Boomerang Distinguisher for the SIMD-512 Compression Function

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Outline

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SHA-3 Competition

- Organized by NIST [Nat07]
- Successor for SHA-1 and SHA-2
- 64 submissions
- 51 round 1 candidates
- 14 round 2 candidates
- 5 finalists

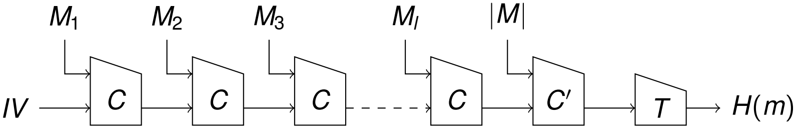
Outline

SIMD Is a Message Digest[LBF08]

- Designed by Gaëtan Leurent, Charles Bouillaguet and Pierre-Alain Fouque
- Round 2 candidate
- Message block
  - SIMD-256: 512 bits
  - SIMD-512: 1024 bits
- Inner state (wide-pipe)
  - SIMD-256: 16 32-bit words
  - SIMD-512: 32 32-bit words

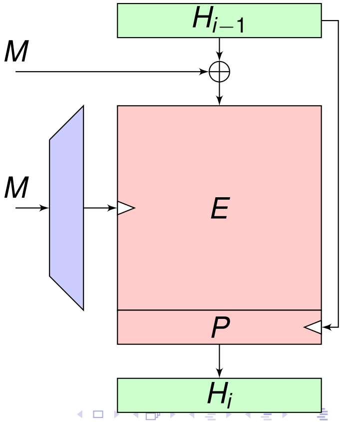
The SIMD Hash Function

- Similar to Chop-MD
- Internal state is twice as large as the output
- Output transformation: truncation T



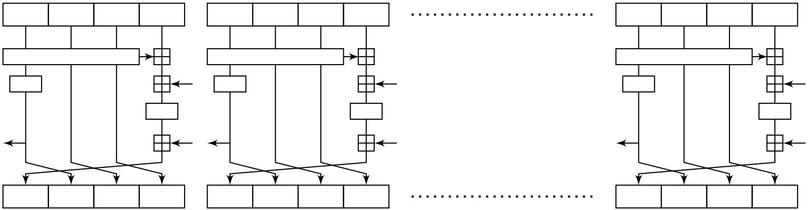
The SIMD Compression Function (1/2)

- Modified Davis-Meyer construction
- Expanded message size: $8 \cdot$ blocksize
- Strong security in the message expansion

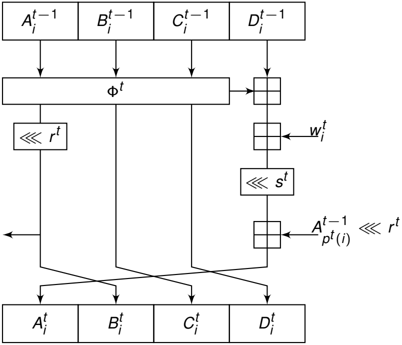


The SIMD Compression Function (2/2)

- Based on a Feistel structure; similar to MD5
- SIMD-256: 4 times the step function in parallel
- SIMD-512: 8 times the step function in parallel
- 32 steps plus 4 steps in the feed-forward



Update Function at Step t



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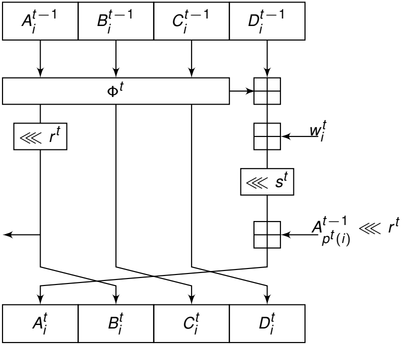

$$A_i^t = (D_i^{t-1} \boxplus w_i^t \boxplus \Phi(A_i^{t-1}, B_i^{t-1}, C_i^{t-1})) \lll s^t \boxplus (A_{p^t(i)}^{t-1} \lll r^t)$$

$$B_i^t = A_i^{t-1} \lll r^t$$

$$C_i^t = B_i^{t-1}$$

$$D_i^t = C_i^{t-1}$$

Φ is either IF or MAJ

Results on SIMD-512

Distinguisher

- Mendel and Nad [MN09]
  - Full compression function (complexity: 2^{427}) $\rightarrow$ tweaked!
- Nikolić et al. [INS10]
  - 12 out of 32 steps (complexity: 2^{236})
- Yu and Wang [YW11]
  - Full compression function (complexity: 2^{398})

Free-start near-collision

- Yu and Wang [YW11]
  - 24 out of 32 steps (complexity: 2^{208})

Our Contribution

Application of Higher-Order Differentials to SIMD-512

- Non-random properties for the permutation of SIMD-512
- Extend technique to overcome the feed-forward of SIMD-512
- Non-random properties for the compression function of SIMD-512

Outline

Higher-Order Differentials

- Introduced by Lai in [Lai94]
- First applied to block ciphers by Knudsen [Knu94]
- Recently applied to SHA-2 [BLMN11] and several SHA-3 candidates
  - BLAKE [BNR11], Hamsi [BC10], Keccak [BC10], Luffa [WHYK10], ...

Higher-Order Differentials: Basic Definitions

Definition

Let $(S, +)$ and $(T, +)$ be abelian groups. For a function $f\colon S \to T$, the derivative at a point $a_1 \in S$ is defined as

$$\Delta_{(a_1)} f(y) = f(y + a_1) - f(y) .$$

The i-th derivative of f at $(a_1, a_2, \ldots, a_i)$ is then recursively defined as

$$\Delta_{(a_1,\ldots,a_i)} f(y) = \Delta_{(a_i)}(\Delta_{(a_1,\ldots,a_{i-1})} f(y)) .$$

Higher-Order Differentials: Basic Definitions

Definition

A differential of order i for a function $f\colon S \rightarrow T$ is an $(i+1)$-tuple $(a_1, a_2, \ldots, a_i; b)$ such that

$$\Delta_{(a_1,\ldots,a_i)} f(y) = b\,.$$

Higher-Order Differential Collision

When applying differential cryptanalysis to a hash function, a collision for the hash function corresponds to a pair of inputs with output difference zero.

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Note that the common definition of a collision for hash functions corresponds to a higher-order differential collision of order $i = 1$.

Complexity

- What is the *query complexity* of a differential collision of order i?
- From the definition before we see that we can freely choose $i+1$ of the input parameters which then fix the remaining ones

$\Rightarrow$ Query complexity: $\approx 2^{n/(i+1)}$

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Note that the complexity might be much higher in practice than this bound for the query complexity.

Higher-Order Differential Collision for Block Cipher based Compression Functions

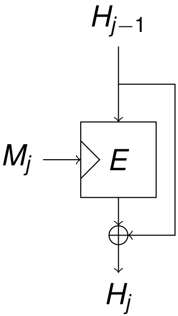
Observation

For any block-cipher-based compression function with which can be written in the form

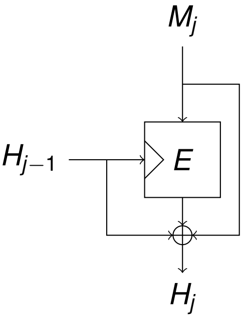
$$f(y) = E(y) + L(y),$$

where L is a linear function with respect to $+$, an i-th-order differential collision for the block cipher transfers to an i-th-order collision for the compression function for $i \geq 2$.

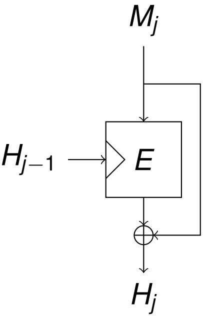
Compression Function Constructions



Davies-Meyer



Miyaguchi-Preneel



Matyas-Meyer-Oseas

Second-order Differential Collision

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$$f(y) - f(y + a_2) + f(y + a_1 + a_2) - f(y + a_1) = 0$$

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$$f(y) - f(y + a_2) + f(y + a_1 + a_2) - f(y + a_1) = 0$$

- Query complexity: $2^{n/3}$
- We are not aware of any algorithm faster than $2^{n/2}$

Basic Attack Strategy

- Split underlying block cipher E into two subparts, $E = E_1 \circ E_0$.
- Assume we are given two differentials for the two subparts:

$$E_0^{-1}(y + \beta) - E_0^{-1}(y) = \alpha \tag{1}$$

and

$$E_1(y + \gamma) - E_1(y) = \delta \tag{2}$$

where the differential in E_0^{-1} holds with probability p_0 and in E_1 holds with probability p_1.

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X

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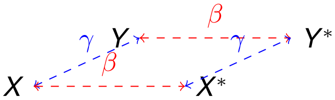

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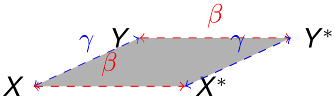

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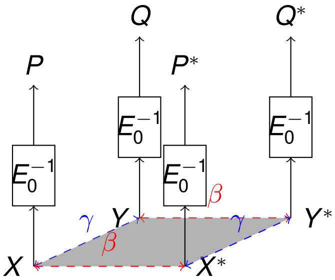

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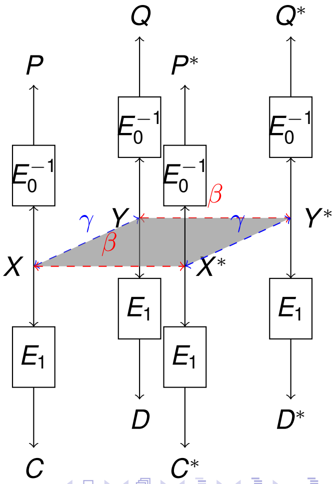

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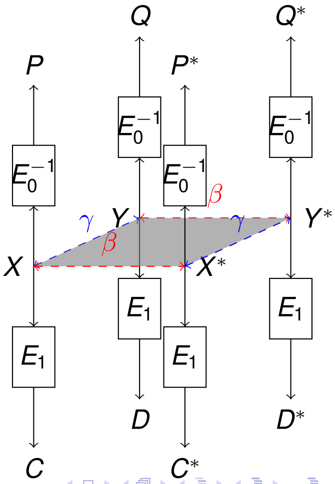
- Check if $P^* - P = Q^* - Q$ and $D - C = D^* - C^*$ is fulfilled.



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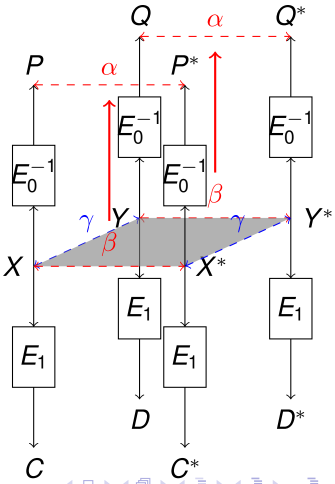

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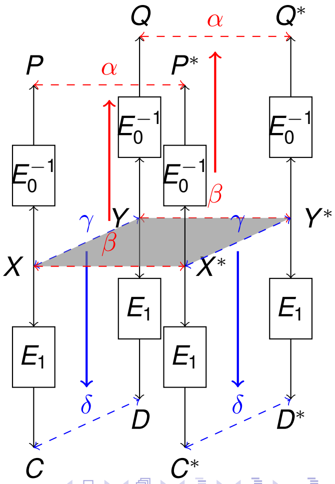

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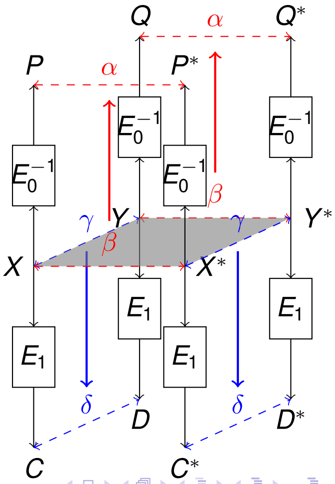
- Check if $P^* - P = Q^* - Q$ and $D - C = D^* - C^*$ is fulfilled.
- Attack succeeds with probability $p_0^2 \cdot p_1^2$.



Related Work

Block Cipher Cryptanalysis

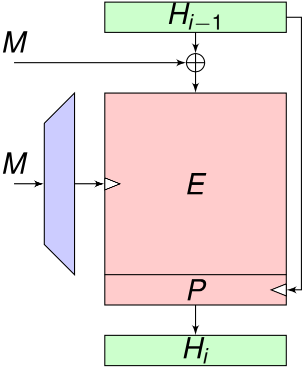
- It stands between the *boomerang attack* and the *inside-out* attack both introduced by Wagner [Wag99]

Hash Functions Cryptanalysis

- A previous application of the boomerang attack to hash functions is due to Joux and Peyrin [JP07]
- The attack bears resemblance with the *rebound attack* introduced by Mendel et al. [MRST09]
- A framework similar to this was independently proposed by Biryukov et al. [BNR11]

Outline

Application to SIMD-512 Permutation



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- Finding the differential characteristics for backward and forward direction is the most difficult part of the attack
- We have two requirements for the differential characteristics:
  - independent
  - high probability

Finding Differential Characteristics

- Linearize the hash function

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  - Modular additions $\rightarrow$ XOR operation
  - Boolean functions f_{IF}, f_{MAJ} $\rightarrow$ 0-function

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  - Modular additions $\rightarrow$ XOR operation
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- Use a probabilistic algorithm from coding theory
- Results
  - Backward: steps 1-18 (probability $2^{-72.04}$)
  - Forward: steps 19-32 (probability $2^{-51.4}$)

Complexity of the Attack

Probability of the Characteristics

- Backward: $2^{-72.04}$
- Forward: $2^{-51.4}$

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- Ignoring conditions at the end [WYY05]

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$\Rightarrow$ improved complexity is $2^{226.52}$

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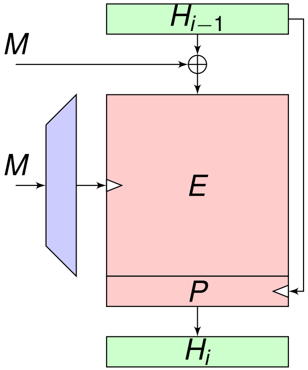
Extending the Attack to the Compression Function



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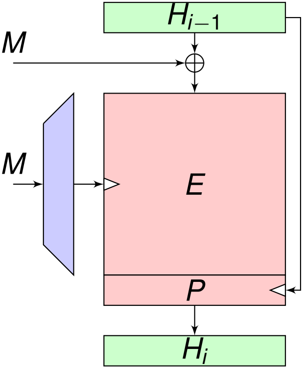

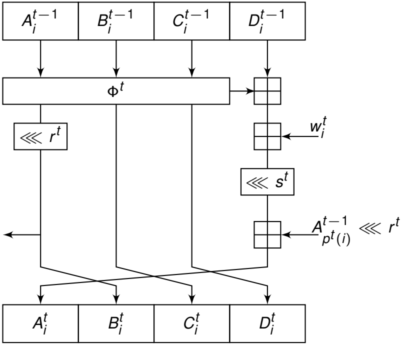

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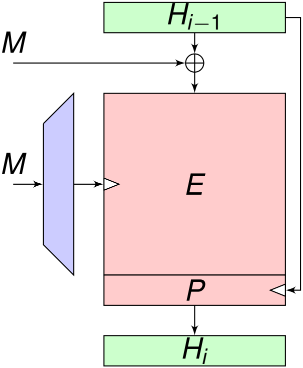

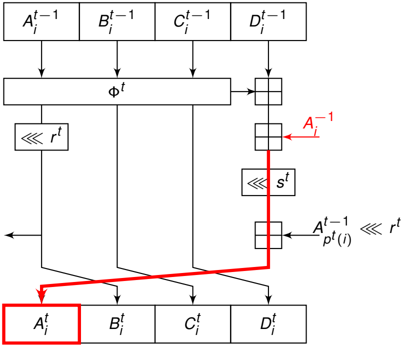

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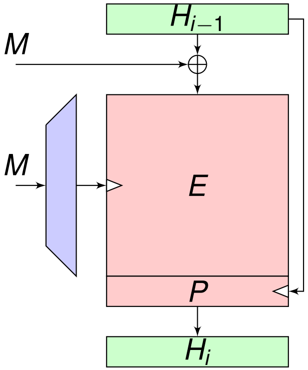

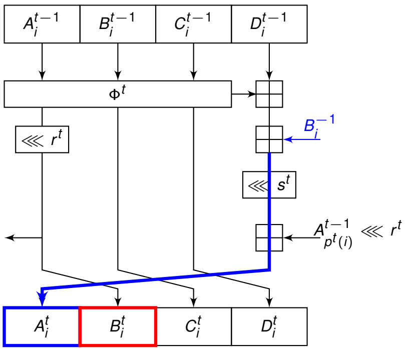

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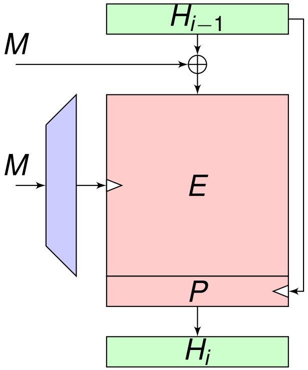

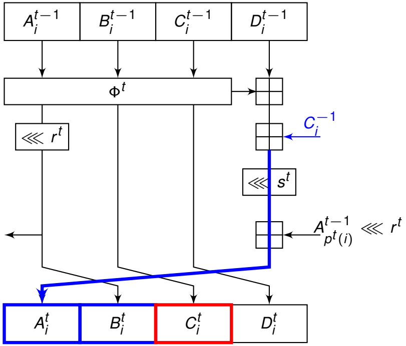

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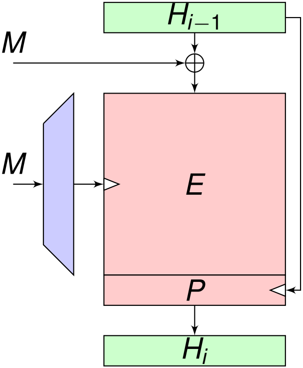
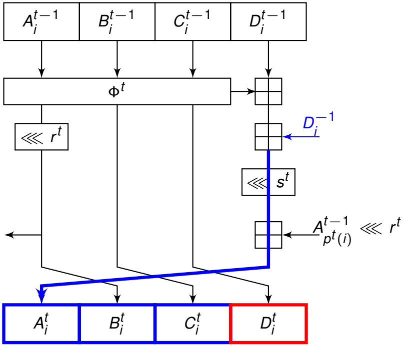
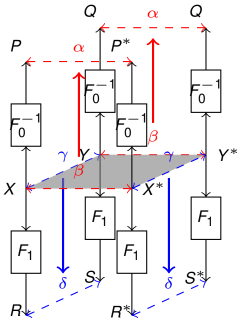




Extended Attack Strategy



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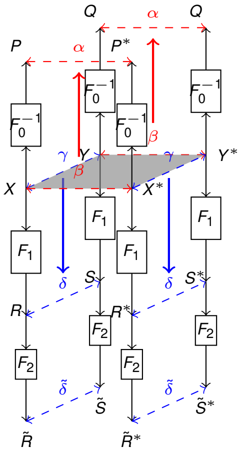

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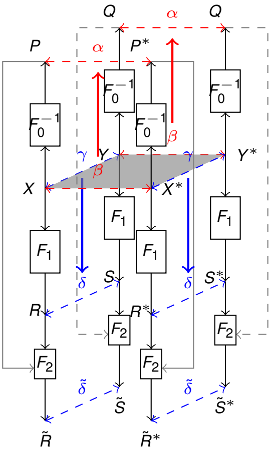

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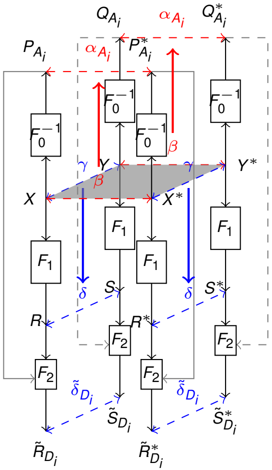

Complexity of the Attack

- Using the same differential characteristic (fix β, γ)
- Backward: only difference in ΔA_6^{-1}
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- Final complexity: $\approx 2^{200.6}$
- Generic complexity: 2^{256}

Outline

Conclusions

- Application of the boomerang attack on SIMD-512
- Using techniques from coding theory to search for two differential characteristics
- Construct a second-order differential collision and define a distinguishing property
- Distinguisher for the full permutation of SIMD-512
- Extend the attack to the full compression function of SIMD-512
  - Best distinguishing attack for SIMD-512 ($2^{200.6}$ vs. 2^{398})

Thank you for your Attention!

Questions?

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