## SCISPACE <br> formerly Typeset

¡ Open access • Journal Article • DOI:10.1162/REST.88.2.348
Borrowing Costs and the Demand for Equity over the Life Cycle - Source link
Steven J. Davis, Felix Kubler, Paul S. Willen
Institutions: University of Chicago, University of Mannheim, Federal Reserve Bank of Boston
Published on: 12 Jun 2006 - The Review of Economics and Statistics (MIT press journals)
Topics: Equity (finance), Rate of return and Expected return

Related papers:

- Consumption and Portfolio Choice over the Life Cycle
- Consumption Over the Life Cycle
- Optimal Life-Cycle Asset Allocation: Understanding the Empirical Evidence
- Optimal Portfolio Choice for Long-Horizon Investors with Nontradable Labor Income
- Portfolio Choice in the Presence of Housing

Share this paper: 1 in in $\square$
View more about this paper here: https://typeset.io/papers/borrowing-costs-and-the-demand-for-equity-over-the-life3q67y6cfog

NBER WORKING PAPER SERIES

# BORROWING COSTS AND THE DEMAND FOR EQUITY OVER THE LIFE CYCLE 

Steven J. Davis<br>Felix Kubler<br>Paul Willen

Working Paper 9331
http://www.nber.org/papers/w9331

NATIONAL BUREAU OF ECONOMIC RESEARCH<br>1050 Massachusetts Avenue<br>Cambridge, MA 02138

November 2002

We thank seminar participants at the 2002 Minnesota Macroeconomics Summer Institute, the 2002 NBER Summer Institute, the 2002 NBER Economic Fluctuations Program Meeting in Chicago, Notre Dame University and the University of Chicago for many helpful comments, Amir Yaron and Muhammet Guvenen for thoughtful remarks, Jonathan Parker for providing parameters of the income processes in Gourinchas and Parker (2002), Nick Souleles for directing us to data on charge-off rates for consumer loans and David Arnold, Jeremy Nalewaik and Stephanie Curcuru for able research assistance. Davis and Willen gratefully acknowledge research support from the Graduate School of Business at the University of Chicago. The views expressed herein are those of the authors and not necessarily those of the National Bureau of Economic Research.
© 2002 by Steven J. Davis, Felix Kubler, and Paul Willen. All rights reserved. Short sections of text, not to exceed two paragraphs, may be quoted without explicit permission provided that full credit, including © notice, is given to the source.

Borrowing Costs and the Demand for Equity Over the Life Cycle
Steven J. Davis, Felix Kubler, and Paul Willen
NBER Working Paper No. 9331
November 2002
JEL No. D91, G11, G12


#### Abstract

We analyze consumption and portfolio behavior in a life-cycle model with realistic borrowing costs and income processes. We show that even a small wedge between borrowing costs and the risk-free return dramatically shrinks the demand for equity. When the cost of borrowing equals or exceeds the expected return on equity - the relevant case according to the data - households hold little or no equity during much of the life cycle. The model also implies that the correlation between consumption growth and equity returns is low at all ages, and that risk aversion estimates based on the standard excess return formulation of the consumption Euler Equation are greatly upward biased. The demand for equity in the model is non-monotonic in borrowing costs and risk aversion, and the standard deviation of marginal utility growth is an order of magnitude smaller than the Sharpe ratio.


Steven J. Davis<br>Graduate School of Business<br>University of Chicago<br>1101 East 58th Street<br>Chicago, IL 60637<br>and NBER<br>steve.davis@gsb.uchicago.edu

Felix Kubler<br>Department of Economics<br>Stanford University<br>Stanford, CA 94305-6010<br>fkubler@stanford.edu

Paul Willen
Graduate School of Business
University of Chicago
1101 East 58th Street
Chicago, IL 60637
paul.willen@gsb.uchicago.edu

## 1 Introduction

This paper analyzes consumption and portfolio behavior in a life-cycle model with borrowing costs that exceed the risk-free investment return. The agents have standard time-separable preferences with modest risk aversion and mild impatience, face realistic income processes, and can invest in risky and risk-free assets.

We show that a wedge between borrowing costs and the risk-free return has several important effects on consumption and portfolio behavior. First, even a modest wedge dramatically shrinks the demand for equity throughout the life cycle. Second, when the borrowing rate equals or exceeds the expected return on equity - the relevant case according to the data - households hold little or no equity until middle age. Third, the correlation between consumption growth and equity returns rises with age, but it is low at all ages for reasonable parameter choices. Fourth, risk aversion estimates based on the standard excess return formulation of the consumption Euler Equation are greatly upward biased. The bias diminishes, but remains large, for "samples" of households with positive equity holdings. Fifth, households borrow to finance consumption but not to finance large equity positions. In each respect, the introduction of a realistic wedge between borrowing costs and the risk-free return greatly improves the fit between theory and empirical evidence.

Table 1 reports data on the size of the wedge. The bottom two rows show that household borrowing costs on unsecured loans exceed the risk-free return by about six to nine percentage points on an annual basis, after adjusting for tax considerations and charge-offs for uncollected loan obligations. Since 1987, roughly two percentage points arise from the asymmetric income tax treatment of household interest receipts and payments. However, the bulk of the wedge arises from transactions costs in the loan market. Despite the evident size of these transaction costs, they have been largely ignored in theoretical analyses of life-cycle consumption and portfolio behavior. They have also been ignored in most empirical studies of asset-pricing behavior.

Aside from the wedge between borrowing costs and the risk-free return, our lifecycle model is entirely standard. None of our results rely on strong risk aversion, a high degree of impatience, habit formation, other types of nonseparability, self-control problems or myopia. Nor do they rely on equity market participation costs, informational barriers, time-varying asset returns, enforcement problems in loan markets, or hard borrowing limits (other than the lifetime budget constraint).

According to our analysis, the cost of borrowing and the shape of the expected life-cycle income profile are key determinants of equity accumulation. The variance of undiversifiable labor income shocks also has important effects on equity holdings, and the risky nature of labor income is an important source of cross-sectional heterogeneity in consumption and asset holdings conditional on age.

Our analysis also highlights several other results. First, equity holdings and market participation rates are non-monotonic in the cost of borrowing, both reaching a minimum where the borrowing cost equals the expected return on equity. As this result suggests, the properties of our model are not "in between" those of the canonical model with equal borrowing and lending rates and alternative models with hard borrowing limits. Second, relative to a model with a no-borrowing constraint, households defer equity market participation when faced with realistic borrowing costs. As a consequence, households accumulate less equity over the life cycle when faced with realistic borrowing costs as compared to a no-borrowing constraint. Third, equity holdings and participation rates are non-monotonic in the degree of risk aversion when borrowing costs exceed the risk-free return. Fourth, once we consider realistic borrowing costs, neither sizable deviations from optimal portfolio shares nor delayed participation in equity markets until, say, age 50 involves large welfare costs.

On the asset-pricing front, our analysis helps resolve two closely related aspects of the equity premium puzzle: Why do people hold so little equity when faced with a substantial equity premium? And, why is the covariance between consumption
growth and asset returns so low? Answers to both question follow from the high borrowing costs reported in Table 1. These data imply a small or negative "leverage" premium on borrowed funds that are invested in the stock market. This fact, when combined with upward sloping income profiles or mildly impatient consumers, means that households accumulate little or no equity until middle age, and they can have modest equity holdings even on the verge of retirement.

The paper proceeds as follows. The balance of the introduction discusses related research and reviews some well-established facts about life-cycle consumption and portfolio behavior. Sections 2 and 3 describe the model and the choice of parameters. Section 4 discusses life-cycle portfolio choice and consumption in our model, and section 5 compares model implications with empirical evidence. Section 6 offers some concluding remarks, and an appendix describes our numerical solution method.

### 1.1 Relationship to the Literature

Our model departs modestly from the seminal work on life-cycle portfolio behavior by Merton (1969) and Samuelson (1969). Indeed, our model is identical to Samuelson's discrete-time setup except for three elements: the wedge between borrowing costs and risk-free returns, the presence of undiversifiable income shocks, and the use of realistic income profiles. The wedge and the undiversifiable labor income shocks necessitate a computational approach to the analysis, which we pursue using the same methods as in Judd, Kubler and Schmedders (2002).

We also build on other research in finance and macroeconomics. Brennan (1971) shows that a wedge between borrowing costs and risk-free returns is easily handled in the standard one-period model of mean-variance portfolio choice. The wedge implies that households cannot attain points above and to the right of the tangency portfolio along the standard capital market line. Higher borrowing costs reduce the demand for equity in the one-period setting, given standard mean-variance preferences. Heaton
and Lucas (1997) show that a borrowing rate that exceeds the risk-free return reduces equity holdings for an infinitely lived agent.

We incorporate key life-cycle elements into a many-period setting with realistic borrowing costs and endogenous wealth accumulation. In our model, unlike Brennan or Heaton-Lucas, higher borrowing costs raise the demand for equity in reasonable circumstances. The causal mechanism behind this result involves the impact of borrowing costs on precautionary savings and life-cycle asset accumulation. More generally, life-cycle factors play a central role in both equity market participation and equity accumulation behavior in our model.

Many researchers have explored the effects of hard borrowing limits on portfolio choice in life-cycle models. For example, Constantinides, Donaldson and Mehra (2002) consider a three-period model with no borrowing, and Gomes and Michaelides (2002) consider a calibrated many-period life-cycle model. We show that allowing households to borrow at a rate above the risk-free return yields more realistic behavior with respect to asset accumulation, equity market participation and borrowing itself than a blanket prohibition on borrowing.

Section 5 shows that our model generates a standard deviation of marginal utility growth that is an order of magnitude smaller than the Sharpe ratio. This result appears inconsistent with Hansen and Jagannathan (1991), who argue that the Sharpe ratio represents a lower bound on the standard deviation of marginal utility growth. Our result should not come as a complete surprise, however. Other researchers (He and Modest, 1995, and Luttmer, 1996) show that the existence of trading frictions can potentially reduce the implied lower bound on the volatility of marginal utility growth. We show that one particular, and easily quantified, friction does indeed reconcile a high Sharpe ratio with low volatility of marginal utility growth.

### 1.2 Some facts of life-cycle consumption and portfolio choice

Three well-established sets of empirical results are relevant to an assessment of our model. First, a large percentage of households hold no equity - a phenomenon sometimes referred to as the "participation puzzle." According to Vissing-Jorgenson (2002), only 44 percent of households held stock in 1994, a big increase over the 28 percent figure for 1984. Participation rates rise with age (Poterba and Samwick, 2001), education and income (Mankiw and Zeldes, 1991, Brav and Geczy, 1995), and self-employed workers are more likely to hold stock (Heaton and Lucas, 2000). To a large degree, low equity market participation can be traced to the fact the many households have little or no financial wealth (Lusardi et al., 2001).

Second, most households that do hold equity hold very little. Vissing-Jorgensen (2002) reports that the median level of equity holdings for stockholding households is about 21 thousand dollars, and the mean is 95 thousand dollars. Ameriks and Zeldes (2001) find that the level of stockholding rises with education, income and age.

Third, at least as far back as Grossman and Shiller (1982), many researchers have observed that the covariance of consumption growth and equity returns is very low, even for most households that hold equity. That is, given standard preferences and a plausible degree of risk aversion, the estimated covariance violates standard consumption-based asset pricing relations. Several researchers have also shown, however, that the covariance is higher for households that hold equity. See, for example, Mankiw and Zeldes (1991), Brav and Geczy (1995), Brav, Constantinides and Geczy (2002) and Attanasio, Banks and Tanner (2002).

## 2 A Life-Cycle Model

We consider a partial equilibrium model of household consumption and portfolio choice. The household life cycle consists of two phases, work and retirement, which
differ with respect to the character of labor income. During the working years, log labor income $\left(\tilde{y}_{t}\right)$ evolves as the sum of a deterministic component $\left(d_{t}\right)$, a random walk component $\left(\tilde{\eta}_{t}\right)$, and an uncorrelated transitory shock $\left(\tilde{\varepsilon}_{t}\right)$ :

$$
\tilde{y}_{t}=d_{t}+\tilde{\eta}_{t}+\tilde{\varepsilon}_{t} .
$$

This type of income process is widely used in life-cycle studies of consumption and asset accumulation.

During the retirement years, a household receives a fraction of its income in the last year of work. Ideally, we would specify retirement income as some fraction of, say, the highest $n$ years of work income - consistent with most defined benefit pension plans and with social security. However, such a structure is computationally burdensome, because it increases the dimensionality of the state space. As a computationally easier alternative, we first calculate the ratio of the average value of $d_{t}$ in the highest $n$ working years to the value of $d$ in the last year of work. We then multiply this ratio by realized income in the last year of work to get the retirement basis. Finally, to get retirement income, we multiply the retirement basis by a number between zero and one called the replacement rate.

To see why this procedure is useful, look at Figure 1, which shows expected labor income profiles for various education groups. Note, for example, that expected labor income in the final year of work is lower relative to lifetime expected income for college-educated households than for households with post-graduate education. Hence, setting retirement income to the same fraction of income in the last working year for all households would effectively assign a relatively low replacement rate to college-educated households. The retirement basis serves to adjust for differences among education groups in the shape of the lifetime expected income profile.

Households can trade three financial assets. They can buy equity with stochastic net return $\tilde{r}_{E}$, save at a net risk-free rate $r_{L}$, and borrow at the rate $r_{B} \geq r_{L}$. Households cannot take short positions in equity, nor can they borrow negative amounts.

Net indebtedness cannot exceed the present value of the household's lowest possible future income stream, discounted at the borrowing rate of interest.

A household chooses a contingency plan for consumption, borrowings and asset holdings at date $t$ to maximize

$$
U\left(c_{t}\right)+\mathrm{E}_{t} \sum_{a=t+1}^{T} \beta^{a-t} U\left(\tilde{c}_{a}\right)
$$

subject to a sequence of budget constraints, where $c_{a}$ is consumption at age $a$, $\mathrm{E}_{t}$ is the expectations operator conditional on time- $t$ information, $\beta$ is a time discount factor, and $U(\cdot)$ is an isoelastic utility function. We solve numerically for the optimal solution using a backward induction algorithm, as described in the appendix.

## 3 Parameter Settings and Discretization

Tables 2 and 3 summarize our parameter settings. We set the coefficient of relative risk aversion to 2 in our baseline specification, but we also report results for other values ranging from 0.5 to 8 . We set the annual time discount factor to $.95 .{ }^{1}$

Following Campbell (1999), we set the annual risk-free investment return to $2 \%$, the expected return on equity to $8 \%$ and the standard deviation of equity returns to $15 \%$. We set the correlation of equity returns and labor income shocks to zero. ${ }^{2}$ In line with Table 1, we set the baseline borrowing rate to $8 \%$.

[^0]For the life-cycle income processes, we adopt parameter values estimated by Gourinchas and Parker (2002) from the Consumer Expenditure Survey (CEX) and the Panel Study of Income Dynamics (PSID). The GP income measure is "after-tax family income less social security tax payments, pension contributions, after-tax asset and interest income" in 1987 dollars. GP also subtract "education, medical care and mortgage interest payments" from their measure of income, because "these categories of expenditure do not provide current utility but rather are either illiquid investments or negative income shocks." ${ }^{3}$ They restrict their sample to male-headed households and attribute the head's age and education to the entire household.

To estimate the deterministic component of income for each education group, GP fit a fifth-order polynomial in the head's age to CEX data on log family income. They also fit a fifth-order polynomial to their entire sample, pooled over the five education groups. To estimate the standard deviation of transitory and permanent income shocks, GP use the longitudinal aspect of the PSID. Since the income measures reported in household surveys contain much measurement error, the raw variance estimates substantially overstate income uncertainty. To adjust for this overstatement, we adopt GP's suggestion to reduce the estimated variance of the transitory shock by one half and the variance of the permanent shock by one third. Table 3 reports the standard deviation of the income shocks after adjusting for measurement error.

The expected income profiles displayed in Figure 1 reflect three elements of the GP income processes: (i) the profile of the deterministic component; (ii) the variance of the transitory shock to log income, which affects the level of expected income; and (iii) the variance of the permanent shock, which affects the level and slope of expected income. The profile for the middle education group, not displayed in Figure 1, is very similar to the profile for the pooled sample.

We discretize the state-space using the Tauchen and Hussey (1991) method. Our

[^1]model has three sources of randomness: a permanent labor income shock, a transitory income shock and an asset return shock. We specify two discrete points for the permanent shock, two points for the transitory shock and three points for the asset return shock, so that the random shocks obey a twelve-state Markov chain.

Our discretization procedure does not generate zero income in any state of nature. In this respect, our specification differs from that of GP. The difference is not innocuous: by assuming that agents have non-zero probability of zero income, GP preclude borrowing. ${ }^{4}$ In our setup, households can and do borrow.

In our numerical analysis below, we often turn off one or both labor income shocks. We do this for two reasons: first, to help understand the impact of income uncertainty on equity demand and, second, to show that many of our results do not rest on income uncertainty. Whenever we shut off one or both income shocks, we also readjust the deterministic income component to preserve the same expected income profile.

## 4 The demand for equity over the life cycle

Before analyzing portfolio choice, we first introduce some terminology and provide intuition. We then explore how four parameters of the household decision problem affect the demand for equity over the life cycle: (1) the borrowing rate, (2) risk aversion, (3) undiversifiable labor income shocks, and (4) the shape of the income profile. Lastly, we turn to the issue of non-participation in equity markets.

Borrowing capacity is the present value of future labor income (including retirement income), when discounted at the borrowing rate, along the lowest possible future

[^2]income path. The equity premium is the difference between the expected return on equity and the risk-free investment return. The leverage premium is the difference between the expected equity return and the borrowing rate. When the cost of borrowing exceeds the risk-free investment return, the equity premium exceeds the leverage premium. Hence, the net return on equity depends on the source of funds invested, as depicted in the following table.

| Source of funds | Opportunity cost | Net equity return |
| :--- | :--- | :--- |
| Liquid wealth | Risk-free return | Equity premium |
| Borrowing capacity | Borrowing rate | Leverage premium |

Given a realistic equity premium (say $6 \%$ ), a household invests all or much of its liquid wealth in equity. When the leverage premium is positive, a household may also borrow to finance equity holdings, but it holds less equity than an otherwise similar household with greater financial wealth. When the leverage premium is negative, households do not draw on borrowing capacity to invest in equity. That is, with a zero or negative leverage premium, equity demand varies closely (often one-forone) with liquid wealth. In turn, liquid wealth depends principally on the shape of the lifetime income profile and the strength of consumption-smoothing motives. For example, a household with a sharply upward-sloping income profile saves little, and thus acquires little equity, early in life. The upshot is that the evolution of liquid wealth over the life cycle is a key determinant of equity demand.

### 4.1 Effect of the borrowing rate

How does the borrowing rate affect the demand for equity over the life cycle? First, a higher borrowing cost lowers borrowing capacity by reducing the present value of labor income. Second, a higher borrowing rate lowers the leverage premium. And third, the borrowing rate affects the evolution of wealth over the life cycle. A low borrowing rate depresses liquid wealth by encouraging greater borrowing for consumption smoothing
purposes and by substituting for precautionary wealth holdings that households would otherwise accumulate to smooth transitory income shocks. But a low borrowing rate can also increase liquid wealth: if the leverage premium is positive, borrowing to invest in equity enables the household to increase wealth over time.

As these remarks suggest, there is a non-monotonic relationship between the cost of borrowing and the demand for equity. For example, consider our baseline specification with no labor income risk for a household from the pooled sample. Figure 2 shows its life-cycle equity holdings (averaged over many draws) for alternative borrowing rates. When the borrowing rate equals the risk-free return of $2 \%$, households invest large amounts in equity throughout the life cycle, a result that is not sensitive to the shape of the income profile. Thus, the standard model with $r_{B}=r_{L}$ implies equity holdings that greatly exceed what we see in the data.

A borrowing rate of $5 \%$ yields much lower equity holdings throughout the life cycle. Why? An increase in the borrowing rate from $2 \%$ to $5 \%$ implies a reduction in the leverage premium from $6 \%$ to $3 \%$ and a decline in borrowing capacity. The effect on a very young household is easily understood: since it has no liquid wealth, a smaller leverage premium and lower borrowing capacity mean lower equity demand. Less obviously, the disparity in equity holdings persists into retirement. Two forces are at work. First, households with a non-zero replacement rate still have borrowing capacity in retirement. As shown in Figure 3, households with a positive leverage premium continue to borrow until the year before death. So even in retirement, the size of the leverage premium affects equity demand. Second, a higher leverage premium earlier in life leads, in expectation, to higher wealth accumulation by retirement, as illustrated in Figure 4. A household with a $2 \%$ borrowing rate has much greater liquid wealth at retirement than a household with a $5 \%$ borrowing rate.

Equity demand behaves differently when the leverage premium is zero or negative. With a borrowing rate equal to $8 \%$, the household from the pooled sample does not
invest in equity until age 54 (Figure 2). That is, it never pays to invest borrowing capacity in equity when the leverage premium is zero or negative. However, the household still borrows when young for consumption-smoothing purposes (Figure 3), provided that the cost of borrowing is not too high, so that liquid wealth is negative during much of the life cycle. Although the household from the pooled sample with a zero leverage premium stops borrowing after age 53, asset accumulation and equity demand are smaller than cases with a positive leverage premium.

At borrowing rates above $8 \%$, households hold more equity than at a borrowing rate equal to $8 \%$ (Figure 2). In this region, higher borrowing rates lead to less borrowing early in life (Figure 3), earlier participation in equity markets (Figure 2) and greater liquid wealth at retirement (Figure 4).

Figure 5 shows average demand for equity as a function of the borrowing rate, computed using the age and education weights in the Gourinchas-Parker CEX sample. As seen in Figure 5, average equity demand is smallest when the borrowing rate equals the expected return on equity. At this trough, equity demand in the model with risky labor income is less than $10 \%$ of its value in an otherwise identical model with borrowing rate equal to the risk-free return. For the model with no labor income risk, average equity demand at $r_{B}=\mathrm{E}\left(\tilde{r}_{E}\right)$ is less than $1 \%$ of its value at $r_{B}=r_{L}$.

Further increases in the cost of borrowing above $\mathrm{E}\left(\tilde{r}_{E}\right)$ raise average equity demand. In particular, higher borrowing costs discourage consumption smoothing through the loan market, so that households begin accumulating wealth earlier in the life cycle. Note, however, that the equity demand function is rather flat to the right of $r_{B}=\mathrm{E}\left(\tilde{r}_{E}\right)$. Hence, when borrowing rates equal or exceed the return on equity, the demand for equity is an order of magnitude smaller than in a standard model with equal borrowing and lending rates.

To sum up, we emphasize three points. First, even a modest wedge between borrowing and lending rates sharply reduces the demand for equity. Second, a borrowing
rate equal to the return on equity minimizes the demand for equity. This result is particularly notable since the borrowing rates reported in Table 1 lie near estimates of the expected return on equity. Third, introducing a realistic borrowing rate into an otherwise standard life-cycle model dramatically shrinks the demand for equity.

### 4.2 Effect of undiversifiable labor income risk

How does undiversifiable labor income risk affect the demand for equity over the life cycle? First, greater income risk makes households with proper preferences effectively more risk averse, which reduces equity demand at given levels of liquid wealth and borrowing capacity. Second, greater income risk intensifies the precautionary saving motive, which encourages wealth accumulation for consumption-smoothing purposes. These two effects work in opposite directions.

Figure 6 shows that the first effect dominates when $r_{B}=r_{L}$, so that labor income uncertainty lowers equity holdings. In contrast, the second effect dominates when $r_{B}=\mathrm{E}\left(\tilde{r}_{E}\right)$. This case differs from the case with $r_{B}=r_{L}$ for two reasons. First, when $r_{B}=\mathrm{E}\left(\tilde{r}_{E}\right)$, younger households hold no equity in the absence of income uncertainty. Hence, they cannot offload risk by reducing their equity holdings, and the first effect is shut off. Second, it is more costly to rely on borrowing as a consumption-smoothing device at a high interest rate, so that the precautionary motive for asset accumulation becomes stronger. As a result, income uncertainty increases equity demand when $r_{B}=\mathrm{E}\left(\tilde{r}_{E}\right) .{ }^{5}$

### 4.3 Effect of relative risk aversion

How does an increase in the relative risk aversion parameter affect the demand for equity over the life cycle? Greater risk aversion lowers a household's appetite for risk,

[^3]and its demand for equity, at a given level of liquid wealth. But risk aversion also has a powerful effect on the evolution of liquid wealth over the life cycle. Higher risk aversion means higher precautionary savings, which raises wealth. Higher risk aversion also means a lower elasticity of substitution under our preference specification, which leads to more borrowing and less wealth accumulation with a rising income profile.

As these remarks suggest, stronger risk aversion can mean higher or lower equity demand, and the effects can vary significantly with age. When the borrowing rate equals the risk-free return, higher risk aversion leads to lower equity holdings throughout the life cycle. When the borrowing rate equals the return on equity, the story is more complicated, as shown in Figure 7. Observe that a household with RRA=1 holds less equity throughout life than a household with $R R A=.5$ or one with $\mathrm{RRA}=4$. A household with $\mathrm{RRA}=8$ holds more equity early in life than any other case shown, but it holds less equity at the end of life.

What drives these results? Since the leverage premium is zero, the main issue is how liquid assets evolve over the life cycle. A household with $\mathrm{RRA}=0.5$ has a high elasticity of substitution, which makes it willing to reduce consumption early in life. As a result, it borrows less early in life than a household with $\mathrm{RRA}=1$. In turn, lower borrowing and more investment lead to higher wealth accumulation throughout the life cycle. A household with $\mathrm{RRA}=4$ accumulates assets early in life because of a strong precautionary motive. The additional savings early in life lead to greater wealth throughout life than the household with $R R A=1$. Finally, what about the household with RRA=8? Early in life, it invests more than the other cases shown, reflecting its strong precautionary saving motive. But all along, unlike the other households, it finds the risk-free asset highly attractive. As it ages, it continues to direct a large fraction of its portfolio to the risk-free asset, so that it experiences a substantially lower return on its asset portfolio.

Figure 8 shows that the average demand for equity is a non-monotonic function
of the risk aversion parameter when borrowing costs exceed the risk-free return. For relative risk aversion below 2 or above 7 , equity demand rises with risk aversion, as predicted by a simpler model with $r_{B}=r_{L}$. For relative risk aversion between 2 and 7 , equity demand is a declining function of the risk aversion parameter. Equity demand is near its minimum for relative risk aversion values near 2 or 3 .

### 4.4 Effect of the expected labor income profile

How does the shape of the expected income profile affect the demand for equity? The answer hinges on the cost of borrowing. When $r_{B}=r_{L}$ and labor income is risky (certain), the shape of the income profile has little (zero) effect on equity demand. In contrast, when $r_{B} \geq \mathrm{E}\left(\tilde{r}_{E}\right)$, the demand for equity is highly sensitive to the shape of the income profile. ${ }^{6}$ The explanation for this sensitivity is straightforward: households borrow only for consumption-smoothing purposes when $r_{B} \geq \mathrm{E}\left(\tilde{r}_{E}\right)$, so they hold no equity until they attain positive financial wealth. The age at which this occurs depends on the shape of the income profile.

Consider the case with $r_{B}=\mathrm{E}\left(\tilde{r}_{E}\right)$. Figure 9 compares life-cycle equity demand for a household from the pooled sample and no labor income risk to an otherwise identical household with a flat income profile. We set income in the flat profile to the simple mean of baseline labor earnings during the working years. The household with a flat profile invests in equity throughout life, whereas the household with the upward-sloping baseline profile waits until age 54. Early investment, compounded by the high return on equity, means that the household with a flat profile accumulates large wealth and equity positions before the baseline household even begins to invest.

Figure 10 shows the effect of the income replacement rate during retirement on equity holdings. In this figure, we vary the replacement rate while holding fixed

[^4]the other income parameters. The more income a household expects to receive in retirement, the less it saves and the less it invests in equity. The nature of retirement income also affect the optimal portfolio mix. When replacement rates are low, or when pensions take the form of defined contribution plans invested heavily in equities, households are more likely to invest in the risk-free asset in addition to equity.

### 4.5 Non-participation in equity markets

When the leverage premium is zero or negative, a household invests in equity if and only if its net liquid wealth position is positive. Since households can borrow in our model, net liquid wealth is often negative or zero, and the household holds no equity. This pattern of borrowing and non-participation in equity markets is fully consistent with rational, time-consistent behavior by patient, mildly risk averse households.

Table 4 shows participation rates for different age groups using the pooled-sample income process and baseline parameter settings. The table highlights several points. First, a borrowing rate above the return on equity raises participation rates conditional on age. This effect on participation arises for the same reason that additional increases in the cost of borrowing above $\mathrm{E}\left(\tilde{r}_{E}\right)$ lead to greater asset holdings.

Second, when permanent income shocks are not too important, a specification with $r_{B}=8 \%$ implies equity market participation rates that are very low for younger households and that then rise gradually after age 40, as seen in the table. In contrast, participation rates are very high throughout life when $r_{B}=99 \%$. Since households do not borrow at a $99 \%$ rate under our parameter settings, this specification is equivalent to a model with a no-borrowing constraint. Thus, a model with realistic borrowing costs delivers much more realistic participation behavior than a model with a noborrowing constraint. ${ }^{7}$

[^5]Third, temporary and permanent income shocks have quite different effects on equity market participation. In other words, the impact of income uncertainty on participation behavior depends on the persistence of the income shocks. We take up this issue in Section 5 below.

Not shown in the table is the ambiguous effect of risk aversion on equity market participation. Participation rates are high for very low levels of risk aversion ( $R R A<$ 1) and for high levels $(R R A>4)$, but they are considerably lower for intermediate levels $(1 \leq R R A \leq 4)$. The explanation for the non-monotonic relationship between participation and risk aversion parallels the explanation given above for the nonmonotonicity in the level of equity holdings.

## 5 Does our model fit the facts?

Section 1 reviewed several facts about life-cycle consumption and portfolio behavior. We now assess the fit between those facts and predictions of the model. Where the model fails to fit the facts, we assess whether the failure is large or small in a welfare sense.

### 5.1 Fact 1: Non-participation in equity markets

To get a better sense of what drives participation behavior in the model, Table 5 shows how average participation rates vary with several aspects of the specification. Two points are immediately clear from Tables 4 and 5 . First, the model can generate low participation rates when borrowing rates exceed the risk-free return. Second, when fit to the GP data on income shock variances, the predicted participation levels exceed those observed in the data.

In evaluating this failure of the model, it is worth emphasizing that we have not

[^6]modelled any liquidity advantage for the risk-free asset. Given the modest liquid wealth positions of many households in our simulations, equity market participation would be much lower if we incorporated some reason to hold small liquid wealth positions in the risk-free asset. We make a related point below when we quantify the welfare costs of non-participation from the vantage point of the model.

Tables 4 and 5 also show that permanent and transitory income shocks affect participation quite differently. The introduction of permanent income shocks raises participation in all scenarios shown, and specifications with only permanent shocks exhibit full participation at all ages. In contrast, transitory income shocks push outcomes away from zero and $100 \%$ participation. Hence, relative to a specification with no income risk, transitory income shocks tend to raise participation at younger ages. But relative to a specification with permanent income shocks, the introduction of transitory income shocks lowers participation at younger ages.

The explanation for these results involves the consumption-smoothing role of borrowing. Borrowing is not helpful in smoothing permanent income shocks, but it is useful for smoothing transitory shocks. In particular, a sufficiently bad transitory shock (or shock sequence) causes the household to draw down liquid wealth and resort to borrowing, at which point it ceases to hold equity. The consumption smoothing role of borrowing is highly sensitive to the borrowing rate - the higher the borrowing rate, the more households act to reduce the likelihood of borrowing. This behavioral response can be seen in Table 4 as a positive relationship between $r_{B}$ and participation, conditional on age, when only transitory income shocks are present. In short, transitory shocks create a motive to hold equity when the household would otherwise hold none, but they also give rise to circumstances in which some households exhaust their asset holdings and turn to borrowing.

Turning to cross-sectional evidence, empirical studies consistently find that equity
market participation rates rise with age. ${ }^{8}$ As seen in Table 4, this empirical regularity is well matched by the predicted life-cycle pattern of participation in the model with transitory income shocks. In this respect, our analysis provides a simple explanation for a widely observed empirical regularity.

Empirical studies also find that participation rates rise with education. In contrast, our specifications predict lower participation rates for the top two education groups, because they face steeper expected income profiles. It is unclear whether this mismatch reflects a failure of the model or an inadequate calibration. For reasons of data availability, our simulations confront all households with the same environment except for the differences among education groups in the income processes. However, small differences among education groups in borrowing costs, risk aversion and time discounting, or small changes in the baseline values for the income shock variances, can alter the predicted relationships between participation rates and education.

### 5.2 Fact 2: Average and life-cycle demand for equity

For realistic borrowing rates, our model predicts small equity holdings, roughly in line with the data. The impact of borrowing costs on average equity demand can be seen in Table 5. The column headed "DKW" shows predicted average equity holdings when $r_{B}=\mathrm{E}\left(\tilde{r}_{E}\right)$, while the column headed "Std" shows average equity holdings in an otherwise identical model with $r_{B}=r_{L}$. Comparing these two columns shows that the borrowing cost wedge dramatically reduces equity demand in every specification.

Focusing now on cases with $r_{B}=\mathrm{E}\left(\tilde{r}_{E}\right)$, some specifications imply tiny values for average equity demand, as illustrated by the row with $\mathrm{RRA}=4$ and no labor income risk. Specifications with risky labor income imply substantially larger equity holdings,

[^7]in line with the discussion in Section 4.2. Table 6 shows that a realistic borrowing rate leads to much lower equity holdings at all ages. Table 6 and Figures 6 and 7 show that equity holdings rise sharply with age until late in the life cycle when $r_{B}=\mathrm{E}\left(\tilde{r}_{E}\right)$.

Some commentators have suggested that our results on equity demand, equity market participation and the covariance between consumption and asset returns would not survive the introduction of margin loans. However, a few observations make clear why the introduction of margin loans would not greatly affect our results. First, initial margin requirements on equity are $50 \%$ or higher. Thus, for a household with one thousand dollars in liquid wealth, a margin loan enables the household to adopt an equity position of no more than two thousand dollars. Second, the data show a large wedge between margin loan rates and risk-free returns. Kubler and Willen (2002) report that as of July 8, 2002, the rates on margin loans of less than $\$ 50,000$ at five major brokerage houses (The Vanguard Group, Fidelity Investments, Charles Schwab, Salomon Smith Barney and UBS Paine Webber) exceed the rate on 90-day U.S. Treasury Bills by 357 to 570 basis points, depending on brokerage house and loan size. Even at these rates, brokerage houses require credit checks and reserve the right to deny margin credit or impose higher margin rates. Finally, the combination of unsecured borrowing and margin loans does not offer an attractive leverage premium. For example, at an $8 \%$ expected return on equity, a risk-free rate of $1.68 \%$ and a $4.63 \%$ margin loan premium, the expected return on a margin-levered equity portfolio is $(1 / .5) 8-(1.68+4.63)=9.69 \%$. Combined with a wedge of 7.5 percentage points on unsecured borrowing, roughly the midpoint of the Table 1 values, the fully levered portfolio offers a leverage premium of $9.69-(1.68+7.5)=.51 \%$. That is, the fully levered portfolio offers an expected return premium of 51 basis points with a standard deviation of $2 \times 15=30 \%$.

### 5.3 Fact 3: Covariance of consumption and equity returns

How does our model fit the facts about the covariance of consumption growth and equity returns? Much better, in three respects, than models with equal borrowing and lending rates. First, our model's implied covariance between minus marginal utility growth and equity returns lies well below the excess return on equity. Second, if one applies standard formulas to calculate risk aversion using data generated by our model, one overestimates risk aversion, often by a factor of ten or more. This implication rationalizes the implausibly large estimates of risk aversion in consumptionbased asset-pricing studies. Third, our model rationalizes the rejection of standard consumption-based asset-pricing relationships in samples that are restricted to households with positive equity holdings. We now develop these three points in turn.

First, consider the covariance between minus marginal utility growth and equity returns. According to standard models with equal borrowing and lending rates,

$$
\begin{equation*}
\mathrm{E}\left(\tilde{r}_{E}\right)-r_{L}=\operatorname{cov}\left(\tilde{r}_{E},-\frac{\Delta \widetilde{M U}}{\mathrm{E}(\Delta \widetilde{M U})}\right) \equiv \operatorname{cov}_{M U} \tag{1}
\end{equation*}
$$

That is, the excess return on equity equals minus the covariance between equity returns and the growth rate of marginal utility. Equation (1) is central to most consumption-based asset pricing studies. It fares poorly in confrontations with the data for standard preference specifications.

As shown in Tables 5 and 6, equation (1) fails to hold in versions of our model with realistic borrowing costs. Table 6 shows that $\operatorname{cov}_{M U}$ is (near) zero for young households and reaches a maximum of 2.3 prior to retirement in the version of the model with risky labor income. In contrast, the standard model implies that $\operatorname{cov}_{M U}$ equals 6.0 at all ages.

We can interpret $\operatorname{cov}_{M U}$ as the equity premium implied by consumption behavior. In particular, suppose we observe a consumption process and know the utility function. In addition, suppose we assume that households face equal borrowing and
lending rates. Then from equation (1), we would infer an equity premium of $0.2 \%$ in our baseline specification with no labor income risk and $1.1 \%$ with risky labor income (first and third rows of Table 5). In other words, we would mistakenly attribute the low covariance of consumption growth and asset returns to a low equity premium, not differential borrowing and lending rates.

We can also follow Hansen and Jagannathan (1991) and interpret the same information in another way. Equation (1) implies that the standard deviation of marginal utility growth is bounded below by the Sharpe ratio. We can use our model to calculate the actual standard deviation of marginal utility growth, reported in Table 5 in the "HJ" column. Note that only when the borrowing cost wedge is zero or very small does the HJ statistic equal or exceed the Sharpe ratio (38\%). For realistic borrowing costs, the model implies that the standard deviation of marginal utility growth is much smaller than the Sharpe ratio.

Second, consider the estimation of risk aversion from consumption-based assetpricing relationships. Recall from Hansen and Singleton (1983) that, if asset returns and consumption are jointly log-normal and individual utility is of the time-separable isoelastic form, then equation (1) implies

$$
\begin{equation*}
\gamma=\frac{\mathrm{E}\left(\tilde{r_{E}}\right)-r_{L}+.5 \operatorname{var}\left(\tilde{r}_{E}\right)}{\operatorname{cov}\left(\tilde{r}_{E}, \Delta \log \tilde{c}\right)} \tag{2}
\end{equation*}
$$

where $\gamma$ is the coefficient of relative risk aversion. The columns headed "Implied RRA" in Tables 5 and 6 calculate risk aversion using equation (2).

Inspecting these tables reveals that equation (2) greatly overestimates risk aversion when borrowing rates exceed lending rates. In the baseline specification with permanent and transitory income shocks (Row 3), the Implied RRA is 13.0 , more than six times the true value. In the baseline specification with certain labor income (Row 1), the Implied RRAA is 60.9 , more than thirty times its true value.

Third, consider the consequences of restricting samples to households with positive equity holdings. Starting with Mankiw and Zeldes (1991), researchers have argued
that equations (1) and (2) are unreasonable descriptions of reality for households that do not participate in equity markets. But, as the argument goes, perhaps equations (1) and (2) approximate reality for households that invest non-zero amounts in equity. This type of argument leads to equity ownership as a sample selection criterion in studies of consumption-based asset pricing relationships.

To evaluate this approach in terms of our model, the bottom row of Table 6 reports statistics for "samples" restricted to equity market participants. This selection criterion leads to a better fit between theory and data, but the theory continues to perform poorly if we maintain that the data are generated by a world with $r_{B}=$ $r_{L}$. For example, restricting the sample to equity market participants lowers the Implied RRA from 55.8 to 15.7 in the specification with no income risk, as compared to a true RRA value of 2.0. If one instead recognizes that the borrowing rate is several percentage points above the risk-free return, then the theory explains why consumers appear so risk averse in consumption-based asset pricing studies, why the implicit equity premium appears so small, and why the Hansen-Jagannathan bound is violated.

### 5.4 Facts we don't match

Our model fails to match the facts on at least two dimensions. First, as we discussed above, versions of the model with large permanent income shocks predict higher equity market participation rates than seen in the data. Second, the model predicts that many households hold equity but not the risk-free asset. Moreover, unless risk aversion is high, the model predicts small portfolio shares in the risk-free asset for households that invest in both assets.

We now investigate the welfare significance of these model failures. Our main message is simple: When borrowing rates are in the neighborhood of the return on equity, the welfare costs of delayed participation in equity markets or the wrong
bond-equity mix are small. Even the right to hold equity can have modest welfare consequences in a life-cycle model with a zero or negative leverage premium. The situation is very different in the standard model. When the borrowing rate equals the risk-free investment return, delayed equity market participation and the wrong bond-equity mix imply large welfare costs. These observations are not intended to downplay the shortcomings of our model. Rather, they suggest that there is ample scope for other factors - participation and transaction costs, alternative preferences, a desire for liquidity, information costs and so on - to strongly influence equity holdings, once we recognize that borrowing costs exceed the risk-free return.

To carry out the welfare comparisons, it is useful to focus on certainty-equivalent consumption levels, which we report in Table 7. To obtain certainty-equivalent consumption, we first calculate lifetime expected utility, $U$, for a given consumption profile. We then find the constant level of consumption, $\bar{c}$, that yields the same level of lifetime expected utility. That is, we solve

$$
\sum_{t=0}^{T} \beta^{t} \frac{\bar{c}^{1-\gamma}}{1-\gamma}=U \quad \text { for } \quad \bar{c}=\left[\frac{1-\gamma}{\sum_{t=0}^{T} \beta^{t}} U\right]^{\gamma-1}
$$

where $\beta$ is the time discount factor, and $\gamma$ is relative risk aversion.
For example, consider a household with less than high school education, no income risk and an $8 \%$ borrowing rate. According to Table 7, that household is indifferent between the optimal lifetime consumption plan and a flat profile with consumption of $\$ 17,850$ per year. If the same household is not allowed to trade equity, certaintyequivalent consumption falls to $\$ 17,800$. In other words, the right to trade equity is worth about $\$ 50$ per year, or less than .3 percent of annual consumption. Table 7 considers three departures from optimal behavior: non-participation in equity markets throughout life, non-participation until age 50 with optimal behavior thereafter, and optimal participation subject to a $50-50$ bond-equity portfolio mix.

Perhaps the most notable feature of Table 7 is the small cost of delayed equity market participation, given realistic borrowing costs. In the specification with no
income risk and an $8 \%$ borrowing rate, the cost of waiting until age 50 to invest in equity is essentially zero. The costs of delayed participation are larger for specifications with risky income, but less than $\$ 400$ per year for every group. Costs on this order are not trivial, but they seem well within range of reasonable values for the costs of participating and transacting in equity markets.

The standard life-cycle model with $r_{B}=r_{L}$ tells a starkly different story. For example, the costs of delayed participation until age 50 when $r_{B}=r_{L}$ range from 14 to 23 thousand dollars per year, which amounts to more than fifty percent of annual consumption. In other words, the standard model implies enormous welfare costs for widely observed departures from theoretically predicted behavior. These costs are far too large to be rationalized by plausible costs of participating and transacting in equity markets.

## 6 Concluding Remarks

Realistic borrowing costs dramatically shrink the demand for equity in the canonical life-cycle model of consumption and portfolio choice. The model's theoretical implications about the average level and life-cycle pattern of equity holdings are roughly in line with the data, given realistic borrowing costs. In addition, a realistic borrowing rate greatly improves the performance of the theory along several dimensions that feature prominently in consumption-based asset pricing studies.

The implications of our model differ in noteworthy respects from those of models that abstract from life-cycle considerations and from models with hard borrowing limits. For example, equity demand and equity market participation rates are nonmonotonic in borrowing costs and risk aversion in our model. The opportunity to borrow at realistic rates in a life-cycle setting also has important consequences for life-cycle wealth accumulation, which in turn drives the demand for equity. Because
households face an upward sloping income profile, they borrow in the early part of the life cycle, which causes a substantial delay in the age at which they first invest in equity or first accumulate substantial equity holdings.

Our analysis points to several directions for future research. We mention three here. First, our model implies that households accumulate little or no liquid wealth until middle age, which is consistent with much empirical evidence (e.g., Lusardi, Cossa and Krupka, 2001). Given its simplicity and its assumption of patient, timeconsistent, rational consumers, our model and analysis challenge claims that households save too little. A natural next step, which we are currently pursuing, is to enrich the model to account for housing consumption and wealth and for the liquidity benefits of safe assets. We plan to evaluate this richer version of the model against a number of facts about consumption, home ownership, wealth accumulation and portfolio choice over the life cycle.

Second, our analysis puts the spotlight on the role of borrowing costs and leverage as key factors in the demand for risky assets. This paper does not consider other ways to leverage equity investments such as margin loans or investments in levered mutual funds. Margin loans provide limited scope for levered equity holdings, but corporate bonds, government securities, real estate and small business wealth are often subject to less stringent restrictions on leverage. Kubler and Willen (2002) consider an extended version of our model to address portfolio choice in a broader setting that encompasses a fuller menu of risky assets and leveraging methods.

Third, our model has strong testable implications for the life-cycle behavior of equity holdings and for the covariance between consumption growth and equity returns. In particular, as a household ages, the model predicts rising equity holdings and an increasing covariance between consumption growth and equity returns. These implications can be tested using now-standard econometric methods applied to (synthetic) panel data on consumption and portfolio holdings.

## References

[1] Altig, David and Steven J. Davis, 1992, The timing of intergenerational transfers, tax policy and aggregate saving, American Economic Review, 82(5), 1199-1220.
[2] Ameriks, John and Stephen P. Zeldes, 2001, How do household portfolio shares vary with age? working paper, Columbia University.
[3] Attanasio, Orazio P., James Banks and Sarah Tanner, 2002, Asset holding and consumption volatility, Journal of Political Economy, 110(4), 771-792.
[4] Brav, Alon, George M. Constantinides and Christopher C. Geczy, 2002, Asset pricing with heterogeneous consumers and limited participation: Empirical evidence, Journal of Political Economy, 110(4), 793-824.
[5] Brav, Alon and Christopher C. Geczy, 1995, "An empirical resurrection of the simple consumption CAPM with power utility," working paper, University of Chicago.
[6] Brennan, Michael J, 1971, Capital market equilibrium with divergent borrowing and lending rates, Journal of Financial and Quantitative Analysis, 6, 1197-1205.
[7] Campbell, John Y, 1999, Asset prices, consumption and the business cycle, in John B. Taylor and Michael Woodford, editors, Handbook of Macroeconomics, Volume 1C. Amsterdam: North-Holland.
[8] Constantinides, George M., John B. Donaldson and Rajnish Mehra, 2002, Junior can't borrow: A new perspective on the equity premium puzzle, Quarerly Journal of Economics, 117(1), 269-296.
[9] Davis Steven J. and Paul S. Willen, 2000, Using financial assets to hedge labor income risks, working paper, University of Chicago.
[10] Deaton, Angus, 1991, Saving and liquidity constraints, Econometrica, 59, 12211248.
[11] Engen, Eric M., William G. Gale, and Cori R. Uccello, 1999, The adequacy of retirement saving, Brookings Papers on Economic Activity, 2, 65-187.
[12] Garcia, C.B. and W.I. Zangwill, 1981, Pathways to Solutions, Fixed Points, and Equilibria. Englewood Cliffs, NJ.: Prentice-Hall.
[13] Gomes, Francisco and Alex Michaelides, 2002, Life cycle asset allocation: A model with borrowing constraints, uninsurable labor income risk and stockmarket participation costs, working paper, London Business School.
[14] Gourinchas Pierre-Olivier and Jonathan A. Parker, 2002, Consumption over the life cycle, Econometrica, 70(1), 47-89.
[15] Grossman Sanford and Robert Shiller, 1982, Consumption correlatedness and risk measurement in economies with non-traded assets and heterogeneous information, Journal of Financial Economics, 10(2), 195-210.
[16] Hansen, Lars Peter and Ravi Jagannathan, 1991, Implications of security market data for models of dynamic economies, Journal of Political Economy, 99(2), 22562.
[17] Hansen, Lars Peter and Kenneth Singleton, 1983, Stochastic consumption, risk aversion, and the temporal behavior of asset returns, Journal of Political Economy, 91, 249-265.
[18] He, Hua and David M. Modest, 1995, Market frictions and consumption-based asset pricing, Journal of Political Economy, 103(1), 94-117.
[19] Heaton, John and Deborah Lucas, 1997, Market frictions, savings behavior, and portfolio choice, Macroeconomic Dynamics 1, 76-101.
[20] Heaton, John and Deborah Lucas, 2000, Portfolio Choice and asset prices: The importance of entrepreneurial risk, Journal of Finance, 55(3), 1163-98.
[21] Judd, Kenneth, Felix Kubler and Karl Schmedders, 2002, Computational methods for dynamic equilibria with heterogeneous agents. Forthcoming in M. Dewatripont, L. P. Hansen and S. J. Turnovsky, editors, Advances in Economics and Econometrics: Theory and Applications, Eighth World Congress. Cambridge University Press.
[22] Kubler, Felix and Paul Willen, 2002, Margin borrowing and life-cycle portfolio choice. Work-in-progress.
[23] Lusardi, Annamaria, Ricardo Cossa and Erin L. Krupka, 2001, Savings of young parents, Journal of Human Resources, 36(4), 762-794.
[24] Luttmer, Erzo G. J., 1996, Asset pricing in economies with frictions, Econometrica, 64(6), 1439-67.
[25] Mankiw, N. Gregory and Stephen P. Zeldes, 1991, The consumption of stockholders and nonstockholders, Journal of Financial Economics, 29, 97-112.
[26] Merton, Robert C, 1969, Lifetime portfolio selection under uncertainty: The continuous time case, Review of Economics and Statistics, 51, 245-257.
[27] Poterba, James M. and Andrew A. Samwick, 2001, Portfolio allocations over the life cycle, in Seiritsu Ogura, Toshiaki Tachibanaki, and A. David Wise, editors, Aging Issues in the United States and Japan. Chicago: University of Chicago Press.
[28] Samuelson, Paul A, 1969, Lifetime portfolio selection by dynamic stochastic programming, Review of Economics and Statistics, 51, 239-243.
[29] Tauchen, George and Robert Hussey, 1991, Quadrature-based methods for obtaining approximate solutions to nonlinear asset pricing models, Econometrica, 59(2), 371-96.
[30] Vissing-Jorgensen, Annette, 2002, Towards an explanation of household portfolio choice heterogeneity: Nonfinancial income and participation cost structures. working paper, University of Chicago.
[31] Watson, L.T., S.C. Billups and A.P. Morgan, 1987, HOMPACK: A suite of codes for a globally convergent homotopy algorithm. ACM Transactions on Mathematical Software, 13, 281-310.

## Appendix: Computational details

We solve the model by backward induction. At age 80, the solution is trivial: consume everything. We then solve for optimal consumption and portfolio choice at age 79 conditional on liquid wealth, income, the state of the world and the (degenerate) policy rule at age 80 . Next, we solve for consumption and portfolio choice at age 78 again conditional on liquid wealth, income, the state of the world and the calculated optimal policy rule at age 79. And so on.

The problem therefore reduces to solving simple two-period optimization problems and to approximating policy rules as functions of some minimal set of state variables (including current age). This appendix focuses on two aspects of the solution procedure: how to reduce the endogenous state space to one variable, and how to solve the two-period problem effectively. First, we use preference homotheticity to simplify the problem and reduce the number of continuous state variables to one. Second, we discuss how we solve the two-period optimization problem.

A little notation will help. Let $z_{t}$ be a Markov chain with finite support $z \in$ $\{1, \ldots, S\}$ and transition $\pi$. Gross equity returns are $\tilde{R}_{E}\left(z_{t}\right)$, and the gross borrowing and lending rates are $R_{B}$ and $R_{L}$, respectively. A date-event $z^{t}$ is a history of shocks $\left(z_{1}, \ldots, z_{t}\right)$. Let $y\left(z^{t}\right)$ denote income at time $t$.

Preference homotheticity allows us to simplify the problem by combining wealth and income into one variable (Deaton, 1991). Suppose we have solved for optimal policy rules from time $t+1$ on. Suppose at date $t$, we are in state $z$ with income $y_{t}$ and wealth $\Xi_{t}$. The optimal policy rule for next period specifies investment of $F_{t+1}^{i}\left(\Xi_{t+1}\left(z^{\prime}\right), y_{t+1}\left(z^{\prime}\right) ; z^{\prime}\right)$ in asset $i=B, L, E$ at time $t+1$. Bellman's principle implies that the solution to the two-period problem below constitutes optimal portfolio choice at $t$ in state $z$ with income $y$ and liquid wealth $\Xi$.

$$
\begin{align*}
& \max _{F^{L}, F^{B}, F^{E}} \frac{c_{t}^{1-\gamma}}{1-\gamma}+\beta \mathrm{E}_{t}\left(\frac{c_{t+1}^{1-\gamma}}{1-\gamma}\right)  \tag{3}\\
& \text { s.t. } \quad c_{t}= y+\Xi-F^{L}+F^{B}-F^{E} ; \\
& c_{t+1}\left(z^{\prime}\right)= y\left(z^{\prime}\right)+\Xi\left(z^{\prime}\right)-F^{L}\left(\Xi\left(z^{\prime}\right), y\left(z^{\prime}\right) ; z^{\prime}\right)+ \\
& F^{B}\left(\Xi\left(z^{\prime}\right), y\left(z^{\prime}\right) ; z^{\prime}\right)-F^{E}\left(\Xi\left(z^{\prime}\right), y\left(z^{\prime}\right) ; z^{\prime}\right), \quad \forall z^{\prime} \in\{1, \ldots, S\} ; \\
& \Xi\left(z^{\prime}\right)= F^{L} R_{L}-F^{B} R_{B}+F^{E} R_{E}\left(z^{\prime}\right), \quad \forall z^{\prime} \in\{1, \ldots, S\} ; \\
& \quad F^{L} \geq 0, \quad F^{B} \geq 0, \quad F^{E} \geq 0 ;
\end{align*}
$$

where we suppress time subscripts on variables other than consumption to reduce notational clutter.

Now divide through by $y_{t}$, define $x_{i}=c_{t} / y_{t}$, and consider the two-period opti-
mization problem:

$$
\begin{align*}
& \max _{f^{L}, f f^{B}, f} E  \tag{4}\\
& \text { s.t. } \quad x_{t}=\xi-f^{L}+f^{B}-f^{E} \\
& x_{t+1}=\frac{y_{t+1}^{1-\gamma}\left(z^{\prime}\right)}{y_{t}}\left[\xi\left(z^{\prime}\right)-f^{L}\left(\xi\left(z^{\prime}\right) ; z^{\prime}\right)+f^{B}\left(\xi\left(z^{\prime}\right) ; z^{\prime}\right)-f^{E}\left(\xi\left(z^{\prime}\right) ; z^{\prime}\right)\right] ; \\
& \xi\left(z^{\prime}\right)=\frac{y_{t+1}\left(z^{\prime}\right)+f^{L} R_{L}-f^{B} R_{B}+f^{E} R_{E}\left(z^{\prime}\right)}{y_{t+1}\left(z^{\prime}\right)} \\
& \quad f^{L} \geq 0, \quad f^{B} \geq 0, \quad f^{E} \geq 0 .
\end{align*}
$$

Observe that the policy rules are now functions of a single endogenous state variable, $\xi$, the ratio of liquid wealth plus current income to current income. This reduction in the dimensionality of the state space greatly simplifies computation.

We can recover the solution to the original problem (3) by multiplying the solution to the transformed problem (4) by current income:

$$
\begin{aligned}
c_{t} & =y_{t} x_{t} \\
F^{L} & =y_{t} f^{L}, \quad F^{B}=y_{t} f^{B}, \quad F^{E}=y_{t} f^{E}
\end{aligned}
$$

To solve the transformed two-period problem, we solve the associated KuhnTucker conditions - a nonlinear system of equations and inequalities that is necessary and sufficient for optimality. Following Garcia and Zangwill (1981, pages 65-68), we use a change of variables to eliminate inequalities in the Kuhn-Tucker conditions and state the optimality conditions as a system consisting solely of equations. The resulting system has 3 unknowns corresponding to the three asset holdings.

In particular, let $\eta_{j} \in \Re$ for $j=1,2,3$, and define the Kuhn-Tucker multiplier for asset $j, \mu_{j}=\left(\max \left\{0,-\eta_{j}\right\}\right)^{3}$. The consumer's holding of asset $j$ is $\theta_{j}=\left(\max \left\{0, \eta_{j}\right\}\right)^{3}$. Note that $\theta$ and $\mu$ are twice continuously differentiable, and that the complementary slackness conditions hold:

$$
\left(\max \left\{0, \eta_{j}\right\}\right)^{3} \geq 0, \quad\left(\max \left\{0,-\eta_{j}\right\}\right)^{3} \geq 0, \quad \text { and }\left(\max \left\{0, \eta_{j}\right\}\right)^{3} \cdot\left(\max \left\{0, \eta_{j}\right\}\right)^{3}=0 .
$$

We implement our solution algorithm using Fortran 90. A simple Newton method usually works well as a nonlinear equation solver when a good starting point is known. In some cases we need to use homotopy methods (as implemented in HOMPACK, see Watson et al (1987)) to solve the system.

Lastly, we draw attention to two practical aspects of our computational solution. First, the range of $f_{t}^{j}(\xi ; z)$ will generally depend on $t$ and $z$. In practice, we set arbitrary bounds on the range that vary only with $t$. We then verify that these
bounds never bind in the simulations. Second, in generating $f_{t}^{j}(\xi ; z)$, we don't solve (4) for every possible value of $\xi$. Instead, we solve (4) for a finite number of values of $\xi$ and use cubic spline interpolation to fill in the rest. See Judd et al. (2002) for details on spline interpolation. Since the true policy functions have non-differentiabilities, we use 50 knots for each spline interpolation to obtain sufficient accuracy.

Maximal relative errors in Euler equations lie below $10^{-6}$. Running times on a Pentium III computer with a 1.2 Ghz processor and 1 GB of RAM clustered around four or five minutes but range from 2 minutes for models with no labor income risk and borrowing rates above the expected return on equity to about 15 minutes for models with labor income risk and borrowing rates below the return on equity.
Table 1: Household Borrowing Costs and Risk-Free Returns, Selected Years [tabx6]

|  | Year | 1972 | 1980 | 1984 | 1987 | 1990 | 1995 | 2001 |
| :--- | :--- | :--- | :--- | :--- | :--- | :---: | :---: | :---: |
| $(1)$ | Average rate on two-year personal loans | 12.5 | 15.5 | 16.5 | 14.2 | 15.5 | 13.9 | 13.2 |
| $(2)$ | Average marginal tax subsidy on borrowing | .181 | .247 | .249 | 0 | 0 | 0 | 0 |
| $(3)$ | After-tax borrowing cost (1 - row 2) ${ }^{*}$ (row 1) | 10.2 | 11.6 | 12.4 | 14.2 | 15.5 | 13.9 | 13.2 |
| $(4)$ | Rate on three-year U.S. Treasury Securities | 5.7 | 11.5 | 11.9 | 7.7 | 8.3 | 6.3 | 4.1 |
| $(5)$ | Average marginal tax rate on interest income | .313 | .428 | .330 | .279 | .250 | .282 | .297 |
| $(6)$ | After-tax risk-free return (1 - row 5) ${ }^{*}($ row 4) | 3.9 | 6.6 | 8.0 | 5.5 | 6.2 | 4.5 | 2.9 |
| $(7)$ | Pre-tax wedge between borrowing cost | 6.7 | 4.0 | 4.6 | 6.5 | 7.2 | 7.7 | 9.1 |
|  | and risk-free return (row 1 - row 4) |  |  |  |  |  |  |  |
| $(8)$ | After-tax wedge between borrowing cost | 6.3 | 5.1 | 4.4 | 8.7 | 9.3 | 9.5 | 10.3 |
|  | and risk-free return (row 3 - row 6) |  |  |  |  |  |  |  |
| $(9)$ | Charge-off rate on loans, net of recoveries |  |  |  | 0.8 | 1.0 | 0.7 | 1.3 |
| $(10)$ | After-tax wedge net of charge offs (row 8 - row 9) |  |  |  | 7.9 | 8.2 | 8.8 | 9.1 |
| $(11)$ | After-tax wedge net of charge offs, credit cards |  |  |  | 9.0 | 8.5 | 7.9 | 6.5 |

[^8]Table 2: Parameter Settings

| Parameter | Baseline | Alternative values |
| :--- | :---: | :---: |
| Relative Risk Aversion | 2 | $0.5,1,4,8$ |
| Annual Discount factor | 0.95 |  |
| Age of labor force entry | 21 |  |
| Age of retirement | 65 |  |
| Age of death | 80 |  |
| $\operatorname{var}(\Delta \tilde{\eta})$ | see Table 3 |  |
| $\operatorname{cov}\left(\Delta \tilde{\eta}, \tilde{r}_{E}\right)$ | 0 |  |
| $\operatorname{var}(\tilde{\varepsilon})$ | see Table 3 |  |
| $\operatorname{cov}\left(\tilde{\varepsilon}, \tilde{r}_{E}\right)$ | 0 |  |
| $\operatorname{Replacement~rate~}$ | $80 \%$ | $20 \%, 40 \%, 60 \%, 100 \%$ |
| $r_{L}$ | $2 \%$ |  |
| $r_{B}$ | $8 \%$ | $2 \%, 5 \%, 8 \%, 20 \%, 99 \%$ |
| $\mathrm{E}\left(\tilde{r}_{E}\right)$ | $8 \%$ |  |
| $\operatorname{std}\left(\tilde{r}_{E}\right)$ | $15 \%$ |  |

Table 3: Income Shock Standard Deviations, percent per year

| Education group | $\sigma(\tilde{\varepsilon})$ <br> (transitory shock) | $\sigma(\Delta \tilde{\eta})$ <br> (permanent shock) |
| :--- | :---: | :---: |
| Some high school | 23 | 12 |
| High school diploma | 15 | 14 |
| Some college | 13 | 12 |
| College degree | 14 | 10 |
| Graduate school | 16 | 9 |
| Pooled Sample | 15 | 12 |

Note: These values are from Gourinchas and Parker (2002), adjusted for measurement error as described in the text.

Table 4: Equity Market Participation Rates [tabx1]

| Age | No income <br> shocks |  | Transitory <br> shocks only |  |  | Permanent and <br> Group |  |  | $r_{B}=$ |  | $r_{B}=$ |  |  | $r_{B}=$ |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 8 | 20 | 99 | 8 | 20 | 99 | 8 | 20 | 99 |  |  |  |  |  |  |  |
| $22-24$ | 0 | 0 | 0 | 0 | 49 | 65 | 65 | 69 | 69 |  |  |  |  |  |  |  |
| $25-29$ | 0 | 0 | 0 | 1 | 57 | 80 | 84 | 93 | 94 |  |  |  |  |  |  |  |
| $30-34$ | 0 | 0 | 0 | 0 | 68 | 85 | 96 | 99 | 99 |  |  |  |  |  |  |  |
| $35-39$ | 0 | 57 | 0 | 1 | 78 | 88 | 100 | 100 | 100 |  |  |  |  |  |  |  |
| $40-44$ | 0 | 100 | 100 | 5 | 88 | 93 | 100 | 100 | 100 |  |  |  |  |  |  |  |
| $45-49$ | 0 | 100 | 100 | 20 | 95 | 97 | 100 | 100 | 100 |  |  |  |  |  |  |  |
| $50-54$ | 20 | 100 | 100 | 54 | 99 | 99 | 100 | 100 | 100 |  |  |  |  |  |  |  |
| $55-59$ | 100 | 100 | 100 | 87 | 100 | 100 | 100 | 100 | 100 |  |  |  |  |  |  |  |
| $60-79$ | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |  |  |  |  |  |  |  |
| All ages | 27 | 64 | 57 | 32 | 82 | 90 | 94 | 96 | 96 |  |  |  |  |  |  |  |

## Notes:

1. Table entries are for the pooled sample and, unless otherwise noted, the baseline parameter settings reported in Tables 2 and 3 .
2. The participation rate is $100 \%$ at all ages for $r_{B} \in\{8,20,99\}$ in the specification with permanent income shocks only.

Table 5: Average Equity Demand and other Statistics, Alternative Specifications [tabx3]

| $r_{B}$ | RRA RR | Labor income shocks? | Income <br> Profile | $\operatorname{cov}_{M U}$ | HJ | $\operatorname{cov}_{C}$ | Equity Demand |  | Implied RRA | $\begin{gathered} \% \\ \text { Ptcp } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  | DKW | Std |  |  |
| - | - - | No | - | 0.2 | 1.5 | 0.1 | 15 | 1871 | 60.9 | 32 |
| - | - - | T | - | 0.3 | 1.9 | 0.1 | 21 | 737 | 47.0 | 39 |
| - | - - | $\mathrm{P}+\mathrm{T}$ | - | 1.1 | 7.1 | 0.5 | 80 | 831 | 13.0 | 90 |
| 2 | - - | No | - | 6.0 | 38.2 | 2.8 | 1871 | 1871 | 2.5 | 100 |
| 2 | - - | T | - | 6.1 | 38.8 | 2.7 | 737 | 737 | 2.6 | 100 |
| 2 | - - | $\mathrm{P}+\mathrm{T}$ | - | 6.1 | 38.7 | 2.7 | 831 | 831 | 2.6 | 100 |
| 99 | - - | No | - | 0.4 | 2.3 | 0.2 | 27 | 1871 | 39.3 | 64 |
| 99 | - - | T | - | 0.6 | 3.8 | 0.3 | 40 | 737 | 23.6 | 91 |
| 99 | - | $\mathrm{P}+\mathrm{T}$ | - | 1.2 | 7.5 | 0.6 | 84 | 831 | 12.3 | 97 |
| - | 4 | No | - | 0.1 | 0.9 | 0.0 | 3 | 589 | 206.9 | 17 |
| - | 4 | $\mathrm{P}+\mathrm{T}$ | - | 2.8 | 18.1 | 0.6 | 99 | 336 | 11.1 | 94 |
| - | 8 - | No | - | 0.0 | 0.2 | 0.0 | 0 | 248 | 1424.1 | 10 |
| - | 8 - | $\mathrm{P}+\mathrm{T}$ | - | 5.0 | 31.8 | 0.6 | 104 | 157 | 11.4 | 98 |
| - | - | No | Flat | 0.9 | 5.8 | 0.5 | 77 | 1920 | 15.3 | 100 |
| - | - 0.2 | No | - | 0.7 | 4.6 | 0.4 | 49 | 1644 | 19.5 | 45 |
| - | 1 | No | - | 0.1 | 0.7 | 0.1 | 7 | 1937 | 123.6 | 26 |

## Notes:

1. Table entries are for the pooled sample and, unless otherwise noted, the baseline parameter settings reported in Tables 2 and 3. RR denotes the income replacement rate during retirement.
2. $\operatorname{cov}_{M U}=\operatorname{cov}\left(\tilde{r}_{E},-\Delta \widetilde{M U} / \mathrm{E}(\Delta \widetilde{M U})\right)$. See Section 5.3 for details.
3. $H J=\operatorname{std}(\Delta \widetilde{M U} / \mathrm{E}(\Delta \widetilde{M U})$. See Section 5.3 for details.
4. $\operatorname{cov}_{C}=\operatorname{cov}\left(\Delta \log \tilde{c}, \tilde{r}_{E}\right)$.
5. DKW denotes our model with indicated value of $r_{B}$. Std denotes the standard model with $r_{B}=r_{L}=2 \%$. The equity demand values are in 1987 dollars.
6. The Implied RRA is calculated using equation (2). See Section 5.3.

Table 6: Equity Demand and other Statistics over the Life Cycle [tabx7]

| Age <br> Group | No labor income shocks |  |  |  | Permanent and transitory shocks |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\operatorname{cov}_{M U}$ | Equity Demand |  | Implied RRA | $\operatorname{cov}_{M U}$ | Equity Demand |  | Implied |
|  |  | DKW | Standard |  |  | DKW | Standard | RRA |
| 23-29 | 0.0 | 0 | 1263 | Inf | 0.1 | 8 | 240 | 147.6 |
| 30-39 | 0.0 | 0 | 1619 | Inf | 0.6 | 32 | 495 | 16.1 |
| 40-49 | 0.0 | 0 | 2117 | Inf | 1.2 | 81 | 884 | 8.7 |
| 50-59 | 0.1 | 9 | 2475 | 116.4 | 1.9 | 154 | 1320 | 5.6 |
| 60-69 | 0.8 | 60 | 2500 | 12.5 | 2.3 | 215 | 1633 | 4.5 |
| 70-79 | 1.1 | 47 | 1668 | 8.7 | 2.2 | 138 | 1138 | 4.5 |
| All ages | 0.2 | 11 | 1891 | 55.8 | 1.1 | 84 | 828 | 9.1 |
| Participants |  |  |  |  |  |  |  |  |
| Only | 0.7 | 41 | 1891 | 15.7 | 1.2 | 88 | 828 | 9.1 |

Note: See notes to Table 5.

Table 7: The Welfare Costs of Sub-Optimal Equity Holdings. [tabx11]
Certainty-equivalent consumption levels, thousands of 1987 dollars.

| $r_{B}$ | Income <br> Shocks | Equity <br> Holdings | Less than <br> High School | High School Degree | Some College | College <br> Degree | Graduate School |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 | No | No | 17.80 | 23.43 | 25.30 | 28.06 | 26.91 |
|  |  | 50/50 | 17.81 | 23.44 | 25.31 | 28.08 | 26.93 |
|  |  | Age 50 | 17.85 | 23.49 | 25.37 | 28.17 | 26.97 |
|  |  | Optimal | 17.85 | 23.49 | 25.37 | 28.17 | 26.97 |
| 2 | No | No | 21.17 | 28.34 | 30.55 | 33.27 | 33.59 |
|  |  | Age 50 | 23.22 | 31.09 | 33.52 | 36.50 | 36.84 |
|  |  | Optimal | 37.70 | 50.48 | 54.42 | 59.28 | 59.84 |
| 8 | Yes, both | No | 13.32 | 16.04 | 18.20 | 22.83 | 22.46 |
|  |  | 50/50 | 13.65 | 16.63 | 18.68 | 23.11 | 22.64 |
|  |  | Age 50 | 13.58 | 16.43 | 18.59 | 23.17 | 22.73 |
|  |  | Optimal | 14.14 | 17.35 | 19.38 | 23.63 | 22.93 |

## Notes:

1. Unless otherwise noted, the table entries are for the baseline parameter settings reported in Tables 2 and 3.
2. " $50 / 50$ " indicates a portfolio mix that is constrained to have $50 \%$ of its value in equity and $50 \%$ in bonds. "Age 50 " means that the household is constrained from participating in the equity market until age 50. See Section 5.4 for details.

Figure 1: Expected labor income profiles. figa7


Figure 2: Life-cycle equity holdings at various borrowing rates. Baseline parameter settings for a household from the pooled sample and no labor income risk. [figa1]


Figure 3: Life-cycle borrowing at various borrowing rates. Baseline parameter settings for a household from the pooled sample and no labor income risk. [figa2]


Figure 4: Life-cycle liquid wealth at various borrowing rates. Baseline parameter settings for a household from the pooled sample and no labor income risk. [figa3]


Figure 5: Average equity demand as a function of borrowing rate. Baseline parameter settings for a household from the pooled sample. [figb1]


Figure 6: Life-cycle equity holdings with and without labor income risks using pooled sample and baseline parameter settings. [figa6]


Figure 7: Life-cycle equity holdings at alternative RRA values for pooled sample with risky labor income and baseline parameter settings. [figa4]


Figure 8: Demand for equity as a function of relative risk aversion. Pooled sample with risky labor income and baseline parameter settings. [figb2]


Figure 9: Life-cycle equity holdings for pooled sample and flat income profiles. [figa9]


Figure 10: Life-cycle equity holdings for alternative income replacement rates during retirement. [figa8]



[^0]:    ${ }^{1}$ Compelling evidence on the value of the subjective discount factor is scarce. See the discussion in Engen et al. (1999). Much, perhaps most, previous research on consumption and portfolio choice over the life cycle adopts values near .95 or .96 . Our model generates realistic asset accumulation behavior for values in this neighborhood. Substantially lower values (e.g., .90) lead to very little wealth accumulation over the life cycle, and substantially higher values (e.g., 1.0) imply much greater wealth accumulation than seen in the data.
    ${ }^{2}$ Davis and Willen (2000) present evidence of non-zero correlations between labor income shocks and equity returns. They also consider the implications of a non-zero correlation for life-cycle portfolio choice in a model with the borrowing rate equal to the risk-free investment return.

[^1]:    ${ }^{3}$ Without these deductions, household income would be about $27 \%$ higher.

[^2]:    ${ }^{4}$ If zero income is possible in the last period of life, households that borrow in the penultimate period run the risk of negative consumption in the final period. With isoelastic utility and relative risk aversion of at least 1 , the possibility of negative consumption, no matter how remote, leads to infinitely negative utility. Thus no households borrow in the penultimate period. The same argument extends to earlier periods of life by induction.

[^3]:    ${ }^{5}$ We discuss the distinct effects of permanent and transitory income shocks on the demand for equity in Sections 5.1 and 5.2 below.

[^4]:    ${ }^{6}$ The shape of the income profile also affects equity demand in the intermediate case with $r_{B} \in$ $\left(r_{L}, \mathrm{E}\left(\tilde{r}_{E}\right)\right)$, but the effect is stronger when $r_{B} \geq \mathrm{E}\left(\tilde{r}_{E}\right)$.

[^5]:    ${ }^{7} \mathrm{~A}$ model that allows for limited borrowing at the risk-free rate also implies very high equity market participation. In addition, it has the unpalatable implication that old households borrow to

[^6]:    finance equity holdings.

[^7]:    ${ }^{8}$ Isolating age effects from time and cohort effects requires an identifying assumption. However, to the best of our knowledge, every study that considers the issue concludes that age has a positive effect on participation in equity markets.

[^8]:    Data for later years as follows: Rows (1) and (4) are from vario of the Federal Reserve Bulletin and the Federal Reserve's Annual Statistical Digest. Row (2) reflects the Tax Reform Act of 1986, which eliminated the tax deductibility of interest payments on non-mortgage loans. Row (5) is from Table 1 in Poterba (2001), which is calculated from the NBER TAXSIM model. Poterba's 1999 value is used for the 2001 entry in row (5). Row (9) is from www.federalreserve.gov/releases/chargeoff/chg_all_sa.txt (visited 3 April 2002). Other rows are calculated by the authors as indicated. Notes: Borrowing costs, returns and charge-offs are expressed as annual percentage rates. Row (9) reports the value of loans removed from the books and discharged against loan loss reserves net of recoveries as a percentage of loans outstanding. Rows (10) and (11) show the difference between the household cost of borrowing and the rate of return on risk-free investments after adjusting for tax considerations and the charge-off rate. Row (11) is calculated in the same manner as row (10), except that it makes use of
    interest rate and charge-off data for credit cards instead of two-year personal loans.

