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REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 2000 Mathematics Subject Classification can be found in print starting with the 1999 annual index of *Mathematical Reviews*. The classifications are also accessible from `www.ams.org/msc/`.

2[65R20, 65L05, 65L20, 65L50, 65L60, 65L80, 45D05]—*Collocation methods for Volterra integral and related functional equations*, by Hermann Brunner, Cambridge University Press, 2004, xiv+597 pp., ISBN 0-89871-572-5, hardcover, US$120.00

This work presents a well thought out and thoroughly written course about collocation methods for Volterra integral and related functional equations, leading from equations with regular solutions to advanced and more complicated topics for equations with various singular solutions. Clarity of exposition, a good number of illustrating examples, comments, and a list of open problems are the distinguished features of the book. The book provides an introduction and an overview to the basic principles and the analysis of collocation methods for initial-value problems for ordinary differential equations, as well as delay differential equations for Volterra integral and related functional equations, and at the same time it gives the current "state of the art" of the field. The list of references is very extensive (over 1300 items) and, together with the Notes at the end of each chapter, it shows that Volterra equations play an important role in time-dependent partial differential equations, boundary integral equations, and in many other areas of mathematics and applications. Within each major topic the exposition is, as a rule, inductive, nearly always proceeding from the statement and the roots of a problem through a theoretical analysis, historical excursus, and suggestive heuristic considerations concerning its solution, toward fundamental ideas of numerical approximations and their analysis.

The book consists of nine chapters and can be divided in a natural way into four parts.

A distinguished feature of Part I (Chapter 1 "The collocation methods for ODEs: an introduction", Chapter 2 "Volterra integral equations with smooth kernels", and Chapter 3 "Volterra integro-differential equations with smooth kernels") is the smoothness of solutions of problems under consideration. This fact allows a reader to concentrate his or her attention on the principal moments of the algorithms and of the analysis (convergence, superconvergence etc.) of collocation methods.

Part II deals with Volterra integral and integro-differential equations containing delay arguments. In Chapter 4, "Initial-value problems with non-vanishing delays", it is shown that non-vanishing delays induce so-called primary discontinuity points at which the regularity of the solution is lower than that of input data. Due to this fact, superconvergence can only occur if the meshes underlying the collocation spaces are adapted to the behavior of the exact solution. Chapter 5, "Initial-value problems with proportional (vanishing) delays", demonstrates a completely different situation as compared with the previous chapter. The solution inherits the regularity of the given data, but the analysis of superconvergence becomes much more complex and a number of problems remain open.

Chapter 6, “Volterra integral equations with weakly singular kernels”, and Chapter 7, “VIDEs with weakly singular kernels”, can be ranged into Part III of the book. The problems considered in these chapters typically have solutions with unbounded derivatives. Due to this fact, the optimal global and local convergence and superconvergence results obtained in Part I for collocation solutions in piecewise polynomial spaces on uniform meshes can no longer be valid. Nevertheless, it is shown that the use of appropriately graded meshes, or of non-polynomial collocation spaces on uniform meshes, are two possible approaches to overcame the reduction of the convergence order.

The last part of the book consists of Chapter 8, “Outlook: integral-algebraic equations and beyond”, and Chapter 9, “Epilogue”, where the titles argue for the substance. In the first of these chapters the author reviews recent and current work on collocation methods for differential-algebraic equations (DAEs) and Volterra-type integral-algebraic equations of index 1. The purpose of the short last chapter is to sketch some possible new approaches for the numerical analysis of collocation solutions to Volterra functional equations which can lie in semigroups and abstract resolvent theory or in C^*-algebra techniques and invertibility of approximating operator sequences.

The subject index rounds up the book and made it very convenient for use.

The book under consideration is aimed primarily at researchers specializing in numerical and applied analysis, at users of collocation methods in the mathematical modelling and engineering, at senior undergraduate and graduate university students and at all those who wish to see both the efficient numerical algorithms on the one hand, and carefully formulated justification theorems and rigorous proofs on the other hand. The book can serve as a support for an introduction to, or special courses on, collocation methods for all who are studying or teaching collocation methods, Volterra integral and related functional equations. The reader will be able to follow the presentation with some previous knowledge from senior-level courses in linear algebra, the theory and numerical analysis of ordinary differential equations, numerical quadrature, and elementary functional analysis.

The researcher can find many open problems and interesting references, in particular giving access to classical as well as to modern results. The book may also serve as a source of topics for M.Sc. and Ph.D. theses.

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3[20X00, 20X04, 20B40, 20C40]—*Handbook of computational group theory*, by Derek F. Holt, Bettina Eick, and Eamonn A. O’Brien, Chapman & Hall/CRC, Boca Raton, Florida, 2005, xiv+514 pp., hardcover, US$89.95, UK£49.99, ISBN 1-58488-372-3

This is a book I am very happy to have, both for the choice of contents and the quality of exposition. Its subject is a very complete and up-to-date review of computational group theory. The main author is Derek Holt, but there are sections written by Bettina Eick and Eamonn O’Brien. Even though the book has 514 pages, the author(s) could not give a detailed account with proofs in this space.

They have chosen to give a thorough and detailed overview of the background and basic algorithms, and to give statements and refer to the literature for proofs or details concerning more advanced topics. There is a very complete bibliography. All together, the book contains of a huge amount of information.

Computational group theory ranges from proving the existence of some of the sporadic finite simple groups to finding the shortest sequence of moves to solve Rubik's cube, and it has applications now in most branches of mathematics.

There are four main ways of representing groups on a computer: The most convenient is as permutations of a finite set. Next are words in a finite set of generators, satisfying the relations given by a finite presentation. Then there is a special case of this situation, which is especially tractable, when the presentation is *polycyclic*. Finally one can represent group elements as matrices over a finite or infinite field. Each of these representations has advantages and drawbacks.

Being permutations of a finite set applies only to finite groups, and even for these there might not exist a small degree faithful permutation representation.

The drawback of finite presentations is that it can be hard to work with the group thus described; for example, it is in general undecidable whether a presentation represents the trivial group.

Polycyclic presentations have no such drawback, but they only apply to polycyclic groups, which are the solvable groups all of whose subgroups are finitely generated. On the other hand for such groups elements can be represented uniquely by integer vectors, and the multiplication algorithm is similar to addition of such vectors, followed by inverting a triangular system, thus is quite efficient.

Finally, matrix groups have the drawback that some computations are rather hard (like testing membership, which is undecidable for matrix groups over the integers), and the mathematics behind some of the theory concerning them can be quite difficult. This is an area where the book is a bit sketchy, referring the reader to the literature for more information.

Let us now review the contents of the book.

After the first chapter, which gives a nice historical introduction to the subject, the second chapter contains 60 pages of a brisk review of the needed background in group theory, including Frattini subgroups, free groups, Schreier transversals, definitions of representations and cohomology and ends by describing Conway polynomials for finite fields.

The 15-page third chapter considers algorithms that are independent of the particular representation of a group. After discussing the corresponding theoretical framework of black-box groups, topics such as the generation of random elements by the product replacement algorithm are discussed.

The fourth chapter, on permutation groups, has 70 pages, which is an indication of the breadth of the subject. Here the basic concept is that of a base and a strong generating set. Assume that G is a group of permutations of the set Ω and denote $C_G(\omega_1, \dots, \omega_n)$ the subgroup which fixes the points $\omega_1, \dots, \omega_n$. A *base* for G is a sequence $\omega_1, \dots, \omega_n$ such that if $G^{(i)} = C_G(\omega_1, \dots, \omega_i)$, then $G = G^{(0)} \not\supseteq G^{(1)} \not\supseteq \cdots \not\supseteq G^{(n)} = 1$. A set of *strong generators* (relative to this basis) is a set S of generators of G such that for i, the set $S \cap G^{(i)}$ generates $G^{(i)}$. The point is that having such data allows efficient answers to many questions, such as the order of G or membership in G. It also gives a compact representation of elements of G, since they are determined by the image of the base under their action. The Schreier-Sims algorithm computes a base and a strong generating set. It is desirable to be as efficient as possible in doing so, and to get a short base: there are variants using random elements, and a variant for solvable groups. Other topics in the chapter are block systems, backtrack searches (which are unfortunately still the best algorithms for problems such as computing normalizers), computation of Sylow subgroups, radicals, etc., and applications.

The 50-page Chapter 5 deals with coset enumeration. This is the basic stuff one does with a presentation, enumerating the cosets of a subgroup (in the case of the trivial subgroup enumerating thus the elements). The book covers the Todd-Coxeter algorithms and the Felsh and other variants and strategies. It then describes how to get presentations of a subgroup, and how to enumerate low-index subgroups.

The 18-page Chapter 6 deals with the "inverse" problem of finding a presentation of a given group.

The 50-page Chapter 7 deals with a lot of material. It contains the main algorithms of representation theory, in particular the Dixon-Schneider algorithm for computing character tables and the meataxe to split representations over finite fields. Other topics include factorization of polynomials over finite fields and computation of the first two cohomology groups. Finally there is an overview of the structural investigation of matrix groups.

The 50-page Chapter 8, written by Bettina Eick, discusses polycyclic groups. After presenting the basic algorithms, it moves on to more advanced topics, such as computing intersections and normalizers then automorphism groups, and ends with a discussion of the structure of finite solvable groups.

The 50-page Chapter 9 deals with quotients of finitely presented groups. One part deals with abelian quotients and surveys such topics such as the Hermite and Smith normal forms of matrices. The other part discusses algorithms such as finding polycyclic presentations of quotients which are p-groups.

The 18-page Chapter 10 describes a recent technique called the *solvable radical* method, whose goal is to try to perform computations more efficiently, such as finding automorphisms or subgroups of finite groups.

The 18-page Chapter 11 describes published libraries and databases, such as the atlas of finite groups and the atlas of Brauer characters, the lists of primitive permutation groups (up to degree 1000), of transitive permutation groups (up to degree 30), of perfect groups (up to 10^6 with 7 missing orders) and of all finite groups of order less than 2000 (with missing order 1024).

The last two chapters deal with automata and word rewriting. The 20-page Chapter 12 gives a presentation of the Knuth-Bendix algorithm, with applications to presentations and rewriting systems for monoids and groups, such as deciding nilpotency of polycyclic groups.

The 35-page Chapter 13 describes automata and automatic groups, and thus overlaps the contents of the excellent book "Word processing in groups" by Epstein, Cannon, Hold, Levy, Paterson and Thurston (Jones and Bartlett, 1992).

In addition to the above, the book contains many worked-out examples and quite a few exercises. I think every mathematician will want this book on his shelf.

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4[15A18, 15A29, 15A90]—*Inverse eigenvalue probems: Theory, algorithms, and applications*, by Moody T. Chu and Gene Golub, Numerical Mathematics and Scientific Computing, Oxford University Press, 2005, xvii + 387 pp., hardcover, US$124.50, UK£60, ISBN 0-19-856664-6

This book is concerned with the study of Inverse Eigenvalue Problems (IEPs), as predicted by reading the title. The different types of IEPs are approached both theoretically and numerically. Theoretical attention is paid to fundamental questions such as solvability, concerning sufficient and necessary conditions under which a solution exists and/or is unique. Numerical attention is paid to various methods for solving the mentioned IEPs, incorporating sensitivity analysis.

Chapters 1 and 2 are concerned with an introduction into IEPs and the exploration of several applications involving IEPs. In Chapters 3 through 9, all the different types of IEPs covered in this book are investigated in detail.

The first chapter introduces the concept of an IEP. It pays attention to the key issues, which will be addressed in the book. It briefly mentions the two constraints involved in the formulation of IEPs, namely the structural constraint, posed on the solution matrix, and the spectral constraint posed on the spectrum of the solution matrix. The four fundamental issues raised in this book are the following ones, for every specific IEP:

- the theory of solvability;
- the practice of computability;
- the analysis of sensitivity;
- the reality of feasibility.

The first three items speak for themselves, the fourth item is merely a matter of differentiation between whether an exact or approximate solution of the problem is needed.

The second chapter deals with different applications in which IEPs arise. One can say that in IEPs, one is concerned with the reconstruction of the physical parameters of the system, once its dynamical behavior is known. The following applications are treated briefly: the pole assignment problem, applied mechanics problems covering the reconstruction of a string with beads knowing its vibration frequencies, applied physics, numerical analysis involving the formulation of preconditioners as an IEP, and signal and data processing.

Starting in Chapter 3, different types of IEPs are discussed, paying attention to the issues raised in Chapter 1. This chapter discusses IEPs for which the structural constraint is regulated by a set of parameters. For example, the linear IEP: given a set of matrices $\{A_i\}_{i=0}^m$, find the parameters c_i such that the matrix $A(c)$,

$$A(c) = A_0 + c_1 A_1 + \cdots + c_m A_m,$$

satisfies a certain spectral constraint. Other variations of this problem are dealt with, such as putting the constraint of being symmetric on the matrix $A(c)$, changing the numbers c_i into matrices. Also the multiplicative and the additive IEP are covered. Results are presented concerning solvability and sensitivity of the mentioned problems. Different algorithms for solving these problems are proposed, among them the projected gradient method, which is used extensively throughout the remainder of the book.

The fourth chapter is concerned with structured inverse eigenvalue problems. These are inverse eigenvalue problems for which a structural constraint is posed on the solution. Focus is placed on the Jacobi inverse eigenvalue problem, in which one searches for a Jacobi matrix obeying certain spectral constraints. The Jacobi IEP finds applications in mechanical oscillation systems. Also the Toeplitz, the nonnegative, stochastic, and unitary Hessenberg IEPs are discussed. Each of these IEPs involves the search for a Toeplitz, nonnegative, stochastic, or unitary Hessenberg matrix having certain demands placed on its spectrum. The most general IEP dealt with is the one in which certain entries of the matrix are described. Finally attention is given to the inverse singular value problem.

In the third and fourth chapters the IEPs were related to satisfying certain constraints of the spectrum. In Chapter 5 the eigenvectors are taken into consideration. Hence the title "Partially described inverse eigenvalue problems", as these problems may consist of partial information involving eigenvalues and/or eigenvectors. Attention is paid to the partially described Toeplitz IEP (PDTIEP). But the partially described quadratic IEP is taken most into consideration with theoretical results as well as numerical algorithms.

The sixth chapter studies least squares IEPs. In previous chapters it was insisted that both spectral and structural constraints need to be satisfied for obtaining a satisfactory solution. Under these conditions, the IEP is, in general, not always solvable, hence one might loosen these conditions. One can apply the least squares idea to either of these constraints. One can search for a solution satisfying the structural constraint and minimizing in the least squares sense the discrepancy between the spectrum of the solution and the initial spectral constraint. Or one can search for a solution satisfying the spectral constraint and the best fit to the desired structure in least squares sense. This idea is used to revisit the linear IEP, for which a lift and projection algorithm is proposed as a numerical method. Moreover, it was proved that both formulations of the least squares IEP, applied to the linear IEP are equivalent in obtaining the final solution.

Chapter 7 is devoted to the construction of the projected gradient method, which can be applied to different types of IEPs. In fact the general covered problem in this chapter is the following one. Search for a matrix X, having specific eigenvalues such that the following function is minimized:

$$F(X) = 1/2\|X - P(X)\|^2,$$

where $P(X)$ denotes the projection of X onto a specific matrix or a subspace. A solution method based on the projected gradient method is proposed. Different IEPs, which were investigated in previous chapters are now revisited, to show that they fit perfectly into this framework, e.g., Toeplitz inverse eigenvalue problem, Jacobi inverse eigenvalue problem, etc. Extensions to other problems, which at first glance do not fit here, are presented, such as, e.g., the orthogonal similarity reductions to a specific form such as Jacobi and also the closest normal matrix problem.

Structured low rank approximation is the subject of Chapter 8. The problem is to approximate a given matrix with a low rank matrix having a certain structure. This can be seen as an IEP, as a number of eigenvalues of the original matrix in the approximation will be set to zero. Several types of this problem are discussed, including structured low rank Toeplitz and circulant approximation, low rank covariance approximation, and so on. This chapter contains numerical methods, theoretical results, and lots of examples.

The ninth and final chapter of the book puts the IEPs as discussed in this book in a more general framework, the IEP can be seen as a general canonical form identification problem. Some basic ideas and problems are presented, aiming to open new channels for tackling IEPs based on this framework.

Overall, one can conclude that the book contains a wealth of material involving IEPs. The book is carefully written. Even though some parts are rather technical and hard to read, the authors succeeded in presenting a rather uniform treatise of this topic. They had to make a selection of all material, and they had to divide it into different chapters and sections. They succeeded in this, not so easy, task. An extensive list of references related to each topic is included and often open problems are formulated, which are certainly meriting this book.

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