Mechanics by opening with the lines

"Gin a body meet a body
Flyin' through the air"

Maxwell played the tartan savage by claiming that

"Ilka problem has its method
By analytics high;
For me, I ken na ane o'them
But what the waur am I?"

Perhaps the greatest artifice in the book is the assumption (that we have so far taken at face value) that it is addressed to an undergraduate desirous of learning Fourier Analysis. That may have been the intent, but the result is an elegant and informative pastiche which should give pleasure to a very wide audience including even card-carrying harmonic analysts. Readers will enjoy the author's quiet sense of fun but should be aware that his didactic purpose is deadly serious. Recalling his reaction to the great blast of heat from infinity; it may well be that we shall never know the name of the rose, but I venture to suggest that the respectable uncle is called Dr. Körner.

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BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 21, Number 2, October 1989

0273-0979/89 $1.00 + $.25 per page

Representations of ∗-algebras, locally compact groups, and Banach ∗-algebraic bundles, by J. M. G. Fell and R. S. Doran. Academic Press, San Diego, 1988. vol. 1, *Basic representation theory of groups and algebras*, xviii + 746 pp., $99.00. ISBN 0-12-252721-6. vol. 2, *Banach ∗-algebraic bundles, induced representations, and the generalized Mackey analysis*, viii + 740 pp., $99.00. ISBN 0-12-252722-4

The theory of group representations has a long history in Mathematics and in Mathematical physics. It has its roots in two lines of mathematical thought. The first concerns the theory of Fourier series and the desire to extend these results, first to noncompact locally compact abelian groups, then to nonabelian compact groups, and finally to general locally compact groups. The second line centers around invariant theory, the Klein Erlanger program, and vector and tensor analysis. A historical account of the latter line may be found in **[13]**. In addition, a long motivational discussion concerning all these areas may be found in the first volume of the work being reviewed.

Let G be a locally compact group whose representation theory one wishes to study. Let λ be a Haar measure on G. The Banach space $L^1(G)$

of λ-summable complex valued functions on G is in fact a Banach algebra with multiplication defined by

$$f * g(s) = \int_G f(t)g(t^{-1}s)\,d\lambda(t), \qquad (f, g \in L^1(G)).$$

Furthermore, there is a natural involution on $L^1(G)$ making it a Banach $*$-algebra. The point is that if π is a unitary representation of G on the Hilbert space H_π, then

$$(\tilde{\pi}(f)\xi, \eta) = \int_G f(s)(\pi(s)\xi, \eta)\,d\lambda(s) \qquad (f \in L^1(G), \xi, \eta \in H_\pi)$$

defines a norm-decreasing $*$-representation $\tilde{\pi}$ of the Banach $*$-algebra $L^1(G)$ on H_π which is nondegenerate in the sense that the closed span of

$$\{\tilde{\pi}(f)\xi \in H_\pi : f \in L^1(G), \xi \in H_\pi\}$$

is all of H_π. Conversely, given a nondegenerate, norm-decreasing $*$-representation L of $L^1(G)$ on H_L, there is a unitary representation η of G on H_L so that $L = \tilde{\eta}$. Thus, the representation theories of G and $L^1(G)$ on Hilbert space are essentially the same. In fact, it is often the case that results concerning unitary representations of G can be obtained most directly by applying general Banach $*$-algebraic techniques to $L^1(G)$.

Naturally, one will eventually be drawn to study the representation theory of Banach $*$-algebras for their own sake, as well as the representation theory of more complicated objects. For example, suppose one has a homomorphism α of a locally compact group G into the $*$-automorphism group of a Banach $*$-algebra A. Such systems are called covariant systems, or dynamical systems. Here the appropriate "representations" of the system are pairs (π, U) consisting of a $*$-representation π of A and a unitary representation U of G on the same Hilbert space with the property that

$$U(s)\pi(a)U(s)^* = \pi(\alpha_s(a)).$$

Still more complicated structures are studied and are discussed in the volumes under review.

One of the principle techniques for constructing representations of groups, or representations of the systems mentioned above, is that of induced representations. Simply put, one hopes to understand the representation theory of the given system by first studying the representation theory of a subgroup (or subsystem) and then use these representations to build representations of the original system. This idea of induced representations originated with Frobenius in 1898 **[4]**. The theory was extended to second countable locally compact groups by Mackey in a series of papers **[6, 7, 8, 9]**, and was extended to nonseparable groups by Loomis **[5]** and Blattner **[1]**. Takesaki **[12]** showed how to extend these notions to covariant systems. In **[10]** Rieffel gave a general procedure for inducing representations between certain C^*-algebras which extends the procedure for groups.

In **[2 and 3]**, Fell introduced the notion of a Banach $*$-algebraic bundle and developed a theory of induced representations for Banach $*$-algebraic bundles. The concept of a Banach $*$-algebraic bundle is a generalization of the observation that $L^1(G)$ may be viewed as a section algebra of $\mathscr{B} = G \times \mathbb{C}$ viewed as a (trivial) bundle over G. In general, a Banach $*$-algebraic bundle is a bundle $\mathscr{B}$ over G whose fibres B_s are Banach spaces. In addition, there is a multiplication and involution defined on $\mathscr{B}$ which satisfies the following covariance condition:

$$B_s B_t \subseteq B_{st} \quad \text{and} \quad (B_s)^* = B_{s^{-1}}$$

for all $s, t \in G$. Using the nonseparable techniques of Loomis and Blattner —rather than the measure theoretic techniques of Mackey—and the Rieffel machinery suitably modified for Banach $*$-algebras, Fell develops a notion of representation for Banach $*$-algebraic bundles which includes all the theories mentioned above. Thus in **[3]**, one finds a general framework in which one can study virtually all the notions of induced representations found in the literature as a single theory. The main focus of **[3]** is to develop the machinery of the Mackey normal subgroup analysis in the context of Banach $*$-algebraic bundles. In fact, the review of **[3]** in **[11]** is particularly relevant to the current review.

The purpose of the volumes under review is to "present the *whole* theory of induced representations, the Imprimitivity Theorem, and the Mackey normal subgroup analysis in the generalized context of Banach $*$-algebraic bundles." Consequently, all of the first volume, and part of the second, is devoted to the functional analysis required to develop the theory of infinite dimensional representations of locally compact groups, of Banach algebras, and of Banach $*$-algebraic bundles. The pace and style is appropriate for a (graduate) text, and could be used as an entry into the field as well as a basic reference.

The remainder of the second volume is devoted to bringing the reader completely up to date with the state of knowledge of the Mackey normal subgroup analysis for Banach $*$-algebraic bundles. While much of this material has appeared before—in **[2]**, and especially in **[3]**—there is much that is new as well. In particular, many of the results in **[3]** which were proved for homogeneous bundles have been extended to cover saturated bundles. However the pace of the first volume is maintained throughout— the material is presented in the style of a textbook rather than a journal article.

Each volume begins with a long introduction which provides copious motivation and historical information. In addition, each chapter ends with a "Notes and Remarks" section which provides the reader with detailed references. Finally, each chapter also provides a long list of exercises ranging from checking routine details omitted in the text to challenging extensions of the theory.

In summary, I think the authors have achieved their goal of providing prospective researchers with a *complete* introduction to the theory of induced representations. The emphasis here is on the word complete. In (almost) all cases, results are presented in as full generality as possible. Separability assumptions are very rare, and hypotheses—such as completeness, the existence of an involution, or the assumption that one is working with a C^*-algebra—are added only as needed. Usually the reader is provided with counterexamples to demonstrate why these hypotheses are required. To offset the burden of maintaining complete generality the authors have been very careful to keep the group case highlighted throughout. Major results are often restated in the group case. There is even an entire chapter devoted to the theory of representations of compact group. Because of this blend of styles between that of a research encyclopedia and that of a textbook, these volumes will make an excellent reference for the student wishing to begin studying representation theory of Banach algebras, as well as for the expert who wishes to check the details of the Mackey normal subgroup analysis in the nonseparable case.

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BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 21, Number 2, October 1989

0273-0979/89 $1.00 + $.25 per page

Combinatorial search, by Martin Aigner. Wiley-Teubner Series in Computer Science, 1988, 368 pp., $44.95. ISBN-0-471-92142-4 (Wiley) ISBN-3-519-02109-9 (Teubner)

Popular combinatorial search problems involve such matters as "twenty questions," weighing to find a counterfeit coin, or locating a word in a dictionary. All these problems can be described using a simple model. Given a search domain consisting of a finite number of points with one