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Boundary Conditions and Calculation of Surface Values for the General Two-Dimensional Electromagnetic Induction Problem

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(Received 1971 Octobe1 4)*

Summary

General boundary conditions for the problem of electromagnetic induction in a two-dimensional model of a conductor with an arbitrary sub-surface conductivity structure are considered. Program subroutines for both *E*-polarization and *H*-polarization cases are given. These boundary condition subroutines can be used to replace the previously presented subroutines and allow the solution of any conductivity configuration within the conducting region by use of the same numerical technique. An example of a particular model with a sub-surface step structure is illustrated. Also, an improved method of calculating the surface values of the tangential component of the *H*-field (*E*-case) and the tangential component of the *E*-field (*H*-case) at the surface of the conducting region is given for the numerical solution. This new method uses a derivative approximated from the true functional form of the fields instead of a linear approximation and may be applied when a layered or subsurface anomaly is modelled. Some general discussion of the numerical method is given.

1. Introduction

At present there is considerable interest in the solution of the problem of electromagnetic induction in the Earth and the local perturbations of the fields when a lateral inhomogeneity is encountered. Jones & Price (1970) considered a two-dimensional problem with a conducting half-space made up of two quarter-spaces of different conductivity, and Jones & Price (1971) considered a surface or buried region of rectangular cross-section of one conductivity surrounded by a region of different conductivity. Jones & Pascoe (1971) extended this work to consider a region of arbitrary shape and of several conductivities surrounded by a region of different conductivity and gave computer programs for the numerical solution of this problem for both the *E*-polarization (*E* parallel to the strike of the structure) and the *H*-polarization (*H* parallel to the strike of the structure) cases.

The programs given by Jones & Pascoe (1971) may be used to consider long cylinders composed of several conductivities and of arbitrary cross-section embedded in a region of uniform conductivity, but cannot be used to solve the problem in which the surrounding region is not uniform. It is important to be able to solve the more general case in which the surrounding medium is a layered one and is not necessarily the same at great distances from the conductivity inhomogeneities on both sides. In

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the previous work (Jones & Pascoe 1971) it was necessary to place the sides of the mesh over which the equations were solved far enough away from any inhomogeneity so that a uniform conducting region could be assumed at these boundaries. It is also true in the more general case presented here that the sides of the mesh must be far from any *vertical* discontinuity in conductivity so that a horizontally layered medium may be assumed at these boundaries. The subroutines for the boundary conditions for the E-polarization and H-polarization cases which we now present can be used in place of the previous boundary condition subroutines (Jones & Pascoe 1971) and allow the solution of any conductivity configuration within the conducting region as long as the above condition is met.

2. The boundary conditions for layered media

Jones (1971) investigated the problem of induction in a two-layered Earth model with a general layer contact topography and derived analytic expressions for the boundary. However, the analytic expressions for the fields at the boundaries in terms of the conductivities and the depth to the interface became cumbersome even for this two-layered case. Therefore, to proceed to a situation which involves more than two layers, a different approach is taken.

(*a*) *E-polarization*

If we consider the same co-ordinate system as before, namely with the origin on the surface, x and y co-ordinates horizontal and the z co-ordinate vertically downward, (Jones & Pascoe 1971), then for a uniformly layered conducting region, $(\partial E_x/\partial y) = 0$ everywhere. The equation which must be solved,

$$\nabla^2 E_x = i\eta^2 E_x$$

where $\eta^2 = 4\pi\sigma\omega$ reduces to

$$\frac{\partial^2 E_x}{\partial z^2} = i\eta^2 E_x.$$

This equation has the solution

$$E_x = D_1 \exp\left(-\eta z\sqrt{(i)}\right) + D_2 \exp\left(\eta z\sqrt{(i)}\right) \quad \text{for } \eta \neq 0$$

and

$$E_x = D_1 + D_2 z \quad \text{for } \eta = 0.$$

If we now consider Fig. 1 and assume that $E_{x|k}$ and $(\partial E_x/\partial z)_{|k}$ are known, then by using the proper functional form in region η_j and the boundary conditions [the continuity of E_x and the tangential component of H, $(\partial E_x/\partial z)$] at the horizontal interface, k, then the constants D_1 and D_2 may be evaluated for layer η_j. Once D_1 and D_2 are known, $E_{x|j}$ and $(\partial E_x/\partial z)_{|j}$ may be calculated. In this way a knowledge of E_x and $(\partial E_x/\partial z)$ on the lowest grid row allows the determination of E_x and $(\partial E_x/\partial z)$ for the remaining grid rows.

(*b*) *H-polarization*

For the H-polarization case a similar form of the solution is encountered:

$$H_x = D_1 \exp\left(-\eta z\sqrt{(i)}\right) + D_2 \exp\left(\eta z\sqrt{(i)}\right) \quad \text{for } \eta \neq 0$$

and

$$H_x = D_1 + D_2 z \quad \text{for } \eta = 0.$$

Again, if $H_{x|k}$ and $(\partial H_x/\partial z)_{|k}$ are known, the constants D_1 and D_2 may be determined by using the conditions of continuity of H and continuity of the tangential

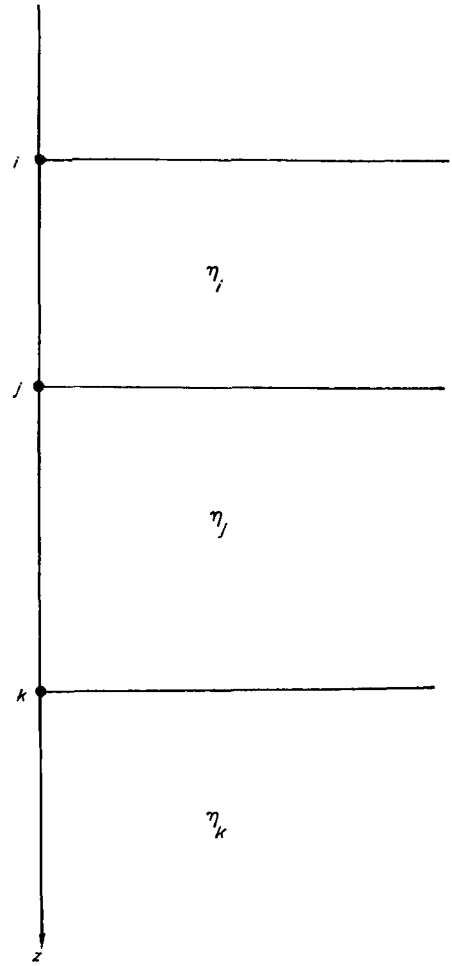

FIG. 1. Notation used to describe boundary of layered medium.

component of E on line k. After D_1 and D_2 are thus determined for η_j, we may calculate $H_{x|j}$ and $(\partial H_x/\partial z)_{|j}$ and proceed as described in the E polarization case.

It should be noted that to be certain that E_x (or H_x) remains finite as $z \to \infty$, we have set the value of E_x (or H_x) on the lowermost grid row according to

$$E_x = \exp(-\eta z \sqrt{(i)})$$
$$(\text{or } H_x = \exp(-\eta z \sqrt{(i)});$$

that is $D_1 = 1$, $D_2 = 0$.

Also, in the H-polarization case, $H_x = H_0$ everywhere in the free-space region (Jones & Price 1970).

In the E-polarization case, it is necessary to take (Jones & Price 1970)

$$H_{y|\text{left-hand surface}} = H_{y|\text{right-hand surface}}$$

and also to place the upper boundary high enough to ensure that any perturbations in H due to discontinuities in the conductor are negligible there.

Furthermore, since the above applies for a horizontally layered conducting medium, we must ensure that the boundaries are far enough away from any vertical discontinuity so that this assumption holds.

3. The boundary value subroutines and example

Figs 2 and 3 give the boundary value subroutine for the *E*-polarization case and Figs 4 and 5 for the *H*-polarization case. In these subroutines complex variables are used directly, and the real and imaginary parts are separated at the end to accommodate the main program.

Fig. 6 gives the conductive configuration for the example illustrated. The model is that of a layered medium with a step discontinuity. The different conductivities are illustrated by the different letters.

Fig. 7 gives the *E*-polarization surface values of the three components, the phase and apparent resistivity. Fig. 8 is the solution for the *H*-polarization case. In the model illustrated, a slightly different method of calculating the surface values using a non-linear approximation of the functional form at the surface is employed.

```
FORTRAN IV G COMPILER        BYCOND          04-13-71         11:56.48        PAGE 0001

0001                 SUBROUTINE BYCOND (N)
0002                 REAL K
0003                 COMMON F(41,41),G(41,41),H(40),K(40),SCALE,FREQ,REGION(40,40)
0004                 COMPLEX E(41,2),EPRIME(41,2),D1(40,2),D2(40,2),ROOTI,ENZ,EMNZ
0005                 DIMENSION AETA(40,2), DIST(41)
               C
               C     SET THE ORIGIN FOR THE Z-AXIS ON GRID ROW 1
               C
0006                 DIST(1)=0.0
0007                 DO 110 I=2,41
0008           110   DIST(I)=DIST(I-1)+K(I-1)
0009                 ROOTI=CMPLX(1./SQRT(2.),1./SQRT(2.))
               C
               C     DETERMINE AETA FOR THE LEFT AND RIGHT BOUNDARIES
               C
0010                 DO 120 I=N,40
0011                 AETA(I,1)=SQRT(REGION(I,1))
0012           120   AETA(I,2)=SQRT(REGION(I,40))
               C
               C     SET THE VALUES OF E AND EPRIME ON THE LAST GRID ROW
               C
0013                 E(41,1)=CEXP(-DIST(41)*AETA(40,1)*ROOTI)
0014                 E(41,2)=CEXP(-DIST(41)*AETA(40,2)*ROOTI)
0015                 EPRIME(41,1)=-AETA(40,1)*ROOTI*CEXP(-DIST(41)*AETA(40,1)*ROOTI)
0016                 EPRIME(41,2)=-AETA(40,2)*ROOTI*CEXP(-DIST(41)*AETA(40,2)*ROOTI)
               C
               C     SOLVE FOR REMAINING BOUNDARY VALUES
               C
0017                 DO 130 J=1,2
0018                 DO 130 I=N,40
0019                 ENZ=CEXP(DIST(41-I+N)*AETA(40-I+N,J)*ROOTI)
0020                 EMNZ=CEXP(-DIST(41-I+N)*AETA(40-I+N,J)*ROOTI)
0021                 D1(40-I+N,J)=((E(41-I+N,J)*AETA(40-I+N,J)*ROOTI*ENZ-EPRIME(41-I+N,
                    1J)*ENZ)/(2.*AETA(40-I+N,J)*ROOTI))
0022                 D2(40-I+N,J)=((EPRIME(41-I+N,J)*EMNZ+E(41-I+N,J)*AETA(40-I+N,J)*RO
                    1OTI*EMNZ)/(2.*AETA(40-I+N,J)*ROOTI))
0023                 ENZ=CEXP(DIST(40-I+N)*AETA(40-I+N,J)*ROOTI)
0024                 EMNZ=CEXP(-DIST(40-I+N)*AETA(40-I+N,J)*ROOTI)
0025                 E(40-I+N,J)=D1(40-I+N,J)*EMNZ+D2(40-I+N,J)*ENZ
0026           130   EPRIME(40-I+N,J)=-AETA(40-I+N,J)*ROOTI*D1(40-I+N,J)*EMNZ+AETA(40-I
                    1+N,J)*ROOTI*D2(40-I+N,J)*ENZ
0027                 L=N-1
0028                 DO 140 J=1,2
0029                 DO 140 I=1,L
0030                 D1(N-I,J)=E(N-I+1,J)-EPRIME(N-I+1,J)*DIST(N-I+1)
0031                 D2(N-I,J)=EPRIME(N-I+1,J)
0032                 E(N-I,J)=D1(N-I,J)+D2(N-I,J)*DIST(N-I)
0033           140   EPRIME(N-I,J)=D2(N-I,J)
               C
               C     NORMALISE TO (1.0,0.0) ON THE SURFACE ON THE LEFT-HAND-SIDE
               C
0034                 ROOTI=E(N,1)
0035                 DO 150 I=1,41
0036                 E(I,1)=E(I,1)/ROOTI
0037           150   EPRIME(I,1)=EPRIME(I,1)/ROOTI
```

FIG. 2. *E*-polarization boundary condition subroutine.

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      C
      C     ADJUST THE RIGHT-HAND-SIDE TO  THE PROPER VALUE
      C
            ROOTI=EPRIME(N,2)/EPRIME(N,1)
            DO 160 I=1,41
            E(I,2)=E(I,2)/ROOTI
        160 EPRIME(I,2)=EPRIME(I,2)/ROOTI
      C
      C     SET THE BOUNDARY OF F & G AND INTERPOLATE LINEARLY ACROSS THE GRID
      C
            DO 170 I=1,41
            F(I,1)=REAL(E(I,1))
            F(I,41)=REAL(E(I,2))
            G(I,1)=AIMAG(E(I,1))
        170 G(I,41)=AIMAG(E(I,2))
            DIST(1)=0.0
            DO 180 I=2,41
        180 DIST(I)=DIST(I-1)+H(I-1)
            DO 190 I=1,41
            DFDY=(F(I,41)-F(I,1))/DIST(41)
            DGDY=(G(I,41)-G(I,1))/DIST(41)
            DO 190 J=2,40
            F(I,J)=F(I,1)+(DFDY*DIST(J))
        190 G(I,J)=G(I,1)+(DGDY*DIST(J))
            RETURN
            END

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11:57.05 4.5C2 RC=0
```

FIG. 3. *E*-polarization boundary condition subroutine.

4. Calculation of the surface values

In the previous work (Jones & Pascoe 1971), it was found that some error was encountered in the calculation of E_y in the H-polarization case and H_y in the E-polarization case. This error is exhibited by a difference between the computed value of the apparent resistivity (ρ_A) on the surface over the uniform conducting regions at the extremities of the mesh and the value expected there.

Since for the H-polarization case

$$E_y = \frac{1}{4\pi\sigma}\frac{\partial H_x}{\partial z}$$

and for the E-polarization case

$$H_y = -\frac{1}{i\omega}\frac{\partial E_x}{\partial z}$$

(Jones & Price 1970), the components E_y and H_y were calculated by taking finite differences in the z direction. This approximation to the derivative is adequate when the grid spacing is not too large. However, a better approximation which is independent of the grid spacing and which uses the true form of the function can be applied.

For the conducting region, the usual method for approximating $(\partial E_x/\partial z)$ (or $(\partial H_x/\partial z)$) at the surface is by using a linear approximation to the derivative. For example, from Fig. 9 we would have

$$\frac{\partial F}{\partial z}\bigg|_0 \simeq \frac{\Delta F}{\Delta z}\bigg|_0 = \frac{F_1 - F_0}{z_1 - z_0},$$

where F equals E_x or H_x. This is a reasonable approximation to the derivative when the grid spacing is small, since the derivative is the value of this gradient in the limit as $z_1 \to z_0$.

```
FORTRAN IV G COMPILER        BYCOND          04-13-71         12:00.35        PAGE 0001

0001                SUBROUTINE BYCOND (N)
0002                REAL K
0003                COMMON F(41,41),G(41,41),H(40),K(40),SCALE,FREQ,REGION(40,40)
0004                COMPLEX E(41,2),EPRIME(41,2),D1(40,2),D2(40,2),ROOTI,ENZ,EMNZ
0005                DIMENSION AETA(41,2),DIST(41)
              C
              C     SET THE ORIGIN FOR THE Z-AXIS ON GRID ROW 1
              C
0006                DIST(1)=0.0
0007                DO 110 I=2,41
0008          110   DIST(I)=DIST(I-1)+K(I-1)
0009                ROOTI=CMPLX(1./SQRT(2.),1./SQRT(2.))
              C
              C     DETERMINE AETA FOR THE LEFT AND RIGHT BOUNDARIES
              C
0010                DO 120 I=N,40
0011                AETA(I,1)=SQRT(REGION(I,1))
0012          120   AETA(I,2)=SQRT(REGION(I,40))
0013                AETA(41,1)=AETA(40,1)
0014                AETA(41,2)=AETA(40,2)
              C
              C     SET THE VALUES OF E AND EPRIME ON THE LAST GRID ROW
              C
0015                E(41,1)=CEXP(-DIST(41)*AETA(40,1)*ROOTI)
0016                E(41,2)=CEXP(-DIST(41)*AETA(40,2)*ROOTI)
0017                EPRIME(41,1)=-AETA(40,1)*ROOTI*CEXP(-DIST(41)*AETA(40,1)*ROOTI)
0018                EPRIME(41,2)=-AETA(40,2)*ROOTI*CEXP(-DIST(41)*AETA(40,2)*ROOTI)
              C
              C     SOLVE FOR REMAINING BOUNDARY VALUES
              C
0019                DO 130 J=1,2
0020                DO 130 I=N,40
0021                ENZ=CEXP(DIST(41-I+N)*AETA(40-I+N,J)*ROOTI)
0022                EMNZ=CEXP(-DIST(41-I+N)*AETA(40-I+N,J)*ROOTI)
0023                D1(40-I+N,J)=((E(41-I+N,J)*AETA(40-I+N,J)*ROOTI*ENZ-((AETA(40-I+N,
                   1J)/AETA(41-I+N,J))**2)*EPRIME(41-I+N,J)*ENZ)/(2.*AETA(40-I+N,J)*RO
                   1OTI))
0024                D2(40-I+N,J)=(((AETA(40-I+N,J)/AETA(41-I+N,J))**2)*EPRIME(41-I+N,
                   1J)*EMNZ+E(41-I+N,J)*AETA(40-I+N,J)*ROOTI*EMNZ)/(2.*AETA(40-I+N,J)*
                   1ROOTI))
0025                ENZ=CEXP(D[redacted](40-I+N)*AETA(40-I+N,J)*ROOTI)
0026                EMNZ=CEXP(-DIST(40-I+N)*AETA(40-I+N,J)*ROOTI)
0027                E(40-I+N,J)=D1(40-I+N,J)*EMNZ+D2(40-I+N,J)*ENZ
0028          130   EPRIME(40-I+N,J)=-AETA(40-I+N,J)*ROOTI*D1(40-I+N,J)*EMNZ+AETA(40-I
                   1+N,J)*ROOTI*D2(40-I+N,J)*ENZ
0029                L=N-1
0030                DO 140 J=1,2
0031                DO 140 I=1,L
0032                EPRIME(N-I,J)=0.0
0033          140   E(N-I,J)=E(N-I+1,J)
              C
              C     NORMALISE TO (1.0,0.0) ON THE SURFACE ON THE LEFT-HAND-SIDE
              C
0034                ROOTI=E(N,1)
0035                DO 150 I=1,41
```

FIG. 4. *H*-Polarization boundary condition subroutine.

```
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0036            E(I,1)=E(I,1)/ROOTI
0037      150   EPRIME(I,1)=EPRIME(I,1)/ROOTI
          C
          C     ADJUST THE RIGHT-HAND-SIDE TO  THE PROPER VALUE
          C
0038            ROOTI=E(N,2)/E(N,1)
0039            DO 160 I=1,41
0040            E(I,2)=E(I,2)/ROOTI
0041      160   EPRIME(I,2)=EPRIME(I,2)/ROOTI
          C
          C     SET THE BOUNDARY OF F & G AND INTERPOLATE LINEARLY ACROSS THE GRID
          C
0042            DO 170 I=1,41
0043            F(I,1)=REAL(E(I,1))
0044            F(I,41)=REAL(E(I,2))
0045            G(I,1)=AIMAG(E(I,1))
0046      170   G(I,41)=AIMAG(E(I,2))
0047            DIST(1)=0.0
0048            DO 180 I=2,41
0049      180   DIST(I)=DIST(I-1)+H(I-1)
0050            DO 190 I=1,41
0051            DFDY=(F(I,41)-F(I,1))/DIST(41)
0052            DGDY=(G(I,41)-G(I,1))/DIST(41)
0053            DO 190 J=2,40
0054            F(I,J)=F(I,1)+(DFDY*DIST(J))
0055      190   G(I,J)=G(I,1)+(DGDY*DIST(J))
0056            RETURN
0057            END

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```

FIG. 5. *H*-polarization boundary condition subroutine.

However, in most instances, the grid spacing is such that the above is only a first approximation to the derivative. If we consider the true form of the function we may obtain a better numerical value for $(\partial F/\partial z)_{|0}$. For Fig. 1 we have the usual functional form for the conducting region (η_1):

$$F(z) = D_1 \exp(-\eta_1 z\sqrt{(i)}) + D_2 \exp(\eta_1 z\sqrt{(i)})$$

where D_1 and D_2 are constants.

If we know $F_{|0}$ and $F_{|1}$, D_1 and D_2 may be calculated numerically. The value of $(\partial F/\partial z)_{|0}$ may then be determined from

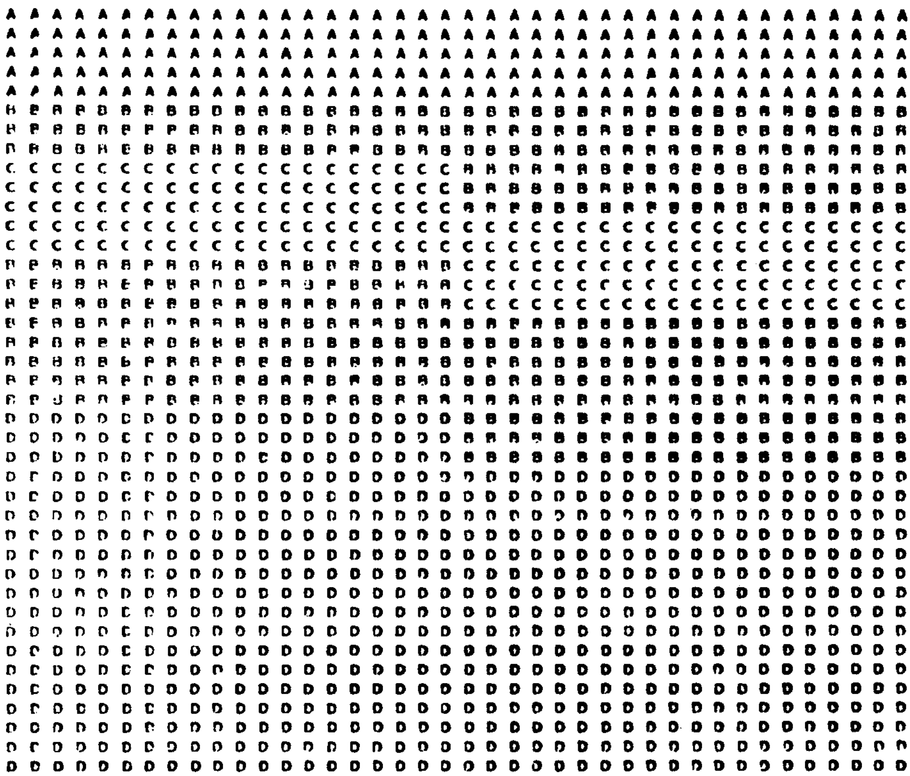
$$\left.\frac{\partial F}{\partial z}\right|_{0} = -\eta_1\sqrt{(i)}\,D_1 + \eta_1\sqrt{(i)}\,D_2,$$

where the origin of the z-axis has been taken at the surface. This will give a more accurate value for $(\partial F/\partial z)_{|0}$ over a uniformly stratified conducting region, and is likely to be at least as accurate as the linear approximation above regions where lateral discontinuities in conductivity occur. It must be applied with care near regions with discontinuities at the surface, since the above functional form may not necessarily apply near such regions.

5. The new surface value subroutine and comparison with the linear approximation

Figs 10 and 11 give the new surface value subroutine for the *H*-polarization case which may be used to replace the previous subroutine (Jones & Pascoe 1971) if the new approximation for the surface values is desired. The altered or inserted statements are those numbered 12–23. The *E*-polarization subroutine would require similar changes. Also, in the new *H*-polarization subroutine a statement (No. 35) is incorporated to indicate where a discontinuity exists at the surface.

/* THE CONDUCTIVE CONFIGURATION */



	SIGMA	SKIN DEPTH
A	0.0	********
B	0.1000E-09	10.00
C	0.5000E-09	4.47
D	0.1000E-08	3.16
E	0.0	********
G	0.0	********
H	0.0	********
K	0.0	********
M	0.0	********
O	0.0	********

FIG. 6. Conductive configuration for example illustrated. Horizontal (H) and vertical (K) grid dimensions and skin depth are in multiples of scale (cm). Frequency used is 0·000253 Hz.

```
                              S     0.0        ********
                              U     0.0        ********
                              W     0.0        ********
                              X     0.0        ********
                              Z     0.0        ********

H VALUES10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.10.

K VALUES90.50.10. 5. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 2. 3. 4. 5. 6. 7. 8. 9.10.11.12.13.14.15.

SCALF =    100000. FREQ =  0.000253
```

FIG. 6. *Continued.*

/* E-POLARISATION */

/* EPS = 0.000100 MAXIMUM NO. OF ITERATIONS = 500*/

/* STOPPED ON ITERATION 238 */

/* SURFACE VALUES */

	ANE	AMHY	AMHZ	DPHASE	DPHAHY	DPHAHZ	APPRES
2	0.999	1.000	0.002	0.006	0.000	-2.743	0.466E 10
3	0.999	1.000	0.000	0.005	-0.001	-2.957	0.465E 10
4	0.999	1.000	0.000	0.004	-0.001	-2.431	0.465E 10
5	0.999	1.000	0.000	0.004	-0.001	-1.665	0.465E 10
6	0.999	1.000	0.000	0.004	-0.001	-1.393	0.466E 10
7	0.999	1.000	0.000	0.004	-0.002	-1.302	0.466E 10
8	1.000	1.000	0.000	0.004	-0.002	-1.273	0.466E 10
9	1.000	1.001	0.000	0.004	-0.002	-1.273	0.466E 10
10	1.000	1.001	0.000	0.004	-0.002	-1.291	0.466E 10
11	1.001	1.001	0.000	0.004	-0.002	-1.325	0.466E 10
12	1.001	1.001	0.000	0.004	-0.002	-1.373	0.466E 10
13	1.002	1.002	0.000	0.003	-0.003	-1.435	0.466E 10
14	1.002	1.002	0.000	0.003	-0.003	-1.507	0.466E 10
15	1.003	1.003	0.001	0.003	-0.004	-1.579	0.466E 10
16	1.004	1.004	0.001	0.002	-0.004	-1.641	0.466E 10
17	1.006	1.006	0.001	0.001	-0.005	-1.692	0.466E 10
18	1.008	1.009	0.003	-0.002	-0.007	-1.700	0.466E 10
19	1.015	1.015	0.008	-0.007	-0.011	-1.514	0.466E 10
20	1.038	1.030	0.030	-0.017	-0.012	-1.072	0.474E 10
21	1.137	1.003	0.052	-0.011	0.003	-0.849	0.594E 10
22	1.251	0.968	0.037	0.016	0.007	-0.838	0.779E 10
23	1.287	0.979	0.012	0.011	0.009	-1.227	0.806E 10
24	1.300	0.987	0.005	0.007	0.007	-1.473	0.809E 10
25	1.305	0.990	0.002	0.005	0.005	-1.566	0.809E 10
26	1.307	0.992	0.001	0.004	0.004	-1.599	0.809E 10
27	1.309	0.994	0.001	0.003	0.003	-1.620	0.809E 10
28	1.310	0.995	0.001	0.002	0.002	-1.637	0.809E 10
29	1.311	0.995	0.000	0.002	0.002	-1.648	0.809E 10
30	1.312	0.996	0.000	0.001	0.001	-1.653	0.809E 10
31	1.312	0.996	0.000	0.001	0.001	-1.654	0.809E 10
32	1.313	0.996	0.000	0.001	0.001	-1.651	0.809E 10
33	1.313	0.997	0.000	0.000	0.001	-1.644	0.809E 10
34	1.313	0.997	0.000	0.000	0.000	-1.627	0.809E 10
35	1.314	0.997	0.000	0.000	0.000	-1.596	0.809E 10
36	1.314	0.998	0.000	-0.000	-0.000	-1.537	0.809E 10
37	1.315	0.998	0.000	-0.000	-0.000	-1.420	0.809E 10
38	1.315	0.999	0.000	-0.000	-0.000	-1.181	0.809E 10
39	1.316	0.999	0.000	-0.000	-0.000	-0.715	0.809E 10
40	1.317	0.999	0.001	0.0	0.0	0.0	0.810E 10

FIG. 7. *E*-polarization surface values. Amplitudes of components normalized, phase differences in radians, apparent resistivity in emu

/* H-POLARISATION */

/* EPS = 0.000100 MAXIMUM NO. OF ITERATIONS = 500*/

/* STOPPED ON ITERATION 167 */

/* SURFACE VALUES */

	AMH	AMEY	AMEZ	DPHASH	DPHAEY	DPHAEZ	APPRES
2	1.000	1.000	0.0	0.0	0.011	0.0	0.470E 10
3	1.000	1.000	0.0	0.0	0.011	0.0	0.470E 10
4	1.000	1.000	0.0	0.0	0.012	0.0	0.470E 10
5	1.000	1.000	0.0	0.0	0.012	0.0	0.470E 10
6	1.000	1.000	0.0	0.0	0.012	0.0	0.470E 10
7	1.000	1.000	0.0	0.0	0.012	0.0	0.470E 10
8	1.000	1.000	0.0	0.0	0.012	0.0	0.470E 10
9	1.000	0.999	0.0	0.0	0.012	0.0	0.470E 10
10	1.000	0.999	0.0	0.0	0.012	0.0	0.469E 10
11	1.000	0.999	0.0	0.0	0.012	0.0	0.469E 10
12	1.000	0.999	0.0	0.0	0.012	0.0	0.469E 10
13	1.000	0.999	0.0	0.0	0.012	0.0	0.469E 10
14	1.000	0.999	0.0	0.0	0.012	0.0	0.469E 10
15	1.000	0.999	0.0	0.0	0.012	0.0	0.469E 10
16	1.000	0.999	0.0	0.0	0.012	0.0	0.469E 10
17	1.000	0.997	0.0	0.0	0.012	0.0	0.469E 10
18	1.000	0.999	0.0	0.0	0.011	0.0	0.469E 10
19	1.000	1.000	0.0	0.0	0.011	0.0	0.470E 10
20	1.000	1.008	0.0	0.0	0.016	0.0	0.478E 10
21	1.000	1.166	0.0	0.0	0.095	0.0	0.639E 10
22	1.000	1.312	0.0	0.0	-0.011	0.0	0.809E 10
23	1.000	1.317	0.0	0.0	-0.001	0.0	0.816E 10
24	1.000	1.318	0.0	0.0	-0.000	0.0	0.817E 10
25	1.000	1.318	0.0	0.0	-0.000	0.0	0.817E 10
26	1.000	1.318	0.0	0.0	-0.000	0.0	0.817E 10
27	1.000	1.318	0.0	0.0	-0.000	0.0	0.817E 10
28	1.000	1.318	0.0	0.0	-0.000	0.0	0.817E 10
29	1.000	1.318	0.0	0.0	-0.000	0.0	0.817E 10
30	1.000	1.318	0.0	0.0	-0.000	0.0	0.817E 10
31	1.000	1.318	0.0	0.0	-0.000	0.0	0.817E 10
32	1.000	1.318	0.0	0.0	-0.000	0.0	0.817E 10
33	1.000	1.318	0.0	0.0	-0.000	0.0	0.817E 10
34	1.000	1.318	0.0	0.0	-0.000	0.0	0.817E 10
35	1.000	1.318	0.0	0.0	-0.000	0.0	0.817E 10
36	1.000	1.318	0.0	0.0	-0.000	0.0	0.817E 10
37	1.000	1.318	0.0	0.0	-0.000	0.0	0.816E 10
38	1.000	1.318	0.0	0.0	-0.000	0.0	0.816E 10
39	1.000	1.318	0.0	0.0	-0.000	0.0	0.816E 10
40	1.000	1.317	0.0	0.0	0.0	0.0	0.816E 10

FIG. 8. *H*-polarization surface values. Amplitudes of components normalized, phase differences in radians, apparent resistivity in emu.

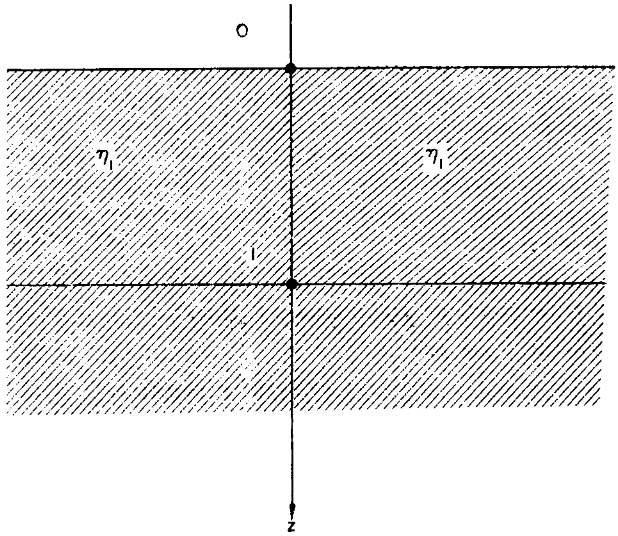


FIG. 9. Notation used to describe surface value calculations.

In Fig. 12 the surface values of apparent resistivity computed by the improved method (solid curves) are compared with the surface values obtained before (dashed curves), (Jones & Pascoe 1971). With the new calculation, the value of apparent resistivity near the boundaries of the mesh approaches the expected value for the uniform subsurface. In the previous calculations, the value of apparent resistivity differed by approximately 6 per cent in the E-polarization case and by approximately 11 per cent in the H-polarization case from the expected values. The profiles also show some change across the conductor surface when the new and old calculations are compared.

6. Dimensional considerations and the iterative procedure

The use of this new method of calculating surface values and the above considerations about accuracy, made it desirable to obtain a better 'feel' for the mode of solution and the relationship between the convergence criteria applied and the various parameters used. To do this, a uniform subsurface case was chosen and the model was run for various convergence conditions and grid-size to skin depth ratios for a uniform square grid. Since in the initialization procedure the boundary values are carried horizontally throughout the mesh, the number of iterations required to satisfy the convergence condition is a measure of the accuracy of the result. Fig. 13 shows a model constructed to illustrate the relationship between the number of iterations (proportional to the height of the rod; terminated at a maximum of 150 iterations), the residual (EPS) and the ratio of grid size to skin depth (GS/SD). It is seen that the iteration should be carried out when the grid size to skin depth ratio is small.

In the solution of the problem the equations are solved by the numerical iterative method over several conductive regions. In each region there is a basic functional form which sets limits on the grid spacing we can use there in order to accommodate the numerical difference equations. For example, in the free-space region the function is approximately linear in both directions and consequently a relatively large grid

```
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              SUBROUTINE SURFVL (L)
              REAL K
              DIMENSION AMH(41), AMEY(41), AMEZ(41), DPHASH(41), DPHAEY(41), DPH
             1AEZ(41), APPRES(41)
              COMMON F(41,41),G(41,41),H(40),K(40),SCALE,FREQ,REGION(40,40)
              COMPLEX ROOTI,H0,H1,DERIVZ
        C
        C
              PI=4.0*ATAN(1.0)
              OMEGA=2.0*PI*FREQ
              WRITE (6,140)
              WRITE (6,150)
              I=L
        C
              DO 110 J=2,40
              AETAL=SQRT(REGION(I,J-1))
              AETAR=SQRT(REGION(I,J))
              AETA=(AETAL+AETAR)/2.0
              ROOTI=CMPLX(1./SQRT(2.),1./SQRT(2.))
              H0=CMPLX(F(I,J),G(I,J))
              H1=CMPLX(F(I+1,J),G(I+1,J))
              DERIVZ=(AETA*ROOTI*(2.0*H1-H0*(CEXP(AETA*K(I)*ROOTI)+CEXP(-AETA*K(
             1I)*ROOTI))))/(CEXP(AETA*K(I)*ROOTI)-CEXP(-AETA*K(I)*ROOTI))
              DPHASH(J)=ATAN2(G(I,J),F(I,J))
              DPHAEY(J)=ATAN2(AIMAG(DERIVZ),REAL(DERIVZ))
              DPHAEZ(J)=0.0
              AMH(J)=SQRT(F(I,J)**2+G(I,J)**2)
              AMEY(J)=(OMEGA/(AETA**2))*CABS(DERIVZ)
              AMEZ(J)=((2.*OMEGA)/(REGION(I,J)+REGION(I,J-1)))*SQRT(((F(I,J+1)-F
             1(I,J-1))/(H(J)+H(J-1)))**2+((G(I,J+1)-G(I,J-1))/(H(J)+H(J-1)))**2)
        110   APPRES(J)=(2.0/FREQ)*((AMEY(J)/AMH(J))**2)
        C
        C     THE COMPONENTS AMEY AND AMEZ ARE NORMALIZED WITH RESPECT TO
        C     THE FIELD AT INFINITY (POINT 2) AND PHASE DIFFERENCES ARE
        C     CALCULATED RELATIVE TO POINT 40
              AME=SQRT(AMEY(2)**2+AMEZ(2)**2)
              DO 120 J=2,40
              AMEY(J)=AMEY(J)/AME
              AMEZ(J)=AMEZ(J)/AME
              DPHASH(J)=DPHASH(J)-DPHASH(40)
              DPHAEY(J)=DPHAEY(J)-DPHAEY(40)
        120   DPHAEZ(J)=DPHAEZ(J)-DPHAEZ(40)
        C
              DO 130 J=2,40
              WRITE (6,160) J,AMH(J),AMEY(J),AMEZ(J),DPHASH(J),DPHAEY(J),DPHAEZ(
             1J),APPRES(J)
              IF (REGION(L,J-1).NE.REGION(L,J)) WRITE (6,170)
        130   CONTINUE
              REWIND 3
              WRITE (3) F,G,H,SCALE,AMH,AMEY,AMEZ,DPHASH,DPHAEY,DPHAEZ,APPRES
              RETURN
        C
        C
        C
        140   FORMAT (1H0,40X,20H/* SURFACE VALUES *///)
```

FIG. 10. *H*-polarization surface value subroutine.

```
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0041.        150    FORMAT (1H0,T9,'AMH',T21,'AMEY',T33,'AMEZ',T45,'DPHASH',T57,'DPHAE   540
                   1Y',T69,'DPHAEZ',T81,'APPRES'//)                                      550
0042         160    FORMAT (1H ,I2,6(2X,F10.3),E12.3)                                     560
0043         170    FORMAT (1H+,T95,'DISCONTINUITY')                                      565
0044                END                                                                    570*

TOTAL MEMORY REQUIREMENTS 000EB8 BYTES
```

FIG. 11. *H*-polarization surface value subroutine.

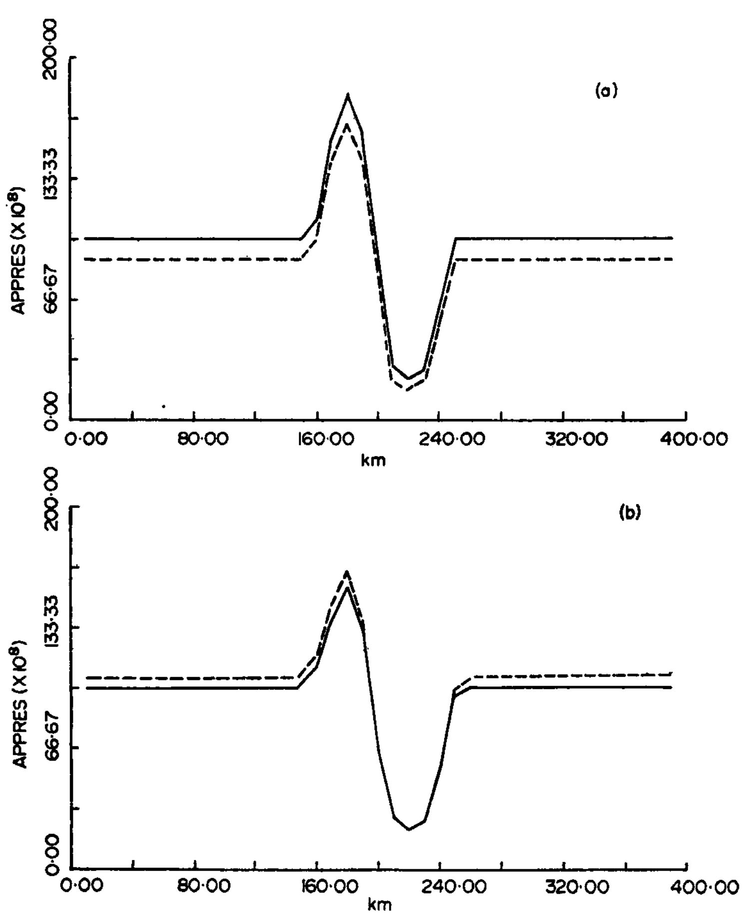

FIG. 12. Apparent resistivity profiles for the previous (dashed lines) calculation and the improved (solid lines) calculation (emu). (a) *H*-polarization. (b) *E*-polarization.

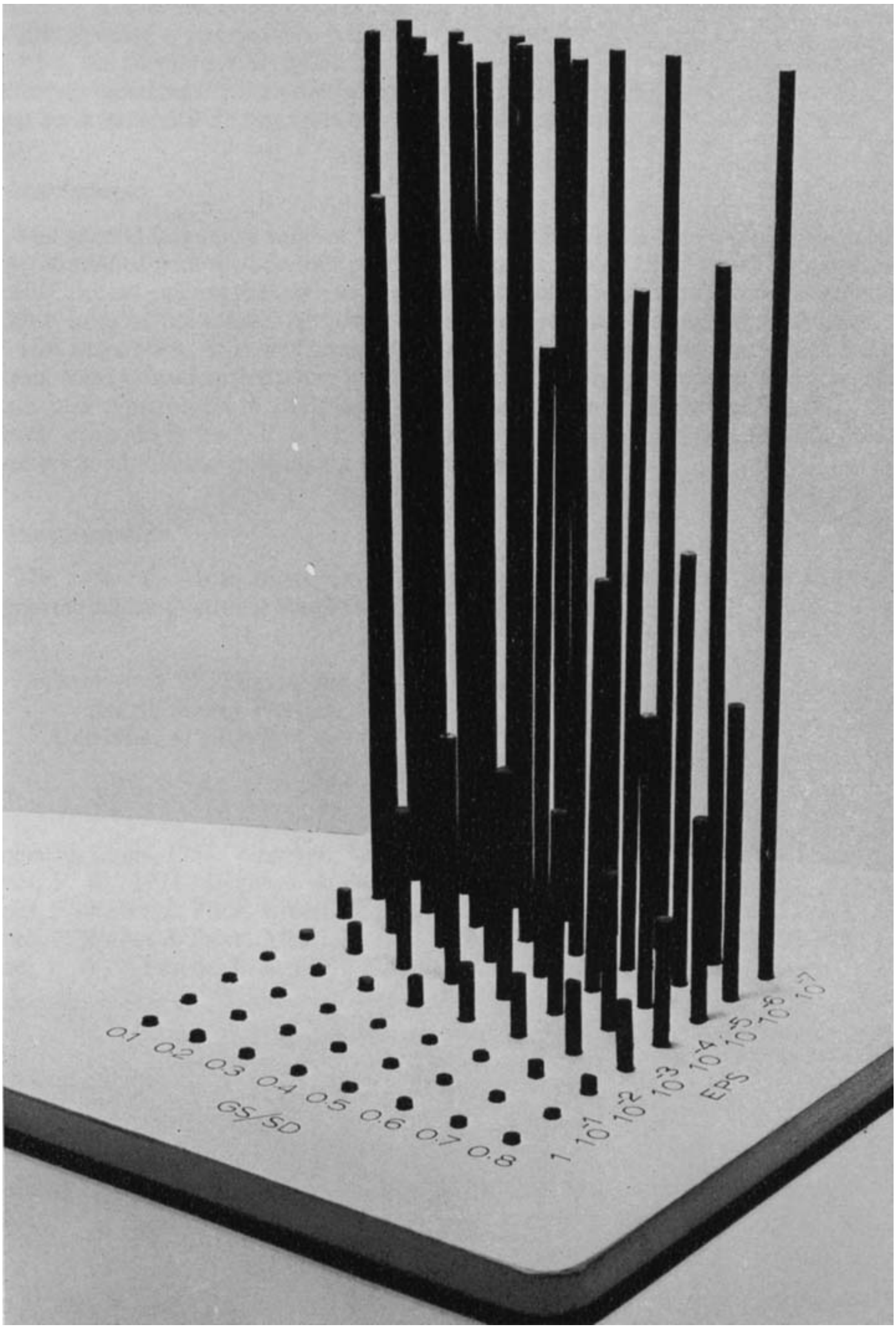

FIG. 13. Model constructed to investigate iteration procedure and relationship of various parameters.

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spacing can be used. In the conducting regions we have taken the phenomenon as approximately linear in the horizontal direction, but exponential in the vertical direction. Therefore, in the conducting region, in order to fit the form of the function the grid spacing is restricted in the vertical direction.

Also, the solution is based on the assumption that as $y \rightarrow \pm\infty$, the subsurface is uniformly stratified. To accommodate this restriction in the model the boundaries must be several skin depths from any vertical discontinuity.

7. Conclusions

The general boundary subroutines as presented provide a two-dimensional model when combined with the previous program (Jones & Pascoe 1971) which is completely general, subject only to the restriction that a uniformly stratified subsurface is required at both sides of the mesh. Although the subsurface must be uniformly stratified at the two extremities, it is not necessary that both sides have identical stratification. In use, some consideration must be given to the relationship between grid size, skin depth and dimensions of the mesh when particular models are considered. This general program is flexible to use and provides a modelling technique which should be of considerable use in studying observed phenomena.

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