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Boundary element method numerical model based on mixture theory of two-phase flow

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Abstract

A boundary element numerical scheme for a flowing mixture of solid particles and a fluid is developed within the context of mixture theory. Major differences between the two in literature on the subject are studied, these are typically employed theories called the averaging approach and mixture theory. A numerical technique applied is the boundary element method based on velocity-vorticity formulation of general equations describing the flow of a two-component mixture of a Newtonian fluid and a granular solid. Integral representations for conservation and field functions, based on parabolic diffusion fundamental solution, are presented. Special attention is focused on the mechanical interaction between the mixture components.

1 Introduction

Fluidised multiphase reactors are of increasing importance in nowadays chemical industries, even though their hydrodynamic behaviour is complex and not yet fully understood. Especially the scale-up from laboratory towards industrial equipment is a problem. For example, equations describing the bubble behaviour in gas-solid fluidised beds are (semi) empirical and often determined under laboratory conditions. For that reason there is little unifying theory describing the bubble behaviour in two-phase two-component flows.

From the other hand computational fluid dynamics (CFD) is becoming more and more an engineering tool to predict flows in various types of apparatus on indus-

trial scale. Although the tools for applying single phase CFD are widely available, application of multiphase CFD is however still complicated from both a physical and numerical point of view. Moreover, experimental validation of multiphase CFD models is still in its infancy because simulations are time consuming and reliable predictions of average flows in large-scale equipment are therefore not readily obtained.

In the past some sets of partial differential equations (PDE) describing complex behaviour of gas-solid multiphase flows were written for primitive variables especially on the basis of kinetic theory of granular flow [1]. With no exception system of these PDE was solved using finite difference method, finite volume method or finite element method. The scheme of Požarnik and Škerget [2] was first known alghoritm where set of governing equations was written for velocity-vorticity variables in combination with a novel boundary domain integral method (Škerget and Rek [3]) to extend the applicability of boundary element method (BEM).

The purpose of this work is to study the differences between two basic approaches and to develop the closed set of PDE for general application in BEM based on mixture theory.

2 Fundamental theories: averaging vs. mixture theory

The large number of articles published concerning fluid-solid flows typically employ one of two theories, averaging or mixture theory (theory of interacting continua). In the averaging approach [4] equations of motion, valid for a single fluid or a single particle, are modified to account the presence of the other components and the interactions between components. These equations are then averaged over time or some suitable volume, which is large compared with a characteristic dimension (for example, particle spacing or the diameter of solid particle) but small compared to the dimesions of the whole system. From the mathematical manipulation of the averaged quantities, a number of terms arise. These terms are usually interpreted as some form of interaction between constituents. Constitutive relations to represent these interactive forces, as well as the stress tensors for each constituent, are required to complete the description.

The other method of modeling multicomponent systems is mixture theory. In this theory the equations and principles of the mechanics of a single continuum are generalized to include any number of superimposed continua. The fundamental assumption of the theory is that, at any instant of time, every point in space is occupied by one particle from each constituent, in a homogenized sense. The details of mixture theory are given in [5]. Like averaging, mixture theory also requires constitutive relations for the stress tensor of each component of the mixture and for the momentum exchange between the components.

Our previous model of solid particles immersed in a fluid (Požarnik and Škerget; [2], [6], [7], and [8]) has relied upon an assumption that the solid particles behave as a linearly viscous fluid with a dynamic viscosity η_s and a pressure field p_s. The meaning of p_s in this context is not entirely clear and it leads to an indeterminacy in the governing equations without adding a number of additional relations for e.g. granular temperature, radial distribution function,... Sometimes this indeterminacy is overcome by assuming a relationship between the solid pressure p_s and the fluid pressure p_f in order to reduce number of unknowns. A typical assumption is that $p_s = p_f$. This assumption may be justified if the mixture is composed of materials like water and steam, but is inappropriate when one component is a granular solid. Some of the two fluid models (TFM) are also inconsistent in that they fail to reduce to the appropriate single phase model in the two extreme limitis.

3 Conservation equations

The continuity equations or mass balances for fluid and solid take the following form

$$\frac{\partial \varrho_P}{\partial \tau} + \frac{\partial}{\partial x_j}(\varrho_P v_{pj}) = 0, \tag{1}$$

where v_{pi} is the ith instantaneous phase velocity component ($p = f$ for fluid and $p = s$ for solid). ϱ_F and ϱ_S are bulk densities (i.e. mass of the constituent per unit volume of the mixture) of the component given by

$$\varrho_F = \varepsilon_f \varrho_f, \qquad \varrho_S = \varepsilon_s \varrho_s, \tag{2}$$

where ϱ_f is density of the pure fluid, ϱ_s is density of the solid grains, ε_f is volume fraction of the fluid phase, and ε_s is volume fraction of the solid grains. Mass exchange between the constituents, e.g. due to chemical reaction or combustion, is not considered. The additional equation giving the relation between volume fraction of the contituents of the mixture is also valid

$$\varepsilon_f + \varepsilon_s = 1. \tag{3}$$

Let S_{fij} and S_{sij} denote the partial stress tensors of the fluid and the solid, respectively. Then the momentum balances for the fluid and solid are given by

$$\frac{\partial}{\partial \tau}(\varrho_P v_{pi}) + \frac{\partial}{\partial x_j}(\varrho_P v_{pi} v_{pj}) = \frac{\partial S_{pij}}{\partial x_j} + \varrho_P b_{pi} \pm f_i, \tag{4}$$

where b_{pi} represents the body force, f_i represents the mechanical interaction called local exchange of momentum, and sign $\pm$ becomes $+$ in case of fluid and $-$ in case of solid phase.

Alternative set of conservation equations is formed applying the definition of Stokes derivative $D(\cdot)/D\tau = \partial(\cdot)/\partial\tau + v_k \partial(\cdot)/\partial x_k$

$$\frac{\partial v_{pj}}{\partial x_j} + \frac{1}{\varrho_P}\frac{D\varrho_P}{D\tau} = 0, \tag{5}$$

$$\varrho_P \frac{Dv_{pi}}{D\tau} = \frac{\partial S_{pij}}{\partial x_j} + \varrho_P b_{pi} \pm f_i, \tag{6}$$

In the following contribution densities of pure constituents are assumed to be constant ($\varrho_p = const.$). Modified viscosities are defined as $\eta_P = \eta_p \varepsilon_p$.

4 Constitutive equations

It is assumed that the fluid and solid phases are dense enough to be modeled as homogeneous continuous media. The fluid constituent behaves as a linearly viscous fluid, whose constitutive equation is

$$S_{fij} = -\varepsilon_f \left(P + \frac{2}{3}\eta_f \mathscr{D}_f \right) \delta_{ij} + 2\eta_F \dot{\varepsilon}_{fij}, \tag{7}$$

where P is the fluid pressure, η_f is fluid dynamic viscosity, $\dot{\varepsilon}_{fij}$ is fluid stretching tensor, δ_{ij} is Kronecker delta function, and $\mathscr{D}_p = div\ \vec{v}_p$. If the fluid is incompressible, then P is one of the unknown quantities in the problem that have to be calculated. If the fluid is compressible, an equation of state is needed for P.

Stress tensor for a granular material is given by

$$S_{sij} = \left(\beta_0 + \frac{1}{\varrho_s^2}\beta_1 \frac{\partial \varrho_S}{\partial x_l}\frac{\partial \varrho_S}{\partial x_l} + \beta_2 \mathscr{D}_s \right) \delta_{ij} + \beta_3 \dot{\varepsilon}_{sij} + \frac{1}{\varrho_s^2}\beta_4 \frac{\partial \varrho_S}{\partial x_i}\frac{\partial \varrho_S}{\partial x_j}, \tag{8}$$

where the terms multiplying δ_{ij} in eq. (8) can be interpreted as the solid pressure p_s similar to the definition in the kinetic theory of granular flow, $\frac{\partial \varrho_S}{\partial x_l}\frac{\partial \varrho_S}{\partial x_l} = grad\ \varrho_S \cdot grad\ \varrho_S$, and $\frac{\partial \varrho_S}{\partial x_i}\frac{\partial \varrho_S}{\partial x_j} = grad\ \varrho_S \otimes grad\ \varrho_S$. The material moduli β_1 and β_4 are material parameters that reflect the distribution of solid particles, and β_0 plays a role analogous to pressure in a compressible fluid and is given by an equation of state. The material modulus β_2 is a viscosity similar to the bulk viscosity in a compressible fluid and β_3 denotes the viscosity (i.e. the resistance of the material to flow) of the granular solids. In general coefficients β_i depend upon the distribution of solid particles and stretching tensor, $\beta_i = \beta_i(\varepsilon_s, \dot{\varepsilon}_{sij})$, but here it is assumed that $\beta_i = \beta_i(\varepsilon_s)$.

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The mechanical interaction between the mixture components f_i is written as

$$f_i = A_1 \frac{\partial \varepsilon_s}{\partial x_i} + A_2 F(\varepsilon_s)(v_{si} - v_{fi}) + A_3 a_{vmi}, \tag{9}$$

where a_{vmi} is a invariant measure of the relative acceleration between the mixture components and $F(\varepsilon_s)$ represents the dependence of the drag coefficient on the volume fraction. A_1, A_2, and A_3 are constants. Terms in eq. (9) reflect the presence of volume fraction gradients, drag, and virtual mass.

5 Summary of governing equations

Combining equations (5), (6), (7), (8), and (9) yields the equations describing the flow of a mixture of a Navier-Stokes fluid

$$\frac{\partial v_{fj}}{\partial x_j} = \mathscr{D}_f = -\frac{1}{\varrho_F}\frac{D\varrho_F}{D\tau}, \tag{10}$$

$$\varrho_F \frac{Dv_{fi}}{D\tau} = -\varepsilon_f \frac{\partial P}{\partial x_i} - P\frac{\partial \varepsilon_f}{\partial x_i} + \frac{\partial}{\partial x_j}\left(2\eta_F \dot{\varepsilon}_{fij} - \frac{2}{3}\eta_F \mathscr{D}_f \delta_{ij}\right) + \varrho_F b_{fi} + f_i, \tag{11}$$

and a granular solid

$$\frac{\partial v_{sj}}{\partial x_j} = \mathscr{D}_s = -\frac{1}{\varrho_S}\frac{D\varrho_S}{D\tau}, \tag{12}$$

$$\varrho_S \frac{Dv_{si}}{D\tau} = \frac{\partial \beta_0}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\left(\frac{\beta_1}{\varrho_s^2}\frac{\partial \varrho_S}{\partial x_l}\frac{\partial \varrho_S}{\partial x_l} + \beta_2 \mathscr{D}_s\right)\delta_{ij} + \beta_3 \dot{\varepsilon}_{sij} + \frac{\beta_4}{\varrho_s^2}\frac{\partial \varrho_S}{\partial x_i}\frac{\partial \varrho_S}{\partial x_j}\right] + \varrho_S b_{si} - f_i. \tag{13}$$

For three-dimensional geometry, the equations (10), (11), (12), (13), and (3) provide nine relations for nine unknowns, v_{fx}, v_{fy}, v_{fz}, v_{sx}, v_{sy}, v_{sz}, ε_f, ε_s, and p. The above field equations have to be solved for appropriate boundary and initial conditions.

By taking into account the extended form of S_{pij} (Škerget and Samec [9]) momentum equations (11) and (13) are reformed in the following for the velocity-vorticity formulation more convenient form

$$\varrho_F \frac{D\vec{v}_f}{D\tau} = -\varepsilon_f \, grad\, p - p \, grad\, \varepsilon_f - rot\,(\eta_F \vec{\omega}_f) + \frac{4}{3} grad\,(\eta_F \mathscr{D}_f)$$

$$+2\, grad\, \vec{v}_f \cdot grad\, \eta_F + 2\, grad\, \eta_F \times \vec{\omega}_f - 2\mathscr{D}_f \, grad\, \eta_F + \varrho_F \vec{b}_f + \vec{f}, \tag{14}$$

$$\varrho_S \frac{D\vec{v}_s}{D\tau} = grad\, \beta_0 - \frac{1}{2} rot\,(\beta_3 \vec{\omega}_f) + grad\,[(\beta_2 + \beta_3)\mathscr{D}_s]$$

$$+grad\,\vec{v}_s \cdot grad\,\beta_3 + grad\,\beta_3 \times \vec{\omega}_s - \mathscr{D}_s\, grad\,\beta_3 + \mathscr{V}_s + \varrho_S \vec{b}_s - \vec{f}, \qquad (15)$$

where $\mathscr{V}_s$ is defined as

$$\mathscr{V}_s = \frac{\partial}{\partial x_j}\left(\frac{\beta_1}{\varrho_s^2}\frac{\partial \varrho_S}{\partial x_l}\frac{\partial \varrho_S}{\partial x_l}\delta_{ij} + \frac{\beta_4}{\varrho_s^2}\frac{\partial \varrho_S}{\partial x_i}\frac{\partial \varrho_S}{\partial x_j}\right). \qquad (16)$$

6 Velocity-vorticity formulation

In BEM, the original set of Navier-Stokes equations is further transformed with the use of the velocity-vorticity variables formulation. Introducing by definition the solenoidal vorticity vector field function $\omega_{pi}(r_j, \tau)$ as a curl of the compatibility velocity field $v_{pi}(r_j, \tau)$ (Škerget and Rek [3]), the fluid and solid motion computation procedure is partitioned into its kinetics and kinematics. The advantages of this approach lie with the numerical separation of kinematic and kinetic aspects of the Navier-Stokes fluid flow and granular solid flow from the pressure computation.

Kinetics of two-phase two-component mixture is given by vorticity transport equations obtained as a curl of momentum balances, equations (14) and (15). It reads for the fluid motion part as

$$\vec{\nabla} \times (\varrho_F \vec{a}_f) = \vec{\nabla} \times \left[-rot\,(\eta_F \vec{\omega}_f) + 2\,grad\,\vec{v}_f \cdot grad\,\eta_F + 2\,grad\,\eta_F \times \vec{\omega}_f - 2\mathscr{D}_f\, grad\,\eta_F + \varrho_F \vec{b}_f + \vec{f}\right], \qquad (17)$$

while kinetics of granular solid flow part is given by

$$\vec{\nabla} \times (\varrho_S \vec{a}_s) = \vec{\nabla} \times \left[-\frac{1}{2} rot\,(\beta_3 \vec{\omega}_f) + grad\,\vec{v}_s \cdot grad\,\beta_3 + grad\,\beta_3 \times \vec{\omega}_s - \mathscr{D}_s\, grad\,\beta_3 + \mathscr{V}_s + \varrho_S \vec{b}_s - \vec{f}\right], \qquad (18)$$

where $\vec{a}_p$ is defined as $\vec{a}_p = D\vec{v}_p/D\tau$. Material properties are given as a sum of a constant and a variable part. In case of Navier-Stokes fluid flow part of a mixture motion modified viscosity and modified denstity are decomposed, while for granular solid flow part coefficients β_i $(i = 1, 2, 3, 4)$ and modified densitiy are rewritten as follows: $\eta_F = \eta_{F0} + \tilde{\eta}_F$, $\varrho_F = \varrho_{F0} + \tilde{\varrho}_F$, $\beta_i = \beta_{i0} + \tilde{\beta}_i$, and $\varrho_S = \varrho_{S0} + \tilde{\varrho}_S$. Finally, nonlinear parabolic diffusion-convective vorticity transport equations suitable for implementation in BEM are obtained

$$\frac{D\omega_{fi}}{D\tau} = \nu_{f0}\frac{\partial^2 \omega_{fi}}{\partial x_j \partial x_j} + \omega_{fj}\frac{\partial v_{fi}}{\partial x_j} - \omega_{fi}\mathscr{D}_f + \frac{1}{\varrho_{F0}} e_{ijk}\frac{\partial F_{fk}}{\partial x_j} + \frac{1}{\varrho_{F0}} e_{ijk}\frac{\partial f_k}{\partial x_j}, \qquad (19)$$

$$\frac{D\omega_{si}}{D\tau} = \frac{\beta_{30}}{2\varrho_{S0}}\frac{\partial^2 \omega_{si}}{\partial x_j \partial x_j} + \omega_{sj}\frac{\partial v_{si}}{\partial x_j} - \omega_{si}\mathscr{D}_s + \frac{1}{\varrho_{S0}} e_{ijk}\frac{\partial F_{sk}}{\partial x_j} - \frac{1}{\varrho_{S0}} e_{ijk}\frac{\partial f_k}{\partial x_j}, \qquad (20)$$

where f_i represents the local exchange of momentum, eq. (9), and F_{fi}, and F_{si} are defined as

$$F_{fi} = \tilde{\varrho}_F \left(b_{fi} - a_{fi}\right) - \tilde{\eta}_F e_{ijk} \frac{\partial \omega_{fk}}{\partial x_j} - e_{ijk}\omega_{fj} \frac{\partial \tilde{\eta}_F}{\partial x_k} + 2 \frac{\partial v_{fi}}{\partial x_j} \frac{\partial \tilde{\eta_F}}{\partial x_j} - 2\mathscr{D}_f \frac{\partial \tilde{\eta_F}}{\partial x_i}, \tag{21}$$

$$F_{si} = \tilde{\varrho}_S \left(b_{si} - a_{si}\right) - \frac{1}{2}\tilde{\beta}_3 e_{ijk} \frac{\partial \omega_{sk}}{\partial x_j} - \frac{1}{2} e_{ijk}\omega_{sj} \frac{\partial \tilde{\beta}_3}{\partial x_k} + \frac{\partial v_{si}}{\partial x_j} \frac{\partial \tilde{\beta}_3}{\partial x_j} - \mathscr{D}_s \frac{\partial \tilde{\beta}_3}{\partial x_i} + \mathscr{V}_s. \tag{22}$$

As a basis to derive the kinematics of two-phase two-component flow of Navier-Stokes fluid and a granular solid reformed continuity equations (10) and (12) are taken. Finally, the kinematics of both components motion is carried out in form of vector elliptic Poisson equation (Škerget and Samec [9])

$$\frac{\partial^2 v_{pi}}{\partial x_j \partial x_j} + e_{ijk} \frac{\partial \omega_{pk}}{\partial x_j} - \frac{\partial \mathscr{D}_p}{\partial x_i} = 0. \tag{23}$$

To increase the stability of the coupled velocity-vorticity iterative scheme on the one side and at the same time coupled Navier-Stokes fluid flow and granular solid flow scheme on the other side kinematics equations (23) are rewritten using the false transient approach

$$\frac{\partial^2 v_{pi}}{\partial x_j \partial x_j} - \frac{1}{\alpha_p} \frac{\partial v_{pi}}{\partial \tau} + e_{ijk} \frac{\partial \omega_{pk}}{\partial x_j} - \frac{\partial \mathscr{D}_p}{\partial x_i} = 0, \tag{24}$$

with α_p as a relaxation parameter. It is obvious that the governing equations (24) are exactly satisfied only at the steady state ($\tau \to \infty$), when the false accumulation term vanishes.

Equations (19), (20), (24), and derivatives of the additional equation defining the volume fraction (3), provide for the three-dimensional geometry fourteen relations for fourteen unknowns, v_{fx}, v_{fy}, v_{fz}, v_{sx}, v_{sy}, v_{sz}, ω_{fx}, ω_{fy}, ω_{fz}, ω_{sx}, ω_{sy}, ω_{sz}, ε_f, and ε_s. The equations present the leading non-linear set of equations governing the two-phase two-component flow to which the weighted residuals technique of the BEM has to be applied.

7 Boundary element model

The advantage of the boundary element method originate from the application of the Green fundamental solutions as particular weighting functions. Since they only describe the linear transport phenomenon, an appropriate selection of a linear differential operator $\mathscr{L}[\cdot]$ is of key importance in establishing a stable and accurate singular integral representation corresponding to the original differential

conservation equation. All differential conservation models can be written as in the following general statement

$$\mathscr{L}[u] + b = 0, \tag{25}$$

where the linear differential operator $\mathscr{L}[\cdot]$ can be either elliptic or parabolic and $u(r_j, \tau)$ is an arbitrary field function while the nonhomogeneous term $b(r_j, \tau)$ is generally used for the nonlinear transport effects or pseudo body forces.

Considering the kinetics in an integral representation one has to take into account the parabolic diffusion-convection character of the vorticity transport equations, (19) and (20). Since only linear parabolic diffusion differential operator is employed, i.e.

$$\mathscr{L}[\cdot] = \kappa_p \frac{\partial^2(\cdot)}{\partial x_j \partial x_j} - \frac{\partial(\cdot)}{\partial \tau}, \tag{26}$$

where the quantity κ_p is a constant modified the vorticity equations can be formulated as a nonhomogeneous parabolic diffusion equations, as follows

$$\mathscr{L}[\omega_{pi}] + b_{pi} = \kappa_p \frac{\partial^2 \omega_{pi}}{\partial x_j \partial x_j} - \frac{\partial \omega_{pi}}{\partial \tau} + b_{pi} = 0, \tag{27}$$

with the following corresponding integral representation written in a time increment form for a time step $\Delta\tau = \tau_F - \tau_{F-1}$

$$c(\xi)\omega_{pi}(\xi, \tau_F) + \kappa_p \int_\Gamma \int_{\tau_{F-1}}^{\tau_F} \omega_{pi} \frac{\partial u_p^\star}{\partial n} d\tau d\Gamma = \kappa_p \int_\Gamma \int_{\tau_{F-1}}^{\tau_F} \frac{\partial \omega_p}{\partial n} u_p^\star d\tau d\Gamma \tag{28}$$

$$+ \int_\Omega b_{pi} u_p^\star d\Omega + \int_\Omega \omega_{pi,F-1} u_{p,F-1}^\star,$$

where $u_p^\star$ is the parabolic diffusion fundamental solution. Assuming constant variation of all field variables within the individual time increment, the time integrals in eq. (28) are evaluated analytically

$$U_p^\star = \kappa_p \int_{\tau_{F-1}}^{\tau_F} u^\star d\tau, \tag{29}$$

enabling us to rewrite eq. (28) in final integral form

$$c(\xi)\omega_{pi}(\xi, \tau_F) + \int_\Gamma \omega_{pi} \frac{\partial U_p^\star}{\partial n} d\Gamma = \int_\Gamma \frac{\partial \omega_{pi}}{\partial n} U_p^\star d\Gamma \tag{30}$$

$$+ \frac{1}{\kappa_p} \int_\Omega b_{pi} U_p^\star d\Omega + \int_\Omega \omega_{pi,F-1} u_{p,F-1}^\star.$$

The eq. (30) represents the vorticity kinetics of Navier-Stokes fluid and granular solid flow mixture in integral form. The domain integral of the nonhomogeneous

nonlinear contribution b_{pi}

$$b_{pi} = -v_{pj}\frac{\partial \omega_{pi}}{\partial x_j} + \omega_{pj}\frac{\partial v_{pi}}{\partial x_j} + \frac{1}{\varrho_{P0}} e_{ijk}\frac{\partial F_{pk}}{\partial x_j} \pm \frac{1}{\varrho_{P0}} e_{ijk}\frac{\partial f_k}{\partial x_j}, \qquad (31)$$

includes the transport, source and exchange effects of vorticity.

The corresponding integral representation of the mixture flow kinematics governed by parabolic equations (24) can be obtained following the same basic idea that each component of the velocity vectors v_{pi} satisfies nonhomogeneous parabolic equation. Therefore again linear parabolic diffusion operator, eq. (26), is employed ($\kappa_p \rightarrow \alpha_p$). The following relation is written

$$\mathscr{L}[v_{pi}] + b_{pi} = \alpha_p \frac{\partial^2 v_{pi}}{\partial x_j \partial x_j} - \frac{\partial v_{pi}}{\partial \tau} + b_{pi} = 0, \qquad (32)$$

rendering the singular boundary integral formulation for the velocity vectors v_{pi} in a time increment form for a time step $\Delta\tau = \tau_F - \tau_{F-1}$

$$c(\xi)\, v_{pi}(\xi, \tau_F) + \alpha_p \int_\Gamma \int_{\tau_{F-1}}^{\tau_F} v_{pi} \frac{\partial u_p^\star}{\partial n} d\tau d\Gamma = \alpha_p \int_\Gamma \int_{\tau_{F-1}}^{\tau_F} \frac{\partial v_{pi}}{\partial n} u_p^\star d\tau d\Gamma \qquad (33)$$

$$+ \int_\Omega b_{pi} u_p^\star d\Omega + \int_\Omega v_{pi,F-1} u_{p,F-1}^\star.$$

Using the same idea as in eq. (29) final integral statement of two-phase two-component kinematics is formed

$$c(\xi)\, v_{pi}(\xi, \tau_F) + \int_\Gamma v_{pi} \frac{\partial U_p^\star}{\partial n} d\Gamma = \int_\Gamma \frac{\partial v_{pi}}{\partial n} U_p^\star d\Gamma \qquad (34)$$

$$+ \frac{1}{\alpha_p} \int_\Omega b_{pi} U_p^\star d\Omega + \int_\Omega v_{pi,F-1} u_{p,F-1}^\star.$$

Linear transport phenomena in eq. (34) is completely represented with the boundary integrals only. The domain integral of the nonhomogeneous nonlinear contribution b_{pi} is represented by vortical and by additional on volume fraction dependent contribution of fluid and granular solid flow

$$b_{pi} = \alpha_p e_{ijk} \frac{\partial \omega_{pk}}{\partial x_j} - \alpha_p \frac{\partial \mathscr{D}_p}{\partial x_i}. \qquad (35)$$

8 Conclusion

Numerical scheme for the computation of two-phase two-component Navier-Stokes fluid-granular solid flows based on mixture theory is presented. Fundamental assumption of the theory is that, at any instant of time, every point in space is occupied by one particle from each constituent, in a homogenized sense. Special emphasis is given to development of velocity-vorticity formulation of governing equations to achieve closed set of PDE. The proposed model do not fail to reduce to the appropriate single phase model in two extreme limits. By taking the curl operator of the momentum equations computational scheme is partitioned in kinematics and kinetics. The advantages of this approach lie with the numerical separation of kinematic and kinetic aspects of the Navier-Stokes fluid flow and granular solid flow from the pressure computation. Numerical technique applied is boundary element method. Integral representations for conservation field functions are based on parabolic diffusion fundamental solution. The presented numerical scheme is prepared for implementation in numerical code BEEAS-BEMFLOW.

References

[1] Bömer, A., Qi, H., Renz, U., Vasquez, S. & Boysan, F., Eulerian computation of fluidized bed hydrodynamics - a comparison of physical models. *Proc. of the 13th Int. Conf. on Fluidized Bed Combustion*, Orlando, pp. 775–781, 1995.

[2] Požarnik, M. & Škerget, L., Physical models for bubbling behaviour in fluidized beds with bdim. *Zeitschrift für angewandte Mathematik und Mechanik*, **80(3)**, pp. 729–730, 2000.

[3] Škerget, L. & Rek, Z., Boundary-domain integral method for using a velocity-vorticity formulation. *Eng anal bound elem*, **15**, pp. 359–370, 1995.

[4] Anderson, T.B. & Jackson, R., Fluid mechanical descriptions of fluidized beds. *Ind Engng Chem Fundam*, **6**, pp. 527–539, 1967.

[5] Atkin, R.J. & Craine, R.E., Continuum theories of mixtures: basic theory and historical development. *Q J Mech Appl Math*, **29**, p. 290, 1976.

[6] Požarnik, M. & Škerget, L., Advanced bdim algorithm for simulation of gas-solid particle flow. *Zeitschrift für angewandte Mathematik und Mechanik*, **81(4)**, pp. 959–960, 2001.

[7] Požarnik, M. & Škerget, L., Simulation of gas-solid particle flows by boundary domain integral method. *Eng anal bound elem*, **26(10)**, pp. 939–949, 2002.

[8] Požarnik, M. & Škerget, L., Robno obmona integralska metoda za numerično modeliranje lebdečih slojev = boundary domain integral method for numerical modeling of fluidized beds. *Stroj vestn*, **48(2)**, pp. 58–72, 2002.

[9] Škerget, L. & Samec, N., Bem for compressible dynamics. *Proc. of the 25th World Conf. on B E M*, Split, p. accepted for publication, 2003.