Bootstrapping Student Understanding of What is Going On in Econometrics

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Abstract

Econometrics is an intellectual game played by rules based on the sampling distribution concept. Most students in our econometrics classes are uncomfortable because they don't know these rules, and so do not understand what is "going on" in econometrics. This article offers some explanations for this phenomenon, and suggestions for how this problem can be addressed. Instructors are encouraged to use "explain how to bootstrap" exercises to promote student understanding of the rules of the game.

Key words: bootstrapping; sampling distribution; Monte Carlo

JEL codes: A20; C10; C15

The majority of students in undergraduate econometrics courses and required graduate econometrics courses feel uncomfortable in those courses. This is not a big secret. Most students readily admit this, even those with good grades in prerequisite introductory statistics courses. Although many feel comfortable doing the math, in their heart of hearts they know that they are surviving in this subject because they are good at math, not because they understand what is going on.

And just what is "going on" in econometrics? To most students econometrics is a bunch of mechanical techniques - running a regression or testing an hypothesis, for example, and they know they can pass the course by remembering how these techniques work. But these mechanical techniques are manifestations of an underlying logic that is the foundation of classical statistics, captured in the sampling distribution concept.[1] What is "going on" in econometrics is that an intellectual game is being played, with the sampling distribution concept serving as the basic rules of the game. If you don't understand the rules you will not do well in this game, and will no doubt dislike it.

The purpose of this article is to defend the view expressed above and prescribe some help from an unexpected source - the bootstrap. I begin by explaining how the sampling distribution concept sets the rules for the game of econometrics, followed by a discussion of why students have not learned these rules, and what should be done about this problem. After expositing the bootstrap, some examples illustrate how explain-how-to-bootstrap exercises can promote student understanding of the sampling distribution concept. Some comments on the opportunity cost of adopting this approach are offered before concluding.

WHAT ARE THE RULES?

How does the sampling distribution concept set the ground rules for the game of econometrics? Most of what econometricians do is either finding "good" estimates for unknown numbers, or testing hypotheses. The sampling distribution concept creates the following underlying logic for these two activities.

1. Using a formula β^* to produce an estimate of β can be conceptualized as the econometrician shutting his or her eyes and obtaining an estimate of β by reaching blindly into the sampling distribution of β^* to obtain a single number. Here β could be a parameter or a value to be forecast.
2. Because of 1 above, choosing between β^* and and a competing formula β^{**} comes down to the following: Would you prefer to produce your estimate of β by reaching

blindly into the sampling distribution of β* or by reaching blindly into the sampling distribution of β**?

3. Because of 2 above, desirable properties of an estimator β* are defined in terms of its sampling distribution. For example, β* is unbiased if the mean of its sampling distribution equals the number β being estimated. This explains why econometricians spend so much algebraic energy figuring out sampling distribution properties, such as mean and variance.
4. The properties of the sampling distribution of an estimator β* depend on the process generating the data, so an estimator can be a good one in one context but a bad one in another. When we move from one textbook topic to another we are moving from one data generating process to another, necessitating a re-examination of the sampling distribution properties of familiar estimators, and development of new estimators designed to have "better" sampling distribution properties. So as we move from textbook topic to textbook topic we play over and over again the same game with sampling distributions.
5. Test statistics have sampling distributions. When we test hypotheses we carefully choose a test statistic which, if the null hypothesis is true, has a sampling distribution described by numbers we know. Many such test statistic sampling distributions are described by tables in the back of statistics textbooks. The explanation of how and why we accept or reject an hypothesis is built on the logic of the sampling distribution. If you understand sampling distributions, the rules of the game, you will understand the logic of hypothesis testing.

My experience has been that the vast majority of students do not understand the rules of the game - they do not understand the basic logic of statistics as captured in the sampling distribution concept. Try asking them what is a sampling distribution and you will find that they think it is a histogram of the sample data. Describe to them the five points above outlining the role of sampling distributions and they will admit that they didn't fully understand how the sampling distribution concept captures what is "going on" in econometrics. My experience is not unique. Hubbard (1997, 11), Zerbolio (1989, 207), and Duggal (1987, 26) are cogent examples confirming my experience.

WHY HAVE THE RULES NOT BEEN LEARNED?

Why has this happened? It is not because our textbooks ignore sampling distributions - they all have plenty of good material on this concept and give it appropriate emphasis. Nor is it because instructors ignore this dimension of textbooks - all instructors swear that they teach this concept thoroughly. I have identified three basic explanations for why this has happened.

1. Inherent Difficulty. The sampling distribution concept is difficult - students can visualize a distribution of sample observations, but a sampling distribution is at a higher level of abstraction, where sample observations yield a single value of a statistic, not an entire distribution. It is important to note that this difficulty is not a mathematical one. In the words of Simon and Bruce (1991, 29), who make the same point in the context of the resampling approach to learning basic statistics, it "requires only hard, clear thinking. You cannot beg off by saying 'I have no brain for math!'"
2. Limited Textbook Exposition. Textbooks typically introduce the sampling distribution concept in the context of the sample mean statistic but do not follow it up adequately when discussing regression, the central focus of econometrics. Furthermore, textbook presentations do not provide the big-picture perspective that the five points outlined above offer; without this perspective it is difficult to realize how important the sampling distribution concept can be in understanding what is "going on."
3. No Assessment. Most importantly, as Hubbard (1997, 1) quotes Resnick "We get what we assess, and if we don't assess it, we won't get it." Students are utility maximizers. Typically their exams require an ability to interpret regressions, calculate t statistics, and perform hypothesis tests, not an ability to explain the concept of a sampling distribution. They can not imagine, and instructors and textbooks seldom provide, examples of exam questions that in any meaningful way probe student understanding of this concept.

This failure to understand the rules of the game has substantive consequences that cumulate over time. According to "constructivism," a popular theory of learning, students bring to the classroom their own ideas, and rather than passively adding to these ideas as material is presented in class, they actively restructure the new information to fit it into their own cognitive frameworks. In this way they are "constructing" their own knowledge, rather than copying knowledge delivered to them through some teaching mechanism. Garfield (1995, 30) provides this explanation:

> *Regardless of how clearly a teacher or book tells them something, students will understand the material only after they have constructed their own meaning for what they are learning. Moreover, ignoring, dismissing, or merely 'disproving' the students' current ideas will leave them intact - and they will outlast the thin veneer of course content.*

> *Students do not come to class as 'blank slates' or 'empty vessels' waiting to be filled, but instead approach learning activities with significant prior knowledge. In learning something new, they interpret the new information in terms of the knowledge they already have, constructing their own meanings by connecting the new information to what they already believe. Students tend to accept new ideas only when their old ideas do not work, or are shown to be inefficient for purposes they think are important.*

If this is indeed what is happening in our classrooms, it should be no surprise that students are uncomfortable - each new topic is the beginning of something completely new, rather than a new application of a familiar paradigm. As time goes on econometrics becomes more and more a collection of mechanical procedures with an accompanying set of formulas, dealt with by relying on mathematical expertise. By fitting each new topic into their mathematical view of statistics, students avoid ever learning properly the sampling distribution concept and so continue to be uncomfortable in their econometrics course. Students must stop viewing econometrics through a mathematical lens and start viewing it through the sampling distribution lens. My own experience discovering this lens was a revelation, akin to the experience I had when I put on my first pair of eyeglasses - suddenly everything was sharp and clear.

WHAT SHOULD BE DONE?

How can students be made to look at econometrics through the sampling distribution lens? Three steps must be taken.

1. <u>Educate Instructors</u>. Good instructors realize that students have mostly forgotten their introductory statistics material, or never learned it properly in the first place, and proceed on the basis that this introductory material needs to be reviewed. Unfortunately, these instructors usually do not realize that what needs to be done in this review is to hammer home the sampling distribution concept, and so their review does not serve to change students' entrenched paradigm - the mathematical lens. Whenever I have commented to instructors that their students probably do not understand the sampling distribution concept they have been surprised, even offended, but do admit that it is something they have never worried about. This impression is reinforced by reading the very limited literature on teaching econometrics. Eric Sowey (1983) and his commentators, for example, ignore this issue. This is indeed a major part of the problem we face - if instructors are not aware of this deficiency in student understanding, they will not try to remedy it.

2) Improve Expositions. Econometrics textbooks are not very helpful. They are in too big a hurry to produce the theorems, proofs, and formulas that define theoretical econometrics. Some review introductory statistics but do so without much emphasis on sampling distributions. I am amazed that econometrics textbooks pay so little attention to the sampling distribution concept, but it is consistent with my claim above that instructors believe that students already understand this concept. Instructors and textbook authors need to make two improvements. First, they should provide an explicit exposition of the sampling distribution concept in the context of regression, rather than confining its exposition to the sample mean statistic. My own efforts in this regard appear in Kennedy (1998a, 1998b, appendix A). And second, they should provide an overview of the role of sampling distributions in econometrics, such as is presented above in the list of five ways in which sampling distributions underlie the rules of the game for econometrics.

3. Assess. Although lecturers and textbook authors like to think otherwise, brilliant expositions seldom cause students fully to understand - such understanding comes through working out problems based on the concept to be learned. But current expositions of the sampling distribution concept, however clear, are seldom accompanied by challenging problems that would force this learning. The main rationale for providing such problems, however, is to motivate students - if they believe there will be final exam questions requiring an understanding of this concept, they will learn the concept. In Garfield's words quoted earlier, "Students tend to accept new ideas only when their old ideas do not work, or are shown to be inefficient for purposes they think are important." Elsewhere (Kennedy 1998a, 1998c) I have suggested that such problems should take the form "explain how to do a Monte Carlo study," and have provided examples in Kennedy (1998a, 1998b, appendix D). The remainder of this article argues that problems that take the form "explain how to bootstrap" can also serve as useful learning problems in this context, and provides examples of such problems.

WHAT IS BOOTSTRAPPING?

Only in simple cases can econometric theory deduce the properties of a statistic's sampling distribution. In most cases theory is forced to use asymptotic algebra, producing results that apply only when the sample size is very large. Although in many cases these asymptotic results provide remarkably good approximations to sampling distributions associated with typical sample sizes, one can never be sure. Because of this econometricians have turned to the computer to discover the sampling distribution properties of statistics in small samples, using a method called Monte Carlo. In the Monte Carlo method the computer is used to mimic the data-generating process, creating several thousand typical samples, calculating for each sample the value of the statistic in question, and then using these thousands of values to characterize the statistic's sampling distribution by estimating its mean, variance, and mean square error, or by drawing a histogram.

When mimicking the data-generating process the computer needs to draw errors from an error distribution, which in reality is unknown. For convenience, in most Monte Carlo studies errors are drawn from a normal distribution. But in many problems the reason we believe that the asymptotic results are not reliable in small samples is because we do not believe that the errors are distributed normally. In such cases traditional Monte Carlo studies are of questionable value. To deal with this problem, we must find a way of drawing errors more representative of the unknown actual error distribution. Bootstrapping is a method for doing this.

Bootstrapping is a variant of Monte Carlo in which the error distribution from which the computer draws errors is an artificial distribution with equal probability on all of the residuals from the initial estimation of the model under investigation. This is typically described as randomly drawing with replacement from the set of ordinary least squares residuals. In effect the actual, unknown distribution of errors is being approximated by this artificial distribution. This bootstrapping procedure has been shown to perform remarkably well - it produces estimates of sampling distributions of statistics that are surprisingly accurate - and so has become increasingly popular in econometric analysis.

In particular, bootstrapping is used for two main purposes.

1. <u>Bootstrapping Test Statistics</u>. Suppose you are using an F test to test some hypothesis, but because you fear that your problem is characterized by nonnormal errors you are worried that for your modest sample size the sampling distribution of your F statistic under the null is not accurately characterized by the numbers in the F table found in the back of statistics texts. In particular you might fear that the 5 percent critical value found in this table may for your problem be more like a 25 percent critical value! By bootstrapping this F statistic under the null hypothesis you can create a description of the F statistic's sampling distribution tailored to your problem. By using this distribution instead of the tabled F distribution you can choose your critical value to ensure that the resulting type I error is indeed 5 percent.
2. <u>Bootstrapping Confidence Intervals</u>. Suppose you are forecasting a variable and wish to produce a confidence interval for your forecast. The usual way of calculating such a forecast interval is to find the standard error of your forecast, multiply it by a suitable

critical value taken from the t distribution found in tables at the back of a statistics text, and then add and subtract the result from your forecast. This procedure could be very inaccurate, however, for several reasons. You may have used a search procedure to develop your econometric specification, the variable being forecast may be a nonlinear function of parameters estimated via this specification, and the errors may not be distributed normally. It is not known how to find the standard error of such a forecast, and even if it was, the forecast would surely not have a t distribution, nor be symmetric. By bootstrapping this entire estimation procedure the actual sampling distribution of the forecast could be estimated, allowing an appropriate confidence interval to be produced.

Because bootstrapping is a means of estimating sampling distributions, "explain how to bootstrap" exercises are suitable vehicles for motivating students to ensure that they understand the sampling distribution concept. An added bonus is that it is not just a theoretical curiosity - bootstrapping has come to be an accepted means of improving applied econometric analysis and so is something with which students should in any event become familiar. Bootstrapping is not a panacea in this regard, however. It requires, for example, special techniques whenever the errors are thought not to be exchangeable[2] (equally likely to be attached to all observations), that the statistic bootstrapped should be pivotal (so that its sampling distribution does not depend on true parameter values), and that least squares residuals be enlarged by multiplying them by the square root of N/(N-K) to account for the fact that although least squares residuals are unbiased estimates of the errors, they underestimate their absolute values (N is sample size, K is number of parameters estimated). Jeong and Maddala (1993) and Li and Maddala (1996) are surveys of bootstrapping techniques.

EXPLAIN-HOW-TO-BOOTSTRAP EXERCISES

"Explain how to Monte Carlo" exercises, as described in Kennedy (1998a, 1998b), should be supplemented with "explain how to bootstrap" exercises. As with with the former, instructors should initiate students to bootstrapping exercises with problems that provide the mechanical steps and ask questions about the results, as illustrated in the following two examples.

Example 1 Suppose you have 25 observations on y and x and believe that $y = \alpha + \beta x + \varepsilon$ where the classical linear regression (CLR) model holds (but not the classical normal linear regression model, which implies that the errors are distributed normally). You run ordinary least squares and obtain estimates 0.50 and 2.25 of α and β, with corresponding estimated variances 0.04 and 0.01, saving the residuals in a vector res. You have programmed a computer to do the following.

i) Draw 25 e values randomly with replacement from the elements of res.

ii) Compute 25 y values as 0.5 + 2.0*x + 1.043*e.

iii) Regress y on x, obtaining an estimate $\hat{\beta}$ of β and its standard error se.

iv) Compute t = ($\hat{\beta}$ - 2)/se and save it.

v) Repeat from i) to obtain 2000 t values.

vi) Order these t values from smallest to largest.

vii) Print the 50th t value t50 and the 1950th t value t1950.

a) Explain what this program is designed to do.

b) Suppose t50 = -2.634 and t1950 = 2.717. What conclusion would you draw?

(Answer: this program is producing 2000 t values[3] that can be used to estimate the sampling distribution, under the null β = 2, of the t statistic for testing β = 2. The t value obtained from the actual data is 2.50; because it lies within the two-tailed 5 percent critical values of -2.634 and 2.717, we fail to reject the null at this significance level.) Appendix A contains computer code for this exercise.

Example 2 Suppose you have 28 observations on y, x, and w, and believe that y = α + βx + δw + ε where the CLR model holds. You run ordinary least squares and obtain estimates 1.0, 1.5 and 3.0 of α, β and δ, saving the residuals in a vector res. You have programmed a computer to do the following.

i) Draw 28 e values randomly with replacement from the elements of res.

ii) Compute 28 y values as 1.0 + 1.5*x + 3.0*w + 1.058*e.

iii) Regress y on x and w, obtaining an estimate $\hat{\beta}$ of β and $\hat{\delta}$ of δ.

iv) Compute r = $\hat{\delta} / \hat{\beta}$ and save it.

v) Repeat from i) to obtain 4000 r values.

vi) Compute and print av, the average of the r values, and var, their variance.

vii) Order these r values from smallest to largest.

Explain how to use these results to produce

a) an estimate of the bias of $\hat{\delta} / \hat{\beta}$ as an estimate of δ/β;

b) an estimate of the standard error of $\hat{\delta} / \hat{\beta}$;

c) a test of the null that the bias is zero; and

d) a 90 percent confidence interval for δ/β.

(Answer: The bias is estimated by av - 2; the square root of var estimates the standard error; a t statistic for testing the null is the estimated bias divided by the square root of its estimated variance, var/4000; and the confidence interval is given by the interval between the 200th and the 3800th r values, adjusted for any bias.[4]) Computer code for this exercise can be found in Appendix A.

After being initiated with this type of exercise, students should be ready to explain on their own the steps for a bootstrapping problem such as the following.

Example 3. Suppose that you believe that the CLR model applies to $y = \alpha + \beta x + \varepsilon$ except that you fear that the error variance is larger for the last half of the data than for the first half. You also fear that the error is not distributed normally, so that the Goldfeld-Quandt statistic will not have an F distribution for your sample size. Given some data, explain in detail how to bootstrap the Goldfeld-Quandt statistic to test the null that the error variances are the same.

This example is generic. Student understanding of all test statistics in the course can be checked by means of a question of this nature. For example, students could be told they have panel data, and be asked to explain how to bootstrap the Hausman test used to determine whether to employ a fixed or a random effects model. As another example, students could be told they have some data along with two competing nonnested specifications, and be asked to explain how to bootstrap the J test. As a third example, students could be asked to explain how to bootstrap the Durbin Watson statistic to remove its indeterminant region and make it applicable to the case of non-normal errors.

Additional Bootstrapping Exercises

Bootstrapping has proved to be of value in a wide variety of econometric problems; Veall (1998) is a recent survey. A variety of other bootstrapping exercises can be based on these applications, such as those described below.

1. When estimating sets of equations via seemingly unrelated estimation (SURE) or three-stage least squares, the (asymptotic) formulas used for estimating the standard errors of the coefficient estimates assume that the contemporaneous variance-covariance matrix of the errors is known. Bootstrapping incorporates the extra uncertainty caused by estimation of the contemporaneous variance-covariance matrix, and so produces more accurate estimates of confidence intervals. See Williams (1986). It also improves

hypothesis testing of cross-equation restrictions such as homogeneity and symmetry restrictions in demand systems. See Taylor, Shonkwiler and Theil (1986).

2. Elasticities are frequently estimated using awkward nonlinear transformations of estimated coefficients. In some instances asymptotic variance formulas have been shown to be quite accurate, but in other instances they are not. To be on the safe side, it is recommended that related test statistics or confidence intervals be bootstrapped. See Green, Rocke and Hahn (1987) and Krinsky and Robb (1991).
3. For some estimating formulas, such as the least absolute deviations estimator, formulas for small-sample standard errors are not available. Bootstrapping can overcome this problem. See Dielman and Pfaffenberger (1986).
4. When forecasting, econometricians frequently assume that explanatory variables for the period in question are known with certainty when in fact their values may be uncertain. It is very difficult to incorporate analytically this uncertainty into forecast confidence intervals, but this can easily be done via bootstrapping. Interest often focusses on forecast error confidence intervals, rather than forecast confidence intervals. This presents an example in which the nonnormality of the error can play a substantive role even in large samples. When estimating parameters (and functions of parameters, such as forecasts), the effects of nonnormal errors are mitigated by the same phenomenon that gives rise to the central limit theorem - in effect they become benign through averaging. But forecast errors incorporate a single error additively; bootstrapping can ensure that these errors represent more adequately the actual error distribution, ensuring more accurate confidence intervals. See Veall (1986), Prescott and Stengos (1987), and Bernard and Veall (1987).
5. Finding a correct specification is a major problem in econometrics, often dealt with via data mining techniques in which searches are made over several explanatory variables to find the ones that seem to be most appropriate. There is no analytical way of incorporating the influence of this searching into the evaluation of test statistics or the production of confidence intervals. By bootstrapping the entire search/estimation procedure, bootstrapping can overcome this problem. Drawing bootstrap errors creates a new data set. This data set is subjected to the traditional econometric "data mining" specification search, resulting in an estimating equation producing parameter estimates, some of which could be zero because some explanatory variables may be dropped out during the specification search. By repeating this procedure many times, a distribution of parameter estimates is created which incorporates the role of the specification search procedure. See Veall (1992).

In my experience, students quickly become comfortable with the mechanics of the procedure - knowing what the computer can do, drawing errors with replacement, and looking for the tails of the distribution, for example. What takes longer for them to master is setting up the structure of the solution, including, remarkably, how to mimic the data-generating process. Next to learning the sampling distribution concept, the biggest payoff to students of working through these problems is an understanding of how data is assumed to be generated by the econometrician when s/he chooses a particular estimating or testing technique. If your students cannot explain how to generate data to represent a simultaneous equation model, a seemingly unrelated equations model, a probit model, or a Tobit model[5], how can you be confident they understand anything you have been explaining to them in the course? For advanced students an instructor may wish to combine Monte Carlo and bootstrapping by asking students to explain how to do a Monte Carlo study to evaluate the merits of bootstrapping for a particular problem. An example is provided in the Appendix B.

OPPORTUNITY COST

An observant reader will notice that my proposal takes the form of creating exercises asking students to "explain how to bootstrap" as opposed to "do a bootstrap." This is deliberate, and follows the advice in Kennedy (1998a, 1998c) that students be asked to explain how to do a Monte Carlo study rather than be asked actually to do a Monte Carlo study. At first glance this appears to violate the old Chinese proverb "I hear, I forget; I see, I remember; I do, I understand." Undoubtedly more would be learned by having students actually do a bootsrap, and this may be suitable as a course project, but the "do" in the Chinese proverb must be interpreted through economists' eyes - students should "do" only to the point at which marginal understanding equals marginal cost.

The issue here is opportunity cost - there are too many topics in econometrics that must be covered to allow time for teaching students programming skills. Although the modern generation of students is supposed to be computer literate, I have discovered that this does not mean they are quick to learn to program. Be content with having them write down a detailed step-by-step set of instructions for a hypothetical research assistant, but be very careful evaluating these instructions. Small mistakes can reflect major misunderstandings. Be particularly on guard for vagueness in these instructions; this is students' favorite way of attempting to conceal lack of understanding. For example, they may not make it completely evident that the data generating process satisfies the null hypothesis, or they may omit describing how to get the first error needed to generate a sequence of bootstrapped first-order autocorrelated errors.

CONCLUSION

The main message of this article is that students must be motivated to learn the sampling distribution concept, and that the way to do this is to provide sample exam questions that require them to demonstrate this understanding. In previous studies I have recommended "explain how to do a Monte Carlo study" exercises for this purpose; I extend that recommendation to "explain how to bootsrap" exercises. An advantage of the latter is that bootstrapping is becoming common in applied econometrics, and so is a topic that instructors should want to teach in any event.

Generating student understanding of the sampling distribution concept is crucial because it facilitates learning the wide array of econometric topics with which students are faced in a typical econometrics course. One of my former students underlined this for me by telling me that at the beginning of his first graduate course in econometrics he was behind the rest of the class because he had to learn from scratch a lot of formulas and algebraic derivations that other students had encountered in their more-traditional undergraduate econometrics courses. But he found that after the first month or so he had become comfortable with the math and suddenly leapfrogged far ahead of the rest of the class because, he said, he understood what was going on but they understood only the mathematics.

NOTES

1. A sampling distribution reflects relative frequencies with which different values of a statistic would be obtained if different errors had been drawn.
2. I do not discuss in this article techniques for bootstrapping when the errors are not exchangeable. An example is heteroskedasticity associated with an explanatory variable, causing large (in absolute value) errors to be more likely to be attached to some observations than others. A very different bootstrap resampling procedure is used to deal with this, in which a bootstrapped sample is formed by drawing with replacement from the set of original observations (where each dependent variable value and its associated independent variables values is a single observation). This points to a potential problem with using bootstrapping for pedagogical purposes - for some applications the bootstrapped samples must be created in imaginative ways; the bootsrap literature should be consulted before devising student exercises based on unfamiliar applications.

3. How many bootstrapped samples are required? This varies from case to case. Efron (1987) suggests that estimation of bias and variance requires only about 200, but estimation of confidence intervals, and thus use for hypothesis testing, requires about 2000.
4. For advanced students the instructor may wish to point out that this example should be bootstrapping a pivotal statistic. This would require bootstrapping a t statistic (where the standard error is calculated using an asymptotic formula) for $\hat{\delta}/\hat{\beta}$; the confidence interval would be calculated by taking the bootstrap critical t values and multiplying them by the bootstrapped estimated standard error.
5. Creating bootstrapped samples for probit or Tobit models are examples in which the bootstrapping mechanism is not easy to deduce; instructors may wish to use only Monte Carlo exercises for these cases.

APPENDIX A: COMPUTER CODE

Shazam computer code for example 1 follows.

```
sample 1 25
set nodoecho
dim t 2000
read (filelocation) y x
?ols y x / resid=res coef=b stderr=se
gen1 tvalue=(b(1)-2)/se(1)
print tvalue
do #= 1, 2000
genr y= b(2) + 2*x + 1.043*samp(res)
?ols y x / coef=b stderr=se
gen1 t(#)=(b(1)-2)/se(1)
endo
sample 1 2000
sort t
print t(50) t(1950)
stop
```

Shazam computer code for example 2 follows.

```
sample 1 28
set nodoecho
dim r 4000
read (filelocation) y x w
?ols y x w / resid=res coef=b
gen1 ratio=b(2)/b(1)
do #=1, 4000
genr y=b(3) + b(1)*x + b(2)*w + 1.058*samp(res)
?ols y x w / coef=b
gen1 r(#)=b(2)/b(1)
endo
sample 1 4000
stat r / mean=av var=var
sort r
gen1 bias=av-ratio
print bias
```

```
gen1 tstat=bias/sqrt(var/4000)
print tstat
gen1 low=r(200)-bias
gen1 high=r(3800)-bias
print low high
stop
```

APPENDIX B: COMBINING MONTE CARLO AND BOOTSTRAPPING

Suppose you have programmed a computer to do the following.

i) Draw 25 x values from a distribution uniform between 4 and 44.

ii) Set ctr = 0.

iii) Draw 25 values from a standard normal distribution and multiply all the negative values by 9 to create 25 e values.

iv) Compute 25 y values as 3 + 2x + e.

v) Regress y on x, saving the intercept estimate as int, the slope estimate as b, the standard error of b as se and the residuals as a vector res.

vi) Compute t# = (b - 2)/se and save it.

vii) Compute 25 y values as int + 2x + 1.043be where be is drawn randomly with replacement from the elements of res.

viii) Regress y on x and compute bt(1) = (b - 2)/se where b is the slope coefficient estimate and se is its standard error.

ix) Repeat from vii) to obtain 200 bt values.

x) Order these bt values from smallest to largest.

xi) Add one to ctr if t# is greater than the 190th ordered bt value.

xii) Repeat from iii to obtain 500 t# values.

xiii) Calculate the fraction of these t# values greater than ____ and compare to _____.

Explain what this program is designed to do, and fill in the blanks.

(Answer: This program is designed to compare the actual type I errors of a traditional one-sided t test and its bootstrapped version. The context is a linear regression in which the error terms have come from an asymmetric distribution, a nominal significance level of 5 percent has been employed, the null is that the slope equals 2, and the alternative is that the slope is greater than 2. The first blank is filled with 1.56, the tabled 5 percent critical value for the t distribution with 23 degrees of freedom. The second blank is filled with ctr/500.) Shazam computer code for this exercise follows.

```
sample 1 25
```

```
dim t 500 tt 200
set nodoecho
set nowarn
genr x= 4 + uni(40)
gen1 bctr=0
gen1 tctr=0
?do #=1, 500
genr e=nor(1)
if (e.lt.0) e=e*9
genr y= 3 + 2*x + e
?ols y x / resid=res coef=b stderr=se
genr t(#) = (b(1)-2)/se(1)
?do $=1, 200
genr y=b(2) + 2*x + 1.043*samp(res)
?ols y x / coef=b stderr=se
genr tt($) = (b(1)-2)/se(1)
?endo
sample 1 200
sort tt
if1 (t(#).gt.tt(190)) bctr=bctr+1
if1 (t(#).gt.1.56) tctr=tctr+1
?endo
gen1 type1b=bctr/500
gen1 type1t=tctr/500
print type1t type1b
stop
```

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