

Bottom Discharge Conduit for Embankment Dams

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Abstract

The structural calculation methods of the conduits that cross the embankment dams can be divided into two approaches. On the one hand, the estimation of the earth's characteristics is done by the multiparameter subgrade model, and on the other hand, the usual finite element software describes the parameters of the earth based on the modulus of elasticity. The conduits have a moment of inertia value for the cross section that is combined with the modulus of elasticity of the material (concrete, steel) resulting in a great rigidity to the assembly. This stiffness, quantified in the stiffness matrix, is much higher than the rigidity of the soil. This work goes in two directions: on the one hand, it argues that the complex methods of calculating the soil characteristics are not relevant for conduits that cross the embankment dams. Second in the longitudinal direction, the conduit has joints that diminish the rigidity of the assembly and whose effect cannot be included in the FEM calculation, as it is usually done in a plane strain model. A calculation method is proposed that contains an inertial moment adjustment that takes into account joints. Finally, a computational method is used in which FEM is used with the empirically estimated momentum variation.

Keywords: bottom discharge conduit, embankment dam, soil-structure interaction, finite element method, multiple-parameter subgrade model, matrix condensation

1. Introduction

Bottom discharge conduits are pipes that cross the body of the dam from the upstream to the downstream. Conduits can be made of reinforced concrete or metal. Less rigid materials can be used for small dams (PEHD, GRP-glass reinforced plastic). Passing a conduit through the body of an embankment dam or beneath its foundation requires a lot of caution and adequate construction measures [1].

The contact surface between the pipe and the embankment is a possible way for infiltration. These are prevented by special shoulders that increase the water infiltration path.

The modeling of the interaction between the structure and the earth filler must solve the following aspects [1]:

- a. Calculating the pressures of the embankment on the conduit
- b. Assessment of the soil (subgrade) reaction

- c. Calculating the displacements and deformations of the structure
- d. Evaluating the state of stress of the cross section and longitudinal section
- e. Cross-sectional and longitudinal cross-sectional composition (joints)

Due to the complexity of the interaction among the structure, the foundation, and the filling, the only current method able to accurately model this phenomenon is the finite element method (FEM).

For the first aspect, (a) the Marston theory of embankment pressures has typically been adopted for calculating loads on a conduit that is partially or fully projecting above the original ground surface [2–4]. Using the Marston theory, vertical load on the conduit is considered to be a combination of the weight of the fill directly above the conduit and the frictional forces, acting either upward or downward, from the adjacent fill.

For aspect (b) assessment of the ground reaction, there are calculation methods in which the behavior of the earth is modeled by springs with behavior not necessarily linearly elastic with or without interaction between them (between springs) [5]. These models introduce more parameters between the linear stiffness of the soil layers and the shear layers. Later [6, 7] the interaction between the structure and the terrain was described by the beam-column analogy as it incorporates the lowest level multiple-parameter subgrade model possible.

For aspects (c) and (d), guidance on the design and construction of conduits are provided in [2, 3, 8, 9]. The references contain accepted methods to design conduits. Reinforced concrete conduits are used for medium and large dams, and precast pipes are used for small dams, urban levees, and other levees where public safety is at risk or substantial property damage could occur [2]. Corrugated metal pipes are acceptable through agricultural levees where the conduit diameter is 900 mm (36 in.) and when levee embankments are no higher than 4 m (12 ft) above the conduit invert. Inlet structures, intake towers, gate wells, and outlet structures should be constructed of cast-in-place reinforced concrete. However, precast concrete or corrugated metal structures may be used in agricultural and rural levees.

Conduit composition in cross-sectional and longitudinal is detailed in [1, 10]. The conduits are made from 10 to 12-m-long sections in order to be able to adapt without cracking with the eventual differentiation of the foundation ground settlement. The outer shape of the cross sections should consider the interaction of the structure with the filler. Curved vault sections are the most recommended. Rectangular outer sections contour lead to stress concentrations and may lead to cracking of the sealing core along the structure.

Next we aim to find the importance of the assessment of the soil (subgrade) reaction. There are many methods of calculating the soil characteristics [5–8]. Which one is suitable for this type of structure? We propose to answer this question.

2. Interaction between soil and conduit

Most structural computing software currently used (e.g., SAP 2000 [11]) included only the spring stiffness connection for the ground displacements similar to the Winkler environment. Computational programs with geotechnical specialization allow the adoption of more complex models of behavior for the earth. However, besides the inability to capture the interaction between the structure and the

ground, another deficiency results from the fact that the loads are obtained by a structural horse considering the fixed supports [5].

Everyone agrees that MEF is the best (exact) calculation method. How do we define the soil in the calculation model?

Most software models the ground through the Winkler springs. Is it accurate enough, or do we need more sophisticated methods of assessing the ground characteristics of the foundation? There are soil-modeling methods [5–7] that quantify the Winkler spring stiffness and a shear effect in the ground.

There are finite element programs (GeoStudio [12]) that model the behavior of the earth using for soil stiffness—Young’s modulus (E). This modulus, which represents the stiffness of the soil, is dependent on the effective confining stress. In FEM (finite element method), finally, an equation is obtained that has unknown displacements and whose terms are stiffness matrices and loads.

Both the use of the multiparameter model subgrade and the use of the modulus of elasticity as a function of stress represent approximations of the effective situation.

In order to evaluate these calculation methods, the structure and earth influence on the rigidity matrix are calculated next. Considering the pipe geometry (large length element), it is possible to approximate the pipe with a beam.

2.1 Stiffness matrix calculation for continuous support

In the traditional method for simulation, the mathematical load-deformation response of a beam in uniaxial bending is a differential equation [6]. The basic form of the matrix formulation for beam flexure is (Eq. (1)):

$$[S]\{d\} = \{q\} \quad (1)$$

where $[S]$ is the stiffness matrix; $\{d\}$ is the displacement vector; $\{q\}$ is the load (force) vector.

The relevance of (Eq. (1)) is that all of the variations in beam behavior can be explained as variations solely in the formulation of the stiffness matrix, $[S]$.

In the Winkler model (**Figure 1**), the flexural behavior of this beam is given by Eq. (2):

$$EI \frac{d^4 w(x)}{dx^4} + p(x) = q(x) \quad (2)$$

where subgrade reaction in one (x -axis) direction only is

$$p(x) = k_w w(x)$$

k_w is the Winkler coefficient of subgrade reaction; E is the modulus of elasticity (Young’s modulus); I is the moment of inertia of beam section.

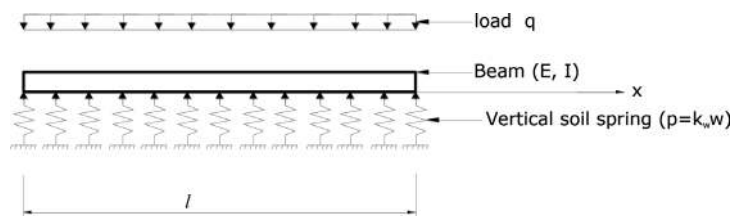


Figure 1.
 The Winkler model.

Solving Eq. (2) by FEM is expressed by Eq. (3):

$$([S_e] + [S_w])\{d\} = \{q\} \quad (3)$$

in which the expressions of the elastic stiffness matrix of the beam $[S_e]$ and subgrade reaction matrix $[S_w]$ are [5].

$$[S_e] = \frac{EI}{l^3} \begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \quad (4)$$

$$[S_w] = \frac{k_w l}{420} \begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix} \quad (5)$$

In the Pasternak model (**Figure 2**), the flexural behavior of this beam is given by Eq. (6) [5, 6]:

$$EI \frac{d^4 w(x)}{dx^4} + p(x) - g \frac{d^2 w(x)}{x^2} = q(x) \quad (6)$$

Solving Eq. (6) by FEM is expressed by Eq. (7):

$$([S_e] + [S_w] + [S_g])\{d\} = \{q\} \quad (7)$$

in which the expressions of the elastic stiffness matrix of the beam $[S_e]$ and subgrade reaction matrix $[S_w]$ are the same like those from Eqs. (4) and (5), and shear matrix $[S_g]$ is given by Eq. (8):

$$[S_g] = \frac{g}{30l} \begin{bmatrix} 36 & 3l & -36 & 3l \\ 3l & 4l^2 & -3l & -l^2 \\ -36 & -3l & 36 & -3l \\ 3l & -l^2 & -3l & 4l^2 \end{bmatrix} \quad (8)$$

Inserting the second parameter for the soil (shear stiffness) has the effect of increasing the stiffness of the beam (increasing the stiffness matrix terms). The stiffness matrices were obtained by considering a continuous bearing of the beam according to **Figure 3** and using a cubic displacement function [13, 14].

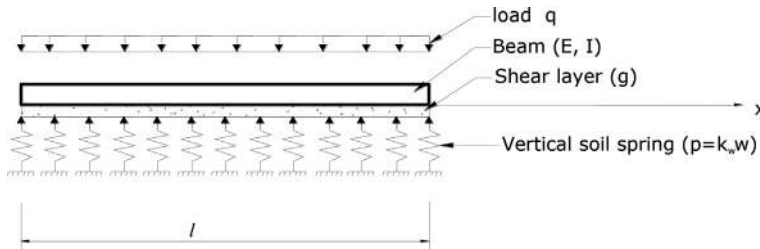


Figure 2.
The Pasternak model.

The meaning of the notations can be inferred from the following equation, which generally defines a stiffness matrix for a finite element with four degrees of freedom (9):

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \quad (9)$$

2.2 Stiffness matrix calculation for support only at the nodes

In the calculation of the beams on elastic support by the finite element method, the determination of the stiffness matrix of the elastic foundation was determined by other authors [15] in the form (10):

$$[S_w] = \frac{k_w l}{2} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (10)$$

Eq. (10) is directly suggested by the calculation scheme of **Figure 4** where it can be seen that only the S_{11} and S_{33} elements of the stiffness matrix have values other than zero (a S_{ij} element of the stiffness matrix is the generalized force that develops in the direction i when a unit displacement or rotation is imposed in the direction j). Eq. (10) can also be obtained by solving Eq. (6) in which the shape functions have the expressions (11):

$$N_1(x) = \begin{cases} 1, & x < \frac{l}{2} \\ 0, & \frac{l}{2} < x < l \end{cases} ; \quad N_2(x) = 0, \quad x \in [0, l] \quad (11)$$

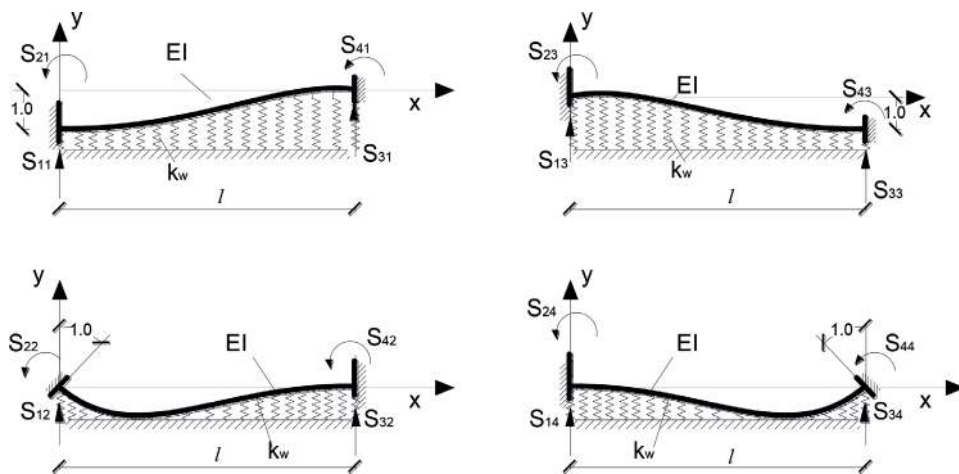


Figure 3.
 Stiffness matrix calculation by continuous bearing.

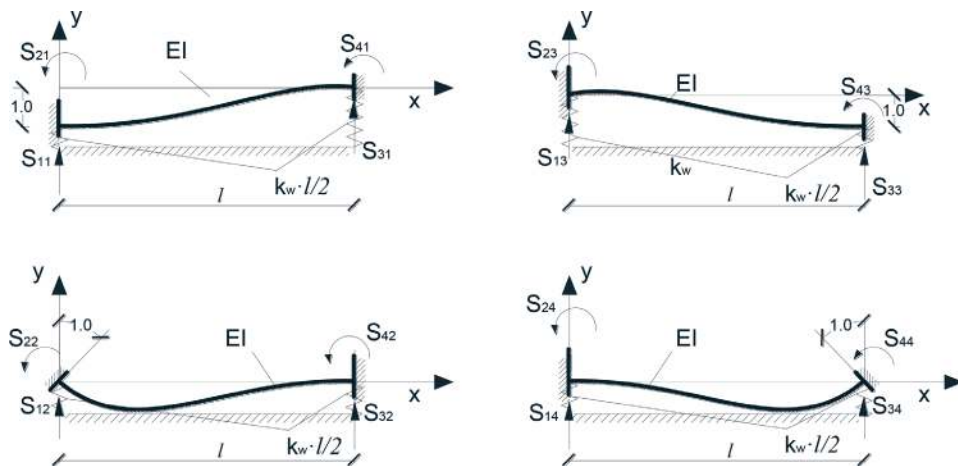


Figure 4.
Stiffness matrix calculation by nodes bearing.

$$N_3(x) = \begin{cases} 0, & x < \frac{l}{2} \\ 1, & \frac{l}{2} < x < l \end{cases} ; \quad N_4(x) = 0, \quad x \in [0, l]$$

With the same expressions (Eq. (10)) for shape functions shear matrix, $[S_g] = 0$. This result it can be understood that the absence of friction between springs considering that the springs are positioned only in the nodes (at a sufficient distance between them).

3. Conduit calculation as a beam

3.1 Calculation scheme

We have noticed that the influence of the earth parameters is quantified in the rigidity matrix of the FEM specific equations (1), (5), and (7). Depending on the multiparameter subgrade model, this influence is variable. Is it also significant for the behavior of the structure, if the values of the matrix elements, which quantify the rigidity of the earth, are comparable to those of the rigidity of the pipe? Further, for comparing the value of the matrix elements and their influence on the final result, we will compute these matrices for a real case [16]. Bottom discharge conduit from an embankment dam is considered as a structural element (Ibaneasa dam from Botosani county—Romania). The pipe is from reinforced concrete with polygonal section (inner rectangle, exterior trapeze) (see **Figure 5**).

The conduit was built in 9 m sections. We will make a calculation of a 9-m-long section in the central area of the dam. It is considered a single finite element between two longitudinal joints of length l (see **Figure 6**). The beam loading and support scheme are shown in **Figure 7**.

Numerical parameters are:

a. For the conduit parameters:

$A = 5.36 \text{ m}^2$, area of concrete section

$I_b = 6.67 \text{ m}^4$, moment of inertia

$E_b = 26 \text{ GPa}$ (for C12/15), modulus of elasticity

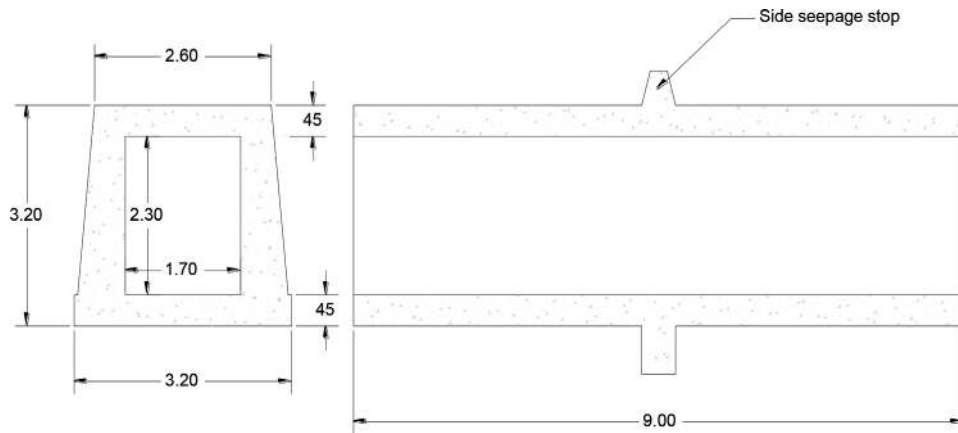


Figure 5.
 Cross and longitudinal section by bottom discharge (reinforced concrete).

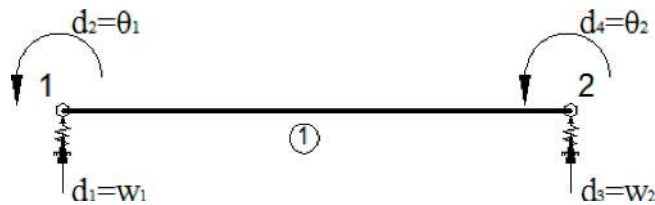


Figure 6.
 Finite element scheme and degrees of freedom.

b. For the ground under conduit:

Each node will be thought of as a spring with its elasticity determined by:

$$k_s = B \cdot k \text{ in which} \quad (12)$$

$B = 3.2 \text{ m}$ is the width of the conduit

The marginal nodes will have the same coefficient of subgrade reaction as the other ones according to [15].

The coefficient of subgrade reaction according to Vesić apud Bowles [15] is given by:

$$k = 0,65 \sqrt[12]{\frac{E_p B^4}{E_b I_b}} \frac{E_p}{B(1 - \mu_p^2)} \quad (13)$$

Ground parameters are (silty clay): $E_p = 35 \text{ MPa}$; $\mu_p = 0.35$; $\gamma_p = 19 \text{ kN/m}^3$.

With these values, it results in $k_s = 3.2 \cdot 5875 = 28.200 \text{ kN/m}$.

Shear modulus for shear layer in foundation is.

$$g = \frac{E_p}{2(1 + \mu_p)} = 13 \text{ MPa} \quad (14)$$

$$g_s = B \cdot g \quad (15)$$

Foundation parameters k and g may be calculated according [6, 7] with the following relations:

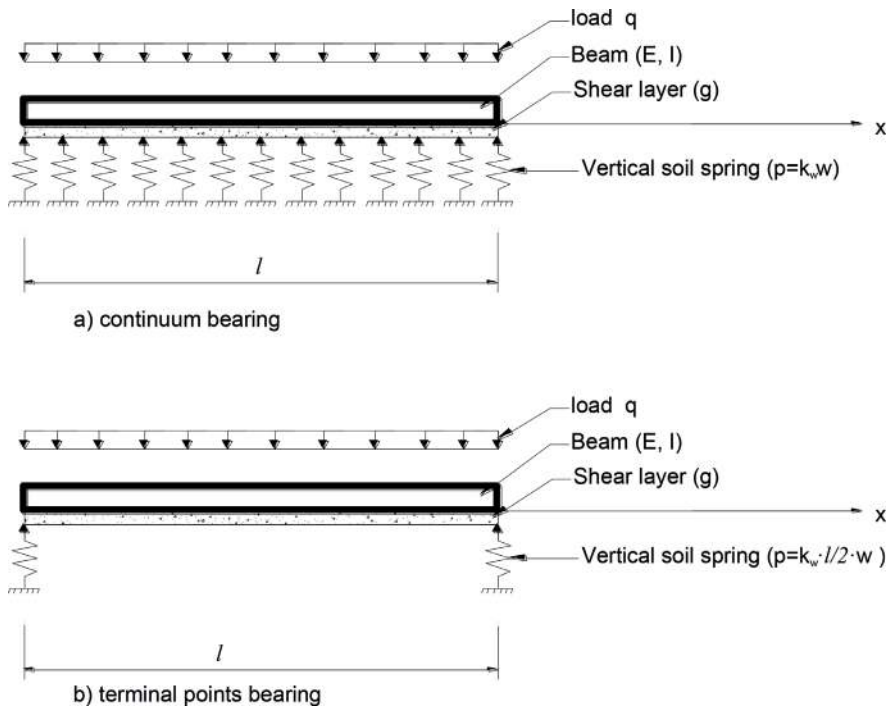


Figure 7. Beam loading and support scheme.

$$k = \frac{E_p}{H} \quad (16)$$

$$g = \frac{E_p}{2(1 + \mu_p)} \frac{H}{2} \quad (17)$$

where H is depth to effective rigid base.

The effective rigid base is defined as the depth at which settlements caused by the structure can be taken to be zero. For decades it has been assumed that the “depth of influence” for settlement equivalent conceptually to the effective depth to rigid base is twice the width of a square loaded area and four times the width of an infinite strip [7].

With this assumptions $H = 6.4$ m; $k = 5468$ kN/m²; $g = 41.5$ MPa.

The load on the conduit from the ground weight can be considered uniformly distributed. The load from the ground with its own pipe weight is 826 kN/m. With these parameters, it is necessary to determine the stresses in conduit schematized by the finite element from **Figures 6** and **7**.

3.2 Solving the FEM equilibrium equation

The matrix equilibrium equation is (7)

$$([S_e] + [S_w] + [S_g])\{d\} = \{q\} \quad (18)$$

which is written in form (1)

$$[S]\{D\} = \{Q\} \quad (19)$$

Solving the equation system (1), (7) is done by partitioning the S-matrices, D and Q separating the displacements according to the free degrees of freedom (2 and 4) by the degrees of freedom with elastic resistances (1 and 3) (see **Figure 6**):

$$\begin{bmatrix} S_{nn} & S_{nr} \\ S_{rn} & S_{rr} \end{bmatrix} \begin{Bmatrix} D_n \\ D_r \end{Bmatrix} = \begin{Bmatrix} Q_n \\ Q_r \end{Bmatrix} + \begin{Bmatrix} R_n \\ R_r \end{Bmatrix} \quad (20)$$

Eq. (20) results in two matrix equations representing the equations of the structure (21):

$$\begin{aligned} S_{nn}D_n + S_{nr}D_r &= Q_n + R_n \\ S_{rn}D_n + S_{rr}D_r &= Q_r + R_r \end{aligned} \quad (21)$$

where

$$D_n = \begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix}; \quad D_r = \begin{Bmatrix} w_1 \\ w_2 \end{Bmatrix} \text{ are displacement subvectors} \quad (22)$$

$$Q_n = q \begin{Bmatrix} -\frac{l^2}{12} \\ l^2 \end{Bmatrix} \quad Q_r = q \begin{Bmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{Bmatrix} \text{ are load subvectors} \quad (23)$$

In solving Eqs. (21), there may be four situations depending on the connections of the structure with the soil [14]:

- a. Fixed connections when $D_r = 0$ and from the first relationship in (21) result displacements and from the second relation (21) result the reactions.
- b. In the case of known displacements of support, D_r has a known value from Eq. (21) that results displacements and reactions.
- c. In the case of elastic connections at the nodes (see **Figures 6 and 7b**).

$$R_n = \begin{Bmatrix} R_2 \\ R_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}; \quad R_r = \begin{Bmatrix} R_1 \\ R_3 \end{Bmatrix} = k_s \begin{Bmatrix} d_1 \\ d_3 \end{Bmatrix} = k_s D_r \text{ are reactions.} \quad (24)$$

Eq. (24) can be written as

$$R_r = \begin{Bmatrix} R_1 \\ R_3 \end{Bmatrix} = \begin{bmatrix} k_s & 0 \\ 0 & k_s \end{bmatrix} \begin{Bmatrix} d_1 \\ d_3 \end{Bmatrix} = [k_s]D_r \quad (25)$$

By replacing (24) in (21), we get

$$\begin{aligned} S_{nn}D_n + S_{nr} \frac{1}{k_s} R_r &= Q_n \\ S_{rn}D_n + \left(S_{rr} \frac{1}{k_s} - I \right) R_r &= Q_r \end{aligned} \quad (26)$$

From the last Eq. (26) results

$$R_r = \left(\frac{1}{k_s} S_{rr} - I \right)^{-1} (Q_r - S_{rn} D_n) \quad (27)$$

which introduced in the first Eq. (26) leading to the calculation of the displacements, which is depicted below

$$\begin{aligned} S_{nn} D_n + S_{nr} \frac{1}{k_s} \left(\frac{1}{k_s} S_{rr} - I \right)^{-1} (Q_r - S_{rn} D_n) &= Q_n \\ S_{nn} D_n + S_{nr} \frac{1}{k_s} \left(\frac{1}{k_s} S_{rr} - I \right)^{-1} Q_r - S_{nr} \frac{1}{k_s} \left(\frac{1}{k_s} S_{rr} - I \right)^{-1} S_{rn} D_n &= Q_n \\ \left[S_{nn} - S_{nr} \frac{1}{k_s} \left(\frac{1}{k_s} S_{rr} - I \right)^{-1} S_{rn} \right] D_n &= Q_n + S_{nr} \frac{1}{k_s} \left(\frac{1}{k_s} S_{rr} - I \right)^{-1} Q_r \end{aligned}$$

Displacements in the directions of unrestrained degrees of freedom are

$$D_n = (S_{nn}^*)^{-1} Q_n^* \quad (28)$$

where

$$S_{nn}^* = S_{nn} - S_{nr} \frac{1}{k_s} \left(\frac{1}{k_s} S_{rr} - I \right)^{-1} S_{rn}; \quad Q_n^* = Q_n + S_{nr} \frac{1}{k_s} \left(\frac{1}{k_s} S_{rr} - I \right)^{-1} Q_r \quad (29)$$

d. In case of elastic connection and continuous support

In this case, (**Figure 7a**, continuum bearing) the rotations are not independent movements; they depend on the rotational stiffness of the ground. In the situation of (c), the rigidity of the displacement matrix had the form

$$[k_s] = \begin{bmatrix} k_s & 0 \\ 0 & k_s \end{bmatrix} \quad (30)$$

In case (d), we will obtain the matrix $[k_s]$ by static condensation of the sum of matrices:

$$[K] = [S_w] + [S_g] \quad (31)$$

For condensation we will use the Guyan method [13]. The condensed matrix K_r is obtained from the matrix K whose terms have the meaning below. DOFs (degrees of freedom) retained are 1 and 3, and DOFs omitted are 2 and 4 (rotation):

$$K = \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{21} & K_{22} & K_{23} & K_{24} \\ K_{31} & K_{32} & K_{33} & K_{34} \\ K_{41} & K_{42} & K_{43} & K_{44} \end{bmatrix} \quad (32)$$

Calculation relation for condensed rigidity matrix of earth parameters is (33) [13].

$$K_r = K_{rr} - K_{ro} K_{oo}^{-1} K_{ro}^T \quad (33)$$

where

$$K_{rr} = \begin{bmatrix} K_{11} & K_{13} \\ K_{31} & K_{33} \end{bmatrix} \quad K_{rn} = \begin{bmatrix} K_{12} & K_{14} \\ K_{32} & K_{34} \end{bmatrix} \quad K_{oo} = \begin{bmatrix} K_{22} & K_{24} \\ K_{42} & K_{44} \end{bmatrix} \quad (34)$$

The condensed rigidity matrix includes the effect of the rigidity of the foundation on the rotation of the beam ends.

3.3 Obtained results

3.3.1 Results for the Winkler (single-parameter) scheme

Stiffness matrix is

$$[S] = [S_e] + [S_w]$$

$$S_e = \begin{pmatrix} 2.855 \times 10^6 & 1.285 \times 10^7 & -2.855 \times 10^6 & 1.285 \times 10^7 \\ 1.285 \times 10^7 & 7.708 \times 10^7 & -1.285 \times 10^7 & 3.854 \times 10^7 \\ -2.855 \times 10^6 & -1.285 \times 10^7 & 2.855 \times 10^6 & -1.285 \times 10^7 \\ 1.285 \times 10^7 & 3.854 \times 10^7 & -1.285 \times 10^7 & 7.708 \times 10^7 \end{pmatrix}$$

$$S_w = \begin{pmatrix} 6.285 \times 10^4 & 7.977 \times 10^4 & 2.175 \times 10^4 & -4.713 \times 10^4 \\ 7.977 \times 10^4 & 1.305 \times 10^5 & 4.713 \times 10^4 & -9.789 \times 10^4 \\ 2.175 \times 10^4 & 4.713 \times 10^4 & 6.285 \times 10^4 & -7.977 \times 10^4 \\ -4.713 \times 10^4 & -9.789 \times 10^4 & -7.977 \times 10^4 & 1.305 \times 10^5 \end{pmatrix}$$

$$S = \begin{pmatrix} 2.917 \times 10^6 & 1.293 \times 10^7 & -2.833 \times 10^6 & 1.28 \times 10^7 \\ 1.293 \times 10^7 & 7.721 \times 10^7 & -1.28 \times 10^7 & 3.844 \times 10^7 \\ -2.833 \times 10^6 & -1.28 \times 10^7 & 2.917 \times 10^6 & -1.293 \times 10^7 \\ 1.28 \times 10^7 & 3.844 \times 10^7 & -1.293 \times 10^7 & 7.721 \times 10^7 \end{pmatrix}$$

The displacements of the conduit ends are:

$$D := \begin{pmatrix} -0.068 \\ -3.78 \times 10^{-4} \\ -0.068 \\ 3.78 \times 10^{-4} \end{pmatrix} = \begin{Bmatrix} w_1 \\ \theta_1 \\ w_2 \\ \theta_2 \end{Bmatrix}$$

Displacements w_1 and w_2 are measured in [m] and rotation in [rad]
 The rigidity matrix (Eq. (30)) has the value

$$k_s = \begin{pmatrix} 3.008 \times 10^4 & 0 \\ 0 & 3.008 \times 10^4 \end{pmatrix}$$

The displacement of the center of the beam (conduit) is

$$w = -0.0689 \text{ [m]}$$

Displacement in the center of the beam is calculated using shape functions according to relation:

$$w(1/2) = [N_1 \ N_2 \ N_3 \ N_4] \{ w_1 \ \theta_1 \ w_2 \ \theta_2 \}^T \quad (35)$$

3.3.2 Results for multiple-parameter subgrade model scheme (Pasternak)

The stiffness matrix is

$$[S] = [S_e] + [S_w] + [S_g] \quad (36)$$

The stiffness matrix values are:

$$S_e = \begin{pmatrix} 2.855 \times 10^6 & 1.285 \times 10^7 & -2.855 \times 10^6 & 1.285 \times 10^7 \\ 1.285 \times 10^7 & 7.708 \times 10^7 & -1.285 \times 10^7 & 3.854 \times 10^7 \\ -2.855 \times 10^6 & -1.285 \times 10^7 & 2.855 \times 10^6 & -1.285 \times 10^7 \\ 1.285 \times 10^7 & 3.854 \times 10^7 & -1.285 \times 10^7 & 7.708 \times 10^7 \end{pmatrix}$$

$$S_w = \begin{pmatrix} 6.285 \times 10^4 & 7.977 \times 10^4 & 2.175 \times 10^4 & -4.713 \times 10^4 \\ 7.977 \times 10^4 & 1.305 \times 10^5 & 4.713 \times 10^4 & -9.789 \times 10^4 \\ 2.175 \times 10^4 & 4.713 \times 10^4 & 6.285 \times 10^4 & -7.977 \times 10^4 \\ -4.713 \times 10^4 & -9.789 \times 10^4 & -7.977 \times 10^4 & 1.305 \times 10^5 \end{pmatrix}$$

$$S_g = \begin{pmatrix} 5.531 \times 10^3 & 4.148 \times 10^3 & -5.531 \times 10^3 & 4.148 \times 10^3 \\ 4.148 \times 10^3 & 4.978 \times 10^4 & -4.148 \times 10^3 & -1.244 \times 10^4 \\ -5.531 \times 10^3 & -4.148 \times 10^3 & 5.531 \times 10^3 & -4.148 \times 10^3 \\ 4.148 \times 10^3 & -1.244 \times 10^4 & -4.148 \times 10^3 & 4.978 \times 10^4 \end{pmatrix}$$

$$S = \begin{pmatrix} 2.923 \times 10^6 & 1.293 \times 10^7 & -2.838 \times 10^6 & 1.28 \times 10^7 \\ 1.293 \times 10^7 & 7.726 \times 10^7 & -1.28 \times 10^7 & 3.843 \times 10^7 \\ -2.838 \times 10^6 & -1.28 \times 10^7 & 2.923 \times 10^6 & -1.293 \times 10^7 \\ 1.28 \times 10^7 & 3.843 \times 10^7 & -1.293 \times 10^7 & 7.726 \times 10^7 \end{pmatrix}$$

The condensed rigidity matrix of earth parameters, according relation (33) is:

$$k_s = \begin{pmatrix} 2.871 \times 10^4 & 472.813 \\ 472.813 & 2.871 \times 10^4 \end{pmatrix}$$

The displacements of the conduit ends are:

$$D := \begin{pmatrix} -0.0657 \\ -3.694 \times 10^{-4} \\ -0.0657 \\ 3.694 \times 10^{-4} \end{pmatrix} = \begin{Bmatrix} w_1 \\ \theta_1 \\ w_2 \\ \theta_2 \end{Bmatrix}$$

Displacements w_1 and w_2 are measured in [m] and rotation in [rad]. The displacement of the center of the beam (conduit) is

$$w = -0.0665 \text{ [m]}$$

3.4 Comments about conduit calculation as a beam

The calculation of rigidity for the conduit and for the terrain is common. Solving equilibrium Eq. (7) for situations (a), (b), and (c) is usual. For continuous support (d) we used condensation of stiffness matrices, the method that I consider new, through which includes the rigidity of rotation of the bar and the rigidity of the terrain in the stiffness of the bar ends. In this way we have reduced the number of unknowns—reactions, and solving the equation in the case (d) becomes similar to the solution in the case (c).

The purpose of calculating rigidity matrices and conduit (beam) displacements, for the two methods of determining the ground parameters (the Winkler and multiple-parameter subgrade model, the Pasternak), is to highlight the small difference between the calculated values. This small difference between displacements (4%) is due to the high rigidity (high moment of inertia) of the conduit.

A first conclusion is that for structures with a high transversal moment of inertia, the method of determining the parameters of the earth is not very important. The difference between the elements of rigidity matrices for the structure and for the Winkler earth is about 100 times and between the Winkler earth and the Pasternak rigidity is about 10 times.

A second conclusion is that the use of the SAP 2000 finite element software which includes the Winkler spring model is acceptable for bottom discharge conduit due to their high rigidity (of the conduit).

From these considerations, we can identify two calculation methods for this type of conduits. In a first method, the pipeline is calculated as a beam supported by the Winkler-type springs. You can use any finite element software that allows you to use springs. The joints between the conduit segments are introduced into the calculation scheme with their specific conditioning (restraints). In a second method the calculation can be done with a geotechnical software (e.g., GeoStudio) in which the modulus of elasticity is introduced as input parameters and the calculation is performed in a plane strain condition [17]. In this calculation method, somehow the influence of the joints between the pipe sections must be simulated. The following relationship is proposed for calculating the equivalent inertia moment of the pipe in which joints are taken into account:

$$I_e = I \frac{1}{2n} \quad (37)$$

where I_e is the equivalent inertial momentum; I is the current section inertial momentum; n is the number of joints.

4. Conduit calculation example

The calculation will be made for the bottom discharge conduit of the Tungeji earth dam in Iasi County, Romania, designed as a cross-sectional structure of three rectangular reinforced concrete cassettes (**Figure 8**) [18]. In the longitudinal profile, the structure has joints between 5 and 10 m. The joints are sealed with PVC tape, and the reinforcement has no continuity in the joint.

The calculation scheme is the Winkler elastic beam and the EngiLab Beam 2D software [19]. The conduit consists of 2 reinforced concrete sections of 5 m and another 11 sections of 10 m length each (**Figure 9**). Between the sections there were joints of 2.5 cm, and the continuity is achieved with the sealing tape.

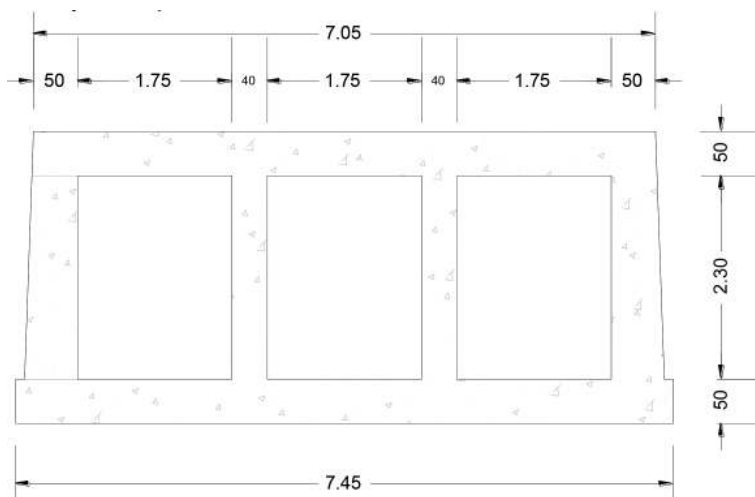


Figure 8.
Transverse section through conduit.

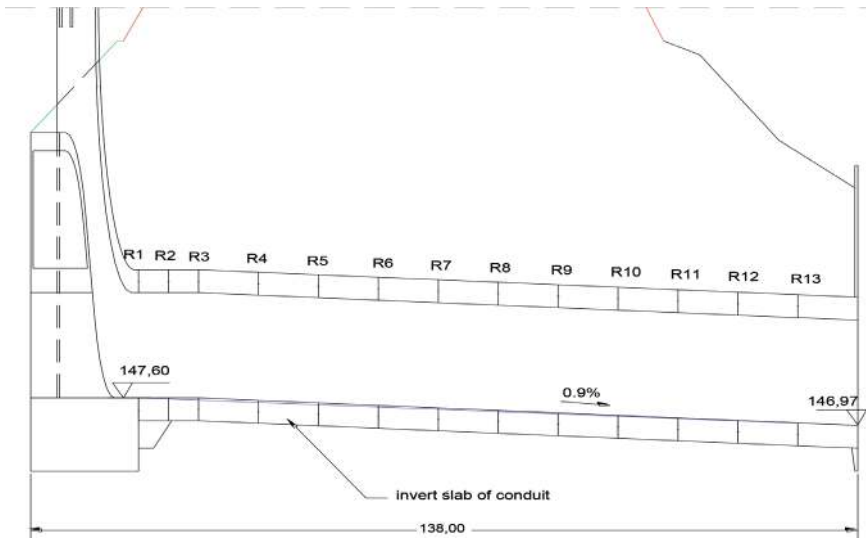


Figure 9.
 Longitudinal section through conduit.

The vertical load on the pipe is after Marston [4]:

$$P_v = C_e \gamma D_e^2, \text{ where:} \quad (38)$$

$$C_e = \frac{e^{2K\mu(H/D_e)} - 1}{2K\mu} \text{ or} \quad (39)$$

$$C_e = \frac{e^{2K\mu(H_e/D_e)} - 1}{2K\mu} + \left(\frac{H}{D_e} - \frac{H_e}{D_e} \right) e^{2K\mu(H_e/D_e)} \quad (40)$$

where $K = \text{tg}^2(45^\circ - j/2)$ is Rankine's lateral pressure coefficient; $\mu = \text{tg} \varphi$ is the coefficient of friction of the earth; H_e is the position of the plan of equal settlement.

Marston determined the existence of a horizontal plane above the conduit where the shearing forces are zero. This plane is called the *plane of equal settlement*. Above this plane, the interior and exterior prisms of soil settle equally.

D_e is the outer diameter (width) of the pipe.

The relation (39) is valid for $H_e > H$ (the plane of equal compression is imaginary) and the relation (40) is valid for $H_e < H$ (the additional load relative to the weight of the earth column) depend on the top-down friction forces appearing in the earth column above the

conduit in the vertical planes tangent to the pipe. In the case of the Tungujei dam, the thickness of the filling of 15 m over a 7.45 m width of the earth column is per linear meter of the pipe. According to the Marston relationship, it follows (considering the plane of equal compaction to the surface $H = H_e$).

$$C_e = 2.64; P_v = 2784 \text{ kN/m}$$

The pipeline calculation parameters are

$A = 11.67 \text{ m}^2$, area of the concrete conduit

$I_b = 16.05 \text{ m}^4$, moment of inertia for section

$E_b = 26 \text{ GPa}$ (for concrete C12/15), modulus of elasticity for concrete

Foundation parameters k may be calculated according Horvath [6] with the following relations:

$$k = \frac{E_p}{H} \quad (41)$$

where $E_p = 18,000 \text{ kN/m}^2$ earth (foundation) modulus; $H = 2 \times 7.5 \text{ m} = 15 \text{ m}$, depth of influence; $k = 1200 \text{ kN/m}^3$.

When loading with the weight of the ground, the weight of the conduit cassette was added.

Two pipeline calculation schemes were used between the R3 and R12 joints (see **Figure 9**). The 5 m sections were removed from the calculation scheme and the terminal sections embedded in the portal.

In the first diagram, the nine sections of the pipe were considered to be articulated at the ends. A maximum of 32.9 cm settlement resulted (**Figures 10–12**).

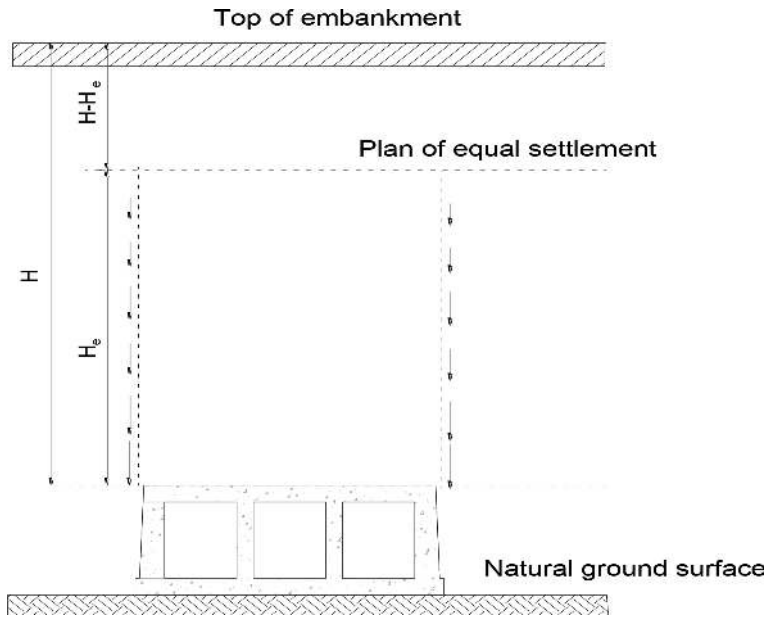


Figure 10.
Vertical loads that the earth filling exerts on the conduit.

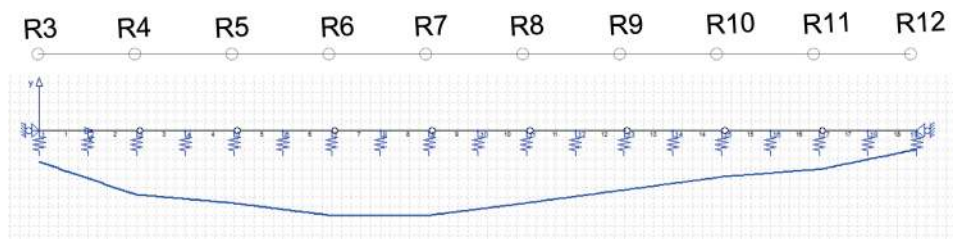


Figure 11.
Conduit displacement with articulations in joints.

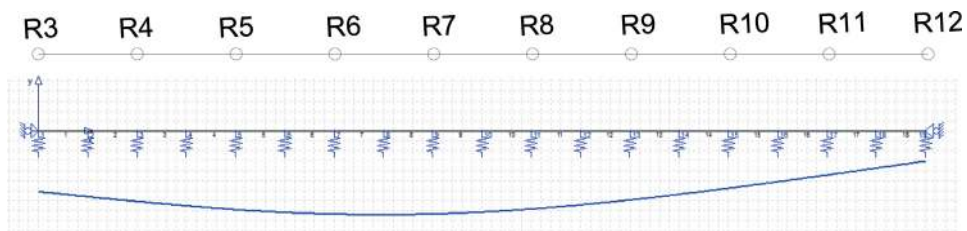


Figure 12.
Conduit displacements without articulations and equivalent moment of inertia.

In the second calculation scheme, the nine continuous pipe sections were considered without interruptions in joints. A maximum of 32.6 cm settlement resulted.

The equivalent moment of inertia is calculated with Eq. (37):

$$I_e = I \frac{1}{2n} = 16.04 \frac{1}{2 \times 8} = 1m^4$$

5. Conclusions

We calculated conduits through FEM using analytical calculation for a finite element and calculation with a software (EngiLab Beam). In the calculation for a finite element, we have highlighted the stiffness matrices to understand the large value difference between the terms of the matrices and the physical significance of this difference.

The low rigidity of the ground slightly influences the state of stresses and deformations in the conduit. It follows that it is sufficient that the parameters of the earth are defined by the Winkler-type springs.

A first conclusion is that computational programs that allow the use of the Winkler-type springs give results with good accuracy (e.g., below 5%) for this type of structure. The overall view at this point is that the Winkler model is outdated.

The difference in conduit displacements calculated with the two methods of estimating the ground characteristics (the Winkler and multiple-parameter subgrade model) is small. It means that the shear stiffness introduced by multiple-parameter subgrade models does not significantly change the result. It is therefore reasonable to use the Winkler parameters for the terrain characteristics. These considerations apply to conduits where the stiffness of the conduit (measured in the stiffness matrix) is 100 times higher than the rigidity of the earth (also expressed by the stiffness matrix).

The calculation of bottom discharge conduit as well as other long hydrotechnical structures is done with reasonable accuracy in the plane strain state. Finite element programs calculate stress state and deformations with this scheme. In the case of conduits, the influence of the joints between the sections must be modeled. If this is not done, the results are not credible. In the calculated example, we used two schemes: one with the pipe provided with joints and with the respective struts and the second with the pipe without joints (continuous) and with the inertia moment adjusted with the relation (37). The difference between the results obtained with these two schemes is small (1%). Models of finite element type are credible if the conduit is made jointless, which is unlikely.

Resuming the findings of this work are:

Using the rigidity matrix condensation, to solve the equilibrium equation for beams with deformable supports, facilitates solving the equation system (21).

We appreciate that the use of the Winkler-type springs is suitable for long rigid structures.

Calculation of conduit sections in plane strain state leads to credible results only if the moment of inertia is adjusted in accordance with pipe joints. For this we propose a relation of calculating the moment of equivalent inertia (37). If the inertia moment is not adjusted, the result obtained is not credible.

Conflict of interest

The author declares that there is no conflict of interest.

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