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Bouncing Cosmologies in Brans-Dicke Theory

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Abstract

In this paper it is shown that in Brans-Dicke (BD) theory, if one considers a non-minimal coupling between the matter and scalar field, it can give rise to a bouncing universe (i.e. an expanding universe preceded by a contracting universe). Two examples of such universes have been considered in a spatially flat FRW universe and their physical properties have been studied.

Keywords: FRW Cosmological model; Brans-Dicke theory; Scalar field; Bounce.

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1 Introduction

Gravitation is the dominant force controlling the large scale behaviour of the universe. Einstein was the first to give a theory of gravitation in a generally covariant form. General Relativity is a geometrical theory of space-time. The fundamental building block is a metric tensor field g_{ij}. The Brans-Dicke (BD) theory [1] (whose original motivation was the search for a theory containing Mach's Principle) is the theory where it is considered that there is a scalar partner for the metric tensor for describing geometry of space-time. In this theory the scalar field is a fundamental part, while in other theories the scalar field is introduced separately in an ad-hoc manner. This theory can pass experimental test from solar system [2, 3]. It has been proposed as an alternative to general relativity, other theories are $f(R)$ gravity [4, 5], massive theories of gravity [6, 7] to name a few. The earliest modification to general relativity was Kaluza-Klein gravity [8] which was intended to unify gravity with electromagnetic force.

The present universe is undergoing an accelerated phase of expansion as confirmed by the observational data regarding the luminosity red-shift relation of type Ia supernovae [9], the cosmic microwave background radiation [10]. In order to explain the cosmic positive acceleration, an exotic form of matter dark energy has been proposed within the frame work of general relativity [11, 12]. Positive acceleration can also be explained through modification of gravity. There have been many attempts to show that BD model can potentially explain the cosmic acceleration. This theory can produce a non-decelerating expansion for low negative value of BD parameter w [13] but this conflicts with lower bound imposed on this parameter by solar system experiments [14]. Some authors propose modifications of Brans-Dicke model by introducing some potential functions for the scalar field [15, 16] or considering a field dependent BD parameter [17].

In a different approach within the framework of general relativity, scalar field is allowed to interact non-minimally with the matter through an arbitrary function of scalar field as a coupling function. This type of field is called Chameleon field [18, 19]. This Chameleon field can be heavy enough in the environment of the laboratory tests so that the local gravity constraints are satisfied. Also, this field can be light enough in low density cosmological scenario to be considered as a candidate for the dark energy. Chameleon field can provide a very smooth transition from a decelerated to accelerated phase of expansion of the universe [20]. In BD theory or its modification, there is an interaction between scalar field and geometry. Chameleon field is non-minimally coupled to normal matter sector. We take a Brans-Dicke framework in which there is a non-minimal coupling of the scalar field with matter. The behaviour of an isotropic cosmological model in the early as well as in late time limits in the Brans-Dicke framework has been investigated by Clifton and Barrow [21]. The non-minimal coupling of a scalar field with both geometry and matter has been used in literature [22, 23, 24, 25, 26].

An unwanted feature of cosmological theories , the initial singularity problem, is being solved by bouncing cosmology [27, 28, 29, 30, 31, 32, 33, 34, 35]. A bouncing universe contracts until a minimal radius is reached, and after that point it expands. The universe, therefore, never goes to a singular point, thus avoiding the initial singularity. The idea of a bouncing universe in GR framework has been examined many time since the 1930's [36], loop-quantum gravity [37, 38], matter bounce theories [39, 40, 41], modified gravity [35, 42] and it offers a consistent description of bouncing cosmologies.

The necessary conditions for bounce in different theories have been investigated by the several authors [43, 44, 45, 46]. Bouncing solutions in cosmological scenarios resulting from effective actions in four dimensions which are, under some assumptions, connected with multi-dimensional, super-gravity and string theories has been discussed in the papers [47, 48]. Farajollahi et al. [49] have considered stability analysis and possibility of Phantom crossing with the assumption that potential of the scalar field and coupling function are in the power-law forms, and bouncing solutions, some cosmological tests are investigated [50]. Recently, bouncing cosmology with future singularities in modified gravity [51], has been discussed with an anstaz for scale factor. Thermodynamics of non-singular bouncing universes governed by GR has been investigated in [52].

The present paper is organised as follows. Section 2 deals with the basic equations of cosmological model. In this section, it is shown that non-minimally coupled BD scalar models can provide a smooth transition from the contracting to expanding model. Section 3 contains two classes of exact solutions for a spatially flat FRW universe. The paper ends with a conclusion given in the Section 4.

2 Basic Equations

We consider the action functional:

$$S = \int \sqrt{-g} d^4x \left[f(\phi) L_m + \frac{1}{2}\left(\phi R - \frac{w}{\phi}\phi_{,i}\phi^{,i} \right) \right] \tag{1}$$

where R is the Ricci scalar, $\phi = \phi(t)$ is the BD Scalar field which is non-minimally coupled to gravity, g is the metric determinant of the metric g_{ij}. Here $f(\phi)$ is the arbitrary function of ϕ and L_m is matter Lagrangian

representing the perfect fluid matter. We take $8\pi G = c = 1$.
Varying the action with respect to metric g_{ij} and ϕ yields the field equations:

$$R_{ij} - \frac{1}{2}Rg_{ij} = -\frac{f(\phi)}{\phi}T_{ij} + \frac{1}{\phi}(g_{ij}\Box - \nabla_i\nabla j)\phi - \frac{w}{\phi^2}(\partial_i\phi\partial_j\phi - \frac{1}{2}g_{ij}(\partial_i\phi)^2) \quad (2)$$

$$\frac{2w+3}{\phi}\Box\phi = \frac{f(\phi)}{\phi}T - 2f^{'}(\phi)L_m \quad (3)$$

where T is the trace of energy-momentum tensor $T_{ij} = \frac{-2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m)}{\delta g^{ij}}$ and prime denotes the derivation with respect to ϕ.

The line element of spatially flat FRW model is

$$ds^2 = dt^2 - a^2(t)\left[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] \quad (4)$$

where $a(t)$ is the scale factor. For perfect fluid, the stress-energy tensor is given by $T_{ij} = (\rho+p)u_iu_j - pg_{ij}$ where, u_i is the 4-velocity vector. Here ρ & p are the energy density and pressure respectively. The field equations for flat FRW universe can be written as:

$$3H^2 = \frac{f(\phi)}{\phi}\rho + \frac{w}{2}\frac{\dot{\phi}^2}{\phi^2} - 3H\frac{\dot{\phi}}{\phi} \quad (5)$$

$$2\dot{H} + 3H^2 = -\frac{f(\phi)}{\phi}p - \frac{w}{2}\frac{\dot{\phi}^2}{\phi^2} - 2H\frac{\dot{\phi}}{\phi} - \frac{\ddot{\phi}}{\phi} \quad (6)$$

where, $H = \frac{\dot{a}}{a}$ is the Hubble parameter . Overhead dot denotes the derivative with respect to t. The dynamical equation for the scalar field ϕ is:

$$(2w+3)(\ddot{\phi} + 3H\dot{\phi}) = f(\rho - 3p) - 3f^{'}\phi\rho \quad (7)$$

The matter conservation equation can be written as

$$\dot{\rho} + 3H(1+\gamma)\rho = -(1+\gamma)\rho\frac{\dot{f}}{f} \quad (8)$$

where, γ is the equation of state parameter defined as $\gamma = \frac{p}{\rho}$. For constant γ, the solution of the above equation can be written as

$$\rho = \frac{\rho_0}{f^{1+\gamma}a^{3(1+\gamma)}} \quad (9)$$

where, ρ_0 is a positive constant. This relation indicates that evolution of matter density depends upon the coupling function f. However if equation of state parameter γ is the function of time then, from equations (5) and (6), the equation of state parameter γ in terms of geometrical quantities can be written as

$$\gamma = \frac{2\dot{H} + 3H^2 + \frac{w}{2}\frac{\dot{\phi}^2}{\phi^2} + 2H\frac{\dot{\phi}}{\phi} + \frac{\ddot{\phi}}{\phi}}{-3H^2 + \frac{w}{2}\frac{\dot{\phi}^2}{\phi^2} - 3H\frac{\dot{\phi}}{\phi}} \tag{10}$$

We take the scale factor $a(t)$ of the form:

$$a(t) = \sqrt{{a_0}^2 + \alpha^2 t^2} \tag{11}$$

where a_0, α are non-zero positive constants. The above scale factor is the temporal analogue of the toy model traversable wormhole [53]. The Hubble parameter for the above scale factor is given by

$$H(t) = \frac{\alpha^2 t}{{a_0}^2 + \alpha^2 t^2} \tag{12}$$

By a bouncing universe, we mean a universe that undergoes a collapse, attains minimum and then subsequently expands. For a successful bounce in FRW model, during contraction phase $a(t)$ is decreasing i.e. $(\dot{a}(t) < 0)$ and then in the expanding phase, scale factor is increasing i.e. $(\dot{a}(t) > 0)$. From Fig.1, it is clear that the above scale factor undergoes contracting to expanding phase. At the bounce point i.e. at $t = t_b$, the minimal necessary condition is (i) $\dot{a}(t_b) = 0$ and (ii) $\ddot{a}(t) > 0$ for $t \in (t_b - \epsilon, t_b) \cup (t_b, t_b + \epsilon)$ for small $\epsilon > 0$. For a non-singular bounce $a(t_b) \neq 0$. These conditions may not be sufficient for a non-singular bounce. For the above scale factor at $t = 0$, we get $\dot{a}(t = 0) = 0$ and $a(0) = a_0$ with $\ddot{a}(t) > 0$ in small neighbourhood of $t = 0$, provided $a_0 > \alpha t$. Therefore, we have a scale factor satisfying necessary condition of bounce.

In terms of geometrical quantities, we have

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 = \frac{(\alpha a_0)^2}{({a_0}^2 + \alpha^2 t^2)^2} \tag{13}$$

Therefore, after the bounce, the universe expands in an accelerated way. The idea of a bounce in flat universe may appear difficult to visualize, but can be understood if we remember that the quantity $\dot{H}_b$ gives a measure of the deviation of the matter world-lines. In this sense, the bounce condition simply means that there exist a phase in which separation between the matter world-lines decreases to a minimum and then increases again. Since, this phenomena is independent of spatial geometry of the space-time, the bounce itself is independent of it [44]. From equations (5) and (6), it is clear that

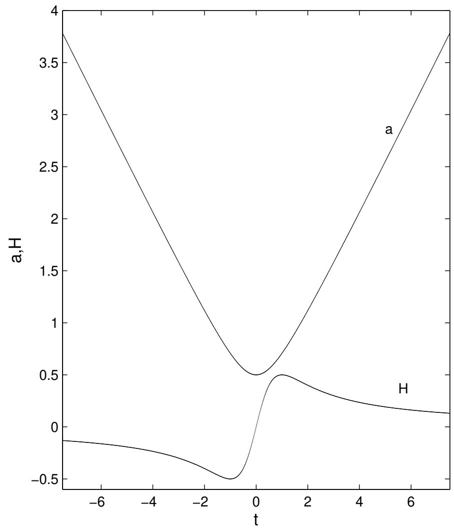

Figure 1: The plot of scale factor (a) and Hubble parameter (H) with t for $a_0 = 0.5$ and $\alpha = 0.5$.

overall dynamics of the universe depends upon the magnitude and sign of derivatives of ϕ. In the next section, we make two anstaz for $\frac{\dot{\phi}}{\phi}$ and discuss the properties of resulting bouncing universe.
In terms of energy density and pressure for the perfect fluid, the energy conditions can be stated as: Null Energy Condition (NEC) is satisfied when $\rho + p \geq 0$.
Weak Energy Condition (WEC) is satisfied when $\rho \geq 0$ and $\rho + p \geq 0$.
Strong Energy Condition (SEC) is satisfied when $\rho + p \geq 0$ and $\rho + 3p \geq 0$.
It is clear that violation of NEC will lead to violation of other energy conditions.

3 Bouncing Universes

In this section we discuss two physically viable cosmologies (i.e. $\frac{\dot{\phi}}{\phi} = -\beta H$ and $\frac{\dot{\phi}}{\phi} = nt$), which have physical interest to describe the scalar field.

3.1 Scalar field as a function of scale factor

Here we make an anstaz:

$$\frac{\dot{\phi}}{\phi} = -\beta H \tag{14}$$

where, β is a non-zero constant. From equations (10), (12) and (14), we get

$$\gamma = \frac{(1-\beta)+(1+\frac{w}{2})\beta^2}{3(\beta-1)+\frac{w}{2}\beta^2} + \frac{(2-\beta)a_0}{\left(3(\beta-1)+\frac{w}{2}\beta^2\right)(\alpha t)^2} \tag{15}$$

From equations (5), (6) and (14), we get

$$\frac{f}{\phi}(\rho+p) = (\beta-2)\dot{H} - (\beta+(1+w)\beta^2)H^2 \tag{16}$$

$$\frac{f}{\phi}(\rho+3p) = 3(\beta-2)\dot{H} + (2-\beta-(2w+3)\beta^2)H^2 \tag{17}$$

At the bouncing point, for $\beta < 2$, NEC and SEC both are violated for $\frac{f}{\phi} > 0$. If $\beta \geq 2$, $\frac{f}{\phi} > 0$, NEC and SEC may not be violated at the bouncing point. The deceleration parameter is given by

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = -\frac{{a_0}^2}{(\alpha t)^2} \tag{18}$$

For any a_0, α, q is always negative and after the bounce, as t increases, q will increase. From equation (13), $\frac{\ddot{a}}{a} > 0$, therefore, after the bounce, universe expands in an accelerated way. From equations (12) and (14), we have

$$\phi = \phi_0({a_0}^2 + \alpha^2 t^2)^{\frac{-\beta}{2}} \tag{19}$$

where, ϕ_0 is a constant of integration. For $w = -\frac{3}{2}$, we have $2w+3 = 0$ and in conformally transformed version of BD theory, it indicates that kinetic energy contribution from the scalar filed is exactly zero [54]. For this choice of w, from equation (7), we have

$$f(\phi) = \frac{\phi_1}{\phi} \tag{20}$$

where, ϕ_1 is a constant of integration. Therefore, in this case, we have

$$\rho = \frac{3\phi_1}{\phi_0}\left(\frac{\beta^2}{4} - \beta + 1\right)\alpha^4 t^2 ({a_0}^2 + \alpha^2 t^2)^{-\beta-2} \tag{21}$$

By using $p = \gamma\rho$, we can get pressure for this bouncing universe with $w = \frac{-3}{2}$. From Fig.2, it is clear that equation of state parameter γ crosses $\gamma = -1$. Therefore, this particular phenomenological model of universe is Quintom model [56]. Quintom is a dynamical model of dark energy. It differs from cosmological constant, Quintessence, Phantom, k-essence and so on, in determination of the cosmological evolution. The behaviour of Brans-Dicke scalar

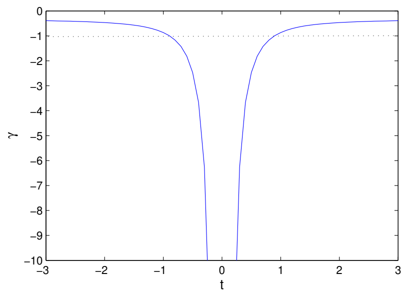

Figure 2: The plot of γ with cosmic time t for $a_0 = 0.5, \beta = -0.5, \alpha = 0.5, w = -3/2$.

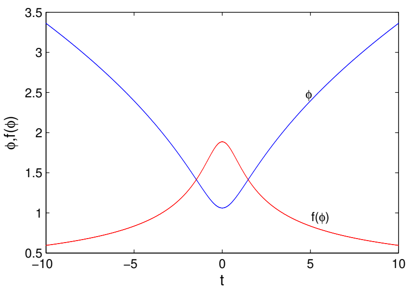

Figure 3: The plot of ϕ and $f(\phi)$ with t for $a_0 = 0.5, \alpha = 0.5, \beta = -0.5, \phi_0 = 1.5, \phi_1 = 2$.

field and the function of scalar field are shown is Fig.3. Here it is observed that the scalar field attains its minima at the bouncing point of the universe. A thermodynamic description of a perfect fluid matter system requires the knowledge of particle flux $N^i = nu^i$ and the entropy flux $S^i = n\sigma u^i$, where, $n = \frac{N}{a^3}$, $\sigma = \frac{S}{N}$ are respectively the concentration and the specific entropy (per particle) of the created/ annihilated particles. Since the energy density of the matter is given by $\rho = nM$, where M is mass of each particle, the appearance of the extra term in the continuity equation means that this extra change of ρ can be attributed to a change of n or M. Here, we assume that mass of each matter particle remains constant and the extra term in the continuity equation only leads to a change of number of number density n [25]. In this case, from continuity equation, we have

$$\dot{n} + 3H(1+\gamma)n = -(1+\gamma)n\frac{\dot{f}}{f} \tag{22}$$

We take $\Gamma = -\frac{\dot{f}}{f}$ as the decay rate. We also assume that overall energy transfer is adiabatic process in which matter particles are continuously created or annihilated while the specific entropy per particle remains constant during the whole process ($\dot{\sigma} = 0$) [55]. This means that

$$\frac{\dot{S}}{S} = \frac{\dot{N}}{N} = \Gamma \tag{23}$$

For $2w + 3 = 0$ case, we have

$$\Gamma = -\frac{\alpha^2 \beta t}{{a_0}^2 + \alpha^2 t^2} \tag{24}$$

Therefore, the second law of thermodynamics, $\dot{S} \geq 0$, can be satisfied if $\beta < 0$. And, $\Gamma \geq 0$ for $\beta < 0$. The second law of thermodynamics, is satisfied at the bouncing point and from equation (23) as $\Gamma \geq 0$ in the expanding universe, the ratio $\frac{\dot{S}}{S}$ will follow the decay rate.

3.2 Scalar field as an exponential form

Here we make an anstaz:

$$\frac{\dot{\phi}}{\phi} = nt \tag{25}$$

where, n is a non-zero constant. Now, from equations (10), (12) and (25), we get

$$\gamma = \frac{{a_0}^2(2\alpha^2 - n{a_0}^2)}{t^2 T} + \frac{\left(\frac{w}{2}+1\right) n^2\alpha^4 t^4 + (\alpha^2 + (w+2)n{a_0}^2)n\alpha^2 t^2 + (\alpha^4 + ({a_0}^2 + \frac{w}{2})n^2{a_0}^2)}{T} \tag{26}$$

where, $T = \frac{w}{2}n^2\alpha^4 t^4 + (wn{a_0}^2 - 3\alpha^2)n\alpha^2 t^2 + (\frac{w}{2}n{a_0}^2 - 3\alpha^2)n{a_0}^2 - 3\alpha^4$. From equations (5), (6) and (25), we get

$$\frac{f}{\phi}(\rho + p) = -2\dot{H} + nHt - (1+w)n^2t^2 - n \tag{27}$$

$$\frac{f}{\phi}(\rho + 3p) = -6\dot{H} - 6H^2 - 3nHt - (2w+3)n^2t^2 \tag{28}$$

At the bouncing point, we have $H = 0$ and $\dot{H} > 0$, therefore for $\frac{f}{\phi} > 0$, $\rho + p < 0$ with $n > 0$ and $\rho + 3p < 0$. Thus in this particular bouncing model, NEC and SEC both are violated for $n > 0$.

As $\frac{\ddot{a}}{a} > 0$, (from equation (13)), we have an accelerating universe after the bounce. For any a_0, α, q is always negative and after the bounce, as t increases, q will increase. From equations (25), we have

$$\phi = \phi_2 \exp\left(\frac{nt^2}{2}\right) \tag{29}$$

where, ϕ_2 is a constant of integration. For $2w+3=0$ case, we have $f(\phi) \propto \frac{1}{\phi}$.
From Fig.4, it is clear that equation of state parameter γ crosses $\gamma = -1$.

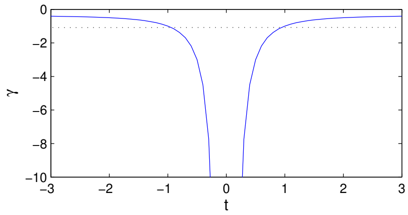

Figure 4: The plot of γ with cosmic time t for $n = 0.0005, \alpha = 0.5, a_0 = 0.5, w = -3/2$.

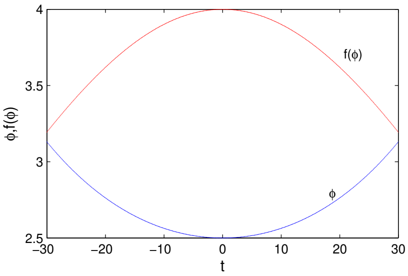

Figure 5: The plot of ϕ and $f(\phi)$ with cosmic time t for $n = 0.0005, \phi_1 = 10, \phi_2 = 2.5$.

The behaviour of scalar field is shown is Fig.5. Here it is observed that, at the bouncing point of the universe, the scalar field and the function of scalar field attain its minima and maxima respectively. Now, we have

$$\rho = \frac{e^{nt^2} t^2}{({a_0}^2 + \alpha^2 t^2)^2} \left(3n\alpha^2 t^2 ({a_0}^2 + \alpha^2 t^2) + 3\alpha^2 - \frac{w}{2} n^2 \right) \tag{30}$$

By using $p = \gamma\rho$, we can get pressure for this bouncing universe with $w = -\frac{3}{2}$. In this case,

$$\Gamma = -\frac{\dot{f}}{f} = nt \tag{31}$$

For $n > 0$, we clearly have $\Gamma \geq 0$. Therefore, from equation (23) and (31), the second law of thermodynamics is satisfied, ($\dot{S} \geq 0$) even at and near the bouncing point.

4 Conclusion

For a spatially flat FRW universe, a contracting model followed by an expanding model through a bouncing point has been constructed. Two specific solutions have been considered and their properties have been studied in detail.

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