



Born–Infeld black holes in 4D Einstein–Gauss–Bonnet gravity

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Abstract A novel four-dimensional Einstein–Gauss–Bonnet gravity was formulated by Glavan and Lin (Phys. Rev. Lett. 124:081301, 2020), which is intended to bypass the Lovelock’s theorem and to yield a non-trivial contribution to the four-dimensional gravitational dynamics. However, the validity and consistency of this theory has been called into question recently. We study a static and spherically symmetric black hole charged by a Born–Infeld electric field in the novel four-dimensional Einstein–Gauss–Bonnet gravity. It is found that the black hole solution still suffers the singularity problem, since particles incident from infinity can reach the singularity. It is also demonstrated that the Born–Infeld charged black hole may be superior to the Maxwell charged black hole to be a charged extension of the Schwarzschild–AdS-like black hole in this new gravitational theory. Some basic thermodynamics of the black hole solution is also analyzed. Besides, we regain the black hole solution in the regularized four-dimensional Einstein–Gauss–Bonnet gravity proposed by Lü and Pang (arXiv:2003.11552).

1 Introduction

As the cornerstone of modern cosmology, Einstein’s general relativity (GR) provides precise descriptions to a variety of phenomena in our universe. According to the powerful Lovelock’s theorem [1,2], the only field equations of a four-dimensional (4D) metric theories of gravity that are second order or less are Einstein’s equations with a cosmological constant. So in order to go beyond Einstein’s theory,

one usually modifies GR by adding some higher derivative terms of the metric or new degrees of freedom into the field equations, by considering other fields rather than the metric, or by extending to higher dimensions, etc. [3,4]. A natural generalization of GR is the Lovelock gravity, which is the unique higher curvature gravitational theory that yields conserved second-order field equations in arbitrary dimensions [1,2]. Besides the Einstein–Hilbert term plus a cosmological constant, there is a Gauss–Bonnet (GB) term \mathcal{G} allowed in Lovelock’s action in higher-dimensional spacetime. The GB term \mathcal{G} is quadratic in curvature and contributes ultraviolet corrections to Einstein’s theory. Moreover, the GB term also appears in the low-energy effective action of string theory [7]. However, it is well known that the GB term is a total derivative in four dimensions, so it does not contribute to the gravitational dynamics. In order to generate a nontrivial contribution, one usually couples the GB term to a scalar field [5,6].

In recent Refs. [8–10], the authors suggested that by rescaling the GB coupling constant $\alpha \rightarrow \alpha/(D-4)$ with D the number of spacetime dimensions, the theory can bypass the Lovelock’s theorem and the GB term can yield a non-trivial contribution to the gravitational dynamics in the limit $D \rightarrow 4$. This theory, now dubbed as the novel 4D Einstein–Gauss–Bonnet (EGB) gravity, will give rise to corrections to the dispersion relation of cosmological tensor and scalar modes, and is practically free from singularity problem in Schwarzschild-like black holes [10]. The novel 4D EGB gravity has drawn intensive attentions recently [11–44].

However, there are also some debates on whether the novel gravity is a consistent and well-defined theory in four dimensions [45–53]. Such as, Gurses et al. pointed out that the novel 4D EGB gravity does not admit a description in terms of a covariantly-conserved rank-2 tensor in four dimensions and the dimensional regularization procedure is ill-defined, since

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one part of the GB tensor, the Lanczos–Bach tensor, always remains higher dimensional [46]. So generally speaking, the theory only makes sense on some selected highly-symmetric spacetimes, such as the FLRW spacetime and static spherically symmetric spacetime. Mahapatra found that in the limiting $D \rightarrow 4$ procedure, the theory gives unphysical divergences in the on-shell action and surface terms in four dimensions [47]. Shu showed that at least the vacuum of the theory is either unphysical or unstable, or has no well-defined limit as $D \rightarrow 4$ [48]. Based on studying the evolution of the homogeneous but anisotropic universe described by the Bianchi type I metric, Tian and Zhu found that the novel 4D EGB gravity with the dimensional-regularization approach is not a complete theory [49]. On the other hand, some regularized versions of 4D EGB gravity were also proposed by [51–53], and the resulting theories belong to the family of Horndeski gravity.

New black hole solutions in the novel 4D EGB gravity were also reported recently. In Ref. [54], Fernandes generalized the Reissner–Nordström (RN) black hole with Maxwell electric field coupling to the gravity. The authors in Refs. [55, 56] considered the Bardeen-like and Hayward-like black holes by interacting the new gravity with nonlinear electrodynamics. The rotating black holes were also investigated in this new theory [57, 58]. Some other black hole solutions were discussed in the Refs. [59–61].

A well-known nonlinear electromagnetic theory, called Born–Infeld (BI) electrodynamics, was proposed by Born and Infeld in 1934 [62]. They introduced a force by limiting the electromagnetic field strength analogy to a relativistic limit on velocity, and regularized the ultraviolet divergent self-energy of a point-like charge in classical dynamics. Since it was found that the BI action can be generated in some limits of string theory [63], the BI electrodynamics has become more popular. As an extension of RN black holes in Einstein–Maxwell theory, charged black hole solutions in Einstein–Born–Infeld (EBI) theory has received some attentions in recent years, see Refs. [64–71] for examples.

In this work, we are interested in generalizing the static spherically symmetric black hole solutions charged by the BI electric field in the novel 4D EGB gravity. The paper is organized as follows. In Sect. 2, we solve the theory to get an exact black hole solution. In Sect. 3, we study some basic thermodynamics of the black hole and analyze its local and global stabilities. In Sect. 4, we regain the black hole solution in the regularized 4D EGB gravity based on the work of Lü and Pang [51]. Finally, brief conclusions are presented.

2 BI black hole solution in the novel 4D EGB gravity

We start from the action of the D -dimensional EGB gravity minimally coupled to the BI electrodynamics in the presence

of a negative cosmological constant $\Lambda = -\frac{(D-1)(D-2)}{2l^2}$,

$$S = \frac{1}{16\pi} \int d^D x \sqrt{-g} \left(R - 2\Lambda + \frac{\alpha}{D-4} \mathcal{G} + \mathcal{L}_{BI} \right), \quad (1)$$

where the GB term is $\mathcal{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$, and the Lagrangian of the BI electrodynamics reads

$$\mathcal{L}_{BI} = 4\beta^2 \left(1 - \sqrt{1 + \frac{F_{\mu\nu}F^{\mu\nu}}{2\beta^2}} \right). \quad (2)$$

Here $\beta > 0$ is the BI parameter and it is the maximum of the electromagnetic field strength. The Maxwell electrodynamics is recovered in the limit $\beta \rightarrow \infty$.

The equations of motion of the theory can be obtained by varying the action with respect to the metric field $g_{\mu\nu}$ and the gauge field A_μ :

$$G_{\mu\nu} + \Lambda g_{\mu\nu} + \frac{\alpha}{D-4} \left(\frac{\delta\mathcal{G}}{\delta g^{\mu\nu}} - \frac{1}{2} g_{\mu\nu} \mathcal{G} \right) + \left(\frac{\delta\mathcal{L}_{BI}}{\delta g^{\mu\nu}} - \frac{1}{2} g_{\mu\nu} \mathcal{L}_{BI} \right) = 0, \quad (3)$$

$$\partial_\mu \left(\frac{\sqrt{-g} F^{\mu\nu}}{\sqrt{1 + \frac{F_{\rho\sigma}F^{\rho\sigma}}{2\beta^2}}} \right) = 0, \quad (4)$$

where

$$\frac{\delta\mathcal{G}}{\delta g^{\mu\nu}} = 2RR_{\mu\nu} + 2R_\mu{}^{\rho\sigma\lambda}R_{\nu\rho\sigma\lambda} - 4R_{\mu\lambda}R^{\lambda\nu} - 4R^{\rho\sigma}R_{\mu\rho\nu\sigma}, \quad (5)$$

$$\frac{\delta\mathcal{L}_{BI}}{\delta g^{\mu\nu}} = \frac{2F_{\mu\lambda}F^{\lambda\nu}}{\sqrt{1 + \frac{F_{\rho\sigma}F^{\rho\sigma}}{2\beta^2}}}. \quad (6)$$

Here we consider a static spherically symmetric metric ansatz in D -dimensional spacetime

$$ds^2 = -a(r)e^{-2b(r)}dt^2 + \frac{dr^2}{a(r)} + r^2 d\Omega_{D-2}^2, \quad (7)$$

where $d\Omega_{D-2}^2$ represents the metric of a $(D-2)$ -dimensional unit sphere. Correspondingly, the vector potential is assumed to be $A = \Phi(r)dt$.

Instead of solving the equations of motion directly, it would be more convenient to start from the following reduced action to get the solution, i.e.,

$$S = \frac{\Sigma_{D-2}}{16\pi} \int dt dr (D-2)e^{-b} \left[\left(r^{D-1} \psi (1 + \alpha(D-3)\psi) + \frac{r^{D-1}}{l^2} \right)' + \frac{4\beta^2 r^{D-2}}{D-2} \left(1 - \sqrt{1 - \beta^{-2} e^{2b} \Phi'^2} \right) \right], \quad (8)$$

where the prime denotes the derivative respect to the radial coordinate r , $\Sigma_{D-2} = 2\pi^{\frac{D-1}{2}}/\Gamma[\frac{D-1}{2}]$ is the area of a unit $(D-2)$ -sphere, and $\psi(r) = (1 - a(r))/r^2$. By varying

the action with respect to $a(r)$, $b(r)$ and $\Phi(r)$, one can easily obtain the field equations and hence the solutions in arbitrary dimensions. Here we are interested in the case of $D = 4$. The solution can be found in a closed form:

$$b(r) = 0, \tag{9}$$

$$\Phi(r) = \frac{Q}{r} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -\frac{Q^2}{\beta^2 r^4}\right), \tag{10}$$

$$a(r) = 1 + \frac{r^2}{2\alpha} \left\{ 1 \pm \left[1 + 4\alpha \left(\frac{2M}{r^3} - \frac{1}{l^2} - \frac{2\beta^2}{3} \left(1 - \sqrt{1 + \frac{Q^2}{\beta^2 r^4}} \right) - \frac{4Q}{3r^3} \Phi(r) \right) \right]^{\frac{1}{2}} \right\}, \tag{11}$$

where we have chosen the integration constants to recover a proper asymptotic limit, and ${}_2F_1$ is the hypergeometric function.

In the limit of $\beta \rightarrow \infty$, the solution recovers the Maxwell charged RN-AdS-like black hole solution in the novel 4D EGB gravity [54],

$$a(r) = 1 + \frac{r^2}{2\alpha} \left[1 \pm \sqrt{1 + 4\alpha \left(\frac{2M}{r^3} - \frac{Q^2}{r^4} - \frac{1}{l^2} \right)} \right]. \tag{12}$$

Moreover, the Schwarzschild-AdS-like black hole solution in the novel 4D EGB gravity is recovered by closing the electric charge [10],

$$a(r) = 1 + \frac{r^2}{2\alpha} \left[1 \pm \sqrt{1 + 4\alpha \left(\frac{2M}{r^3} - \frac{1}{l^2} \right)} \right]. \tag{13}$$

On the other hand, in the large distance, the metric function $a(r)$ (11) has the asymptotic behavior

$$a(r) = 1 + \frac{r^2}{2\alpha} \left(1 \pm \sqrt{1 - \frac{4\alpha}{l^2}} \right) \pm \frac{2Mr - Q^2}{r^2 \sqrt{1 - \frac{4\alpha}{l^2}}} + \mathcal{O}(r^{-4}). \tag{14}$$

So in order to have a real metric function in large distance, it is clear that we require $0 < \alpha \leq l^2/4$ or $\alpha < 0$. Further, by taking the small α limit, the solution of the plus-sign branch reduces to a RN-AdS solution with a negative gravitational mass and imaginary charge, and only the minus-sign branch can recover a proper RN-AdS limit,

$$a(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} + \frac{r^2}{l^2} + \mathcal{O}(\alpha). \tag{15}$$

So we only focus on the solution of the minus-sign branch in the rest of the paper.

We illustrate the minus-sign branch of the metric function $a(r)$ in Fig. 1 for some parameter choices, where the solutions of the BI charged black hole in GR and the Maxwell charged

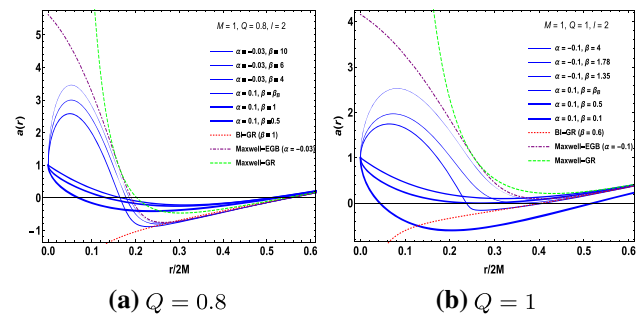


Fig. 1 The metric function $a(r)$ for some parameter choices

black holes in the novel 4D EGB gravity and GR are also plotted for comparison. As shown in the figures, similar to the RN black hole, the solution (11) can have zero, one, or two horizons depending on the parameters. However, the metric function $a(r)$ always approaches a finite value $a(0) = 1$ at the origin $r \rightarrow 0$ in the novel 4D EGB gravity. This property is different from the BI charged AdS black hole in EBI theory [70], where there are different types of solutions relying on the parameters: a ‘‘Schwarzschild-like’’ type for $\beta < \beta_B$ (only one horizon with $a(0) \rightarrow -\infty$), an ‘‘RN’’ type for $\beta > \beta_B$ (naked singularity, one or two horizons with $a(0) \rightarrow +\infty$), and a ‘‘marginal’’ type for $\beta = \beta_B$ (naked singularity or one horizon with a finite value $a(0) = 1 - 2\beta_B Q$). Interestingly, as shown in Fig. 1(b), even the solution coupled to the Maxwell field becomes naked singularities, the ones coupled to the BI field are still regular black holes.

In the small distance limit, the hypergeometric function behaves like

$${}_2F_1\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -\frac{Q^2}{\beta^2 r^4}\right) = \sqrt{\frac{\beta}{\pi Q}} \frac{\Gamma^2(1/4)}{4} r - \frac{\beta}{Q} r^2 + \mathcal{O}(r^6). \tag{16}$$

Then the formula in the square root of $a(r)$ can be rearranged as

$$1 + 4\alpha \left[\frac{2M}{r^3} \left(1 - \sqrt{\frac{\beta}{\beta_B}} \right) + \frac{4\beta Q}{3r^2} - \frac{1}{l^2} - \frac{2\beta^2}{3} \left(1 - \sqrt{1 + \frac{Q^2}{\beta^2 r^4}} \right) \right] + \mathcal{O}(r^2), \tag{17}$$

where $\beta_B \equiv \frac{36\pi M^2}{\Gamma(1/4)^4 Q^3}$. One special case appears at $\beta = \beta_B$, where the metric function $a(r)$ (11) in the small distance limit behaves like

$$a(r) = 1 - \sqrt{\frac{2\beta_B Q}{\alpha}} r + \frac{r^2}{2\alpha} + \mathcal{O}(r^3), \tag{18}$$

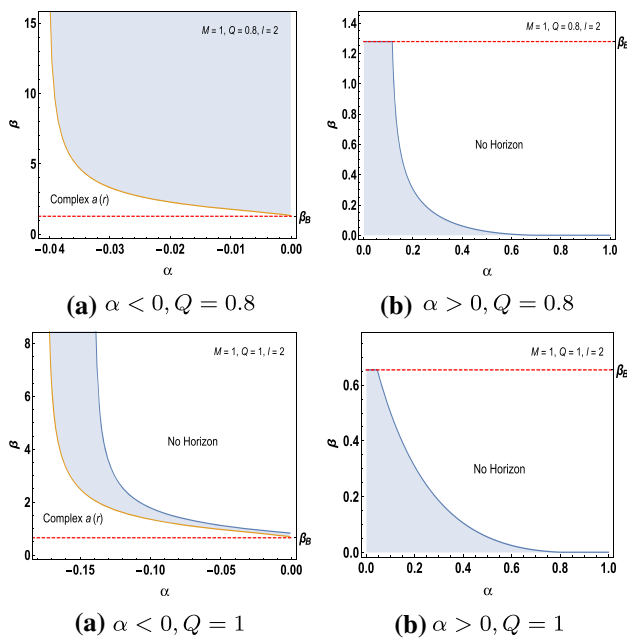


Fig. 2 The parameter space for the case of $M = 1, Q = 0.8, 1,$ and $l = 2$, where the shaded area represents available parameter region

and the Ricci scalar $R \approx \sqrt{\frac{2\beta_B Q}{\alpha}} \frac{6}{r}$ is divergent. However, if $\beta \neq \beta_B, a(r)$ behaves like

$$a(r) = 1 - \frac{\sqrt{K} r^{\frac{1}{2}}}{\alpha} - \frac{\beta Q r^{\frac{3}{2}}}{\sqrt{K}} + \frac{r^2}{2\alpha} + \mathcal{O}(r^{\frac{5}{2}}), \tag{19}$$

where $K = 2M\alpha \left(1 - \sqrt{\frac{\beta}{\beta_B}}\right)$. Now the Ricci scalar reads $R \approx \frac{15}{8\alpha} \frac{\sqrt{K}}{r^{3/2}}$. So, by considering Eqs. (18) and (19), we require $\alpha > 0, 0 < \beta \leq \beta_B$ or $\alpha < 0, \beta > \beta_B$ in order to have a real metric function.

The above constraints $0 < \alpha \leq l^2/4, 0 < \beta \leq \beta_B$ or $\alpha < 0, \beta > \beta_B$ are not enough to guarantee a real metric function in the whole parameter space. More accurate constraints can be worked out by requiring the formula under the square root of the metric function $a(r)$ (11) to be positive. Moreover, in order to ensure the existence of the black hole horizon, there are further constraints on α and β , as shown in Fig. 1. Due to the complicated form of $a(r)$, we fail to get the analytic full constraints. Nevertheless, we can numerically solve the constraints for fixed Q, M , and l . For example, we show the results for the cases of $Q = 0.8$ and $Q = 1$ with $M = 1$ and $l = 2$ in Fig. 2. As shown in the figures, regular black hole solution only exists in the shaded parameter region, and the solution turns into a naked singularity above the shaded region. Especially, on the border, the two horizons degenerate and the solution corresponds to an extremal black hole.

Since the leading term $8\alpha M/r^3$ under the square root of Eq. (13) is negative at short radius for $\alpha < 0$, a real metric function is present only for positive α [10]. However, for the

charged solution (12), the leading term $-4\alpha Q^2/r^4$ under the square root is always negative at small radius for positive α , so one requires $\alpha < 0$ to get a real metric function. Therefore, by “throwing” electric charges into the Schwarzschild-AdS-like black hole (13) with positive α , it would not be deformed into the Maxwell charged RN-AdS-like black hole (12) with positive α . However, as illustrated in Fig. 2, the BI charged black hole solution (11) can be real for both positive and negative α . In the limit of $Q \rightarrow 0, \beta_B = \frac{36\pi M^2}{\Gamma(1/4)^4 Q^3} \rightarrow \infty$, the BI charged black hole with positive α can be continuously deformed into the Schwarzschild-AdS-like black hole (13). This is a hint that the BI charged black hole may be superior to the Maxwell charged black hole in the novel 4D EGB gravity.

Note that the metric (19) approaches a finite value $a(0) = 1$ as $r \rightarrow 0$, and it is similar to the behavior of (13) in Ref. [10] for positive α . So in this case, an infalling particle would feel a repulsive gravitational force when it approaches the singular point $r = 0$. However, for the solution with negative α , the infalling particle would feel an attractive gravitational force in short distance. Interestingly, for the critical case (18) with $\alpha > 0$ and $\beta = \beta_B$, there is a maximal repulsive gravitational force when the particle approaches the singular point. However, whether the particle can reach the singularity or not depends on its initial conditions.

When the particle starts at rest at the radius R and freely falls radially toward the black hole, its velocity is given by [72]

$$\frac{dr}{d\tau} = \pm \sqrt{a(R) - a(r)}, \tag{20}$$

where $a(R) = \tilde{E}^2$ with \tilde{E} the energy per unit rest mass, and the plus (minus) sign refers to the infalling (outgoing) particle. For simplicity we consider the case of asymptotically Minkowski space, i.e., $l \rightarrow \infty$. As illustrated in Fig. 3, if the radius R is finite, the particle can not reach the singularity for the solution with positive α , but it can reach the singularity in short distance for the solution with negative α , since the particle feels an attractive gravitational force when it gets close to the singularity. However, if the particle starts at rest at infinity, i.e., $\tilde{E}^2 = a(R \rightarrow \infty) = 1$, since $a(r \rightarrow 0) = 1$ as seen in Eqs. (18) and (19), it will just reach the singularity with zero speed [50]. So if the particle has a kinetic energy at infinity, namely, $\tilde{E}^2 = a(R \rightarrow \infty) > 1$, it will reach the singularity with a nonzero speed. Therefore, in this sense, the black hole solution still suffers the singularity problem.

3 Thermodynamics

Black holes are widely believed to be thermodynamic objects, which possess the standard thermodynamic variables

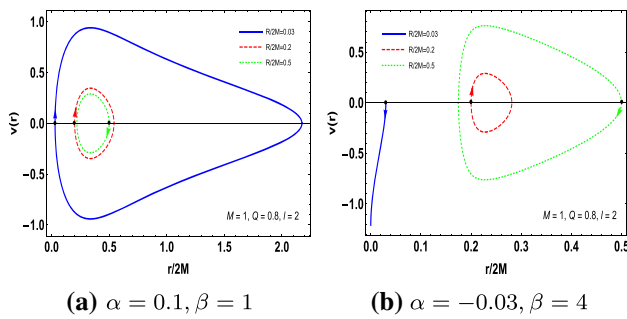


Fig. 3 The velocity $v(r)$ of an infalling particle for starting at rest at different radii R , where the dot represents the initial position and the arrow represents the direction in which the velocity starts to increase

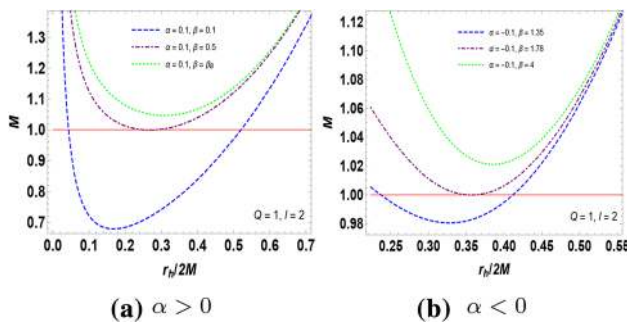


Fig. 4 The mass of the black hole for positive α (a) and negative α (b)

and satisfy the four laws of black hole thermodynamics. In this section, we explore some basic thermodynamics of the BI charged black hole in the novel 4D EGB gravity.

By setting $a(r) = 0$ in Eq. (11), the black hole mass can be expressed with the radius of the event horizon r_h , namely,

$$M = \frac{r_h}{2} \left[1 + \frac{r_h^2}{l^2} + \frac{\alpha}{r^2} + \frac{2\beta^2 r_h^2}{3} \left(1 - \sqrt{1 + \frac{Q^2}{\beta^2 r_h^4}} \right) + \frac{4Q^2}{3r^2} {}_2F_1 \left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -\frac{Q^2}{\beta^2 r_h^4} \right) \right], \tag{21}$$

where the horizon radius has to satisfy $r_h > \sqrt{-2\alpha}$ for negative α , since the square root of metric function (11) must be positive. So there is a minimum horizon radius $r_h^* = \sqrt{-2\alpha}$ for negative α . As shown in Fig. 4, for a given mass, the number of horizons depends on the parameters α and β , and the corresponding parameter space is explicitly shown in Fig. 2. The critical case, i.e., an extremal black hole, just happens when $M'(r_h) = 0$, which yields

$$1 + \frac{3r_h^2}{l^2} - \frac{\alpha}{r_h^2} + 2\beta^2 r_h^2 \left(1 - \sqrt{1 + \frac{Q^2}{\beta^2 r_h^4}} \right) = 0. \tag{22}$$

In the presence of the GB term and BI electrodynamics, the dimensionful GB coupling parameter α and BI parameter β should be also treated as new thermodynamic variables [70].

Now the generalized Smarr formula is given by

$$M = 2(TS - VP + A\alpha) + \Phi Q - B\beta. \tag{23}$$

The electrostatic potential Φ given in (10) is measured at the horizon r_h in this formula.

The Hawking temperature can be obtained from the relation $T = \frac{\kappa}{2\pi}$ with the surface gravity $\kappa = -\frac{1}{2} \frac{\partial g_{tt}}{\partial r} \Big|_{r=r_h}$, and reads

$$T = \frac{1}{4\pi r_h} \left[\frac{3r_h^4 - 3\alpha l^2 + 2\beta^2 l^2 r_h^4 \left(1 - \sqrt{1 + \frac{Q^2}{\beta^2 r_h^4}} \right)}{l^2(r_h^2 + 2\alpha)} + 1 \right]. \tag{24}$$

The temperature vanishes for the extremal black hole, which can be easily proved with the critical condition (22). In the large β and small α limits, the temperature reduces to

$$T = \frac{1}{4\pi r_h} \left(1 - \frac{Q^2}{r_h^2} + \frac{3r_h^2}{l^2} \right) - \frac{\alpha}{4\pi r_h} \left(\frac{6}{l^2} + \frac{3}{r_h^2} - \frac{2Q^2}{r_h^4} \right) + \frac{Q^4}{16\pi\beta^2 r_h^7} + \mathcal{O} \left(\alpha^2, \frac{\alpha}{\beta^2}, \frac{1}{\beta^4} \right), \tag{25}$$

where the first term is the standard Hawking temperature of the RN-AdS black hole, the second term is the leading order correction from the Gauss-Bonnet term, and the third term is the leading order Born-Infeld correction.

By following the approach of Ref. [73], the entropy of the black hole can be worked out from

$$S = \int \frac{1}{T} \left(\frac{\partial M}{\partial r_h} \right)_{P,Q,\alpha,\beta} dr_h + S_0, \tag{26}$$

where S_0 is an integration constant. Then it yields

$$S = \pi r_h^2 + 2\pi\alpha \ln r_h^2 + S_0 = \frac{A_h}{4} + 2\pi\alpha \ln \frac{A_h}{A_0}, \tag{27}$$

where $A_h = 4\pi r_h^2$ is the horizon area and A_0 is a constant with dimension of area, which is not determined from first principle [51]. By considering that the black entropy is generally independent of the black hole charge and cosmological constant but is relevant to the GB coupling parameter α , which has the dimension of area, we simply fix the undetermined constant as $A_0 = 4\pi|\alpha|$ [19]. Here a logarithmic term associated with the GB coupling parameter α arises as a sub-leading correction to the Bekenstein-Hawking area formula, which is universal in some quantum theories of gravity [74]. Since the novel 4D EGB gravity is considered as a classical modified gravitational theory [10], the logarithmic correction appears at classical level here.

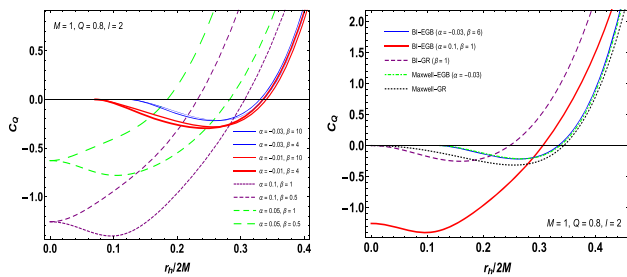


Fig. 5 The specific heat **a** for some parameter choices and **b** for the BI charged and Maxwell charged solutions in the novel 4D EGB gravity and GR

The pressure $P = \frac{3}{8\pi l^2}$ is associated with the cosmological constant [75, 76], and the corresponding thermodynamical volume V is given by

$$V = \left(\frac{\partial M}{\partial P} \right)_{S,Q,\alpha,\beta} = \frac{4\pi r_h^3}{3}. \tag{28}$$

By noting that the entropy is associated with the horizon r_h and GB coupling parameter α , the conjugate quantity \mathcal{A} of α can be obtained from Eqs. (21) and (27) as

$$\mathcal{A} = \frac{1}{2r_h} + \left[\frac{3r^3}{2l^2} + \frac{r^2 - \alpha}{3r} + \beta^2 r^3 \left(1 - \sqrt{1 + \frac{Q^2}{\beta^2 r^4}} \right) \right] \times \frac{1 - \ln \frac{r^2}{|\alpha|}}{r^2 + 2\alpha}. \tag{29}$$

Finally, we can calculate the BI vacuum polarization \mathcal{B} from Eq. (21):

$$\mathcal{B} = \frac{2r_h^3}{3} \left(1 - \sqrt{1 + \frac{Q^2}{\beta^2 r_h^4}} \right) + \frac{Q^2}{3\beta r} {}_2F_1 \left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -\frac{Q^2}{\beta^2 r_h^4} \right). \tag{30}$$

With all the above thermodynamic quantities at hand, it is easy to verify the validity of Smarr formula (23). Further, it is also straightforward to verify that the first law of black hole

$$dM = TdS - VdP + \mathcal{A}d\alpha + \Phi dQ - \mathcal{B}d\beta \tag{31}$$

holds as expected.

The specific heat is helpful to analyze the local stability of a black hole solution, and it can be evaluated by

$$C_Q = \left(\frac{\partial M}{\partial T} \right)_Q = \left(\frac{\partial M}{\partial r_h} \right)_Q \left(\frac{\partial r_h}{\partial T} \right)_Q. \tag{32}$$

Instead of bothering to list the cumbersome result, we plot the specific heat in Fig. 5. As is shown in Fig. 5a, it is evident that

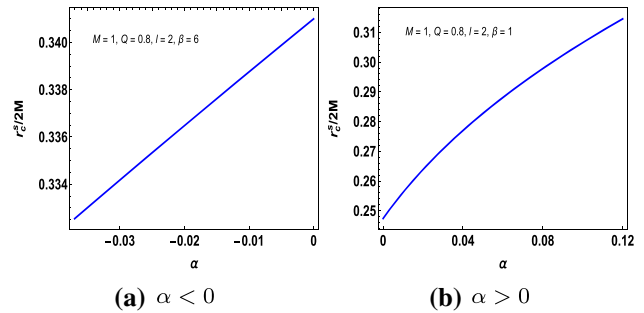


Fig. 6 The critical size r_c^S of specific heat **a** for negative α and **b** for positive α in the novel 4D EGB gravity

the black hole has a negative specific heat when the horizon radius is smaller than some critical size r_c^S . So only large black holes are stable against fluctuations. A black hole with positive α has larger stable region than that with negative α . Moreover, for a given α , the smaller the parameter β , the larger the stable region is. We also show the specific heats for the BI charged and Maxwell charged solutions in the novel 4D EGB gravity and GR in Fig. 5b. What they have in common is that large black holes are stable against fluctuations in these theories, since all kinds of black hole solutions shown in Fig. 5b are asymptotic AdS and have the behavior of $a(r) \propto r^2$ at large r .

In order to investigate the effect of the GB term on the specific heat, we depict the critical size r_c^S with varying GB parameter α in Fig. 6. When the GB term is switched off, i.e., $\alpha = 0$, the BI black hole solution $a(r)$ in the novel 4D EGB gravity will recover the BI black hole solution obtained in EBI theory [66]. So as shown in Fig. 6, the BI black hole solution with a negative α in the novel 4D EGB gravity has a larger stable region than that in EBI theory, but the solution with a positive α has a smaller region than that in EBI theory.

We further evaluate the Gibb’s free energy of the black hole in order to investigate its global stability, which in the canonical ensemble is written as

$$F = M - TS = \frac{r_h}{2} + \frac{r_h^3}{2l^2} + \frac{\alpha}{2r_h} + \frac{\beta^2 r_h^3}{3} \left(1 - \sqrt{1 + \frac{Q^2}{\beta^2 r_h^4}} \right) + \frac{2Q^2}{3r_h} {}_2F_1 \left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -\frac{Q^2}{\beta^2 r_h^4} \right) - \left[\frac{r_h}{2} + \frac{r_h^3}{2l^2} - \frac{\alpha}{2r_h} + \beta^2 r_h^3 \left(1 - \sqrt{1 + \frac{Q^2}{\beta^2 r_h^4}} \right) \right] \frac{r_h^2 + 2\alpha \ln \frac{r_h^2}{|\alpha|}}{2(r_h^2 + 2\alpha)}. \tag{33}$$

The behavior of the free energy is illustrated in Fig. 7 for different parameter choices and different black hole solutions. A black hole with a negative free energy is globally stable, but a black hole with a positive one is globally unstable. So as shown in Fig. 7a, for the black hole solution with

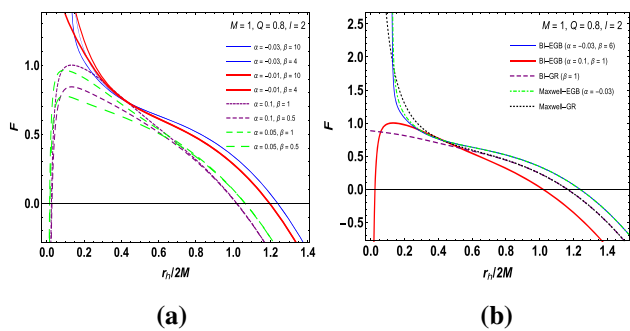


Fig. 7 The Gibb’s free energy **a** for some parameter choices and **b** for the BI charged and Maxwell charged solutions in the novel 4D EGB gravity and GR

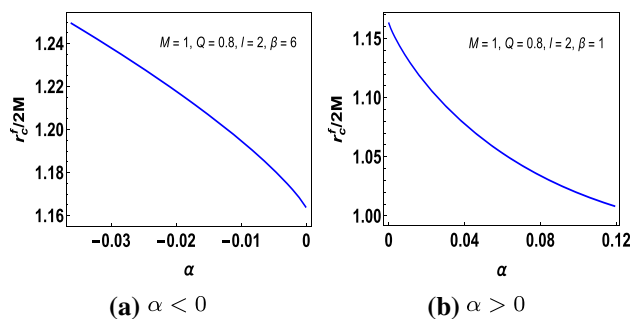


Fig. 8 The critical size r_c^f of free energy **a** for negative α and **b** for positive α in the novel 4D EGB gravity

$\alpha < 0$, only when its radius is larger than some critical size r_c^f , the black hole is globally stable, while for the solution with $\alpha > 0$, both larger and smaller black holes are globally stable. However, smaller black holes with positive α are locally unstable due to their negative specific heat, as is shown in Fig. 5a. Moreover, black holes with positive α have larger stable region than that with negative α . Further, as shown in Fig. 7b, BI charged black holes with positive α in the novel 4D EGB gravity have the largest stable region than the others.

In particular, we illustrate the relationship between the critical size r_c^f and the GB parameter α in Fig. 8, where a non-vanishing α corresponds to the BI black hole in the novel 4D EGB gravity and a vanishing α corresponds to the BI black hole in EBI theory. Therefore, it is shown that the BI black hole solution with $\alpha > 0$ in the novel 4D EGB gravity has a larger stable region than that in EBI theory, but the solution with $\alpha < 0$ has a smaller region than that in EBI theory.

4 Regain the solution in the regularized 4D EGB gravity

In order to get a well defined action principle, Lü and Pang proposed a procedure for $D \rightarrow 4$ limit of EGB gravity [51].

They started from a D -dimensional EGB gravity on a maximally symmetric space of $(D-4)$ -dimensions with the metric

$$ds_D^2 = ds_4^2 + e^{2\phi} d\Sigma_{D-4,\lambda}^2, \tag{34}$$

where the breathing scalar ϕ depends only on the external 4-dimensional coordinates, ds_4^2 is the 4-dimensional line element, and $d\Sigma_{D-4,\lambda}^2$ is the line element of the internal maximally symmetric space with the curvature tensor $R_{abcd} = \lambda(g_{ac}g_{bd} - g_{ad}g_{bc})$. Further, by redefining the GB coupling parameter α as $\alpha \rightarrow \frac{\alpha}{D-p}$, and then taking the limit $D \rightarrow 4$, they obtained a special scalar-tensor theory that belongs to the family of Horndeski gravity. In this section, we show that the black hole solution (9), (10) and (11) we obtained in the novel 4D EGB gravity is also the solution of this regularized 4D EGB gravity.

The 4-dimensional regularized gravitational action is given by [51]

$$S_{\text{reg}} = \int d^4x \sqrt{-g} \left[R + \frac{6}{l^2} + \alpha \left(\phi \mathcal{G} + 4G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - 2\lambda R e^{-2\phi} - 4(\partial\phi)^2 \square\phi + 2((\partial\phi)^2)^2 - 12\lambda(\partial\phi)^2 e^{-2\phi} - 6\lambda^2 e^{-4\phi} \right) \right]. \tag{35}$$

By including the Lagrangian of the BI electrodynamics (2), and substituting the ansatzes for the scalar field $\phi = \phi(r)$ and the 4D static spherically symmetric metric

$$ds^2 = -a(r)e^{-2b(r)} dt^2 + \frac{dr^2}{a(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{36}$$

into the action, the effective Lagrangian reads

$$L_{\text{eff}} = e^{-b} \left[2 \left(1 + \frac{3r^2}{l^2} - a - ra' \right) + \frac{2}{3} \alpha \phi' \left(3r^2 a^2 \phi'^3 - 2rf\phi'^2(4a + ra' - 2rab') - 6a\phi'(1 - a - ra' + 2rab') + 6(1 - a)(a' - 2ab') \right) - 6\alpha\lambda^2 r^2 e^{-4\phi} - 4\alpha\lambda e^{-2\phi}(1 - a - ra' - r^2 a' \phi' + 2r^2 ab' \phi') + 3r^2 a \phi'^2 \right] + 4\beta^2 r^2 \left(1 - \sqrt{1 - \beta^{-2} e^{2b} \Phi'^2} \right). \tag{37}$$

The corresponding equations of motion can be obtained by varying the effective action with respect to $a(r)$, $b(r)$, $\phi(r)$ and $\Phi(r)$. Further, it is easy to see that $b'(r)$ satisfies the equation

$$b' = \frac{\left[a(r\phi' - 1)^2 - \lambda r^2 e^{-2\phi} - 1 \right] (\phi'' + \phi'^2)}{\phi' \left[1 - a(3 + r\phi'(r\phi' - 3)) \right] + \lambda r \frac{(r\phi' - 1)}{e^{2\phi}} + \frac{r}{2\alpha}}. \tag{38}$$

So following the similar analysis in Ref. [51], $b(r) = 0$ is a consistent truncation. In this case, $a(r)$, $\phi(r)$ and $\Phi(r)$

satisfy the equations

$$1 - a - ra' + \frac{2\alpha(a-1)a'}{r} - \frac{\alpha(a-1)^2}{r^2} + \frac{3r^2}{l^2} + 2\beta^2 r^2 \left(1 - \sqrt{1 + \frac{Q^2}{r^4 \beta^2}} \right) = 0, \quad (39)$$

$$1 + r^2 \lambda e^{-2\phi} - a(r\phi' - 1)^2 = 0, \quad (40)$$

$$2\beta^2 \phi' - 2\phi'^3 + r\beta^2 \phi'' = 0. \quad (41)$$

It is obviously that $b'(r) = 0$ when Eq. (40) holds, hence $b(r) = 0$ is indeed a consistent choice. By solving the equations, we have the solution

$$\Phi(r) = \frac{Q}{r} {}_2F_1 \left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -\frac{Q^2}{\beta^2 r^4} \right), \quad (42)$$

$$a(r) = 1 + \frac{r^2}{2\alpha} \left\{ 1 \pm \left[1 + 4\alpha \left(\frac{2M}{r^3} - \frac{1}{l^2} - \frac{2\beta^2}{3} \left(1 - \sqrt{1 + \frac{Q^2}{\beta^2 r^4}} \right) - \frac{4Q}{3r^3} \Phi \right) \right]^{\frac{1}{2}} \right\} \quad (43)$$

$$\phi(r)_{\pm} = \ln \frac{r}{L} + \ln \left[\cosh \psi \pm \sqrt{1 + \lambda L^2} \sinh \psi \right], \quad (44)$$

where L is an arbitrary integration constant and $\psi(r) = \int_{r_h}^r \frac{du}{u\sqrt{a|u|}}$ [51]. So we regain the black hole solution in this regularized 4D EGB gravity. Note that the electrostatic potential $\Phi(r)$ and the metric function $a(r)$ are independent of the parameter λ , which is associated with the curvature of the internal maximally symmetric space.

5 Conclusions

In this work, we obtained the BI electric field charged black hole solution in the novel 4D EGB gravity with a negative cosmological constant. In order to have a real black hole solution, the GB coupling parameter α and BI parameter β have to be constrained in some regions. It is known that the Maxwell charged solution is real only for negative α . While the BI charged black hole solution found in this paper can be real for both positive and negative α . Therefore, the BI charged black hole may be superior to the Maxwell charged black hole to be a charged extension of the Schwarzschild-AdS-like black hole in the novel 4D EGB gravity. The black hole has zero, one, or two horizons depending on the parameters. However, since particles incident from infinity can reach the singularity, the black hole solution still suffers the singularity problem. We also explored some simple thermodynamic properties of the BI charged black hole solution. The Smarr formula and the first law of black hole thermodynamics were verified. By evaluating the specific heat and Gibb's free energy, we showed that the black hole is thermodynamically stable when

the horizon radius is large, but is unstable when it is small. This is a well-known property of the AdS black holes. Moreover, a black hole with positive α has larger stable region than that with negative α . At last, we also regained the black hole solution in the regularized 4D EGB gravity proposed by H. Lü and Y. Pang.

Black hole thermodynamics in AdS space has been of great interest since it possesses some interesting phase transitions and critical phenomena as seen in normal thermodynamic systems. Further thermodynamic properties of the BI charged black hole in the novel 4D EGB gravity is left for our future investigations.

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References

1. D. Lovelock, The Einstein tensor and its generalizations. *J. Math. Phys.* **12**, 498 (1971)
2. D. Lovelock, The four-dimensionality of space and the Einstein tensor. *J. Math. Phys.* **13**, 874 (1972)
3. T. Clifton, P.G. Ferreira, A. Padilla, C. Skordis, Modified gravity and cosmology. *Phys. Rep.* **513**, 1 (2012). [arXiv:1106.2476](https://arxiv.org/abs/1106.2476)
4. M. Ishak, Testing general relativity in cosmology. *Living Rev. Rel.* **22**, 1 (2019). [arXiv:1806.10122](https://arxiv.org/abs/1806.10122)
5. S. Odintsov, V. Oikonomou, F. Fronimos, Rectifying Einstein-Gauss-Bonnet Inflation in View of GW170817. [arXiv:2003.13724](https://arxiv.org/abs/2003.13724)
6. S. Odintsov, V. Oikonomou, Swampland implications of GW170817-compatible Einstein-Gauss-Bonnet gravity. *Phys. Lett. B* **805**, 135437 (2020). [arXiv:2004.00479](https://arxiv.org/abs/2004.00479)
7. B. Zwiebach, Curvature squared terms and string theories. *Phys. Lett. B* **156**, 315 (1985)
8. Y. Tomozawa, Quantum corrections to gravity. [arXiv:1107.1424](https://arxiv.org/abs/1107.1424)
9. G. Cognola, R. Myrzakulov, L. Sebastiani, S. Zerbini, Einstein gravity with Gauss-Bonnet entropic corrections. *Phys. Rev. D* **88**, 024006 (2013). [arXiv:1304.1878](https://arxiv.org/abs/1304.1878)

10. D. Glavan, C. Lin, Einstein-Gauss-Bonnet gravity in 4-dimensional space-time. *Phys. Rev. Lett.* **124**, 081301 (2020). [arXiv:1905.03601](#)
11. M. Guo, P.-C. Li, The innermost stable circular orbit and shadow in the novel 4D Einstein-Gauss-Bonnet gravity. *Eur. Phys. J. C* **80**, 588 (2020). [[arXiv:2003.02523](#)]
12. K. Hegde, A. Naveena Kumara, C. A. Rizwan, A. K. M., M. S. Ali, Thermodynamics, Phase Transition and Joule Thomson Expansion of Novel 4-D Gauss Bonnet AdS Black Hole. [arXiv:2003.08778](#)
13. Y.-P. Zhang, S.-W. Wei, Y.-X. Liu, Spinning Test Particle in Four-dimensional Einstein-Gauss-Bonnet Black Hole. [arXiv:2003.10960](#)
14. D. V. Singh, S. Siwach, Thermodynamics and P-V Criticality of Bardeen-AdS Black Hole in 4-D Einstein-Gauss-Bonnet Gravity. [arXiv:2003.11754](#)
15. R. Konoplya, A. Zhidenko, (In)stability of black holes in the 4D Einstein-Gauss-Bonnet and Einstein-Lovelock gravities. [arXiv:2003.12492](#)
16. C.-Y. Zhang, P.-C. Li, M. Guo, Greybody factor and power spectra of the Hawking radiation in the novel 4D Einstein-Gauss-Bonnet de-Sitter gravity. [arXiv:2003.13068](#)
17. R. Roy, S. Chakrabarti, A study on black hole shadows in asymptotically de Sitter spacetimes. [arXiv:2003.14107](#)
18. S. A. Hosseini Mansoori, Thermodynamic geometry of novel 4-D Gauss Bonnet AdS black hole. [arXiv:2003.13382](#)
19. S.-W. Wei, Y.-X. Liu, Extended thermodynamics and microstructures of four-dimensional charged Gauss-Bonnet black hole in AdS space. *Phys. Rev. D* **101**, 104018 (2020). [arXiv:2003.14275](#)
20. D. V. Singh, S. G. Ghosh, S. D. Maharaj, Clouds of string in the novel 4D Einstein-Gauss-Bonnet black holes. [arXiv:2003.14136](#)
21. M. Churilova, Quasinormal modes of the Dirac field in the novel 4D Einstein-Gauss-Bonnet gravity. [arXiv:2004.00513](#)
22. S. U. Islam, R. Kumar, S. G. Ghosh, Gravitational lensing by black holes in 4D Einstein-Gauss-Bonnet gravity. [arXiv:2004.01038](#)
23. A. K. Mishra, Quasinormal modes and strong cosmic censorship in the novel 4D Einstein-Gauss-Bonnet gravity. [arXiv:2004.01243](#)
24. C. Liu, T. Zhu, Q. Wu, Thin accretion disk around a four-dimensional Einstein-Gauss-Bonnet black hole. [arXiv:2004.01662](#)
25. R. A. Konoplya, A. F. Zinhailo, Grey-body factors and Hawking radiation of black holes in 4D Einstein-Gauss-Bonnet gravity. [arXiv:2004.02248](#)
26. M. Heydari-Fard, M. Heydari-Fard, H. Sepangi, Bending of light in novel 4D Gauss-Bonnet-de Sitter black holes by Rindler-Ishak method. [arXiv:2004.02140](#)
27. B. Eslam Panah, K. Jafarzade, 4D Einstein-Gauss-Bonnet AdS Black Holes as Heat Engine. [arXiv:2004.04058](#)
28. A. Naveena Kumara, C. A. Rizwan, K. Hegde, M. S. Ali, A. K. M, Rotating 4D Gauss-Bonnet black hole as particle accelerator. [arXiv:2004.04521](#)
29. A. Aragon, R. Becar, P. Gonzalez, Y. Vasquez, Perturbative and nonperturbative quasinormal modes of 4D Einstein-Gauss-Bonnet black holes. [arXiv:2004.05632](#)
30. M. Cuyubamba, Stability of asymptotically de Sitter and anti-de Sitter black holes in 4D regularized Einstein-Gauss-Bonnet theory. [arXiv:2004.09025](#)
31. C.-Y. Zhang, S.-J. Zhang, P.-C. Li, M. Guo, Superradiance and stability of the novel 4D charged Einstein-Gauss-Bonnet black hole. [arXiv:2004.03141](#)
32. S.-J. Yang, J.-J. Wan, J. Chen, J. Yang, Y.-Q. Wang, Weak cosmic censorship conjecture for the novel 4D charged Einstein-Gauss-Bonnet black hole with test scalar field and particle. [arXiv:2004.07934](#)
33. S. Ying, Thermodynamics and weak cosmic censorship conjecture of 4D Gauss-Bonnet-Maxwell black holes via charged particle absorption. [arXiv:2004.09480](#)
34. D. D. Doneva, S. S. Yazadjiev, Relativistic stars in 4D Einstein-Gauss-Bonnet gravity. [arXiv:2003.10284](#)
35. D. Malafarina, B. Toshmatov, N. Dadhich, Dust collapse in 4D Einstein-Gauss-Bonnet gravity. *Phys. Dark Univ.* **30**, 100598 (2020). [arXiv:2004.07089](#)
36. S.-L. Li, P. Wu, H. Yu, Stability of the Einstein Static Universe in 4D Gauss-Bonnet Gravity. [arXiv:2004.02080](#)
37. T. Kobayashi, Effective scalar-tensor description of regularized Lovelock gravity in four dimensions. [arXiv:2003.12771](#)
38. A. Casalino, A. Colleaux, M. Rinaldi, S. Vicentini, Regularized Lovelock gravity. [arXiv:2003.07068](#)
39. J. Rayimbaev, A. Abdujabbarov, B. Turimov, F. Atamurotov, Magnetized particle motion around 4-D Einstein-Gauss-Bonnet black hole. [arXiv:2004.10031](#)
40. P. Liu, C. Niu, C.-Y. Zhang, Instability of the novel 4D charged Einstein-Gauss-Bonnet de-Sitter black hole. [arXiv:2004.10620](#)
41. P. Liu, C. Niu, X. Wang, C.-Y. Zhang, Thin-shell wormhole in the novel 4D Einstein-Gauss-Bonnet theory. [arXiv:2004.14267](#)
42. J. Bonifacio, K. Hinterbichler, L. A. Johnson, Amplitudes and 4D Gauss-Bonnet Theory. [arXiv:2004.10716](#)
43. X.-X. Zeng, H.-Q. Zhang, H. Zhang, Shadows and photon spheres with spherical accretions in the four-dimensional Gauss-Bonnet black hole. [arXiv:2004.12074](#)
44. X.-H. Ge, S.-J. Sin, Causality of black holes in 4-dimensional Einstein-Gauss-Bonnet-Maxwell theory. [arXiv:2004.12191](#)
45. W.-Y. Ai, A note on the novel 4D Einstein-Gauss-Bonnet gravity. [arXiv:2004.02858](#)
46. M. Gurses, T. C. Sisman, B. Tekin, Is there a novel Einstein-Gauss-Bonnet theory in four dimensions? [arXiv:2004.03390](#)
47. S. Mahapatra, A note on the total action of 4D Gauss-Bonnet theory. [arXiv:2004.09214](#)
48. F.-W. Shu, Vacua in novel 4D Einstein-Gauss-Bonnet gravity: Pathology and instability. [arXiv:2004.09339](#)
49. S. Tian, Z.-H. Zhu, Comment on "Einstein-Gauss-Bonnet Gravity in Four-Dimensional Spacetime". [arXiv:2004.09954](#)
50. J. Arrechea, A. Delhom, A. Jimnez-Cano, Yet another comment on four-dimensional Einstein-Gauss-Bonnet gravity. [arXiv:2004.12998](#)
51. H. Lu, Y. Pang, Horndeski Gravity as D Limit of Gauss-Bonnet. [arXiv:2003.11552](#)
52. P. G. Fernandes, P. Carrilho, T. Clifton, D. J. Mulryne, Derivation of Regularized Field Equations for the Einstein-Gauss-Bonnet Theory in Four Dimensions. [arXiv:2004.08362](#)
53. R. A. Hennigar, D. Kubiznak, R. B. Mann, C. Pollack, On Taking the $D \rightarrow 4$ Limit of Gauss-Bonnet Gravity: Theory and Solutions. [arXiv:2004.09472](#)
54. P.G.S. Fernandes, Charged black holes in AdS spaces in 4D Einstein Gauss-Bonnet gravity. *Phys. Lett. B* **805**, 135468 (2020). [arXiv:2003.05491](#)
55. A. Kumar, S. G. Ghosh, Hayward black holes in the novel 4D Einstein-Gauss-Bonnet gravity. [arXiv:2004.01131](#)
56. A. Kumar, R. Kumar, Bardeen black holes in the novel 4D Einstein-Gauss-Bonnet gravity. [arXiv:2003.13104](#)
57. S.-W. Wei, Y.-X. Liu, Testing the nature of Gauss-Bonnet gravity by four-dimensional rotating black hole shadow. [arXiv:2003.07769](#)
58. R. Kumar, S. G. Ghosh, Rotating black holes in the novel 4D Einstein-Gauss-Bonnet gravity. [arXiv:2003.08927](#)
59. S. G. Ghosh, R. Kumar, Generating black holes in the novel 4D Einstein-Gauss-Bonnet gravity. [arXiv:2003.12291](#)
60. S. G. Ghosh, S. D. Maharaj, Radiating black holes in the novel 4D Einstein-Gauss-Bonnet gravity. [arXiv:2003.09841](#)
61. S. G. Ghosh, S. D. Maharaj, Noncommutative inspired black holes in regularised 4D Einstein-Gauss-Bonnet theory. [arXiv:2004.13519](#)
62. M. Born, L. Infeld, Foundations of the new field theory. *Proc. R. Soc. Lond. A* **144**, 425 (1934)

63. E. Fradkin, A. Tseytlin, Non-linear electrodynamics from quantized strings. *Phys. Lett. B* **163**, 123 (1985)
64. D.A. Garcia, I.H. Salazar, J.F. Plebanski, Type-d solutions of the Einstein and Born-Infeld nonlinear-electrodynamics equations. *Nuovo Cimento B* **84**, 65 (1984)
65. M. Demianski, Static electromagnetic geon. *Found. Phys.* **16**, 187 (1986)
66. T.K. Dey, Born-Infeld black holes in the presence of a cosmological constant. *Phys. Lett. B* **595**, 484 (2004). [arXiv:hep-th/0406169](https://arxiv.org/abs/hep-th/0406169)
67. R.-G. Cai, D.-W. Pang, A. Wang, Born-Infeld black holes in (A)dS spaces. *Phys. Rev. D* **70**, 124034 (2004). [arXiv:hep-th/0410158](https://arxiv.org/abs/hep-th/0410158)
68. M. Aiello, R. Ferraro, G. Giribet, Exact solutions of Lovelock-Born-Infeld black holes. *Phys. Rev. D* **70**, 104014 (2004). [arXiv:gr-qc/0408078](https://arxiv.org/abs/gr-qc/0408078)
69. O. Miskovic, R. Olea, Thermodynamics of Einstein-Born-Infeld black holes with negative cosmological constant. *Phys. Rev. D* **77**, 124048 (2008). [arXiv:0802.2081](https://arxiv.org/abs/0802.2081)
70. S. Gunasekaran, R.B. Mann, D. Kubiznak, Extended phase space thermodynamics for charged and rotating black holes and Born-Infeld vacuum polarization. *JHEP* **11**, 110 (2012). [[arXiv:1208.6251](https://arxiv.org/abs/1208.6251)]
71. A. Dehyadegari, A. Sheykhi, Reentrant phase transition of Born-Infeld-AdS black holes. *Phys. Rev. D* **98**, 024011 (2018). [arXiv:1711.01151](https://arxiv.org/abs/1711.01151)
72. C. W. Misner, K. S. Thorne, J. A. Wheeler, *Gravitation*, WH Gravitation-San Francisco (1973)
73. R.-G. Cai, K.-S. Soh, Topological black holes in the dimensionally continued gravity. *Phys. Rev. D* **59**, 044013 (1999). [arXiv:gr-qc/9808067](https://arxiv.org/abs/gr-qc/9808067)
74. R.-G. Cai, L.-M. Cao, N. Ohta, Black holes in gravity with conformal anomaly and logarithmic term in black hole entropy. *JHEP* **04**, 082 (2010). [arXiv:0911.4379](https://arxiv.org/abs/0911.4379)
75. D. Kastor, S. Ray, J. Traschen, Enthalpy and the mechanics of AdS black holes class. *Quant. Grav.* **26**, 195011 (2009). [arXiv:0904.2765](https://arxiv.org/abs/0904.2765)
76. D. Kubiznak, R.B. Mann, P-V criticality of charged AdS black holes. *JHEP* **07**, 033 (2012). [arXiv:1205.0559](https://arxiv.org/abs/1205.0559)