): $\beta < 0$, $\tau_\pi > 1$,

2. Countercyclical Macroeconomic Shocks and Passive Monetary Policy (CP): $\beta < 0$, $\tau_\pi \leq 1$,

3. Procyclical Macroeconomic Shocks and Active Monetary Policy (PA): $\beta \geq 0$, $\tau_\pi > 1$.

I allow the standard deviation of the inflation target innovations $\sigma_{x\pi}$ to differ across regimes. In particular, I assume that while $\sigma_{x\pi}$ is largest under passive monetary policy regime, it is assumed to be smallest under procyclical inflation regime.[21] The restriction is summarized by

$$\sigma_{x\pi}(CP) > \sigma_{x\pi}(CA) > \sigma_{x\pi}(PA).$$

Since real endowment process is exogenously specified in this economy, I assume that policy response to real growth is identical across regimes, that is, $\tau_c(A) = \tau_c(P)$.

Finally, to reduce the number of estimated parameters, a subset of parameters, under (1), (2), and (3) in Table 2, is fixed based on Schorfheide, Song, and Yaron (2016). I also assume that the monetary policy shock is not serially correlated, that is., $\rho_i = 0$. This restriction is conservative in terms of the fit of the model because it effectively reduces the number of persistent state variables from three to two: real growth and inflation target.

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[21]The restriction on $\sigma_{x\pi}$ helps identify the different regimes. However, I find that even without this restriction the identified regimes are qualitatively the same.

Table 2: Posterior Median Estimates

(1) Preference		(2) Consumption		(3) Dividend		(4) Factor Shocks		(5) Monetary Policy	
δ	0.999	μ_c	0.0016	μ_d	0.0016	$\beta(-)$	-2.50 $[-3.81, \quad -1.32]$	$\tau_0(A)$	0.0041 $[0.0021,0.0047]$
ψ	2	ρ_c	.99	ϕ	3	$\beta(+)$	1.00 $[0.08, \quad 2.13]$	$\tau_0(P)$	0.0043 $[0.0031,0.0052]$
γ	8	σ_c	0.0024	σ_d/σ_c	6.25	σ_{xc}	0.00015 $[0.00011,0.00018]$	$\tau_\pi(A)$	1.40 $[1.11, \quad 1.75]$
						$\sigma_{x\pi}(CA)$	0.00019 $[0.00017,0.00023]$	$\tau_\pi(P)$	0.94 $[0.89, \quad 0.99]$
						$\sigma_{x\pi}(CP)$	0.00039 $[0.00034,0.00047]$	$\tau_c(A,P)$	0.48 $[0.38, \quad 0.57]$
						$\sigma_{x\pi}(PA)$	0.00011 $[0.00008,0.00014]$		
						σ_{xi}	0.0020 $[0.0013, \quad 0.0027]$		

Notes: The estimation results are based on monthly data from 1963:M1 to 2014:M12. A subset of parameters under (1), (2), and (3) is fixed based on Schorfheide, Song, and Yaron (2016). I show the posterior interquartile range (5%, 95%) in brackets.

4.4   Parameter Estimates and Regime Probabilities

Parameter Estimates. The priors for the parameters are fairly agnostic and are shown in Appendix D. Percentiles for the posterior distribution are reported in Table 2. The most important results for the subsequent analysis are provided in (14) and in the fourth and fifth columns of Table 2.

$$
\Pi = \begin{bmatrix}
\underset{[0.85,0.97]}{0.907} & \underset{[0.035,0.067]}{0.045} & \underset{[0.042,0.071]}{0.050} \\
\underset{[0.042,0.071]}{0.050} & \underset{[0.85,0.97]}{0.911} & \underset{[0.042,0.070]}{0.045} \\
\underset{[0.037,0.068]}{0.049} & \underset{[0.035,0.067]}{0.046} & \underset{[0.85,0.97]}{0.905}
\end{bmatrix}. \tag{14}
$$

First, (14) reports posterior estimates of the Markov-chain transition probabilities. Below each posterior median parameter estimate, we show the posterior interquartile range (5%, 95%) in brackets. The regimes are ordered by CA, CP, and PA. The respective unconditional regime probabilities are 0.35, 0.33, and 0.32. This result can be interpreted as an indication that the risks of experiencing countercyclical macroeconomic shocks are substantial, as indicated by the sum of the probability of the CA and CP regimes, 0.68. The unconditional probability of staying in the active monetary policy regime, as indicated by the sum of the probability of the CA and PA regimes, is around 0.67, which is twice as large as that of the passive monetary policy regime.

Figure 3: Regime Probabilities
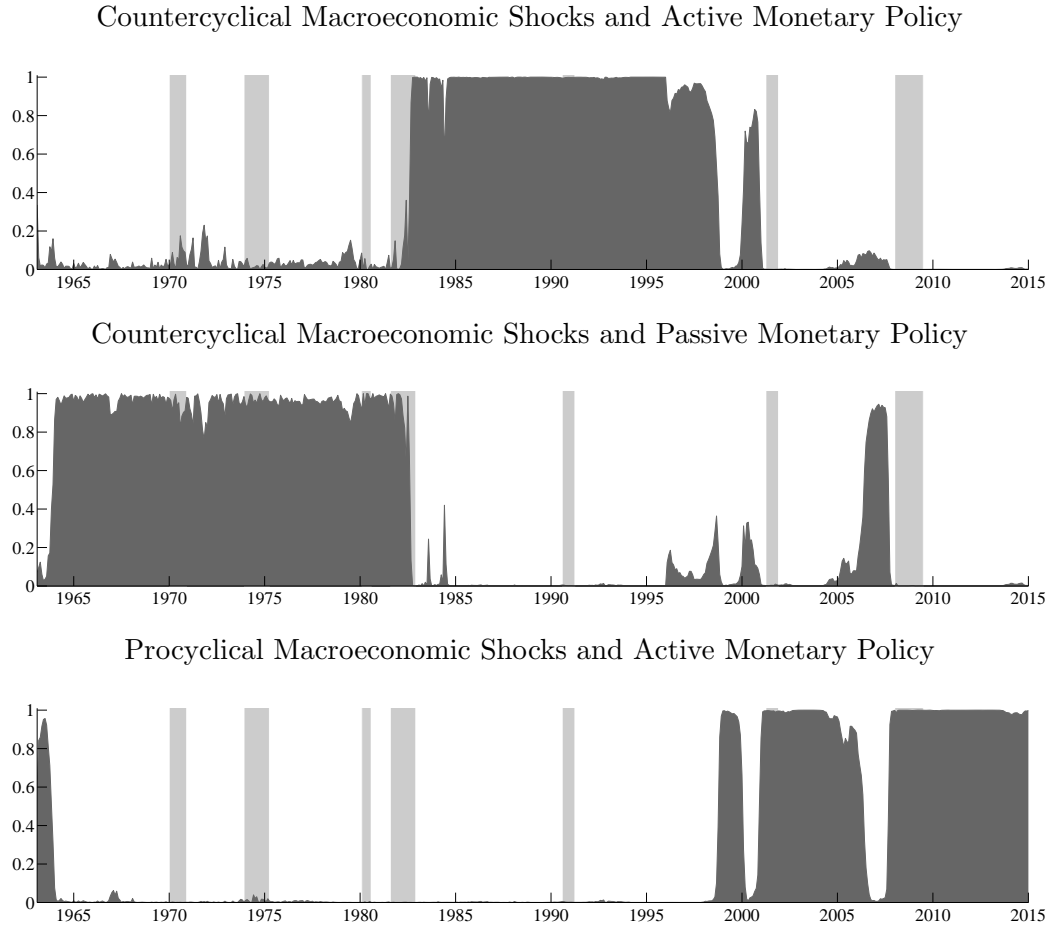
Countercyclical Macroeconomic Shocks and Active Monetary Policy

Countercyclical Macroeconomic Shocks and Passive Monetary Policy

Procyclical Macroeconomic Shocks and Active Monetary Policy

Notes: The dark-gray-shaded areas represent posterior medians of regime probabilities. The light-gray-shaded bars indicate the NBER recession dates.

Second, strong evidence suggests parameter instability in the dynamics of the long-run components. Most prominently, the sign change in the correlation structure is notable: the posterior median estimate of β is -2.5 in the countercyclical macroeconomic shock regime and 1.0 in the procyclical macroeconomic shock regime; and the correlation between real growth and inflation target $\beta\sigma_{xc}/\sqrt{\beta^2\sigma_{xc}^2 + \sigma_{x\pi}^2}$ is -0.9, -0.7, and 0.8 under the CA, CP, and PA regimes, respectively.

Third, two very different posterior estimates of the monetary policy rule in the fifth column of Table 2 support the view of Clarida, Gali, and Gertler (2000) that there has been a substantial change in the way monetary policy is conducted. Active regime is associated with a larger monetary policy rule coefficient, 1.40, which implies that the central bank will more aggressively respond to

short-run inflation fluctuations. Passive regime is characterized by a less responsive monetary policy rule, in which I find much lower loading, 0.94. Given the posterior transition probabilities, I verify that the estimated monetary policy coefficients fall into the admissible ranges consistent with the long-run Taylor principle in Figure 2.

Regime Probabilities. Figure 3 depicts the smoothed posterior regime probabilities. The estimation identifies inflation as countercyclical from the early 1970s through the late 1990s and as procyclical from the late 1990s onward. This is consistent with the evidence provided in Figure 1. Figure 3 also suggests that the switch is not a permanent event, but is an occasional one.[22] The historical paths of the monetary policy stance are also consistent with the empirical monetary literature: Active monetary policy appeared in the early 60s but was largely dormant until the early 80s; it became active after the appointment of Paul Volcker as Chairman of the Federal Reserve and remained active throughout the sample.[23]

4.5   Implications for Macroeconomic Aggregates and Asset Prices

While asset pricing moments implicitly enter the likelihood function of the state-space model, it is instructive to examine the extent to which sample moments implied by the estimated state-space model mimic the sample moments computed from our actual data set. To do so, I report percentiles of the posterior predictive distribution for various sample moments based on simulations from the posterior distribution of the same length as the data.[24] The posterior predictive distributions are obtained conditional on posterior median estimates of the parameters and only reflect sampling uncertainty. Except for a few cases, to facilitate comparison across different regimes, I only report the median values in the tables. Broadly speaking, I compare two cases: one in which regimes are fixed and the other in which regime switching is allowed in the economy. When regime switching is allowed, the solutions of the model account for "across-regime" risks through the iterated expectation over the regimes. When simulation allows for actual transition to different regimes, the results are reported under "Mix". If simulation results under regime switching are reported under a specific regime identity, for example, "CA", this means that, even though agents take into account possible regime switches in their expectations formation, regime switches do not occur along the simulated paths ex post. By comparing these outcomes to those from the fixed-regime economy, I am able to isolate the expectations effect on asset prices.

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[22]This evidence is also supported by David and Veronesi (2013).

[23]The smoothed paths for monetary policy are broadly consistent with those found in Bianchi (2012), Clarida, Gali, and Gertler (2000), Ang, Boivin, Dong, and Loo-Kung (2011), Bikbov and Chernov (2013), and Coibon and Gorodnichenko (2011).

[24]This is called a posterior predictive check; see Geweke (2005) for a textbook treatment.

Table 3: Model-Implied Moments: Macroeconomic Aggregates

| | Data | | Consumption | | | | | |
| | | Regime-Switching Model | | | | Fixed-Regime Model | | |
		Mix	CA	CP	PA	CA	CP	PA
$E(\Delta c)$	1.90	1.93	1.93	1.86	1.87	1.94	1.90	1.90
$\sigma(\Delta c)$	0.93	0.85	0.86	0.85	0.85	0.88	0.88	0.88
$corr(\Delta c, \pi)$	-0.05	-0.04	-0.18	-0.09	0.14	-0.18	-0.14	0.14

| | Data | | Inflation | | | | | |
| | | Regime-Switching Model | | | | Fixed-Regime Model | | |
		Mix	CA	CP	PA	CA	CP	PA
$E(\pi)$	3.71	3.58	3.54	4.24	3.26	3.26	3.93	3.29
$\sigma(\pi)$	1.10	1.30	1.41	1.63	0.74	1.42	1.81	0.76
$AC(\pi)$	0.62	0.84	0.86	0.90	0.56	0.87	0.93	0.51

| | Data | | Expected Inflation | | | | | |
| | | Regime-Switching Model | | | | Fixed-Regime Model | | |
		Mix	CA	CP	PA	CA	CP	PA
1-Year Ahead	3.60	3.32	3.42	3.30	3.26	3.21	3.44	3.27
5-Year Ahead		3.33	3.33	3.33	3.33	3.21	3.44	3.27

Notes: The 1-year-ahead inflation forecasts for the Survey of Professional Forecasters are provided by the Federal Reserve Bank of Philadelphia and are available from 1970 to 2014 at `https://https://www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters/historical-data/inflation-forecasts`. I report the mean of the forecasts. Detailed descriptions of the data are provided in Appendix C.

Macroeconomic Aggregates. The model-implied distributions for the first and second moments of the macroeconomic aggregates are provided in Table 3. By construction, the sample moments for consumption do not differ across regimes. Yet the sample moments for inflation are quite different across regimes: inflation in the CP regime is most volatile and persistent, while inflation in the PA regime is the exact opposite; the sample correlation between consumption and inflation is positive only in the PA regime; and the averages of inflation are also lowest in the PA regime, which is about 1% lower than those in the CP regime. Based on the regime probabilities in Figure 3, I find that the model is able to account for significant changes in the inflation dynamics observed in the data (see Figure 1). As the economy shifted from the CP regime to the CA regime and to the PA

regime, the variance and persistence of inflation decreased substantially. Overall, the model finds that inflation has become procyclical and less risky as the economy shifted towards active monetary policy and experienced procyclical macroeconomic shocks.

Importantly, the model shows that monetary policy does impact the expected inflation process (since shocks themselves do not impact the expectations). The one-year-ahead expected inflation in the CP regime is largest in a fixed-regime economy, but not in a regime-switching economy. Since real growth is a mean-reverting process and inflation target expectation is based on the random-walk forecast (hence is identical across regimes), the long-run expectation of inflation converges in a regime-switching economy. In sum, the first and second moments for consumption and inflation implied by the model replicate the actual counterparts well.

Bond Yields, Term Premiums, and Bond Risk Premiums. Now, to evaluate whether the model can reproduce key bond market features in the data, the model-implied distributions of bond yields, term premiums, and bond risk premiums are reported in Table 4. The model performs well along this dimension since the model-implied median values are fairly close to their data estimates. Yet important distinctions arise across regimes. Let's first focus on the fixed-regime case. Under the estimated parameter configuration, I find that the term structure is upward- (downward-) sloping and term premiums and bond risk premiums are positive (negative) in the CA regime and the CP (PA) regime consistent with the implications in Table 1. Once regime switching is allowed, the striking feature is that the model is able to generate an upward-sloping term structure in all regimes, including the PA regime. Agents are aware of the possibility of encountering countercyclical inflation in the future (either the CA or CP regime) and demand compensations for holding nominal bonds that might be exposed to future inflation risks. Since risks and compensations are greater for longer-term bonds, the model leads to an upward-sloping nominal yield curve in all regimes. Note also that term premium for the 5-year maturity bond becomes positive. I find that the magnitudes of term premiums and bond risk premiums become much smaller in the CA and CP regimes because the PA regime risks are mixed together through the iterated expectation over the regimes. Overall, the key takeaway from this exercise is that "across-regime" risks are quantitatively important risk factors and bring about very different asset market implications.

Excess Bond Return Predictability. Under the expectations hypothesis (EH), the expected holding returns from long-term and short-term bonds should be the same (strong form) or should only differ by a constant (weak form). However, even the weak form has been consistently rejected by empirical researchers. For example, Campbell and Shiller (1991), Dai and Singleton (2002), Cochrane and Piazzesi (2005), and Bansal and Shaliastovich (2013) all argue that the EH neglects the risks inherent in bonds and provide strong empirical evidence for predictable changes in future

Table 4: Model-Implied Moments: Bond Market

		Average Bond Yield						
	Data	Model						
		Regime-Switching				Fixed-Regime		
Maturity		Mix	CA	CP	PA	CA	CP	PA
1-Year Bond	5.26	5.28	5.18	5.33	4.92	4.89	5.05	4.47
2-Year Bond	5.47	5.43	5.27	5.40	4.92	4.99	5.34	4.28
3-Year Bond	5.65	5.58	5.30	5.47	4.97	5.24	5.59	4.11
4-Year Bond	5.81	5.76	5.38	5.52	5.01	5.46	5.79	3.94
5-Year Bond	5.92	5.84	5.41	5.63	5.52	5.69	6.01	3.75

		Average Term Premiums						
	Data	Model						
		Regime-Switching				Fixed-Regime		
Maturity		Mix	CA	CP	PA	CA	CP	PA
1-Year Bond	0.16	0.17	0.09	0.17	-0.10	0.20	0.24	-0.15
2-Year Bond	0.39	0.22	0.16	0.27	-0.11	0.42	0.50	-0.34
3-Year Bond	0.58	0.30	0.22	0.37	-0.09	0.63	0.74	-0.52
4-Year Bond	0.75	0.37	0.27	0.42	-0.01	0.85	0.95	-0.69
5-Year Bond	0.90	0.42	0.34	0.55	0.04	1.05	1.16	-0.86

		Average Bond Risk Premiums						
	Data	Model						
		Regime-Switching				Fixed-Regime		
Maturity		Mix	CA	CP	PA	CA	CP	PA
1-Year Bond		0.08	0.08	0.07	-0.07	0.10	0.13	-0.08
2-Year Bond		0.20	0.18	0.18	-0.16	0.24	0.29	-0.18
3-Year Bond		0.34	0.28	0.30	-0.25	0.38	0.45	-0.27
4-Year Bond		0.49	0.39	0.43	-0.33	0.52	0.61	-0.36
5-Year Bond		0.65	0.51	0.55	-0.41	0.66	0.77	-0.45

Notes: Detailed descriptions of the data are provided in Appendix C. Treasury term premia estimates for maturities from one to five years from 1963 to the present are based on Adrian, Crump, and Moench (2013) and are available at `https://www.newyorkfed.org/research/data_indicators/term_premia.html`.

excess returns. The first panel of Table 5 compares model-implied distributions for the slope coefficient to the corresponding data estimates. Since the presence of regime switching gives rise to

Table 5: Model-Implied Moments: Bond Excess Return Predictability

Campbell-Shiller Regression: Slope				
	Data	Regime-Switching Model		
Maturity		5%	50%	95%
2-Year Bond	-0.62	-0.63	-0.49	-0.37
3-Year Bond	-1.01	-1.22	-1.00	-0.68
4-Year Bond	-1.42	-1.89	-1.50	-0.96
5-Year Bond	-1.45	-2.44	-1.96	-1.34
Cochrane-Piazzesi Regression: R^2				
	Data	Regime-Switching Model		
Maturity		5%	50%	95%
2-Year Bond	0.19	0.32	0.42	0.49
3-Year Bond	0.20	0.20	0.27	0.33
4-Year Bond	0.23	0.15	0.22	0.30
5-Year Bond	0.21	0.13	0.19	0.28

time variations in risk premiums, I only focus on simulation results that allow for actual transition to different regimes (which correspond to results under "Mix" in previous Tables). Here, I also report the posterior interquartile range (5%, 95%), in addition to median values. The first thing to note is that the model generates very comparable results. The model produces slope coefficients that are significantly negative, lower than unity, and whose absolute magnitudes rise over maturities, like in the data. Another exercise consists of running regressions that predict excess bond returns. Following Cochrane and Piazzesi (2005), I focus on regressing the excess bond return of a n-maturity bond over the 1-year bond on a linear combination of forward rates that includes a constant term, a 1-year bond yield, and four forwards rates with maturities of 2 to 5 years. The model-implied R^2 values (in percents) from the regression are provided in the second panel of Table 5 and are comparable to (but slightly overshoot) the corresponding data estimates.

Market Returns and Log Price-Dividend Ratio. Now, I examine the stock market implications of the model in Table 6. The regime-switching model with actual transitions to different regimes (values under "Mix") is able to explain large part of the equity premium $E[r_{m,t+1} - r_{f,t}] + 1/2\sigma(r_m)^2 \approx (0.049 - 0.017) + 0.5 \times 0.1789^2 \approx 5\%$ (the mean of the model-implied risk-free rate is

Table 6: Model-Implied Moments: Stock Market

		Average Stock Market Moments						
	Data	Model						
		Regime-Switching				Fixed-Regime		
Maturity		Mix	CA	CP	PA	CA	CP	PA
$E(r_m)$	5.75	4.92	4.78	4.25	4.85	4.94	4.80	4.85
$\sigma(r_m)$	15.48	17.89	11.76	11.94	11.73	11.78	11.80	11.78
$E(pd)$	3.60	3.53	3.47	3.65	3.46	3.53	3.53	3.54
$\sigma(pd)$	0.37	0.19	0.16	0.17	0.16	0.17	0.16	0.17
		Average (Conditional) Stock-Bond Return Correlation						
	Data	Model						
		Regime-Switching				Fixed-Regime		
Maturity		Mix	CA	CP	PA	CA	CP	PA
1-Year Bond	0.08	0.16	0.71	0.47	-0.72	0.83	0.68	-0.92
2-Year Bond	0.08	0.18	0.71	0.50	-0.72	0.84	0.68	-0.92
3-Year Bond	0.09	0.19	0.72	0.52	-0.70	0.84	0.68	-0.91
4-Year Bond	0.10	0.20	0.73	0.53	-0.70	0.84	0.68	-0.91
5-Year Bond	0.10	0.20	0.73	0.54	-0.69	0.85	0.68	-0.91

Notes: Detailed descriptions of the data are provided in Appendix C.

around 1.7%). The standard deviation of the market returns $\sigma(r_m) = 17.89\%$ and price-dividend $\sigma(pd) = 0.19\%$ are comparable to their data counterparts. This feature owes, in part, to the fact that the regime-switching model can accommodate some non-Gaussian features in the data.

Stock-Bond Return Correlation. Since the estimated model is quite successful in explaining several bond and stock market phenomena, I can proceed to examine the interactions between the two. The second panel of Table 6 reports the averages of the model-implied conditional stock-bond return correlation. When transitions to different regimes are allowed (values under "Mix"), the model generates mildly positive stock-bond return correlation. It is because inflation risks are substantial in the model: the unconditional probability of experiencing countercyclical macroeconomic shocks, as indicated by the sum of the probability of the CA and CP regimes, is 0.68. Now I look at the conditional stock-bond return correlation implied by each regime. This experiment is

useful because it isolates "within-regime" risks in the stock-bond return correlation. I find that the active monetary policy stance tends to generate stronger positive stock-bond return correlation. My results are consistent with the findings in Campbell, Pflueger, and Viceira (2015) in which they argue that a more aggressive response of the central bank to inflation fluctuations increases the stock-bond return correlation. Through the estimation, I have identified that (while the stance of the monetary policy remained active) the economy faced changes in the covariance between the inflation target and real growth shocks, that is, transition from the countercyclical to the procyclical macroeconomic shock regime, in the late 90s. From Table 1, we learned that as the economy shift towards active monetary policy and experience procyclical macroeconomic shocks, nominal bonds become hedges and nominal bonds behave qualitatively similarly to real bonds. Simulation results in Table 6 confirm that quantitatively the effects are strong enough to generate negative stock-bond return correlation.

How is it, then, that the model generates negative risk premium and stock-bond return correlation and at the same time produces an upward-sloping nominal yield curve? Under the estimated parameter configuration, I find that the risks of moving across regimes have a disproportionately larger impact on the slope of the yield curve than on the risk premium and the stock-bond return correlation. Thus, while the risk premium and the stock-bond return correlation become much less negative in magnitude, only the slope of the yield curve switches sign. Specifically, I can express the slope of the nominal yield curve by

$$E\big(y_{n,t}^{\$} - y_{1,t}^{\$}\big) \approx C_{1,0}^{\$}(S_t) - \frac{1}{n}C_{n,0}^{\$}(S_t). \tag{15}$$

With regime switching, through the iterated expectations over the regimes, $\frac{1}{n}C_{n,0}^{\$}(PA)$ can actually be smaller than $C_{1,0}^{\$}(PA)$ and the corresponding slope of the yield curve becomes positive. For a fixed-regime economy, that is, without the expectations formation effects, the slope is always negative in the PA regime. The key takeaway is that the expectations formation effect can go a long way in modifying equilibrium outcomes and is quantitatively very important risk factor in the bond market.

4.6 Robustness Checks

The model is successful in quantitatively accounting for both new and old bond market stylized facts. The key ingredients of the model include preference for an early resolution of uncertainty, time variation in expected real consumption growth and inflation target, and especially regime switches in the monetary policy action, as well as in the distribution of macroeconomic shocks.

Having said that, there are two potential caveats that need to be taken into account in this paper. First, as recently pointed out by Duffee (2015), standard term structure models (especially these type of long-run risks models with recursive preferences) have known problem of embedding too much inflation risks in the model. I investigate how my model performs along this dimension and demonstrate that allowing for regime switching could be an economically appealing way of modeling inflation dynamics. Second, the empirical results heavily rely on the identification of the regimes. One may question the validity of the identification of the monetary policy regimes defined in a simple endowment economy. To provide robustness of the identification of monetary policy regimes, I estimate a prototypical New Keynesian model.

Inflation Risks and Bond Yields. Duffee (2015) suggests that the role of news about expected future inflation in driving the variation of nominal yields has to be small for well-behaved term structure models. Duffee (2015) decomposes shocks to nominal bond yields $\epsilon_{y^\$,n,t}$ into news about expected future inflation $\epsilon_{\pi,n,t}$, news about expected future real short rates $\epsilon_{y_1,n,t}$, and expected excess returns $\epsilon_{x,n,t}$. A yield shock is the sum of news

$$\epsilon_{y^\$,n,t} = \epsilon_{\pi,n,t} + \epsilon_{y_1,n,t} + \epsilon_{x,n,t}, \tag{16}$$

where

$$
\begin{aligned}
\epsilon_{y^\$,n,t} &= y^\$_{n,t} - E_{t-1}\left(y^\$_{n,t}\right) \\
\epsilon_{\pi,n,t} &= E_t\left(\frac{1}{n}\sum_{i=1}^{n}\pi_{t+i}\right) - E_{t-1}\left(\frac{1}{n}\sum_{i=1}^{n}\pi_{t+i}\right) \\
\epsilon_{y_1,n,t} &= E_t\left(\frac{1}{n}\sum_{i=1}^{n}y_{1,t+i-1}\right) - E_{t-1}\left(\frac{1}{n}\sum_{i=1}^{n}y_{1,t+i-1}\right) \\
\epsilon_{x,n,t} &= E_t\left(\frac{1}{n}\sum_{i=1}^{n}rx^\$_{n-i+1,t+i}\right) - E_{t-1}\left(\frac{1}{n}\sum_{i=1}^{n}rx^\$_{n-i+1,t+i}\right)
\end{aligned}
\tag{17}
$$

denotes the news.[25] The calculation of (17) is explained in detail in Appendices B.9, B.10, and B.11.

Duffee (2015) defines a measure of inflation risk by

$$\text{inflation risk} = \frac{Var(\epsilon_{\pi,n,t})}{Var(\epsilon_{y^\$,n,t})} \tag{18}$$

and provides both survey- and model-based measures of (18) that are estimated to be around 10 to 20 percent. He argues that this magnitude of inflation risk is strongly at odds with values implied

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[25]The accounting identity lets us decompose the n-maturity nominal bond into future expected average inflation, real rates, and excess log returns: $y^\$_{n,t} = \frac{1}{m}\sum_{i=1}^{m}E_t(\pi_{t+i}) + \frac{1}{m}\sum_{i=1}^{m}E_t(y_{1,t+i-1}) + \frac{1}{m}\sum_{i=1}^{m}E_t(rx^\$_{n-i+1,t+i})$.

Table 7: Decompositions of Variances of Yield Innovations: Expected Inflation News

| | Without Measurement Errors | | | | | |
| | Regime-Switching Model | | | Fixed-Regime Model | | |
Maturity	CA	CP	PA	CA	CP	PA
1-Year Bond	1.30	1.15	0.59	1.27	1.19	0.55
2-Year Bond	1.28	1.13	0.61	1.25	1.18	0.57
3-Year Bond	1.26	1.12	0.63	1.23	1.17	0.58
4-Year Bond	1.23	1.11	0.64	1.20	1.16	0.60
5-Year Bond	1.22	1.10	0.65	1.18	1.15	0.62
	With Measurement Errors					
	Regime-Switching Model			Fixed-Regime Model		
Maturity	CA	CP	PA	CA	CP	PA
1-Year Bond	0.42	0.52	0.13	0.46	0.63	0.11
2-Year Bond	0.42	0.53	0.14	0.46	0.63	0.11
3-Year Bond	0.42	0.53	0.14	0.46	0.62	0.11
4-Year Bond	0.41	0.53	0.14	0.45	0.62	0.11
5-Year Bond	0.41	0.54	0.14	0.45	0.61	0.11

Notes: The size of the measurement error variance is less than 5% of the sample variance.

by standard equilibrium models of inflation and bond yields. In particular, the standard term structure models (with recursive preferences and long-run risks) counterfactually imply too high of an inflation risk, which often exceeds one.

Table 7 shows the model-implied inflation risks (18), which exceed one in the CA and CP regimes and are less than one in the PA regime. The results are expected since inflation is countercyclical and risky in the CA and CP regimes. Based on the unconditional probabilities of regimes, I can compute the averages of inflation risks to be around one. This evidence speaks against the empirical validity of my model. However, if the measure of a yield shock, $\epsilon_{y^\$,n,t} = y_{n,t}^\$ - E_{t-1}(y_{n,t}^\$)$, includes measurement errors (whose variance is less than 5% of the sample variance), model-implied inflation risks (18) become significantly smaller.[26]

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[26]Bauer and Rudebusch (2015) and Cieslak and Povala (2015) show that the presence of small yield measurement errors can have a large impact on the spanning ability of interest rates and on the analysis of term premium, respectively.

Figure 4: Regime Probabilities: New Keynesian Model
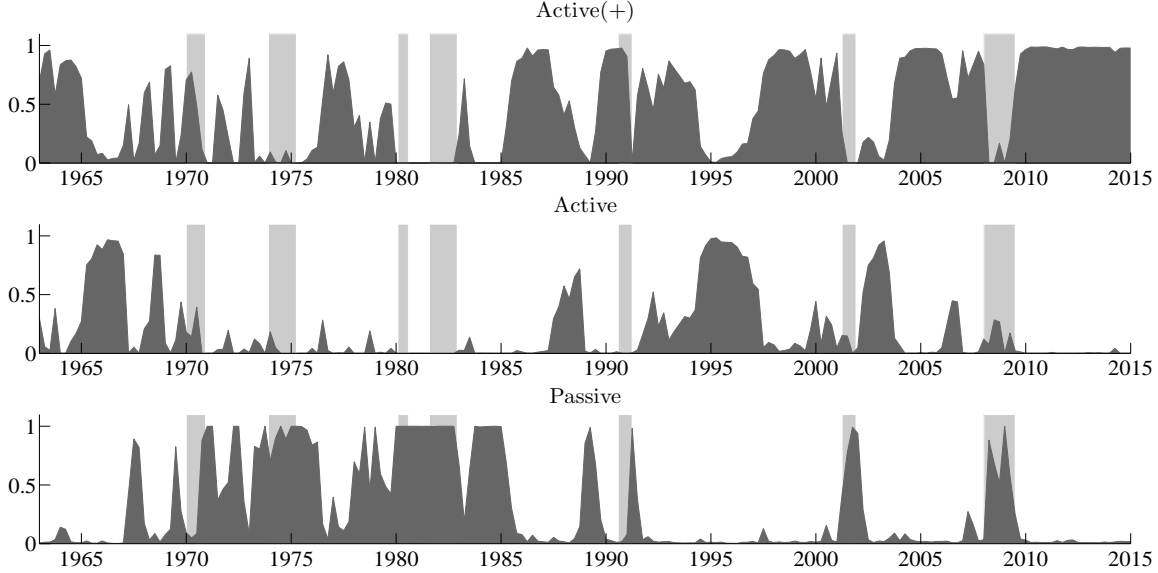
Notes: The dark-gray-shaded areas represent posterior medians of regime probabilities. The light-gray-shaded bars indicate the NBER recession dates.

Having said that, I can draw two important lessons from this exercise. First, it is important to relax the constant (time-invariant) parameter assumption since it overemphasizes the role of inflation risk in the yield curve. Second, allowing for regime switching is an economically appealing way of modeling inflation dynamics. Since each regime corresponds to a different level of inflation risk, a richer description of inflation dynamics is possible. As a result, it provides us a more comprehensive understanding of the sources of risk behind the yield curve.

Identification of Monetary Policy Regimes. To provide robustness in identifying the monetary policy regimes, I estimate the New Keynesian model proposed by Campbell, Pflueger, and Viceira (2015). The model has the following three structural equations (IS curve, Phillips Curve, and Monetary Policy rule) and law of motion for time-varying inflation target

1. IS Curve : $x_t = \rho^{x-}(S_t)x_{t-1} + \rho^{x+}(S_t)E_t x_{t+1} - \psi(S_t)(i_t - E_t \pi_{t+1})$,

2. Phillips Curve : $\pi_t = \rho^{\pi}(S_t)\pi_{t-1} + (1 - \rho^{\pi}(S_t))E_t \pi_{t+1} + \kappa(S_t)x_t + u_t^{PC}$,

3. Monetary Policy : $i_t = \rho^i(S_t)i_{t-1} + (1 - \rho^i(S_t))\big(\gamma^x(S_t)x_t + \gamma^{\pi}(S_t)(\pi_t - \pi_t^{TG}) + \pi_t^{TG}\big) + u_t^{MP}$,

4. Inflation Target : $\pi_t^{TG} = \pi_{t-1}^{TG} + u_t^{TG}$,

where x_t is the log output gap, π_t is the inflation rate, i_t is the log yield of a one month maturity at time t, and π_t^{TG} is inflation target. Note that I am using the same notations like in Campbell, Pflueger, and Viceira (2015). To satisfy the Lucas critique, I assume that all model coefficients are regime dependent and there are three distinct monetary policy regimes in total. The identification restriction is that γ^π is less than one in the passive regime; and greater than one in the other two active regimes. It is possible to believe that within the active monetary policy regimes, the central bank might respond more aggressively in one regime than the other. In this regard, I assume that the magnitude of γ^π is greater in "Active(+)" regime than that in "Active" regime. The vector of shocks, $u_t = [u_t^{PC}, u_t^{MP}, u_t^{TG}]'$, is independently and conditionally normally distributed with mean zero and diagonal variance-covariance matrix, $E_{t-1}[u_t u_t'] = \Sigma_u(S_t)$. I refer to Campbell, Pflueger, and Viceira (2015) for a detailed description of the model. I employ the solution algorithm proposed by Farmer, Waggoner, and Zha (2011) and use Bayesian method to make inference on model coefficients and regime probabilities. The details are explained in Appendix E.

There are several takeaways from this exercise. First, the estimated monetary policy coefficients (reported in Appendix E) and the smoothed regime probabilities in Figure 4 are broadly consistent with Table 2 and Figure 3 in that the central bank's response to inflation (and to the output gap) has been active since the mid-1980s. Second, the variances of the structural innovations (reported in Appendix E) were largest from the 1970s to the mid-1980s (passive regime) which are consistent with Table 2 (captured by larger inflation innovation variance). Third, data do not strongly support the existence of the third monetary policy regime. It suffices to have a single active regime in addition to the passive regime.

Overall, I find that the paths for monetary policy under the New Keynesian model are broadly consistent with my proposed model.[27]

5   Conclusion

The paper studies the behavior of the nominal U.S. Treasury yield curve and the changing stock-bond return correlations in a model that allows for regime switches in the aggressiveness of monetary policy and in the conditional covariance of macroeconomic shocks. The model follows the long-run risk literature for the real side of the economy and extends it to add the nominal sector and changing regimes. The estimation identifies inflation as countercyclical from the early 1970s through the late

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[27]It is important to note that the regimes in Figure 4 are estimated with output gap, inflation, and the federal funds rate (without asset data). It is possible that the regime probabilities might change with the inclusion of asset data.

1990s and as procyclical from the late 1990s onward. This is overlaid with the "active" monetary policy regime that dominates most of the sample outside of the 1970s period, which is classified as "passive." The model is used to study the key moments of the yield curve and the correlation between bond-stock returns. It approximately matches the timing during which the stock-bond correlation switches signs from positive to negative in the late 1990s.

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Online Appendix

A Piazzesi and Schneider (2006) Revisited

Following Piazzesi and Schneider (2006), I assume that the vector of inflation and consumption growth has the following state space representation

$$z_t = s_{t-1} + \varepsilon_t, \quad z_t = [\pi_t, \Delta c_t]' \tag{A.1}$$
$$s_t = \phi s_{t-1} + \phi K \varepsilon_t, \quad \varepsilon_t \sim N(0, \Omega).$$

The state vector s_t is two dimensional and contains expected inflation and consumption; ϕ is the 2×2 autoregressive matrix; and K is the 2×2 gain matrix, where,

$$\phi = \begin{bmatrix} \phi_1 & \phi_{12} \\ \phi_{21} & \phi_2 \end{bmatrix}, \quad K = \begin{bmatrix} k_1 & k_{12} \\ k_{21} & k_2 \end{bmatrix}, \quad \Omega = \begin{bmatrix} \Omega_1 & \Omega_{12} \\ \Omega_{12} & \Omega_2 \end{bmatrix}.$$

Using the Bayesian method, I estimate this system with data for consumption and inflation. Table A-1 provides the details of parameter prior and posterior distributions. Because the complete estimation information in the tables can be difficult to absorb fully, I briefly present aspects of the results in a more revealing way. The parameters to be estimated are those in the transition equation ϕ, K and those in the covariance matrix Ω. Hence, I simply display the estimated transition equation and the estimated Ω matrices.

1. From 1959:Q1 to 1997:Q4,

$$s_t = \begin{bmatrix} 0.96 & 0.14 \\ \scriptstyle[0.92,0.98] & \scriptstyle[0.03,0.25] \\ -0.06 & 0.52 \\ \scriptstyle[-0.10,-0.02] & \scriptstyle[0.36,0.69] \end{bmatrix} s_{t-1} + \begin{bmatrix} 0.63 & 0.25 \\ \scriptstyle[0.57,0.73] & \scriptstyle[0.07,0.50] \\ -0.21 & 0.27 \\ \scriptstyle[-0.22,-0.16] & \scriptstyle[0.11,0.57] \end{bmatrix} \epsilon_t$$

$$\epsilon_t \sim N(0, \begin{bmatrix} 2.35 & -0.14 \\ \scriptstyle[2.13,2.60] & \scriptstyle[-0.21,-0.05] \\ -0.14 & 2.68 \\ \scriptstyle[-0.21,-0.05] & \scriptstyle[2.40,2.96] \end{bmatrix}), \quad var(\phi K \epsilon_t) = \begin{bmatrix} 1.06 & -0.14 \\ \scriptstyle[0.68,2.10] & \scriptstyle[-0.26,0.52] \\ -0.14 & 0.32 \\ \scriptstyle[-0.26,0.52] & \scriptstyle[0.14,1.02] \end{bmatrix}.$$

2. From 1998:Q1 to 2014:Q4,

$$s_t = \begin{bmatrix} 0.41 & 0.26 \\ \scriptstyle[0.28,0.55] & \scriptstyle[0.12,0.39] \\ 0.07 & 0.83 \\ \scriptstyle[-0.03,0.18] & \scriptstyle[0.72,0.91] \end{bmatrix} s_{t-1} + \begin{bmatrix} 0.33 & 0.43 \\ \scriptstyle[0.12,0.69] & \scriptstyle[0.14,0.86] \\ -0.02 & 0.71 \\ \scriptstyle[-0.20,0.24] & \scriptstyle[0.48,1.02] \end{bmatrix} \epsilon_t$$

$$\epsilon_t \sim N(0, \begin{bmatrix} 5.42 & -0.01 \\ \scriptstyle[4.63,6.37] & \scriptstyle[-0.09,0.07] \\ -0.01 & 1.10 \\ \scriptstyle[-0.09,0.07] & \scriptstyle[0.93,1.30] \end{bmatrix}), \quad var(\phi K \epsilon_t) = \begin{bmatrix} 0.78 & 0.29 \\ \scriptstyle[0.08,4.14] & \scriptstyle[0.01,2.28] \\ 0.29 & 0.55 \\ \scriptstyle[0.01,2.28] & \scriptstyle[0.42,1.77] \end{bmatrix}.$$

Table A-1: Posterior Estimates

		Prior			1959:Q1 - 1997:Q4 Posterior			1998:Q1 - 2014:Q4 Posterior		
	Distr.	20%	80%		20%	50%	80%		20%	50%	80%
ϕ_1	N^T	[-.35	.99]		0.92	0.96	0.98		0.28	0.41	0.55
ϕ_{12}	N	[-.82	.82]		0.03	0.14	0.25		0.12	0.26	0.39
ϕ_{21}	N	[-.82	.82]		-0.10	-0.06	-0.02		-0.03	0.07	0.18
ϕ_2	N^T	[-.35	.99]		0.35	0.52	0.69		0.72	0.83	0.91
k_1	N	[.15	1.81]		0.63	0.71	0.80		0.53	0.87	1.26
k_{12}	N	[-.82	.82]		0.07	0.18	0.29		0.22	0.53	0.90
k_{21}	N	[-.82	.82]		-0.44	-0.32	-0.20		-0.26	-0.10	0.02
k_2	N	[.15	1.81]		0.33	0.55	0.83		0.68	0.81	0.95
Ω_1	IG	[0.80	5.78]		2.13	2.35	2.60		4.63	5.42	6.37
Ω_{12}	N	[-.82	.82]		-0.21	-0.14	-0.05		-0.09	-0.01	0.07
Ω_2	IG	[0.80	5.78]		2.40	2.68	2.96		0.93	1.10	1.30

Notes: The estimation results are based on (annualized) quarterly consumption growth data and inflation data from 1959:Q1 to 2014:Q4. N, N^T, and IG are normal, truncated (outside of the interval $(-1, 1)$) normal, and inverse gamma distributions, respectively.

Here, I mention a few of many noteworthy aspects of the results. First, the autoregressive matrix ϕ estimates are quite different across the two periods. More specifically, I find a large decline in the persistence of the expected inflation process. Also, the lagged inflation used to predict negative future consumption, but in the last fifteen years it positively forecasts consumption. Second, the sign of the estimated covariance (in the reduced-form covariance matrix $\mathrm{var}(\phi K \varepsilon_t)$) changed from negative to positive during the recent periods.

B   Asset Pricing Solution of a Regime-Switching Model

B.1   Derivation of Approximate Analytical Solutions

The Euler equation for the economy is

$$1 = E_t \left[\exp \left(m_{t+1} + r_{k,t+1} \right) \right], \quad k \in \{c, m\}, \tag{A.2}$$

where $m_{t+1} = \theta \log \delta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{c,t+1}$ is the log stochastic discount factor, $r_{c,t+1}$ is the log return on the consumption claim, and $r_{m,t+1}$ is the log market return. All returns are given by the approximation of Campbell and Shiller (1988a):

$$
\begin{aligned}
r_{c,t+1} &= \kappa_{0,c} + \kappa_{1,c} z_{c,t+1} - z_{c,t} + g_{c,t+1}, & \text{(A.3)} \\
r_{m,t+1} &= \kappa_{0,m} + \kappa_{1,m} z_{m,t+1} - z_{m,t} + g_{d,t+1}.
\end{aligned}
$$

The first-order condition of the agent's expected utility maximization problem yields the Euler equations

$$
\mathbb{E}_t \left[\exp\left(m_{t+1} + r_{k,t+1} \right) \right] = 1, \quad k \in \{c, m\}, \quad \text{Real Assets}, \tag{A.4}
$$

$$
p_{n,t}^{\$} = \log \mathbb{E}_t [\exp(m_{t+1} - \pi_{t+1} + p_{n-1,t+1}^{\$})], \quad \text{Nominal Assets}, \tag{A.5}
$$

where $r_{c,t+1}$ is the log return on the consumption claim, $r_{m,t+1}$ is the log market return, and $p_{n,t}^{\$}$ is the nominal n-maturity log bond price. The solutions to (A.4) and (A.5) depend on the joint dynamics of consumption, dividend growth, and inflation.

Asset prices are determined from the approximate analytical solution described by Bansal and Zhou (2002). Let I_t denote the current information set $\{S_t, X_t\}$ and define $I_{t+1} = I_t \cup \{S_{t+1}\}$, which includes information regarding S_{t+1} in addition to I_t. The derivation of (A.4) follows Bansal and Zhou (2002), who repeatedly use the law of iterated expectations. For example, real asset returns are determined by

$$
\begin{aligned}
1 &= \mathbb{E}\left(\mathbb{E}\left[\exp\left(m_{t+1} + r_{m,t+1} \right) \mid I_{t+1} \right] \mid I_t \right) \\
&= \sum_{j=1}^{3} \Pi_{ij} \mathbb{E}\left(\exp\left(m_{t+1} + r_{m,t+1} \right) \mid S_{t+1} = j, X_t \right) \\
0 &= \sum_{j=1}^{3} \Pi_{ij} \underbrace{\left(\mathbb{E}\left[m_{t+1} + r_{m,t+1} \mid S_{t+1} = j, X_t \right] + \frac{1}{2} \mathbb{V}\left[m_{t+1} + r_{m,t+1} \mid S_{t+1} = j, X_t \right] \right)}_{B}.
\end{aligned}
$$

The first line uses the law of iterated expectations; the second line uses the definition of Markov chain; and the third line applies log-linearization (i.e., $\exp(B) - 1 \approx B$) and a log-normality assumption. The derivation of (A.5) can be done analogously.

B.2   Real Endowments

With regime switching coefficients, the joint consumption and dividend dynamics are

$$
\begin{aligned}
G_{t+1} &= \mu + \varphi X_t + \Sigma \eta_{t+1}, \quad \eta_t \sim N(0, I), \\
X_{t+1} &= \Phi(S_{t+1}) X_t + \Omega(S_{t+1}) \Sigma_x(S_{t+1}) \eta_{x,t+1}, \quad \eta_{x,t} \sim N(0, I),
\end{aligned}
$$

where $G_t = [\Delta c_t, \Delta d_t]'$, $\mu = [\mu_c, \mu_d]'$, $\eta_t = [\eta_{c,t}, \eta_{d,t}]'$, $X_t = [x_{c,t}, x_{\pi,t}, x_{i,t}]'$, $\eta_{x,t} = [\eta_{xc,t}, \eta_{x\pi,t}, \eta_{xi,t}]'$ and

$$\varphi = \begin{bmatrix} 1 & 0 & 0 \\ \phi & 0 & 0 \end{bmatrix}, \quad \Sigma = \begin{bmatrix} \sigma_c & 0 \\ 0 & \sigma_d \end{bmatrix},$$

$$\Phi = \begin{bmatrix} \rho_c & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \rho_i \end{bmatrix}, \quad \Omega = \begin{bmatrix} 1 & 0 & 0 \\ \beta & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \Sigma_x = \begin{bmatrix} \sigma_{xc} & 0 & 0 \\ 0 & \sigma_{x\pi} & 0 \\ 0 & 0 & \sigma_{xi} \end{bmatrix}.$$

It is important to mention that I conveniently decompose $\eta_{x,t+1}$ in (3), which is (A) below, into

$$\underbrace{\eta_{x,t+1}}_{(A)} = \Omega(S_{t+1})\Sigma_x(S_{t+1})\underbrace{\eta_{x,t+1}}_{(B)}. \tag{A.6}$$

η_{xt} in (B) are now orthogonalized.

B.3 Real Consumption Claim

If the conjectured solution to log price-consumption ratio is

$$z_{c,t} = A_0(S_t) + A_1(S_t)X_t,$$

then the return on the consumption claim can be written as

$$
\begin{aligned}
r_{c,t+1} &= \kappa_0 + \mu_c + \kappa_1 A_0(S_{t+1}) - A_0(S_t) + \left(e_1 + \kappa_1 A_1(S_{t+1})\Phi(S_{t+1}) - A_1(S_t)\right)X_t \\
&+ \kappa_1 A_1(S_{t+1})\Omega(S_{t+1})\Sigma_x(S_{t+1})\eta_{x,t+1} + e_1\Sigma\eta_{t+1}.
\end{aligned}
$$

The solutions for the A's that describe the dynamics of the price-consumption ratio are determined from

$$E_t(m_{t+1} + r_{c,t+1}) + \frac{1}{2}Var_t(m_{t+1} + r_{c,t+1}) = 0.$$

$$
\begin{bmatrix} A_1(1)' \\ A_1(2)' \\ A_1(3)' \end{bmatrix} = \begin{bmatrix} I - p_{11}\kappa_1\Phi(1) & -p_{12}\kappa_1\Phi(2) & -p_{13}\kappa_1\Phi(3) \\ -p_{21}\kappa_1\Phi(1) & I - p_{22}\kappa_1\Phi(2) & -p_{23}\kappa_1\Phi(3) \\ -p_{31}\kappa_1\Phi(1) & -p_{32}\kappa_1\Phi(2) & I - p_{33}\kappa_1\Phi(3) \end{bmatrix}^{-1} (1 - \frac{1}{\psi}) \begin{bmatrix} e_1' \\ e_1' \\ e_1' \end{bmatrix},
$$

$$
\begin{bmatrix} A_0(1) \\ A_0(2) \\ A_0(3) \end{bmatrix} = (I - \kappa_1\Pi)^{-1}\Pi \begin{bmatrix} \log\delta + \kappa_0 + (1 - \frac{1}{\psi})\mu_c + \frac{\theta}{2}(1 - \frac{1}{\psi})^2 e_1\Sigma\Sigma'e_1' + \frac{\theta}{2}\Psi(1)\Psi(1)' \\ \log\delta + \kappa_0 + (1 - \frac{1}{\psi})\mu_c + \frac{\theta}{2}(1 - \frac{1}{\psi})^2 e_1\Sigma\Sigma'e_1' + \frac{\theta}{2}\Psi(2)\Psi(2)' \\ \log\delta + \kappa_0 + (1 - \frac{1}{\psi})\mu_c + \frac{\theta}{2}(1 - \frac{1}{\psi})^2 e_1\Sigma\Sigma'e_1' + \frac{\theta}{2}\Psi(3)\Psi(3)' \end{bmatrix},
$$

$$\Psi(S_t) = \kappa_1 A_1(S_{t+1})\Omega(S_{t+1})\Sigma_x(S_{t+1}).$$

The log pricing kernel is

$$
\begin{aligned}
m_{t+1} \;=\;& \theta \log \delta + (\theta - 1)\big(\kappa_0 + \kappa_1 A_0(S_{t+1}) - A_0(S_t)\big) - \gamma \mu_c \\
& - \frac{1}{\psi} e_1 X_t + (\theta - 1)\Big((1 - \frac{1}{\psi})e_1 + \kappa_1 A_1(S_{t+1})\Phi(S_{t+1}) - A_1(S_t)\Big)X_t \\
& - \gamma e_1 \Sigma \eta_{t+1} + (\theta - 1)\kappa_1 A_1(S_{t+1})\Omega(S_{t+1})\Sigma_x(S_{t+1})\eta_{x,t+1}.
\end{aligned}
$$

B.4 Real Dividend Claim/Market Return

Analogously, the conjectured solution to log price-dividend ratio is

$$
z_{m,t} \;=\; A_{0,m}(S_t) + A_{1,m}(S_t)X_t,
$$

and the return on the dividend claim can be written as

$$
\begin{aligned}
r_{m,t+1} \;=\;& \kappa_{0,m} + \mu_d + \kappa_{1,m} A_{0,m}(S_{t+1}) - A_{0,m}(S_t) + \Big(\phi e_1 + \kappa_{1,m} A_{1,m}(S_{t+1})\Phi(S_{t+1}) - A_{1,m}(S_t)\Big)X_t \\
& + \kappa_{1,m} A_{1,m}(S_{t+1})\Omega(S_{t+1})\Sigma_x(S_{t+1})\eta_{x,t+1} + e_2 \Sigma \eta_{t+1}.
\end{aligned}
$$

The solutions for A_ms are

$$
\begin{bmatrix} A_{1,m}(1)' \\ A_{1,m}(2)' \\ A_{1,m}(3)' \end{bmatrix}
=
\begin{bmatrix}
I - p_{11}\kappa_{1,m}\Phi(1) & -p_{12}\kappa_{1,m}\Phi(2) & -p_{13}\kappa_{1,m}\Phi(3) \\
-p_{21}\kappa_{1,m}\Phi(1) & I - p_{22}\kappa_{1,m}\Phi(2) & -p_{23}\kappa_{1,m}\Phi(3) \\
-p_{31}\kappa_{1,m}\Phi(1) & -p_{32}\kappa_{1,m}\Phi(2) & I - p_{33}\kappa_{1,m}\Phi(3)
\end{bmatrix}^{-1}
(\phi - \frac{1}{\psi})
\begin{bmatrix} e_1' \\ e_1' \\ e_1' \end{bmatrix},
$$

$$
\begin{bmatrix} A_{0,m}(1) \\ A_{0,m}(2) \\ A_{0,m}(3) \end{bmatrix}
= (I - \kappa_{1,m}\Pi)^{-1}\Big(\Pi
\begin{bmatrix}
(\theta - 1)\kappa_1 A_0(1) + \frac{1}{2}\Psi_m(1)\Psi_m(1)' \\
(\theta - 1)\kappa_1 A_0(2) + \frac{1}{2}\Psi_m(2)\Psi_m(2)' \\
(\theta - 1)\kappa_1 A_0(3) + \frac{1}{2}\Psi_m(3)\Psi_m(3)'
\end{bmatrix}
+
\begin{bmatrix} \Xi_m(1) \\ \Xi_m(2) \\ \Xi_m(3) \end{bmatrix}
\Big),
$$

$$
\Xi_m(S_t) \;=\; \theta \log \delta + (\theta - 1)(\kappa_0 - A_0(S_t)) - \gamma \mu_c + \kappa_{0,m} + \mu_d + \frac{1}{2}\Big(\gamma^2 e_1 \Sigma \Sigma' e_1' + e_2 \Sigma \Sigma' e_2'\Big),
$$

$$
\Psi_m(S_t) \;=\; \Big((\theta - 1)\kappa_1 A_1(S_{t+1}) + \kappa_{1,m} A_{1,m}(S_{t+1})\Big)\Omega(S_t)\Sigma_x(S_t).
$$

B.5 Linearization Parameters

Let $\bar{p}_j = \sum_{i \in 3} \bar{p}_i \Pi_{ij}$. For any asset, the linearization parameters are endogenously determined by the following system of equations

$$
\begin{aligned}
\bar{z}_i \;&=\; \sum_{j=1}^{3} \bar{p}_j A_{0,i}(j), \\
\kappa_{1,i} \;&=\; \frac{\exp(\bar{z}_i)}{1 + \exp(\bar{z}_i)}, \\
\kappa_{0,i} \;&=\; \log(1 + \exp(\bar{z}_i)) - \kappa_{1,i}\bar{z}_i.
\end{aligned}
$$

The solution is numerically determined by iteration until reaching a fixed point of $\bar{z}_i$ for $i \in \{c, m\}$.

B.6 Endogenous Inflation Process under a Regime-Switching Monetary Policy Rule

The inflation target augmented monetary policy rule is

$$i_t = \tau_0(S_t) + \tau_c(S_t)x_{c,t} + \tau_\pi(S_t)(\pi_t - \Gamma_0(S_t) - x_{\pi,t}) + x_{\pi,t} + x_{i,t}, \tag{A.7}$$

and the conjectured solution for inflation process is

$$\pi_t = \Gamma_0(S_t) + \underbrace{\left[\ \Gamma_{1,c}(S_t),\quad \Gamma_{1,\pi}(S_t),\quad \Gamma_{1,i}(S_t)\ \right]}_{\Gamma_1(S_t)} X_t. \tag{A.8}$$

Combining equations (A.7) and (A.8), I rewrite the monetary policy rule as

$$i_t = \tau_0(S_t) + \left[\ \tau_c(S_t) + \tau_\pi(S_t)\Gamma_{1,c}(S_t),\quad 1 - \tau_\pi(S_t) + \tau_\pi(S_t)\Gamma_{1,\pi}(S_t),\quad 1 + \tau_\pi(S_t)\Gamma_{1,i}(S_t)\ \right] X_t. \tag{A.9}$$

Setting equation (A.9) equal to

$$i_t = -E_t(m_{t+1} - \pi_{t+1}) - \frac{1}{2}Var_t(m_{t+1} - \pi_{t+1}),$$

we can solve for

$$\begin{bmatrix} \Gamma_{1,c}(1) \\ \Gamma_{1,c}(2) \\ \Gamma_{1,c}(3) \end{bmatrix} = \left(\begin{bmatrix} \tau_\pi(1) & 0 & 0 \\ 0 & \tau_\pi(2) & 0 \\ 0 & 0 & \tau_\pi(3) \end{bmatrix} - \Pi \begin{bmatrix} \rho_c(1) & 0 & 0 \\ 0 & \rho_c(2) & 0 \\ 0 & 0 & \rho_c(3) \end{bmatrix} \right)^{-1} \begin{bmatrix} \frac{1}{\psi} - \tau_c(1) \\ \frac{1}{\psi} - \tau_c(2) \\ \frac{1}{\psi} - \tau_c(3) \end{bmatrix},$$

$$\begin{bmatrix} \Gamma_{1,\pi}(1) \\ \Gamma_{1,\pi}(2) \\ \Gamma_{1,\pi}(3) \end{bmatrix} = \left(\begin{bmatrix} \tau_\pi(1) & 0 & 0 \\ 0 & \tau_\pi(2) & 0 \\ 0 & 0 & \tau_\pi(3) \end{bmatrix} - \Pi \begin{bmatrix} \rho_\pi(1) & 0 & 0 \\ 0 & \rho_\pi(2) & 0 \\ 0 & 0 & \rho_\pi(3) \end{bmatrix} \right)^{-1} \begin{bmatrix} \tau_\pi(1) - 1 \\ \tau_\pi(2) - 1 \\ \tau_\pi(3) - 1 \end{bmatrix},$$

$$\begin{bmatrix} \Gamma_{1,i}(1) \\ \Gamma_{1,i}(2) \\ \Gamma_{1,i}(3) \end{bmatrix} = -\left(\begin{bmatrix} \tau_\pi(1) & 0 & 0 \\ 0 & \tau_\pi(2) & 0 \\ 0 & 0 & \tau_\pi(3) \end{bmatrix} - \Pi \begin{bmatrix} \rho_i(1) & 0 & 0 \\ 0 & \rho_i(2) & 0 \\ 0 & 0 & \rho_i(3) \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix},$$

and the constant

$$\begin{bmatrix} \Gamma_0(1) \\ \Gamma_0(2) \\ \Gamma_0(3) \end{bmatrix} = \Pi^{-1}\left(\begin{bmatrix} \Psi_\pi(1) \\ \Psi_\pi(2) \\ \Psi_\pi(3) \end{bmatrix} \right) + \begin{bmatrix} \Xi_\pi(1) \\ \Xi_\pi(2) \\ \Xi_\pi(3) \end{bmatrix},$$

where

$$
\begin{aligned}
\Xi_\pi(S_t) &= (\theta - 1)\kappa_1 A_0(S_t) \\
&+ \frac{1}{2}\left\{\left((\theta - 1)\kappa_1 A_1(S_t) - \Gamma_1(S_t)\right)\Omega(S_t)\Sigma_x(S_t)\right\}\left\{\left((\theta - 1)\kappa_1 A_1(S_t) - \Gamma_1(S_t)\right)\Omega(S_t)\Sigma_x(S_t)\right\}', \\
\Psi_\pi(S_t) &= \tau_0(S_t) + \left(\theta\log\delta + (\theta - 1)\kappa_0 - \gamma\mu_c + \frac{\gamma^2}{2}e_1\Sigma\Sigma'e_1'\right) - (\theta - 1)A_0(S_t).
\end{aligned}
$$

B.7 Nominal Bond Prices

The log nominal pricing kernel is

$$
\begin{aligned}
m_{t+1}^\$ &= m_{t+1} - \pi_{t+1} \\
&= \theta\log\delta + (\theta - 1)\left(\kappa_0 + \kappa_1 A_0(S_{t+1}) - A_0(S_t)\right) - \gamma\mu_c - \Gamma_0(S_{t+1}) \\
&- \left(\frac{1}{\psi}e_1 + \Gamma_1(S_{t+1})\Phi(S_{t+1})\right)X_t + (\theta - 1)\left((1 - \frac{1}{\psi})e_1 + \kappa_1 A_1(S_{t+1})\Phi(S_{t+1}) - A_1(S_t)\right)X_t \\
&- \gamma e_1\Sigma\eta_{t+1} + \left((\theta - 1)\kappa_1 A_1(S_{t+1}) - \Gamma_1(S_{t+1})\right)\Omega(S_{t+1})\Sigma_x(S_{t+1})\eta_{x,t+1}.
\end{aligned}
$$

The nominal n-maturity log bond price satisfies

$$
\begin{aligned}
p_{n,t}^\$(S_t) &= C_{n,0}^\$(S_t) + C_{n,1}^\$(S_t)X_t, \\
&= E_t(p_{n-1,t+1}^\$(S_{t+1}) + m_{t+1} - \pi_{t+1}) + \frac{1}{2}Var_t(p_{n-1,t+1}^\$(S_{t+1}) + m_{t+1} - \pi_{t+1}),
\end{aligned}
$$

where the bond loadings follow the recursions

$$
\begin{bmatrix} C_{n,1}^\$(1,:) \\ C_{n,1}^\$(2,:) \\ C_{n,1}^\$(3,:) \end{bmatrix} = \Pi \begin{bmatrix} \{C_{n-1,1}^\$(1,:) - \Gamma_1(1)\}\Phi(1) \\ \{C_{n-1,1}^\$(2,:) - \Gamma_1(2)\}\Phi(2) \\ \{C_{n-1,1}^\$(3,:) - \Gamma_1(3)\}\Phi(3) \end{bmatrix} + \frac{1}{\psi}\begin{bmatrix} e_1 \\ e_1 \\ e_1 \end{bmatrix}
$$

$$
\begin{bmatrix} C_{n,0}^\$(1) \\ C_{n,0}^\$(2) \\ C_{n,0}^\$(3) \end{bmatrix} = \Pi \begin{bmatrix} C_{n-1,0}^\$(1) - \Gamma_0(1) + (\theta - 1)\kappa_1 A_0(1) + \frac{1}{2}\Psi_{n-1,c}(1)\Psi_{n-1,c}(1)' \\ C_{n-1,0}^\$(2) - \Gamma_0(2) + (\theta - 1)\kappa_1 A_0(2) + \frac{1}{2}\Psi_{n-1,c}(2)\Psi_{n-1,c}(2)' \\ C_{n-1,0}^\$(3) - \Gamma_0(3) + (\theta - 1)\kappa_1 A_0(3) + \frac{1}{2}\Psi_{n-1,c}(3)\Psi_{n-1,c}(3)' \end{bmatrix} + \begin{bmatrix} \Xi_c(1) \\ \Xi_c(2) \\ \Xi_c(3) \end{bmatrix}
$$

$$
\begin{aligned}
\Xi_c(S_t) &= \theta\log\delta + (\theta - 1)(\kappa_0 - A_0(S_t)) - \gamma\mu_c + \frac{1}{2}\gamma^2 e_1\Sigma\Sigma'e_1' \\
\Psi_{n-1,c}(S_t) &= \{C_{n-1,1}^\$(S_t) + (\theta - 1)\kappa_1 A_1(S_t) - \Gamma_1(S_t)\}\Omega(S_t)\Sigma_x(S_t).
\end{aligned}
$$

The loadings on nominal bond yields are

$$
B_{n,0}^\$ = -\frac{1}{n}C_{n,0}^\$, \quad B_{n,1}^\$ = -\frac{1}{n}C_{n,1}^\$.
$$

The log return to holding a n-maturity nominal bond from t to $t+1$ is

$$r^\$_{n,t+1} = C^\$_{n-1,0}(S_{t+1}) - C^\$_{n,0}(S_t) + \left(C^\$_{n-1,1}(S_{t+1})\Phi(S_{t+1}) - C^\$_{n,1}(S_t) \right) X_t$$

$$+ C^\$_{n-1,1}(S_{t+1})\Omega(S_{t+1})\Sigma_x(S_{t+1})\eta_{x,t+1}.$$

The log return to holding a n-maturity nominal bond from t to $t+1$ in excess of the log return to a one-period nominal bond is

$$rx^\$_{n,t+1} = C^\$_{n-1,0}(S_{t+1}) - C^\$_{n,0}(S_t) + C^\$_{1,0}(S_t) + \left(C^\$_{n-1,1}(S_{t+1})\Phi(S_{t+1}) - C^\$_{n,1}(S_t) + C^\$_{1,1}(S_t) \right) X_t$$

$$+ C^\$_{n-1,1}(S_{t+1})\Omega(S_{t+1})\Sigma_x(S_{t+1})\eta_{x,t+1}.$$

The one-period expected excess return on nominal bonds can be written in the following form

$$E(rx^\$_{n,t+1}|S_t = k) + \frac{1}{2}Var(rx^\$_{n,t+1}|S_t = k) = -Cov(m^\$_{t+1}, rx^\$_{n,t+1}|S_t = k)$$

$$= -\Pi(k,:) \times \left(\begin{bmatrix} \Lambda_1(k,1)\Lambda_2(k,1) + \Lambda_3(k,1) \\ \Lambda_1(k,2)\Lambda_2(k,2) + \Lambda_3(k,2) \\ \Lambda_1(k,3)\Lambda_2(k,3) + \Lambda_3(k,3) \end{bmatrix} \right)$$

$$\approx -\Pi(k,:) \times \begin{bmatrix} \Lambda_3(k,1) \\ \Lambda_3(k,2) \\ \Lambda_3(k,3) \end{bmatrix}$$

where the approximation is exact in a fixed-regime economy and

$$\Lambda_1(k,j) = K'_t C(j)' - \Pi(k,:) \begin{bmatrix} K'_t C(1)' \\ K'_t C(2)' \\ K'_t C(3)' \end{bmatrix}$$

$$\Lambda_2(k,j) = M(j)K_t - \Pi(k,:) \begin{bmatrix} M(1)K_t \\ M(2)K_t \\ M(3)K_t \end{bmatrix}$$

$$\Lambda_3(k,j) = \left((\theta-1)\kappa_1 A_1(j) - \Gamma_1(j) \right)\Omega(j)\Sigma_x(j)\Sigma_x(j)'\Omega_x(j)'(C^\$_{n-1,1}(j))'$$

and

$$M(j) = \begin{bmatrix} (\theta-1)\kappa_1 A_0(j) - \Gamma_0(j) & (\theta-1)\kappa_1 A_1(j)\Phi(j) - \Gamma_1(j)\Phi(j) \end{bmatrix}$$

$$C(j) = \begin{bmatrix} C^\$_{n-1,0}(j) & C^\$_{n-1,1}(j)\Phi(j) \end{bmatrix}$$

$$K_t = \begin{bmatrix} 1 \\ X_t \end{bmatrix}.$$

Intuitively, I consider the limiting case in which the regime is fixed. The one-period expected excess return is $E_t(rx_{n,t+1}^\$) + \frac{1}{2}Var_t(rx_{n,t+1}^\$)$

$$
\begin{aligned}
&= -\big((\theta-1)\kappa_1 A_1 - \Gamma_1\big)\Omega\Sigma_x\Sigma_x'\Omega_x'(C_{n-1,1}^\$)' \\
&= -(n-1)\sigma_{xc}^2\left\{(\gamma-\frac{1}{\psi})\frac{\kappa_1}{1-\kappa_1\rho_c}+\frac{1/\psi-\tau_c}{\tau_\pi-\rho_c}\right\}\left\{\left(\frac{1/\psi\tau_\pi-\rho_c\tau_c}{\tau_\pi-\rho_c}\right)\frac{1}{n-1}\left(\frac{1-\rho_c^{n-1}}{1-\rho_c}\right)+\beta\right\} \\
&\quad -(n-1)\sigma_{xc}^2\left\{\left(\frac{1/\psi\tau_\pi-\rho_c\tau_c}{\tau_\pi-\rho_c}\right)\frac{1}{n-1}\left(\frac{1-\rho_c^{n-1}}{1-\rho_c}\right)\right\}\beta-(n-1)\sigma_{xc}^2\left(\beta^2+\frac{\sigma_{x\pi}^2}{\sigma_{xc}^2}\right)
\end{aligned}
$$

where

$$
\begin{aligned}
-\big((\theta-1)\kappa_1 A_1 - \Gamma_1\big) &= \left[\begin{array}{cc} (\gamma-\frac{1}{\psi})\frac{\kappa_1}{1-\kappa_1\rho_c}+\frac{1/\psi-\tau_c}{\tau_\pi-\rho_c}, & 1 \end{array}\right] \\
\Omega\Sigma_x\Sigma_x'\Omega_x' &= \left[\begin{array}{cc} \sigma_{xc}^2 & \beta\sigma_{xc}^2 \\ \beta\sigma_{xc}^2 & \beta^2\sigma_{xc}^2+\sigma_{x\pi}^2 \end{array}\right] \\
C_{n-1,1}^\$ &= \left[\begin{array}{cc} -\left(\frac{1/\psi\tau_\pi-\rho_c\tau_c}{\tau_\pi-\rho_c}\right)\left(\frac{1-\rho_c^{n-1}}{1-\rho_c}\right), & -(n-1) \end{array}\right].
\end{aligned}
$$

After some tedious algebra, the sign of the one-period expected excess return can be expressed as

$$
sign\left(E_t(rx_{n,t+1}^\$) + \frac{1}{2}Var_t(rx_{n,t+1}^\$)\right) = -sign\left[(B_{n-1,1,c}^\$+\beta)\left\{\frac{(\gamma-1/\psi)\kappa_1}{1-\kappa_1\rho_c}+\frac{1/\psi-\tau_c}{\tau_\pi-\rho_c}+\beta\right\}+\frac{\sigma_{x\pi}^2}{\sigma_{xc}^2}\right].
$$

B.8 Real Stock and Nominal Bond Return Correlation

The return on dividend claim (i.e., market return) is

$$
\begin{aligned}
r_{m,t+1} &= \kappa_{0,m}+\mu_d+\kappa_{1,m}A_{0,m}(S_{t+1})-A_{0,m}(S_t)+\Big(\phi e_1+\kappa_{1,m}A_{1,m}(S_{t+1})\Phi(S_{t+1})-A_{1,m}(S_t)\Big)X_t \\
&\quad + \kappa_{1,m}A_{1,m}(S_{t+1})\Omega(S_{t+1})\Sigma_x(S_{t+1})\eta_{x,t+1}+e_2\Sigma\eta_{t+1}.
\end{aligned}
$$

The conditional covariance between the two is

$$
\begin{aligned}
Cov(r_{n,t+1}^\$,r_{m,t+1}|S_t=k) &= \Pi(k,:)\times\left(\left[\begin{array}{c} \Lambda_1(k,1)\Lambda_2(k,1)+\Lambda_3(k,1) \\ \Lambda_1(k,2)\Lambda_2(k,2)+\Lambda_3(k,2) \\ \Lambda_1(k,3)\Lambda_2(k,3)+\Lambda_3(k,3) \end{array}\right]\right) \\
&\approx \Pi(k,:)\times\left[\begin{array}{c} \Lambda_3(k,1) \\ \Lambda_3(k,2) \\ \Lambda_3(k,3) \end{array}\right]
\end{aligned}
$$

where the approximation is exact in a fixed-regime economy and

$$
\Lambda_1(k,j) = K_t'C(j)' - \Pi(k,:) \begin{bmatrix} K_t'C(1)' \\ K_t'C(2)' \\ K_t'C(3)' \end{bmatrix}
$$

$$
\Lambda_2(k,j) = A_m(j)K_t - \Pi(k,:) \begin{bmatrix} A_m(1)K_t \\ A_m(2)K_t \\ A_m(3)K_t \end{bmatrix}
$$

$$
\Lambda_3(k,j) = \kappa_{1,m}A_{1,m}(j)\Omega(j)\Sigma_x(j)\Sigma_x(j)'\Omega_x(j)'(C^{\$}_{n-1,1}(j))'
$$

and

$$
A_m(j) = \begin{bmatrix} \kappa_{1,m}A_{0,m}(j) & \kappa_{1,m}A_{0,m}(j)\Phi(j) \end{bmatrix}
$$

$$
C(j) = \begin{bmatrix} C^{\$}_{n-1,0}(j) & C^{\$}_{n-1,1}(j)\Phi(j) \end{bmatrix}
$$

$$
K_t = \begin{bmatrix} 1 \\ X_t \end{bmatrix}.
$$

I consider the limiting case in which the regime is fixed. The stock-bond return covariance is

$$
Cov_t(r^{\$}_{n,t+1}, r_{m,t+1}) = -\kappa_{1,m}\sigma^2_{xc}\left(\frac{\phi - 1/\psi}{1 - \kappa_{1,m}\rho_c}\right)(n-1)\left(B^{\$}_{n-1,1,c} + \beta\right),
$$

from which I can deduce that

$$
sign(Cov_t(r^{\$}_{n,t+1}, r_{m,t+1})) = -sign\left(B^{\$}_{n-1,1,c} + \beta\right).
$$

Since $rx^{\$}_{n,t+1} = r^{\$}_{n,t+1} - y^{\$}_{1,t}$,

$$
sign(Cov_t(rx^{\$}_{n,t+1}, r_{m,t+1})) = -sign\left(B^{\$}_{n-1,1,c} + \beta\right).
$$

B.9 k Step ahead Expectations

Any variable K_{t+1} that can be expressed as

$$
K_{t+1} = \Lambda_0(S_{t+1}) + \Lambda_1(S_{t+1})X_{t+1}
$$

$$
X_{t+1} = \Phi(S_{t+1})X_t + \Omega(S_{t+1})\Sigma_x(S_{t+1})\eta_{x,t+1}, \quad \eta_{x,t} \sim N(0, I)
$$

has the following k-step-ahead expectation form of

$$
E(K_{t+k}|S_t) = \underbrace{E(\Lambda_0(S_{t+k})|S_t)}_{\Lambda_0^{(k)}} + \underbrace{E(\Lambda_1(S_{t+k})\Phi(S_{t+k})\Phi(S_{t+k-1})\ldots\Phi(S_{t+1})|S_t)}_{\Lambda_1^{(k)}} X_t \quad (A.10)
$$

$$
\begin{aligned}
K_{t+k} =& \; \Lambda_0(S_{t+k}) + \Lambda_1(S_{t+k})\Phi(S_{t+k})\Phi(S_{t+k-1})\ldots\Phi(S_{t+1})X_t \\
&+ \Lambda_1(S_{t+k})\Omega(S_{t+1})\Sigma_x(S_{t+1})\eta_{x,t+k} \\
&+ \sum_{i=0}^{k-2} \Lambda_1(S_{t+k}) \prod_{j=0}^{i} \Phi(S_{t+k-j})\Omega(S_{t+k-i-1})\Sigma_x(S_{t+k-i-1})\eta_{x,t+k-i-1}.
\end{aligned}
$$

We can characterize the constant and the slope coefficients as

$$
\Lambda_0^{(k)}(j) = \begin{bmatrix} \Lambda_0(1) & \Lambda_0(2) & \Lambda_0(3) \end{bmatrix}
\begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix}^{(k-1)}
\times
\begin{bmatrix} p_{1j} \\ p_{2j} \\ p_{3j} \end{bmatrix},
$$

$$
\Lambda_1^{(k)}(j) = \begin{bmatrix} \Lambda_1(1) & \Lambda_1(2) & \Lambda_1(3) \end{bmatrix}
\begin{bmatrix} p_{11}\Phi(1) & p_{12}\Phi(1) & p_{13}\Phi(1) \\ p_{21}\Phi(2) & p_{22}\Phi(2) & p_{23}\Phi(2) \\ p_{31}\Phi(3) & p_{32}\Phi(3) & p_{33}\Phi(3) \end{bmatrix}^{(k-1)}
$$

$$
\times
\begin{bmatrix} \Phi(1) & 0 & 0 \\ 0 & \Phi(2) & 0 \\ 0 & 0 & \Phi(3) \end{bmatrix}
\begin{bmatrix} p_{1j} \\ p_{2j} \\ p_{3j} \end{bmatrix}.
$$

The cumulative k-step-ahead expectation is

$$
\sum_{i=0}^{k-1} E(K_{t+i}|S_t = j) = \left(\Lambda_0^{(0)}(j) + \Lambda_0^{(1)}(j) + \ldots + \Lambda_0^{(k-1)}(j) \right) \quad (A.11)
$$

$$
+ \left(\Lambda_1^{(0)}(j) + \Lambda_1^{(1)}(j) + \ldots + \Lambda_1^{(k-1)}(j) \right) X_t.
$$

B.10   Yield Decomposition

The log yield for n-maturity nominal bond can be expressed as

$$
y_{n,t}^{\$} = -\frac{1}{n} C_{n,0}^{\$}(S_t) - \frac{1}{n} C_{n,1}^{\$}(S_t) X_t.
$$

The term premium for n-maturity bond is

$$
\begin{aligned}
\xi_{n,t} =& \; y_{n,t}^{\$} - \frac{1}{n} \sum_{i=0}^{n-1} E(y_{1,t+i}^{\$}|S_t = j), \quad (A.12) \\
=& \; \frac{1}{n}\left(C_{1,0}^{\$,(0)}(j) + \ldots + C_{1,0}^{\$,(n-1)}(j) - C_{n,0}^{\$}(j) \right) + \frac{1}{n}\left(C_{1,1}^{\$,(0)}(j) + \ldots + C_{1,1}^{\$,(n-1)}(j) - C_{n,1}^{\$}(j) \right) X_t.
\end{aligned}
$$

The real rate for n-maturity bond is

$$y_{n,t} \;=\; y_{n,t}^{\$} - \frac{1}{n}\sum_{i=1}^{n} E(\pi_{t+i}|S_t = j) - \xi_{n,t}. \tag{A.13}$$

B.11   News and Shock Decomposition

For illustrative purposes, I assume that $S_{t-1} = m$ and $S_t = j$.

$$
\begin{aligned}
\eta_{K,t}^{(k)} \;=\;& E_t\!\left(\frac{1}{k}\sum_{i=1}^{k} K_{t+i}\right) - E_{t-1}\!\left(\frac{1}{k}\sum_{i=1}^{k} K_{t+i}\right) \tag{A.14} \\[4pt]
=\;& E\!\left(\frac{1}{k}\sum_{i=1}^{k} K_{t+i}\Big|S_t = j\right) - E\!\left(\frac{1}{k}\sum_{i=1}^{k} K_{t+i}\Big|S_{t-1} = m\right) \\[4pt]
=\;& \frac{1}{k}\!\left(\Lambda_0^{(1)}(j) + \ldots + \Lambda_0^{(k)}(j)\right) + \frac{1}{k}\!\left(\Lambda_1^{(1)}(j) + \ldots + \Lambda_1^{(k)}(j)\right)X_t \\
& -\frac{1}{k}\!\left(\Lambda_0^{(2)}(m) + \ldots + \Lambda_0^{(k+1)}(m)\right) - \frac{1}{k}\!\left(\Lambda_1^{(2)}(m) + \ldots + \Lambda_1^{(k+1)}(m)\right)X_{t-1} \\[4pt]
=\;& \frac{1}{k}\!\left(\Lambda_0^{(1)}(j) + \ldots + \Lambda_0^{(k)}(j)\right) + \frac{1}{k}\!\left(\Lambda_1^{(1)}(j) + \ldots + \Lambda_1^{(k)}(j)\right)\!\left(\Phi(j)X_{t-1} + \Omega(j)\Sigma_x(j)\eta_{x,t}\right) \\
& -\frac{1}{k}\!\left(\Lambda_0^{(2)}(m) + \ldots + \Lambda_0^{(k+1)}(m)\right) - \frac{1}{k}\!\left(\Lambda_1^{(2)}(m) + \ldots + \Lambda_1^{(k+1)}(m)\right)X_{t-1} \\[4pt]
=\;& \frac{1}{k}\!\left(\Lambda_0^{(1)}(j) + \ldots + \Lambda_0^{(k)}(j)\right) - \frac{1}{k}\!\left(\Lambda_0^{(2)}(m) + \ldots + \Lambda_0^{(k+1)}(m)\right) \\
& +\frac{1}{k}\!\left(\Lambda_1^{(1)}(j) + \ldots + \Lambda_1^{(k)}(j)\right)\Phi(j)X_{t-1} - \frac{1}{k}\!\left(\Lambda_1^{(2)}(m) + \ldots + \Lambda_1^{(k+1)}(m)\right)X_{t-1} \\
& +\frac{1}{k}\!\left(\Lambda_1^{(1)}(j) + \ldots + \Lambda_1^{(k)}(j)\right)\Omega(j)\Sigma_x(j)\eta_{x,t} \\[4pt]
=\;& \frac{1}{k}\!\left(\left\{\Lambda_0^{(1)}(j) - \Lambda_0^{(2)}(m)\right\} + \ldots + \left\{\Lambda_0^{(k)}(j) - \Lambda_0^{(k+1)}(m)\right\}\right) \\
& +\frac{1}{k}\!\left(\left\{\Lambda_1^{(1)}(j)\Phi(j) - \Lambda_1^{(2)}(m)\right\} + \ldots + \left\{\Lambda_1^{(k)}(j)\Phi(j) - \Lambda_1^{(k+1)}(m)\right\}\right)X_{t-1} \\
& +\frac{1}{k}\!\left(\Lambda_1^{(1)}(j) + \ldots + \Lambda_1^{(k)}(j)\right)\Omega(j)\Sigma_x(j)\eta_{x,t}
\end{aligned}
$$

denotes the news.

$$
\begin{aligned}
\varepsilon_{K,t} \;=\;& K_t - E\big(K_t|S_{t-1}\big) \tag{A.15} \\[4pt]
=\;& \Lambda_0(j) + \Lambda_1(j)X_t - \Lambda_0^{(1)}(m) - \Lambda_1^{(1)}(m)X_{t-1} \\[4pt]
=\;& \Lambda_0(j) + \Lambda_1(j)\!\left(\Phi(j)X_{t-1} + \Omega(j)\Sigma_x(j)\eta_{x,t}\right) - \Lambda_0^{(1)}(m) - \Lambda_1^{(1)}(m)X_{t-1} \\[4pt]
=\;& \left(\Lambda_0(j) - \Lambda_0^{(1)}(m)\right) + \left(\Lambda_1(j)\Phi(j) - \Lambda_1^{(1)}(m)\right)X_{t-1} + \Lambda_1(j)\Omega(j)\Sigma_x(j)\eta_{x,t}
\end{aligned}
$$

denotes the shocks.

If $S_{t-1} = j$ and $S_t = j$, we can express $\eta_{K,t}^{(k)}$ and $\varepsilon_{K,t}$ by

$$
\begin{aligned}
\eta_{K,t}^{(k)} &= \frac{1}{k}\left(\left\{\Lambda_0^{(1)}(j) - \Lambda_0^{(2)}(j)\right\} + \ldots + \left\{\Lambda_0^{(k)}(j) - \Lambda_0^{(k+1)}(j)\right\}\right) \qquad\qquad\text{(A.16)}\\
&\quad + \frac{1}{k}\left(\left\{\Lambda_1^{(1)}(j)\Phi(j) - \Lambda_1^{(2)}(j)\right\} + \ldots + \left\{\Lambda_1^{(k)}(j)\Phi(j) - \Lambda_1^{(k+1)}(j)\right\}\right)X_{t-1}\\
&\quad + \frac{1}{k}\left(\Lambda_1^{(1)}(j) + \ldots + \Lambda_1^{(k)}(j)\right)\Omega(j)\Sigma_x(j)\eta_{x,t}\\
&\approx \frac{1}{k}\left(\Lambda_1^{(1)}(j) + \ldots + \Lambda_1^{(k)}(j)\right)\Omega(j)\Sigma_x(j)\eta_{x,t},\\
\varepsilon_{K,t} &= \left(\Lambda_0(j) - \Lambda_0^{(1)}(j)\right) + \left(\Lambda_1(j)\Phi(j) - \Lambda_1^{(1)}(j)\right)X_{t-1} + \Lambda_1(j)\Omega(j)\Sigma_x(j)\eta_{x,t}\\
&\approx \Lambda_1(j)\Omega(j)\Sigma_x(j)\eta_{x,t}.
\end{aligned}
$$

C Data

Table A-2: Data (Annualized)

| Consumption		Inflation	
$E(\Delta c)$	1.90	$E(\pi)$	3.71
$\sigma(\Delta c)$	0.93	$\sigma(\pi)$	1.10
$corr(\Delta c, \pi)$	-0.05	$AC(\pi)$	0.62
Log Price-Dividend Ratio		**Stock Market Returns**	
$E(pd)$	3.60	$E(r_m)$	5.75
$\sigma(pd)$	0.37	$\sigma(r_m)$	15.48
Bond Yields		**Stock-Bond Return Correlation**	
$E(y_1^\$)$	5.26	$E(corr(r_m, y_1^\$))$	0.08
$E(y_2^\$)$	5.47	$E(corr(r_m, y_2^\$))$	0.08
$E(y_3^\$)$	5.65	$E(corr(r_m, y_3^\$))$	0.09
$E(y_4^\$)$	5.81	$E(corr(r_m, y_4^\$))$	0.10
$E(y_5^\$)$	5.92	$E(corr(r_m, y_5^\$))$	0.10

Notes: The data moments for consumption are based on the measurement-error-free monthly consumption growth series from Schorfheide, Song, and Yaron (2016). I use daily stock market returns and k-year bond returns to compute the realized conditional correlation, $corr_t(r_{m,t+1}, r_{ky,t+1}^\$)$, where $k = 1, ..., 5$. The sample ranges from 1963:M1 to 2014:M12.

Figure A-1: Data (Annualized)
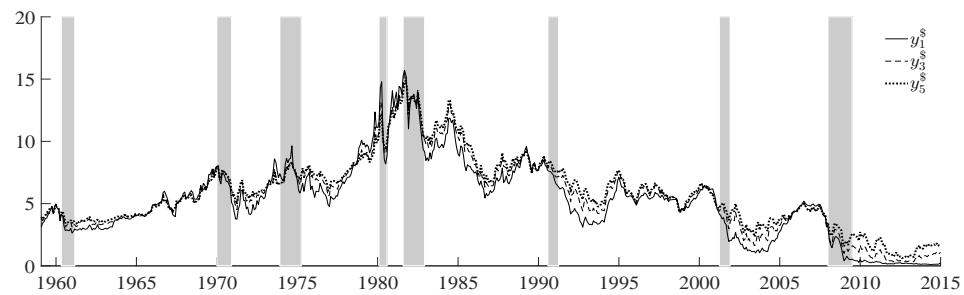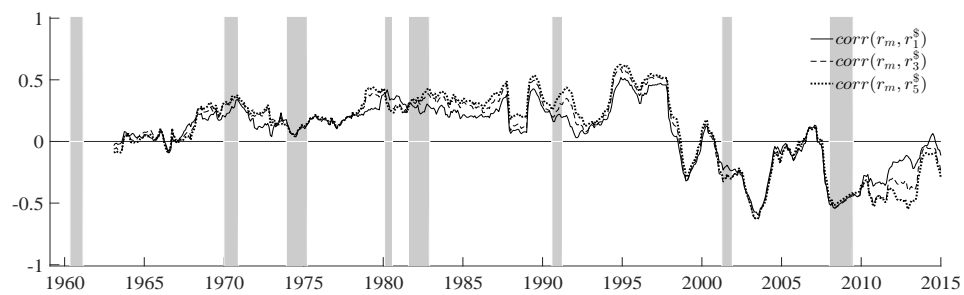
Consumption Growth and Inflation
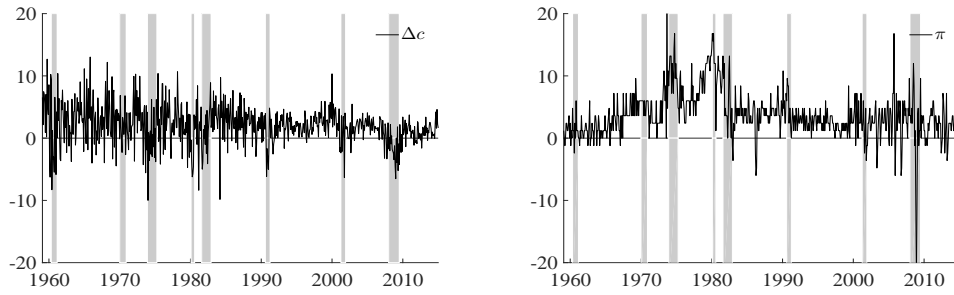
Log Price-Dividend Ratio and Market Returns
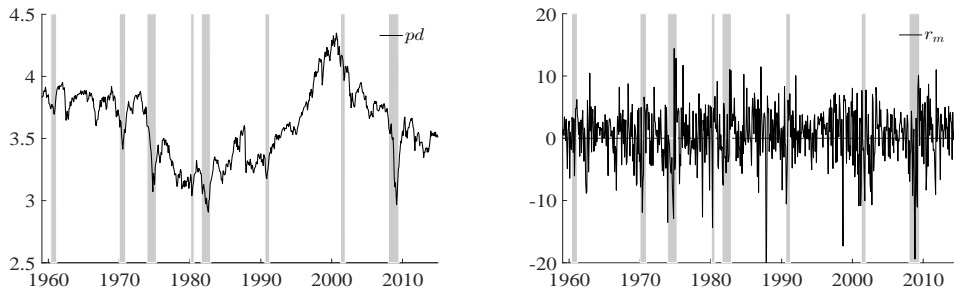
Bond Yields

Stock-Bond Return Correlation

Notes: All data are annualized. The light-gray shaded bars indicate the NBER recession dates.

D   Bayesian Inference

Posterior inference is implemented with a Metropolis-within-Gibbs sampler, which builds on the work of Carter and Kohn (1994) and Kim and Nelson (1999). $Y_{1:T}$ denotes the sequence of observations, where

$$Y_t = (\Delta c_t, \pi_t, pd_t, y_{1,t}, y_{2,t}, y_{3,t}, y_{4,t}, y_{5,t}).$$

Moreover, let $S_{1:T}$ be the sequence of hidden states, and let $\Theta = (\Theta_1, \Theta_2)$, where

$$
\begin{aligned}
\Theta_1 &= (\delta, \gamma, \psi), \\
\Theta_2 &= (\mu_c, \mu_d, \rho_c, \rho_i, \phi, \sigma_c, \sigma_d, \sigma_{xc}, \sigma_{x\pi}, \sigma_{xi}, \beta(-), \beta(+), \tau_0(P), \tau_0(A), \tau_c(P), \tau_c(A), \tau_\pi(P), \tau_\pi(A)), \\
\Phi &= (\{\Pi_{ij}\}_{i,j=\{1,2,3\}}).
\end{aligned}
$$

The Metropolis-within-Gibbs algorithm involves iteratively sampling from three conditional posterior distributions. To initialize the sampler, I start from (Θ^0, Φ^0).

Algorithm: Metropolis Sampler

For $i = 1, \ldots, N$:

1. Draw $S_{1:T}^{i+1}$ conditional on Θ^i, Φ^i, and $Y_{1:T}$. This step is implemented using the multi-move simulation smoother described in Section 9.1.1 of Kim and Nelson (1999).

2. Draw Φ^{i+1} conditional on Θ^i, $S_{1:T}^{i+1}$, and $Y_{1:T}$. If the dependence of the distribution of the initial state S_1 on Φ is ignored, then it can be shown that the conditional posterior of Φ is of the Dirichlet form. I use the resultant Dirichlet distribution as a proposal distribution in a Metropolis-Hastings step.

3. Draw Θ^{i+1}, conditional on Φ^{i+1}, $S_{1:T}^{i+1}$, and $Y_{1:T}$. Since the prior distribution is nonconjugate, I am using a random-walk Metropolis step to generate a draw from the conditional posterior of Θ. The proposal distribution is $N(\Theta^i, c\Omega)$.

I obtain the covariance matrix Ω of the proposal distribution in Step 2 as follows. Following Schorfheide (2005), I maximize the posterior density,

$$p(\Theta|Y_{1:T}) \propto p(Y_{1:T}|\Theta)p(\Theta),$$

to obtain the posterior mode $\tilde{\Theta}$. I then construct the negative inverse of the Hessian at the mode. I choose the scaling factor c to obtain an acceptance rate of approximately 40 percent. We initialize our algorithm choosing (Θ^0) in the neighborhood of $(\tilde{\Theta})$ and use it to generate $N = 100,000$ draws from the posterior distribution.

Table A-3: Prior Distributions

	Distr.	5%	50%	95%		Distr.	5%	50%	95%
		Factor Shocks				Transition Probability			
$\beta(+)$	U	0.5	5.0	9.5	p_{11}	Dir	0.85	0.90	0.95
$\beta(-)$	U	-9.5	-5.0	-0.5	p_{21}	Dir	0.02	0.05	0.09
σ	IG	0.0008	0.0019	0.0061	p_{31}	Dir	0.02	0.05	0.09
		Monetary Policy			p_{12}	Dir	0.02	0.05	0.09
τ_0	U	0.0014	0.0055	0.0095	p_{22}	Dir	0.85	0.90	0.95
$\tau_\pi(A)$	U	1.10	2.00	2.90	p_{32}	Dir	0.02	0.05	0.09
$\tau_\pi(P)$	U	0.05	0.50	0.95	p_{13}	Dir	0.02	0.05	0.09
τ_c	U	0.05	0.50	0.95	p_{23}	Dir	0.85	0.90	0.95
					p_{33}	Dir	0.02	0.05	0.09

Notes: Dir, IG, and U denote the dirichlet, inverse gamma, and uniform distributions, respectively.

E   Campbell, Pflueger, and Viceira (2015) Revisited

I estimate the New Keynesian model proposed by Campbell, Pflueger, and Viceira (2015). The model has three structural equations (IS curve, Phillips curve, and Monetary policy rule) and uses the law of motion for time-varying inflation target.

1. IS curve : $x_t = \rho^{x-}(S_t)x_{t-1} + \rho^{x+}(S_t)E_t x_{t+1} - \psi(S_t)(i_t - E_t\pi_{t+1})$

2. Phillips curve : $\pi_t = \rho^\pi(S_t)\pi_{t-1} + (1 - \rho^\pi(S_t))E_t\pi_{t+1} + \kappa(S_t)x_t + u_t^{PC}$

3. Monetary policy : $i_t = \rho^i(S_t)i_{t-1} + (1 - \rho^i(S_t))\big(\gamma^x(S_t)x_t + \gamma^\pi(S_t)(\pi_t - \pi_t^{TG}) + \pi_t^{TG}\big) + u_t^{MP}$

4. Inflation target : $\pi_t^{TG} = \pi_{t-1}^{TG} + u_t^{TG}$,

where x_t is the log output gap, π_t is the inflation rate, i_t is the log yield of a one month maturity at time t, π_t^{TG} is inflation target, and $u_t = [u_t^{PC}, u_t^{MP}, u_t^{TG}]'$, $E_{t-1}[u_t u_t'] = \Sigma_u$ is diagonal and homoskedastic.

I employ the solution algorithm proposed by Farmer, Waggoner, and Zha (2011) (minimal state variable solutions to Markov-switching rational expectations models)

$$
\begin{aligned}
\alpha_t &= \left[x_t, E_t x_{t+1}, \pi_t, E_t \pi_{t+1}, i_t, \pi_t^{TG} \right]' \\
\eta_t &= \left[x_t - E_{t-1} x_t, \pi_t - E_{t-1} \pi_t \right] \\
\Gamma_0(s_t)\alpha_t &= \Gamma_1(s_t)\alpha_{t-1} + \Psi(s_t)u_t + \Pi(s_t)\eta_t
\end{aligned}
$$

and cast the model into state-space representation

$$
\begin{aligned}
Y_t &= \Lambda \alpha_t, \quad Y_t = [x_t, 4\pi_t, 4i_t]', \\
\alpha_t &= \Phi(s_t)\alpha_{t-1} + \Sigma(s_t)u_t.
\end{aligned}
$$

I use the Bayesian method (described in the previous section) to make inference on model coefficients and the regime probabilities

$$
\begin{aligned}
\Theta|(s_t = i) &= \left\{ \rho^{x-}, \rho^{x+}, \psi, \rho^\pi, \kappa, \rho^i, \gamma^x, \gamma^\pi, \Sigma_u \right\}, \\
Pr(s_t = i), \quad i \in & \quad \{\text{Passive, Active, Active}(+)\}.
\end{aligned}
$$

The state space is estimated with output gap, inflation, and the federal funds rate (without asset data).

Table A-4: Posterior Estimates: New Keynesian Model

	Prior			Posterior								
				Passive			Active			Active(+)		
	Distr.	5%	95%	50%	5%	95%	50%	5%	95%	50%	5%	95%
Preferences												
ρ^{x-}	U	[0.03	0.47]	0.258	[0.184	0.340]	0.072	[0.010	0.145]	0.087	[0.017	0.173]
ρ^{x+}	U	[0.54	1.45]	0.777	[0.699	0.856]	0.967	[0.896	1.045]	0.945	[0.852	1.018]
ψ	U	[0.01	2.25]	0.031	[0.016	0.053]	0.059	[0.030	0.100]	0.036	[0.021	0.063]
ρ^{π}	B	[0.74	0.99]	0.924	[0.867	0.965]	0.959	[0.902	0.990]	0.816	[0.752	0.884]
κ	U	[0.03	0.47]	0.101	[0.039	0.195]	0.045	[0.010	0.099]	0.027	[0.005	0.063]
Monetary Policy Rule Coefficients												
ρ^{i}	B	[0.74	0.99]	0.833	[0.814	0.868]	0.852	[0.837	0.880]	0.938	[0.910	0.960]
γ^{π}	U	[0.01	2.25]	0.719	[0.676	0.752]	1.379	[1.215	1.453]	1.439	[1.356	1.557]
γ^{x}	U	[0.01	2.25]	0.419	[0.339	0.494]	0.635	[0.514	0.733]	0.672	[0.564	0.812]
Standard Deviations of Macro Shocks												
σ^{PC}	IG	[0.05	1.25]	0.373	[0.319	0.441]	0.130	[0.097	0.201]	0.210	[0.178	0.259]
σ^{MP}	IG	[0.05	1.25]	0.361	[0.325	0.404]	0.113	[0.090	0.149]	0.061	[0.050	0.073]
σ^{TG}	IG	[0.05	1.25]	0.716	[0.568	0.902]	0.302	[0.248	0.356]	0.192	[0.147	0.238]

Notes: B, U, and *IG* denote the beta, uniform, and inverse gamma distributions, respectively.