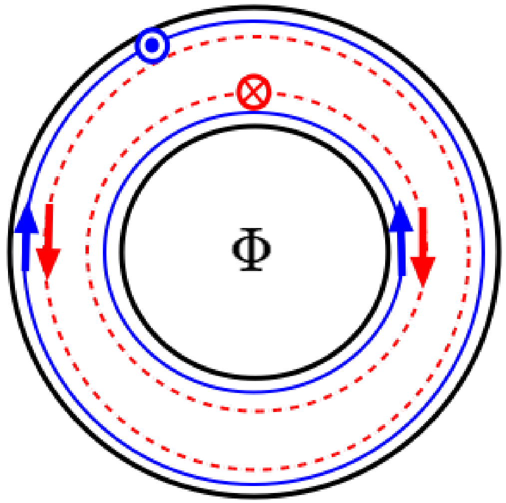
Capri Spring School on Transport in Nanostructures, 12.04.11

Bound states and persistent currents in topological insulator rings

Phys. Rev. B 83, 125420 (2011)



Emmy Noether-Programm
Deutsche Forschungsgemeinschaft
DFG

QuickTime™ and a decompressor are needed to see this picture.

Paolo Michetti
Universität Würzburg

In collaboration with: Patrik Recher (Uni Würzburg)

Mesoscopic Rings

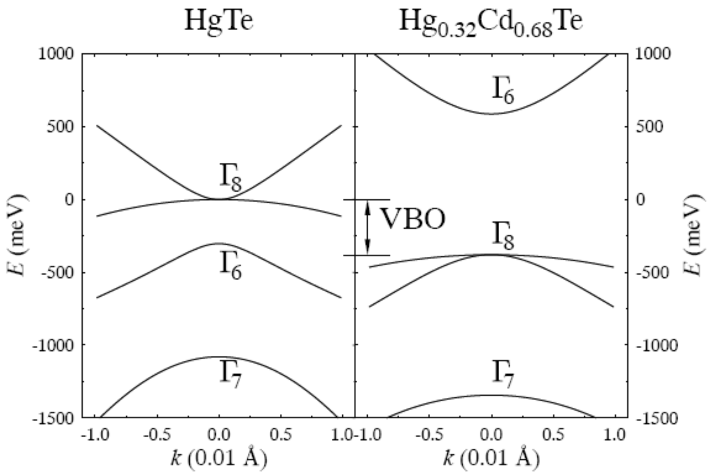
- Phase QM effects: A-B effect, persistent currents

- PCs observed in metallic and semiconducting rings

- Many-electrons to few-electrons quantum rings

- Quantum rings of Dirac systems: graphene

Band structure of HgTe QWs



B.A. Bernevig et al., Science (2006), M. König et al., Science (2007)

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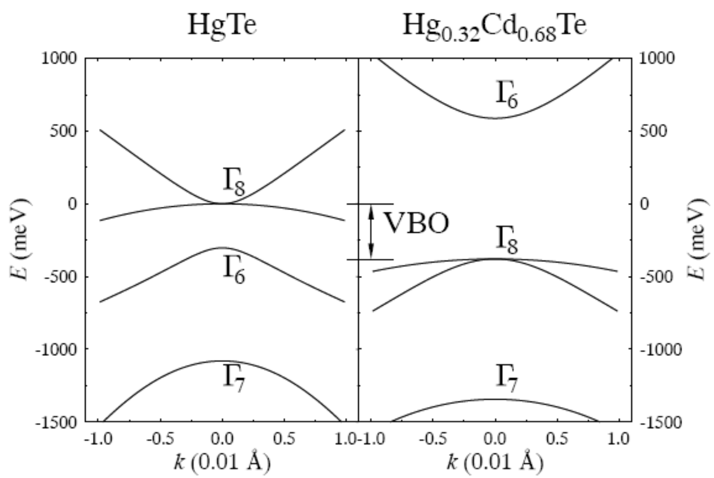

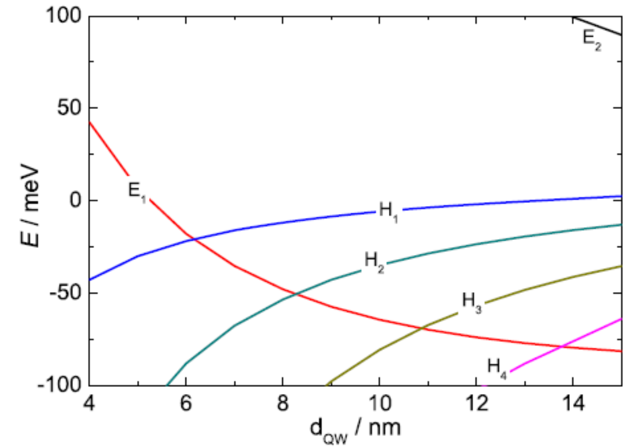

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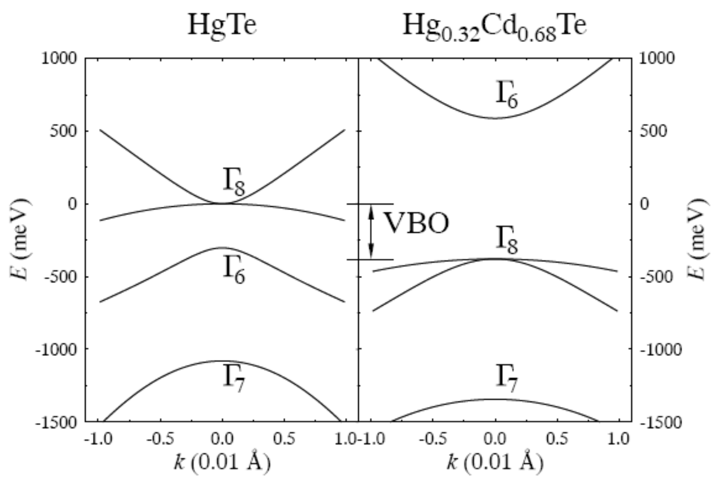

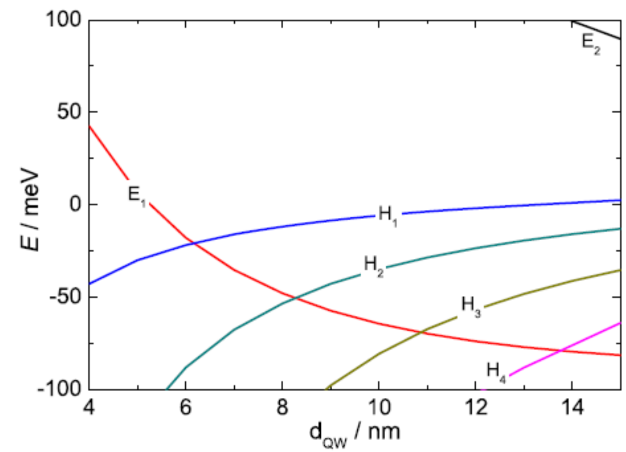

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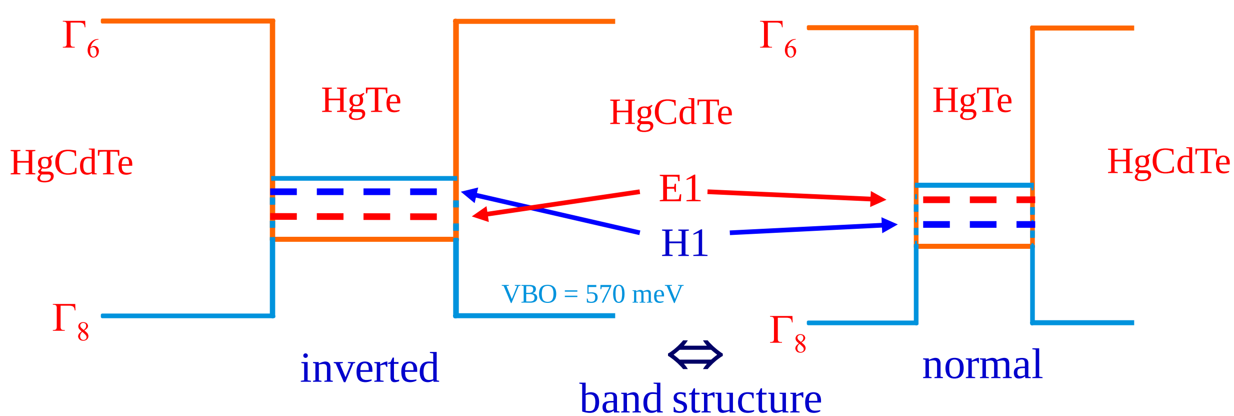

Effective model near Γ point

$$H = \begin{pmatrix} h(\mathbf{k}) & 0 \\ 0 & h^*(-\mathbf{k}) \end{pmatrix}$$

$$h(\mathbf{k}) = \varepsilon(\mathbf{k}) + \sum_a d_a(\mathbf{k})\sigma_a$$

Bernevig, Hughes, Zhang, Science (2006)

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Bernevig, Hughes, Zhang, Science (2006)

$$\varepsilon(\mathbf{k}) = C - Dk^2$$

$$\mathbf{d}(\mathbf{k}) = (Ak_x, -Ak_y, M - Bk^2)$$

basis states:

$$\{|E_1 +\rangle, |H_1 +\rangle, |E_1 -\rangle, |H_1 -\rangle\}$$

±: degenerate Kramers partners

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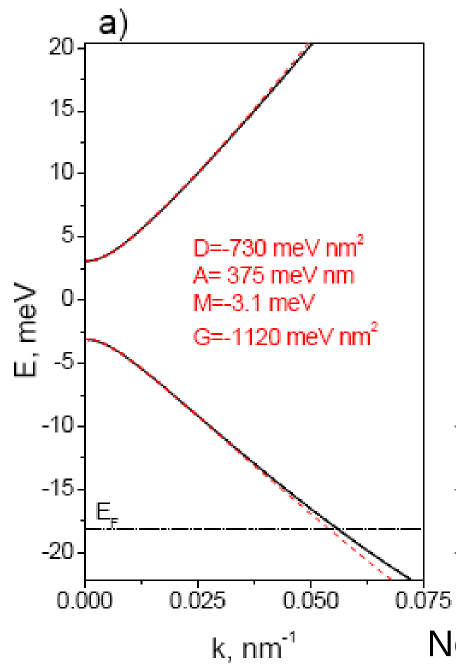

comparison to 8-band Kane model

Novik et al. PRB 2005

Schmidt et al. PRB 2009

Hamiltonian in polar coordinates

$$H = C + M\tau_z\sigma_z + (D + B\tau_z\sigma_z)\Delta - iAe^{i\sigma_z\theta}\Pi$$

$$\Delta(r,\theta) = \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right)$$

σ_a : pseudo spin (E1,H1)

τ_a : Kramer's partners (+,-)

$$\Pi(r,\theta) = \left(\frac{\partial}{\partial r}\sigma_x - \frac{1}{r}\frac{\partial}{\partial\theta}\sigma_y\right) \longrightarrow$$

similar to rings in graphene,
P. Recher et al. PRB (2007)

- inclusion of magnetic flux Φ : $\frac{\partial}{\partial\theta} \rightarrow \frac{\partial}{\partial\theta} + i\frac{\Phi}{\Phi_0}$

- change of basis: $\{|E_1+\rangle, |H_1+\rangle, -i|H_1-\rangle, i|E_1-\rangle\}$

Symmetries

operators:

$$\hat{l}_z = -i\hbar\partial_\theta$$ (angular AM)

$$\hat{S}_z = \frac{\hbar}{2}\tau_0\sigma_z$$ (pseudospin)

$$\hat{j}_z = \hat{l}_z - \hat{S}_z$$ ("total" pseudo AM)

$$\hat{T} = -\tau_x\sigma_y\hat{K}$$ (TR)

$$\hat{J}_z = \hat{l}_z + (\hbar\tau_z - \hat{S}_z)$$

(real total angular momentum)

Symmetries

operators:

$$\hat{l}_z = -i\hbar\partial_\theta$$ (orbital AM)

$$\hat{S}_z = \frac{\hbar}{2}\tau_0\sigma_z$$ (pseudospin)

$$\hat{j}_z = \hat{l}_z - \hat{S}_z$$ ("total" pseudo AM)

$$\hat{T} = -\tau_x\sigma_y\hat{K}$$ (TR)

$$\hat{J}_z = \hat{l}_z + (\hbar\tau_z - \hat{S}_z)$$

(real total angular momentum)

$\left[H, \hat{j}_z\right] = 0$ and also $\left[H, \tau_z\right] = 0 \Rightarrow$

$$H_{m,\tau}\psi_{m,\tau} = E\psi_{m,\tau}$$

$$\hat{j}_z\psi_{m,\tau} = \hbar m\psi_{m,\tau}$$

$$\hat{\tau}_z\psi_{m,\tau} = \tau\psi_{m,\tau}$$

Symmetry: $E_{m,\tau} = E_{-m,-\tau}$

but: $E_{m,\tau} \neq E_{-m,\tau}$

$i\sigma_y\hat{K}$ - symmetry broken by $M(k)$

Eigenstates of the ring

$$\psi_{m,\tau}(r,\theta) = \frac{e^{im\theta}}{\sqrt{2\pi}} \begin{pmatrix} \chi_1^{m,\tau}(r) e^{i\frac{\theta}{2}} \\ \chi_2^{m,\tau}(r) e^{-i\frac{\theta}{2}} \end{pmatrix}$$

$$m = \pm\frac{1}{2}, \pm\frac{3}{2}, \ldots$$

- Solutions (χ_1, χ_2) : **$\chi_{m\pm 1/2}(Kr)$** Besselfunctions (of order $m \pm 1/2$, resp.) and with the condition:

$$K^2 = -F \pm \sqrt{F^2 - Q^2}$$

$$Q^2 = \frac{M^2 - E^2}{B^2 - D^2}$$

$$F = \frac{A^2 - 2(BM + DE)}{2(B^2 - D^2)}$$

- typical values for HgTe-QWs

$$A = 375\,\text{meVnm}$$

$$B = -1.12\,\text{eVnm}^2$$

$$D = -730\,\text{meVnm}^2$$

Edge states of the ring

- conditions:

$$|E| < |M|$$

and

$$\frac{A^2}{B^2 - D^2} > \frac{4M}{B} > 0$$

$\Longrightarrow$ (all 4-solutions for K are imaginary in the gap: $K = \pm i K_{1,2}$)

Edge states of the ring

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$$|E| < |M|$$ and $$\frac{A^2}{B^2 - D^2} > \frac{4M}{B} > 0$$

$\Longrightarrow$ (all 4-solutions for K are imaginary in the gap: $K = \pm iK_{1,2}$)

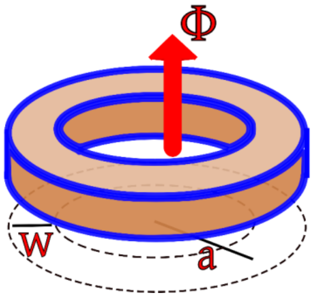
• Solving hard-wall boundary conditions:

$$\Psi = \sum_i c_i \begin{pmatrix} \chi_1(K_i r) \\ \chi_2(K_i r) \end{pmatrix} = 0 \qquad r = a \pm \frac{W}{2}$$

$\Longrightarrow$ discrete bound states: $E_{n,m,\tau}$

$n = \pm 1, \pm 2, \cdots$ denotes radial quantum number

Dispersion of edge states

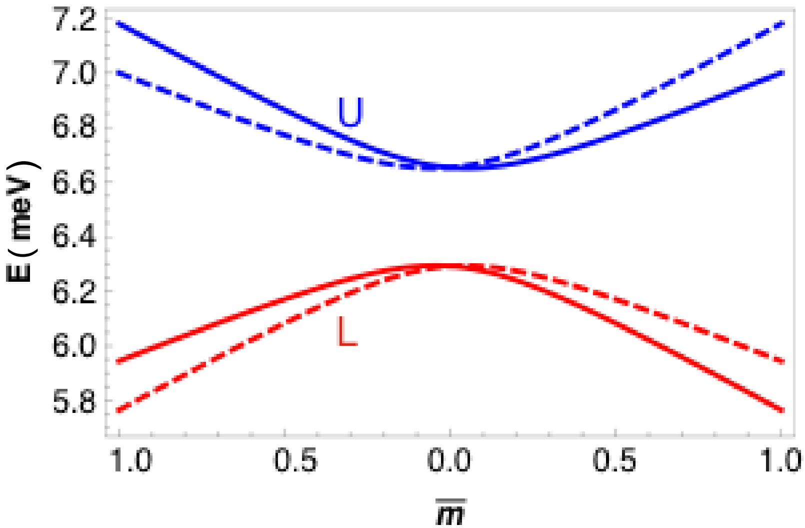


$a = 500\,\mathrm{nm}$

$W = 200\,\mathrm{nm}$

$M = -10\,\mathrm{meV}$

(inverted)

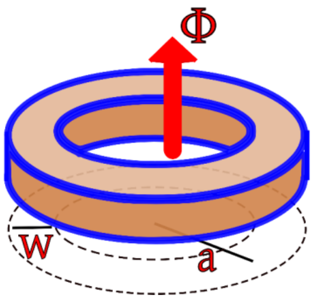


$$\bar{m} = m + \frac{\Phi}{\Phi_0}$$

solid line: $\tau = +1$

dashed line: $\tau = -1$

Dispersion of edge states



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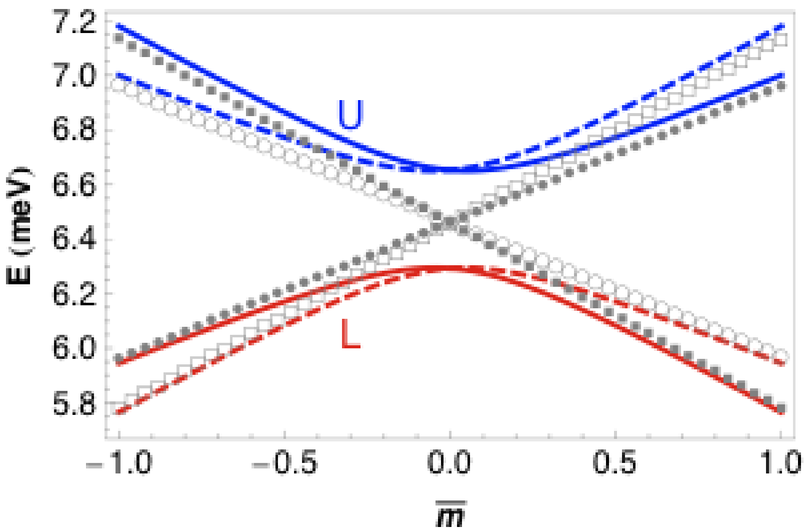

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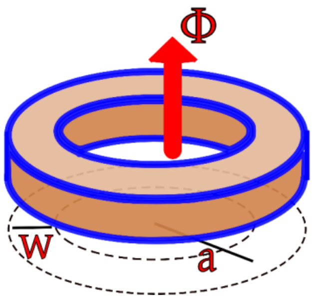

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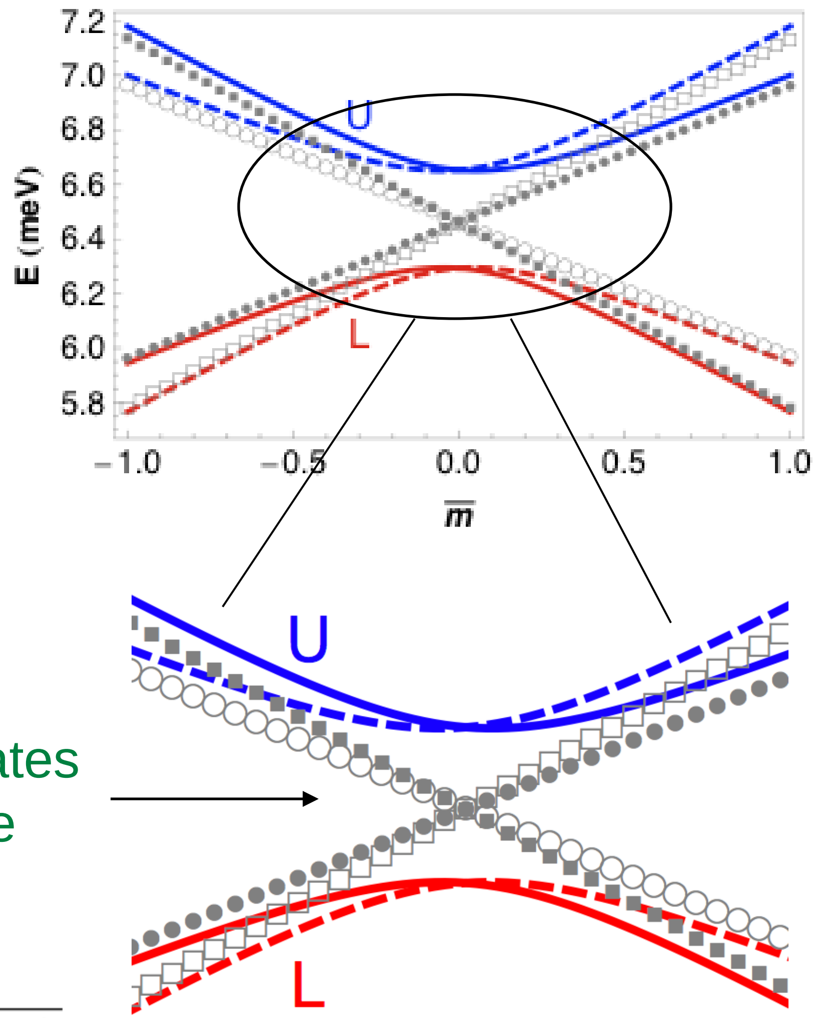


$$\bar{m} = m + \frac{\Phi}{\Phi_0}$$

—— $\tau = +1$

- - - - $\tau = -1$

Helical edge states of disk and hole

□ (internal boundary)

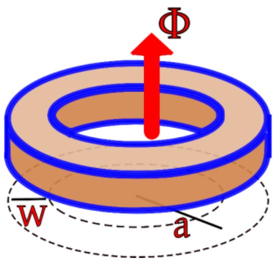
○ (external boundary)

$\tau = +1$ (filled)

$\tau = -1$ (empty)

$$\text{slope} \sim \frac{A}{r}\sqrt{\frac{B^2 - D^2}{B^2}}$$

Spin-selective persistent current

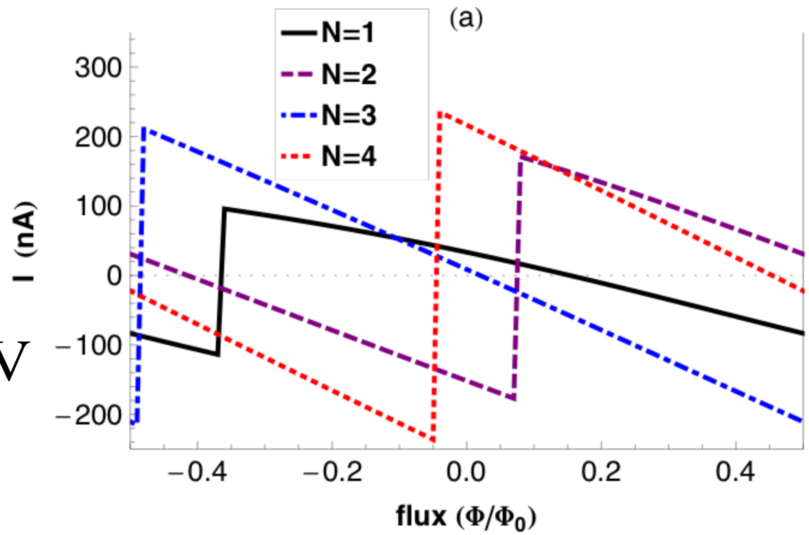


$a = 100\,\mathrm{nm}$

$W = 75\,\mathrm{nm}$

$M = -10\,\mathrm{meV}$

(inverted)

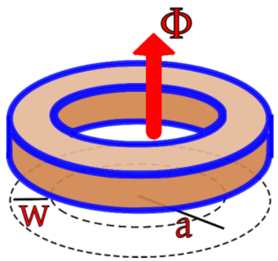


$$I = -\sum_{n,m,\tau} \frac{\partial E_{n,m,\tau}}{\partial \Phi}$$

only ONE spin-block:

$I(\Phi = 0) \neq 0$

Spin-selective persistent current



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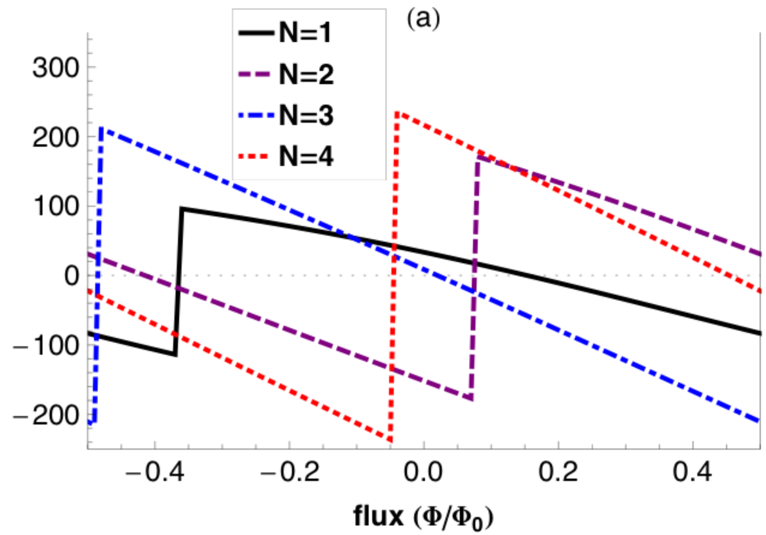

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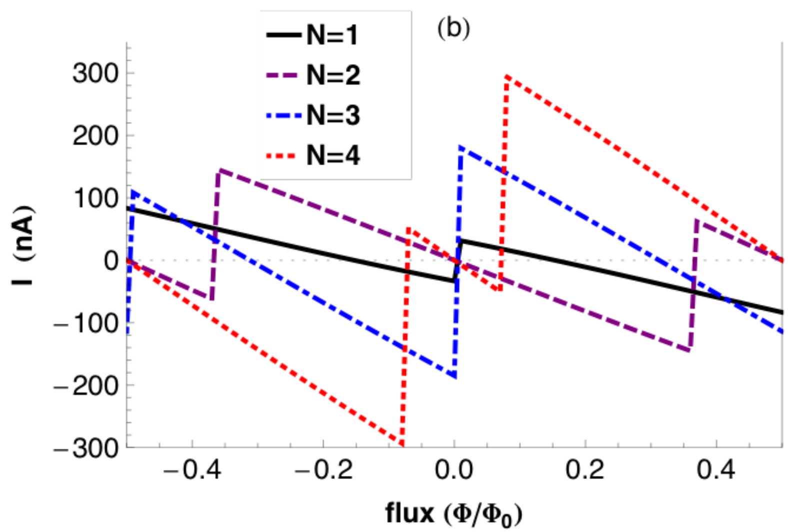
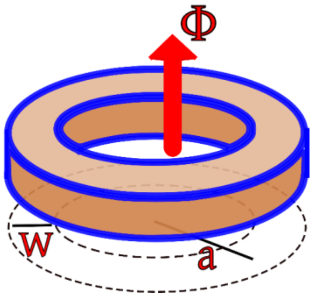


BOTH spin-blocks:
$I(\Phi = 0) = 0$ (TRS)

HOWEVER: $I_{+} - I_{-} \neq 0$

Localization properties of ring states



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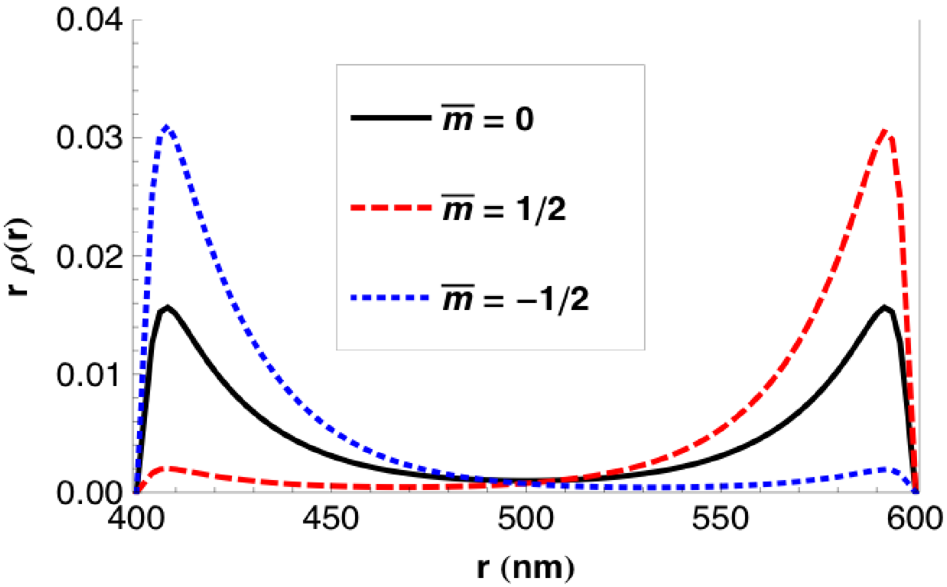
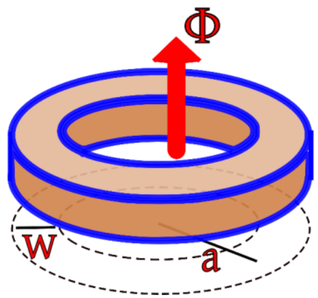


$\tau = +1$

$\rho(r) = |\Psi(r)|^2$

- tunable localization properties by means of flux Φ

Localization properties of ring states



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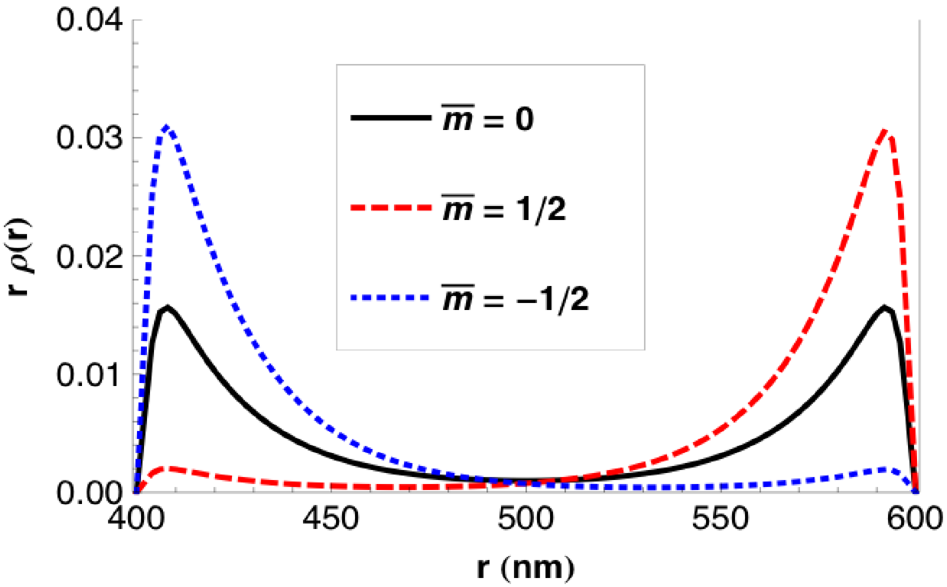
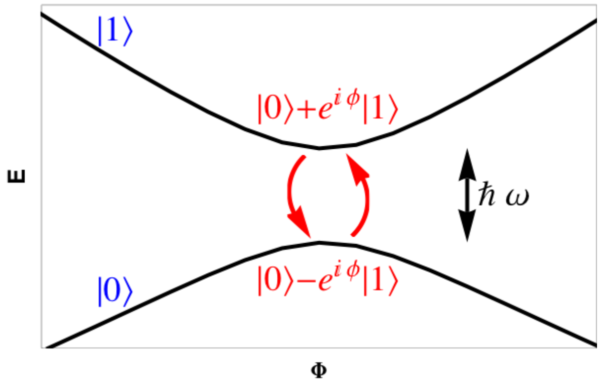


$\tau = +1$

$\rho(r) = |\Psi(r)|^2$

- tunable localization properties by means of flux Φ
- two-kind of tunable qubits:
  - edge qubits (inner ⇔ outer edge)
  - spin qubits ($\tau = \pm 1$)



Rashba spin-orbit interaction

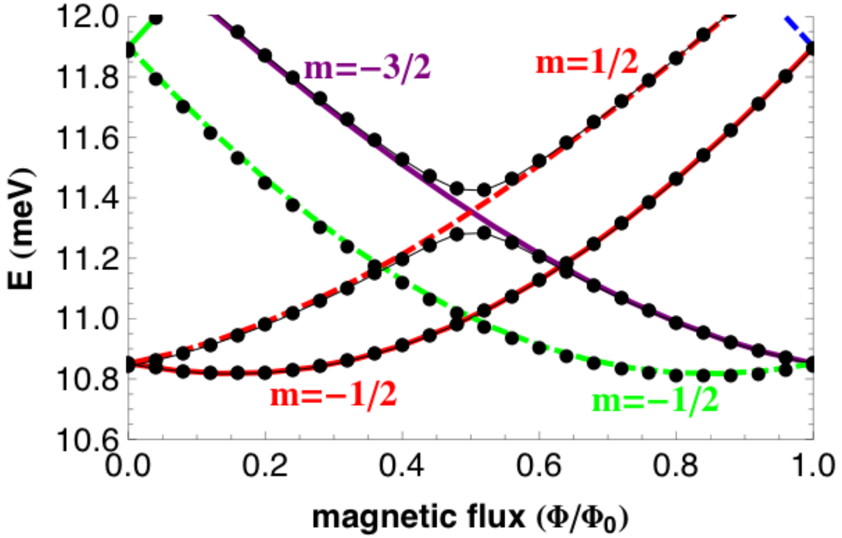
- Inversion symmetry breaking terms couple two blocks $\tau = \pm 1$

$$H_R = \begin{pmatrix} h(\mathbf{k}) & -\frac{i}{2} R_0 k_-(\sigma_0 + \sigma_z) \\ \frac{i}{2} R_0 k_+(\sigma_0 + \sigma_z) & h^*(-\mathbf{k}) \end{pmatrix}$$

$$k_\pm = k_x \pm i k_y$$

$a = 100$ nm $\quad R_0 = 20$ meV $\cdot$ nm

$W = 75$ nm $\quad M = -10$ meV



$$\left[H_R, \hat{J}_z \right] = 0$$

$$\hat{J}_z = \hat{l}_z + (\hbar \tau_z - \hat{S}_z)$$

$$\Rightarrow \Delta m = 2$$

- spin-rotation by Rashba S.O.*

  tunable from 0 to 30 meV by gate

* S. Debald and C. Emary, PRL (2004)

Conclusions

- Four-band model for 2D topological insulator HgTe QWs
  [or thin films of 3D TIs, like Bi_2Te_3 and Bi_2Se_3, Liu et al. PRB (2010)]

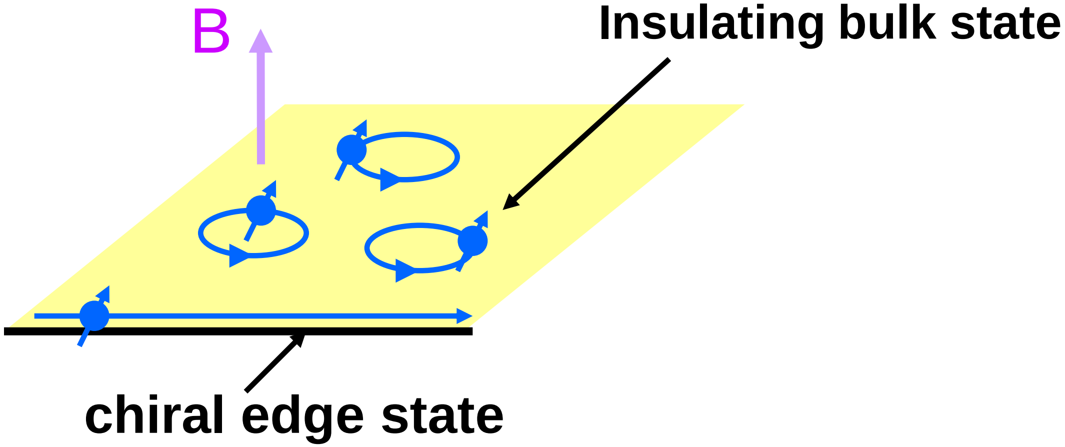
- Bound states of 2D topological insulator ring
  (like HgTe QWs) threaded by magnetic flux

- Spin-sensitive persistent currents $\rightarrow$ single-spin detector

- Helical edge states allow for tunable mixing of edge
  and spin degrees of freedom

Quantum Hall effects

Quantum Hall Effect

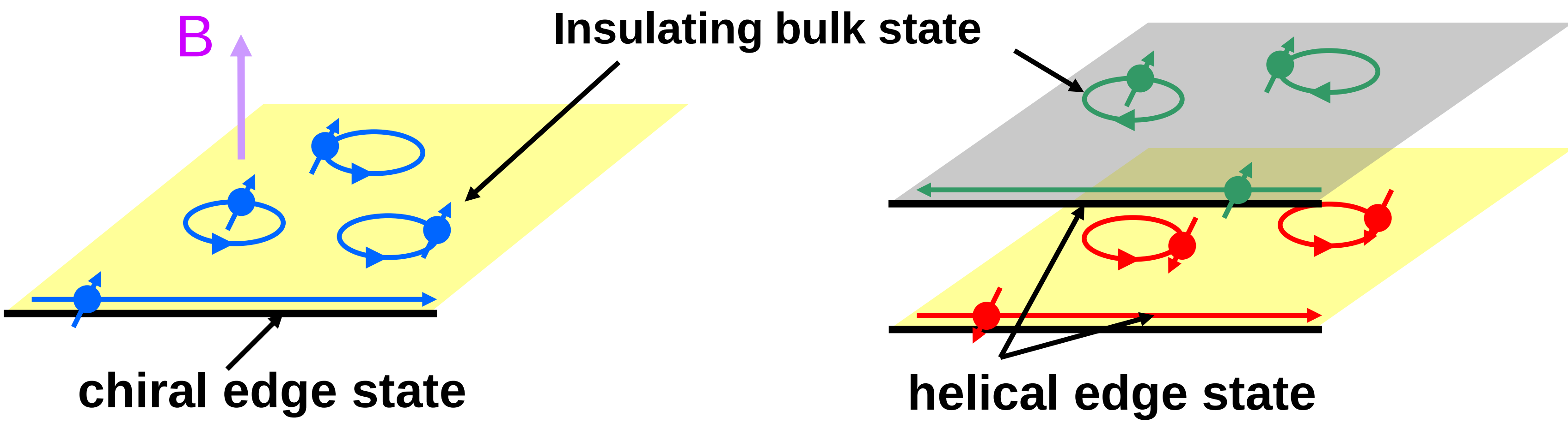


breaks TR, magnetic field B

Quantum Hall effects

Quantum Hall Effect

Quantum Spin Hall Effect



breaks TR, magnetic field B

preserves TR, SOC