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Boundary element tools for the finite body unilateral contact problem

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ABSTRACT

A numerical solution of the three dimensional frictionless contact problem and its data parallel implementation on the Connection Machine system CM-2 is presented. The numerical solution is obtained by means of boundary element discretization of a variational inequality and related extremum principle; the associated Green's function is approximated by means of standard direct boundary element procedure. Important new results for the three dimensional finite body contact problem are reported. The results clearly illustrate the distinct ability of the method to capture the influence of the body shape and loading on the contact area and the pressure acting in it.

1 INTRODUCTION

In recent years, much interest has been devoted to the mathematical formulation of structural problems involving unilateral constraints (Kikuchi and Oden, 1988; Alliney et al., 1990). An important class of these problems is the frictionless contact between two elastic bodies.

A rigorous numerical treatment of contact problems, has to start from the variational inequalities, Fichera (1972), Kikuchi (1979). The first variational formulations defined on the contact area were proposed in the seventies. A variety of variational principles for contact problems and an extensive bibliography is given in Kalker (1975). A systematic and rigorous mathematical treatment of the variational boundary formulations in terms of Green's function is reported in Kikuchi and Oden (1988). Recently, Panagiotopoulos and Lazaridis (1987), and Alliney et al. (1990), have proposed a numerical scheme, similar to the one presented in this paper, for the contact problem with unilateral constraints for two-dimensional homogeneous bodies. We have formulated, previously, Rabinovich et al. (1992), the numerical method for the three-dimensional homogeneous body frictionless contact problem. The method is based on the rigorous variational formulation and application of a direct boundary element procedure for the approximation of the related Green's functions. The method has been implemented in a data parallel programming environment - the Connection Machine System (CM-2) - Rabinovich and Sipcic (1992). Recently, we have extended the formulation to the nonhomogeneous composite body contact problem, Sipcic and Rabinovich (1993).

The present investigation is undertaken to further document the theoretical and numerical aspects of the proposed method. Examples chosen illustrate the distinct ability of the method to capture the influence of the body shape and loading on the contact area and the pressure acting in it. An important new result for the three dimensional finite body contact problem, namely, that of the rigid punch indenting a corner of an elastic unit cube subjected to a bending loading is presented.

2 FORMULATION OF THE PROBLEM

The boundary value problem governed by the Navier-Cauchy equations of linear elasticity subjected to the natural, essential and frictionless contact conditions is defined. An equivalent reciprocal variational formulation for the problem is outlined. Two partial boundary value problems, whose solution preceed the solution of the variational problem, are introduced. The boundary integral formulation for both partial problems is developed.

2.1 Boundary Value Problem

Two three dimensional linearly elastic bodies, $\bar{\Omega}^\alpha$ $(\alpha = 1, 2)$, are pressed together as in Figure 1. Note that upper Greek indices will be used throughout the paper to denote bodies, and that no summation convention is assumed on a repeated Greek index. The boundary $\Gamma^\alpha \equiv \partial\Omega^\alpha$ is assumed to be piecewise continuous, and composed of three mutually disjoint open manifolds $\Gamma^\alpha_D, \Gamma^\alpha_F$, and Γ^α_C. We demand that on Γ^α_F the surface traction $\boldsymbol{t}^\alpha$ be specified and that on Γ^α_D the displacement be specified. The contact area is assumed to be small so that only displacements perpendicular to the common tangential plane at a representative contact point are taken into account. Furthermore, the two surfaces $\Gamma^\alpha_C, (\alpha = 1, 2)$, can be identified by

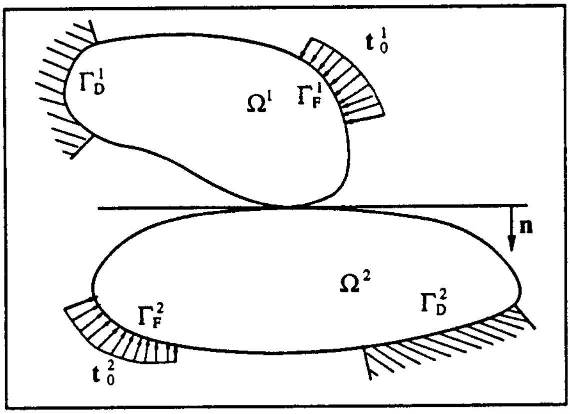

Figure 1: A model of two-body contact problems.

their projection Γ_C to the tangential plane (contact plane). We will assume that the normal $\boldsymbol{n}$ to the tangential plane is oriented from the region Ω^1 to the region Ω^2 as shown in Figure 1.

Assuming that the bodies are in equilibrium, the problem becomes one of finding the displacement field u_i^α, stress field σ_{ij}^α, and the contact pressure p, that satisfy the equilibrium conditions

$$-\left(E_{ijkm}^\alpha\, u_{m,k}^\alpha\right)_{,j} = 0 \ \text{ in } \ \Omega^\alpha \tag{1}$$

The associated natural, essential, and frictionless contact conditions are given as,

$$t_i^\alpha = (t_i^\alpha)_0 \ \text{ on } \ \Gamma_F^\alpha \tag{2}$$

$$u_i^\alpha = 0 \quad \text{ on } \ \Gamma_D^\alpha \tag{3}$$

$$\left.\begin{array}{lll} \sigma_{T_i} = 0 & & \\ p = 0 & \text{for} & u_n^R < g \\ p \le 0 & \text{for} & u_n^R = g \end{array}\right\} \ \text{ on } \ \Gamma_C \tag{4}$$

respectively, where σ_{T_i} denotes the tangential component of stress vector, p is the contact pressure, $u_n^R = u_n^1 - u_n^2$ is the relative displacement perpendicular to the common tangential plane at a contact point, and g is the "gap" function between the two surfaces Γ_C^1 and Γ_C^2, as measured along the normal $\boldsymbol{n}$ before the deformation. Since the contact zone is not known in advance the problem is nonlinear.

2.2 Equivalent Variational Formulation

An equivalent reciprocal variational formulation for the problem considered is outlined here, the details are presented in Rabinovich and Sipcic (1992) and Rabinovich et al. (1992). Suppose that for each elastic body Ω^α, ($\alpha = 1,2$) and given essential boundary conditions (3) we have determined the Green's matrices $\boldsymbol{G}^\alpha(\boldsymbol{x},\boldsymbol{\eta})$ such that the resultant displacement $\boldsymbol{u}^\alpha$ may be decomposed as

$$\boldsymbol{u}^\alpha(\boldsymbol{x}) = \hat{\boldsymbol{u}}^\alpha(\boldsymbol{x}) + \int_{\Gamma_C} \boldsymbol{G}^\alpha(\boldsymbol{x},\boldsymbol{\eta}) \cdot \boldsymbol{n}(\boldsymbol{\eta})\, p(\boldsymbol{\eta})\, dS_\eta \tag{5}$$

where $\hat{\boldsymbol{u}}^\alpha$ are the displacement fields in the bodies $\bar{\Omega}^\alpha$ constrained at Γ_D^α and loaded with the boundary tractions $\boldsymbol{t}^\alpha$ along Γ_F^α. Then the normal component of the boundary value problem under consideration is equivalent to the minimization of the complementary energy functional

$$\min_{p\leq 0} \Phi(p) = \frac{1}{2}\int_{\Gamma_C} p(\boldsymbol{x}) \int_{\Gamma_C} G(\boldsymbol{x},\boldsymbol{\eta})\, p(\boldsymbol{\eta})\, dS_\eta\, dS_x - \int_{\Gamma_C} p(\boldsymbol{x})\, \hat{g}(\boldsymbol{x})\, dS_x \tag{6}$$

where $G(\boldsymbol{x},\boldsymbol{\eta}) = \boldsymbol{n}(\boldsymbol{x}) \cdot (\boldsymbol{G}^1(\boldsymbol{x},\boldsymbol{\eta}) - \boldsymbol{G}^2(\boldsymbol{x},\boldsymbol{\eta})) \cdot \boldsymbol{n}(\boldsymbol{\eta})$ is the relative normal component of the Green's matrices $\boldsymbol{G}^\alpha$, and where the gap function is modified by the relative normal displacement $\hat{u}_n^R$, i.e.,

$$\hat{g} = g - \hat{u}_n^R \tag{7}$$

The key point of this formulation is that the minimization problem (6) has been obtained by using Green's matrix for the elastic problem without the contact condition. This amounts to considering the solution of the complete problem as the superposition of two partial solutions. First, is a solution for $\hat{g}$, or, according to (7), solution for the displacement fields $\hat{\boldsymbol{u}}^\alpha$ in the bodies $\bar{\Omega}^\alpha$ constrained at Γ_D^α and loaded with the boundary tractions $\boldsymbol{t}^\alpha$ along Γ_F^α. Second, is a solution for Green's matrix $\boldsymbol{G}(\boldsymbol{x},\boldsymbol{\eta})$, which, according to (5), amounts to the necessity of solving for the displacement fields in the bodies Ω^α constrained at Γ_D^α and loaded with unit pressure at $\boldsymbol{\eta} \in \Gamma_C^\alpha$. We define the boundary integral formulation for both partial problems.

2.3 Boundary Integral Equation

It is well known, Cruse (1969), that the mixed boundary value problem of finding a displacement field $\boldsymbol{u}$ in the linear elastic body $\bar{\Omega}$, with the boundary Γ, constrained at Γ_D and loaded with the boundary traction $\boldsymbol{t}$ along Γ_F, has a unique solution that admits the representation

$$\frac{1}{2}\boldsymbol{u}(\boldsymbol{\eta}) + \int_\Gamma \boldsymbol{T}(\boldsymbol{x},\boldsymbol{\eta}) \cdot \boldsymbol{u}(\boldsymbol{x})\, d\Gamma_x = \int_\Gamma \boldsymbol{U}(\boldsymbol{x},\boldsymbol{\eta}) \cdot \boldsymbol{t}(\boldsymbol{x})\, d\Gamma_x \tag{8}$$

The matrix components of $\boldsymbol{U}$ and $\boldsymbol{T}$ in (8) are given by

$$U_{mn} = \frac{1}{16\pi\mu(1-\nu)}\frac{1}{r}\left[(3-4\nu)\delta_{mn} + r_{,m}r_{,n}\right] \tag{9}$$

$$T_{mn} = -\frac{1-2\nu}{8\pi(1-\nu)}\frac{1}{r^2}\left[\frac{\partial r}{\partial n}\left(\delta_{mn} + \frac{3}{1-2\nu}r_{,m}r_{,n}\right) - n_n r_{,m} + n_m r_{,n}\right] \quad (10)$$

with $1 \leq m, n \leq 3$. Here λ and μ are Lame coefficients, and $\nu = \frac{\lambda}{2(\lambda-\mu)}$ is the Poisson ratio, of a body Ω. In addition r is the distance between the field point $\boldsymbol{x}$ and the load point $\boldsymbol{\eta}$, and the comma indicates derivative; note that all differentiations are with respect to the field point.

Equation (8) can be viewed as a constrained equation relating surface tractions to surface displacements. We establish an integral equation (8) for each of the partial problems defined in Section 2.2 and use the direct boundary element procedure for their numerical approximation.

3 NUMERICAL SOLUTION

A boundary element technique was adopted as a method for discretization of the continuous variational problem (6), leading to the finite dimensional quadratic programming problem. Boundary integral equations (8) for the partial displacements (5) are approximated using a standard direct boundary element procedure.

3.1 Approximation of the Variational Inequality

We partition the candidate contact surface Γ_C into M_C flat polygonal regions $\{\Gamma_C^i\}_{i=1}^{M_C}$. We approximate the smooth function $p \in H^{-\frac{1}{2}}(\Gamma_C)$ by its finite dimensional subspace of all linear combinations of the zero-order shape functions which are constant over any boundary element Γ_C^i in Γ_C, namely

$$p(\boldsymbol{x}) = \sum_{i=1}^{M_C} \phi_i(\boldsymbol{x})\, p_i \quad (11)$$

where $p_i \leq 0 \in \mathcal{R}$, and $\phi_i(\boldsymbol{x})$ is an indicator function for the boundary element Γ_C^i

$$\phi_i(\boldsymbol{x}) = \begin{cases} 1 & \text{if } \boldsymbol{x} \in \Gamma_C^i \\ 0 & \text{if } \boldsymbol{x} \notin \Gamma_C^i \end{cases} \quad (12)$$

For a chosen approximation p, the minimization problem (6) turns into the following finite dimensional quadratic programming problem,

$$\min_{\boldsymbol{p}\in \boldsymbol{K}^{M_C}} \Phi_{M_C}(\boldsymbol{p}) = \frac{1}{2}\sum_{i=1}^{M_C}\sum_{j=1}^{M_C} A_{ij}p_i p_j - \sum_{i=1}^{M_C} B_i p_i \quad (13)$$

where $\boldsymbol{K}^{M_C} = \{\boldsymbol{p} \in \mathcal{R}^{M_C} \mid \forall\, i\ p_i \leq 0\}$ and

$$A_{ij} = \int_{\Gamma_C} \phi_i(\boldsymbol{x})\, w_j(\boldsymbol{x})\, dS_x \simeq \mathrm{mes}(\Gamma_i)\, w_j(\boldsymbol{x}_i) \quad (14)$$

$$B_i = \int_{\Gamma_C} \phi_i(\boldsymbol{x})\,\hat{g}(\boldsymbol{x})\,dS_x \simeq \mathrm{mes}(\Gamma_i)\,\hat{g}(\boldsymbol{x}_i) \tag{15}$$

Here

$$w_j(\boldsymbol{x}) = \int_{\Gamma_C} G(\boldsymbol{x},\eta)\,\phi_j(\eta)\,dS_\eta \tag{16}$$

denotes the relative normal displacement at $\boldsymbol{x} \in \Gamma_C$ due to a unit pressure along Γ_C^j. Furthermore, the functions $w_j(\boldsymbol{x})$ and $\hat{g}(\boldsymbol{x})$ have been approximated by the piecewise step functions.

Thus the problem becomes one of finding the displacements $w_j(\boldsymbol{x}_i)$ and $\hat{\boldsymbol{u}}_j^\alpha(\boldsymbol{x}_i)$ for $\alpha = 1,2$ and $i,j = 1,\ldots,M_C$. We utilize a direct boundary element method for the numerical evaluation of these displacements.

3.2 Boundary Element Formulation

Let us start with the numerical approximation of the integral equation for the displacement fields $\hat{\boldsymbol{u}}^\alpha$ in the bodies $\bar{\Omega}^\alpha$, $\alpha = 1,2$, constrained at Γ_D^α and loaded with the boundary tractions $\boldsymbol{t}^\alpha$ along Γ_F^α.

Consider discretizing the boundary Γ^α into M^α plane triangular elements, $\left\{\Gamma^l\right\}_{l=1}^{M^\alpha}$, such that $\{\Gamma_C^i\} \subset \{\Gamma^l\}$. Let the boundary data be approximated by,

$$\hat{\boldsymbol{u}}^\alpha(\boldsymbol{x}) = \sum_{l=1}^{M^\alpha} \phi_l(\boldsymbol{x})\,\hat{\boldsymbol{u}}^\alpha(\boldsymbol{x}_l) \tag{17}$$

$$\boldsymbol{t}^\alpha(\boldsymbol{x}) = \sum_{l=1}^{M^\alpha} \phi_l(\boldsymbol{x})\,\boldsymbol{t}^\alpha(\boldsymbol{x}_l) \tag{18}$$

where $\hat{\boldsymbol{u}}^\alpha(\boldsymbol{x}_l), \boldsymbol{t}^\alpha(\boldsymbol{x}_l) \in \mathcal{R}^3$ are displacement and traction at the centroid $\boldsymbol{x}_l$ of the boundary element Γ^l, and the indicator function for the boundary element Γ^l is defined as in (12). Substituting these functions into (8), written for the centroid η_k of the boundary element Γ^k, we arrive at a linear algebraic system of the form

$$\frac{1}{2}\hat{\boldsymbol{u}}^\alpha(\eta_k) + \sum_{l=1}^{M^\alpha} \boldsymbol{T}_{lk}^\alpha \cdot \hat{\boldsymbol{u}}^\alpha(\boldsymbol{x}_l) = \sum_{l=1}^{M^\alpha} \boldsymbol{U}_{lk}^\alpha \cdot \boldsymbol{t}^\alpha(\boldsymbol{x}_l) \tag{19}$$

The integrals

$$\boldsymbol{T}_{lk}^\alpha = \int_{\Gamma^\alpha} \phi_l(\boldsymbol{x})\,\boldsymbol{T}^\alpha(\boldsymbol{x},\eta_k)\,d\Gamma_x \tag{20}$$

$$\boldsymbol{U}_{lk}^\alpha = \int_{\Gamma^\alpha} \phi_l(\boldsymbol{x})\,\boldsymbol{U}^\alpha(\boldsymbol{x},\eta_k)\,d\Gamma_x \tag{21}$$

are calculated exactly from the size, orientation, and location of the element Γ^l and the point η_k, according to Cruse (1969). The unknown data in equation (19) are the displacements $\hat{\boldsymbol{u}}^\alpha$ along $\mathrm{int}(\Gamma^\alpha - \Gamma_D^\alpha)$ and traction along Γ_D^α. Consequently, the solution of the mixed boundary value problem is obtained by first appropriately rearranging the columns in equation (19) so that all the unknown data appear in the vector $\boldsymbol{x}^\alpha$,

$$\boldsymbol{A}^\alpha \cdot \boldsymbol{x}^\alpha = \boldsymbol{B}^\alpha \tag{22}$$

Solving equations (22), for bodies $\alpha = 1, 2$, one obtains the boundary displacements $\hat{\boldsymbol{u}}^\alpha(\boldsymbol{x}_i)$ in the candidate contact surface $\{\Gamma_C^i\}_{i=1}^{M_C}$ and the approximation for the modified gap function $\hat{g}(\boldsymbol{x}_i)$ from (7).

Consider, now, the numerical approximation for the relative normal displacement $w_j(\boldsymbol{x}_i)$ at the centroid $\boldsymbol{x}_i$ of the boundary element Γ_C^i due to a unit pressure along Γ_C^j. In order to apply an integral equation (8) to this case we set the boundary traction to

$$\boldsymbol{t}_j^\alpha(\boldsymbol{x}) = \phi_j(\boldsymbol{x})\, \boldsymbol{n}^\alpha(\boldsymbol{x}) \tag{23}$$

and, following the boundary-integral solution strategy that led to equation (22), we derive a set of equations

$$\boldsymbol{A}^\alpha \cdot \boldsymbol{x}_j^\alpha = \boldsymbol{B}_j^\alpha \tag{24}$$

for the unknown displacements along $\mathrm{int}(\Gamma^\alpha - \Gamma_D^\alpha)$ and traction along Γ_D^α. Solving equations (24), for bodies $\alpha = 1, 2$, one obtains the boundary displacements in the candidate contact surface $\{\Gamma_C^i\}_{i=1}^{M_C}$ and the approximation for the relative normal displacement $w_j(\boldsymbol{x}_i)$ from (16). In order to establish equation (16) for all $\boldsymbol{t}_j^\alpha$, ($j = 1, \ldots, M_C$, $\alpha = 1, 2$), as required by (14), equations (24) ought to be solved $2 \times M_C$ times, with defined boundary conditions on Γ_D^α. The fact that $\boldsymbol{A}^\alpha$ matrices in equations (22) and (24) are equal, allows for an efficient numerical solution of the problem involving only one matrix inversion per body.

3.3 Algorithm

An algorithm for the solution of the quadratic programming problem (13) is as follows:

(i) Discretize the boundaries Γ^α, $\alpha = 1, 2$, into M^α plane triangular elements and calculate the integrals $\boldsymbol{T}_{lk}^\alpha$ and $\boldsymbol{U}_{lk}^\alpha$ using equations (20) and (21) respectively.

(ii) Using the matrices $\boldsymbol{T}_{lk}^\alpha$, $\boldsymbol{U}_{lk}^\alpha$, with the given essential boundary conditions (3), assemble and invert the matrices $\boldsymbol{A}^\alpha$, $\alpha = 1, 2$.

(iii) Using the matrices $\boldsymbol{T}_{lk}^\alpha$, $\boldsymbol{U}_{lk}^\alpha$, with the given natural boundary conditions (2), assemble the $\boldsymbol{B}^\alpha$ vector and obtain the solution vector by multiplying $(\boldsymbol{A}^\alpha)^{-1} \cdot \boldsymbol{B}^\alpha$, $\alpha = 1, 2$. Extract the displacements $\hat{\boldsymbol{u}}^\alpha(\boldsymbol{x}_i)$

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in the candidate contact surface $\{\Gamma_C^i\}_{i=1}^{M_C}$ from the solution vector $\boldsymbol{x}^\alpha$. For a given gap function $g(\boldsymbol{x}_i)$ and calculated displacements $\hat{\boldsymbol{u}}^\alpha(\boldsymbol{x}_i)$ obtain the modified gap function $\hat{g}(\boldsymbol{x}_i)$, from (7), and then the linear part of the functional Φ_{M_C}(boldmath$p$), B_i, using (15).

(iv) Using the matrices $\boldsymbol{T}_{lk}^\alpha$, $\boldsymbol{U}_{lk}^\alpha$ assemble the vectors $\boldsymbol{B}_j^\alpha$ from (24) and obtain the solution by multiplying $(\boldsymbol{A}^\alpha)^{-1} \cdot \boldsymbol{B}_j^\alpha$. Extract from the solution vector $\boldsymbol{x}_j^\alpha$ the boundary displacements in the candidate contact surface $\{\Gamma_C^i\}_{i=1}^{M_C}$ and obtain the approximation for the relative normal displacement $w_j(\boldsymbol{x}_i)$ from (16). Knowing the displacements $w_j(\boldsymbol{x}_i)$ obtain the quadratic part of the functional Φ_{M_C}(boldmath$p$), A_{ij}, from (14). Repeate this step for all $j = 1, \ldots, M_C$.

(v) Solve the quadratic programming problem (13).

3.4 Data Parallel Implementataion

In a data parallel environment, the implementation of a computationally intensive algorithm such as the boundary element method, involves a judicious choice for the logical unit for the data. All processors of the computing system can operate on such logical unit concurrently. The general strategy for the problem implementation described herein was to use the local network capabilities as much as possible and to use a high-speed parallel storage system (Data Vault) for the extensive storage requirements associated with the strategy. Consequently, the computing system is configured as a two dimensional lattice of processors, and the logical unit on which the lattice of processors operate is an element-load point pair.

The general division of labor between the front end computer, (Sun 4/480), and the CM-2 in this application is as follows: the problem is initiated on the front end computer, steps (i) through (iv) of the algorithm defined in Section 3.2 are performed in parallel on the CM-2, while the variational part of the solution procedure, step (v), is performed on the front end.

4 APPLICATIONS

Previously, a variety of contact problems including Hertzian and non-Hertzian were investigated numerically see Rabinovich and Sipcic (1992), Rabinovich et al. (1992), and Sipcic and Rabinovich (1993) for details. The results of our calculations are in good agreement with existing analytical and numerical results. Presented here is an important new result for the three dimensional finite bodies contact problem, namely, that the rigid punch is indenting a corner of the elastic unit cube subjected to the bending loading

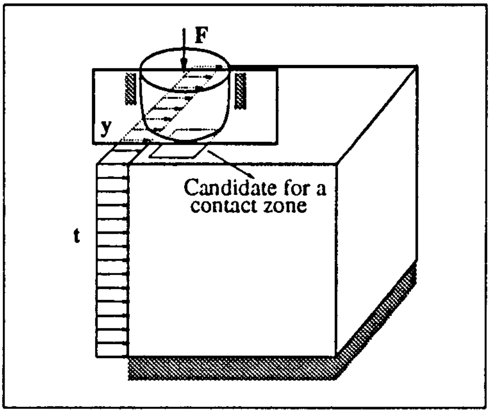

Figure 2: Indentation of an elastic unit cube by a rigid punch with the shape of fourth order paraboloid.

as shown in Figure 2. Required data includes the material properties, the surface element arrangement in general and in the candidate contact surface Γ_C in particular. The known surface tractions and displacements are assumed constant over each surface element. In the case of a rigid punch problem the total compressive force or the indentation is required. The results consist of the contact area, the pressure acting in it, as well as stresses and displacements where required.

4.1 Finite Bodies Contact Problem

Consider first the indentation of a rigid punch into the center and corner of a unloaded unit elastic cube. The profile of the punch is a fourth-degree paraboloid and the depth of indentation is $d = 1.64 \times 10^{-5}$. The contact pressures are given in Figures 3 and 4 respectively. Numerical results are plotted along with the analytical solution obtained for the half-space by Lur'e (1964), pp. 279–284. Note that the calculated pressure in the case of an indentation into the center is in a good agreement with the Lur'e solution, indicating that the influence of the free boundary is negligible. In the case of an indentation into the corner, one can see that the pressure is lower at the points close to the free surface, as what should be expected on the physical ground; the material is "softer" close to the free boundary.

In order to examine the influence of the bending loading on the contact pressure, an indentation into the corner of a unit elastic cube loaded with the traction along one its side is considered, see Figure 2. The contact pressure, for the same indentation as in all previous cases, is given in Figure 5, showing

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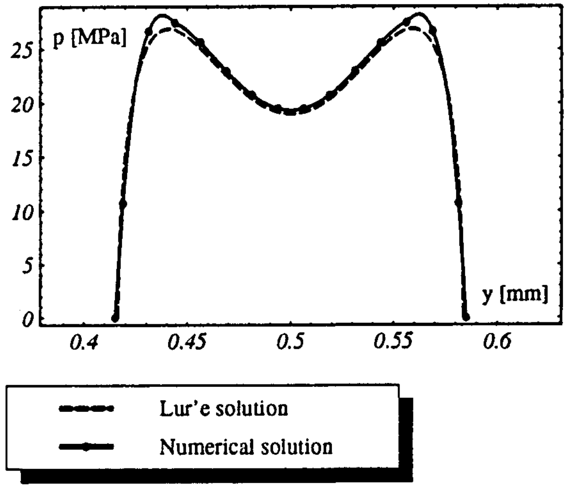

Figure 3: Indentation of an elastic unit cube by a rigid punch with the shape of fourth order paraboloid: contact pressure.

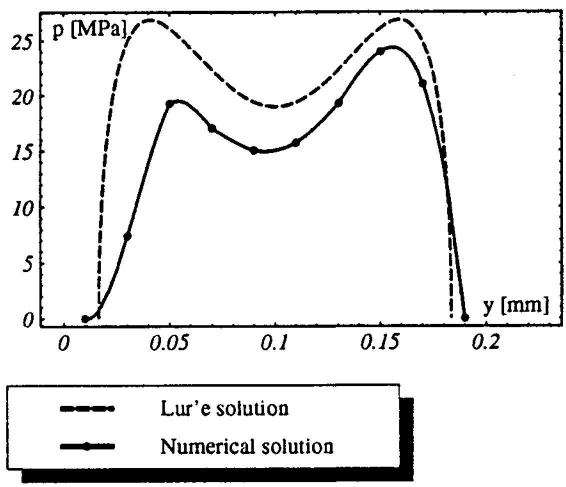

Figure 4: Contact pressure in y plane of Figure 2: illustration of the influence of the free boundary.

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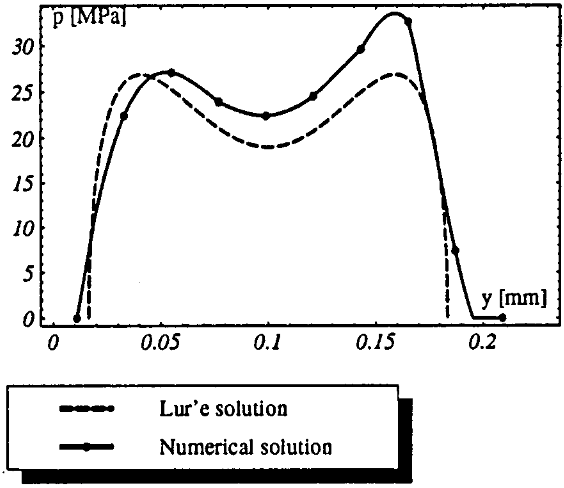

Figure 5: Contact pressure in y plane of Figure 2: illustration of the influence of the bending traction.

overall increase compared to the unloaded case.

In Figure 6 a contact area and several isobars are given for the case of an indentation into the corner; the corresponding pressure distribution is given in Figure 4. It is seen that the contact zone is symmetric with respect to the diagonal plane and that the region of higher pressure is shifted towards the bulk of the material. Note that the slight irregularities in the shape of the contact zone are the consequence of the discretization.

The influence of the bending traction on a structure of the contact zone is illustrated in Figure 7; the corresponding pressure distribution is given in Figure 5. Note that the region of higher pressure is shifted towards the side of the cube loaded by the traction.

5 CONCLUSION

A method for the numerical solution of the frictionless contact between two elastic bodies having arbitrary shape and loading is presented. The method has a reciprocal variational formulation for a strating point. The numerical solution is obtained by means of a boundary element discretization of the variational inequality and related extremum principle. The numerical procedure is highly efficient involving only one matrix inversion per contacting body.

The method is highly suited for a data parallel implementation, and a

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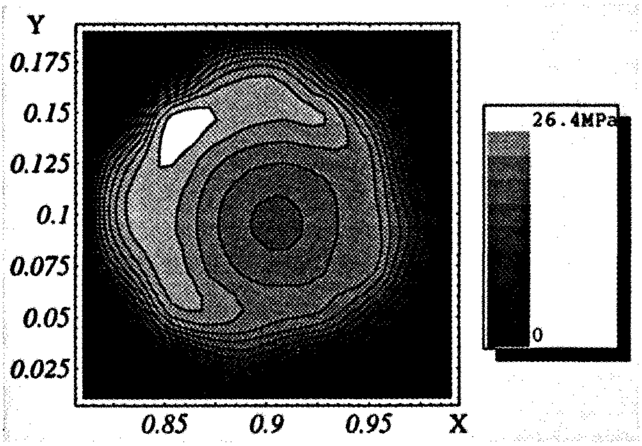

Figure 6: Contact zone for the case given in Figure 4.

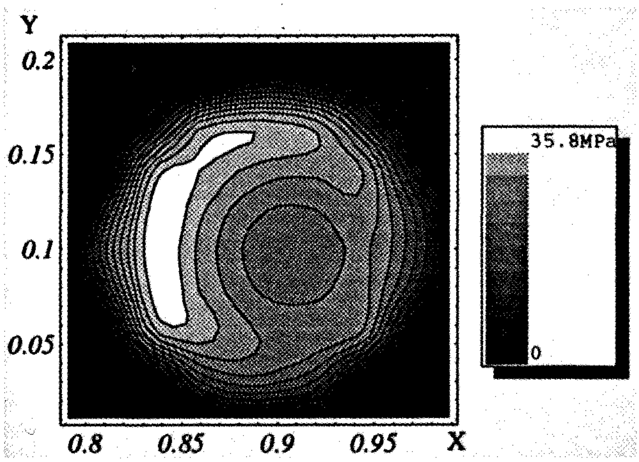

Figure 7: Contact zone for the case given in Figure 5.

methodology for implementation is outlined. The computing system is configured as a two dimensional lattice of processors, and the logical unit on which the lattice of processors operate is an element-load point pair.

The results of our numerical calculations are in good agreement with existing analytical results. Presented here are important new results which illustrate the distinct ability of the method to capture the influence of the body size and loading on the contact area and the pressure acting in it.

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