

УДК 517.9; 539.3

BOUNDARY HOMOGENIZATION AND REDUCTION OF DIMENSION IN THE KIRCHHOFF – LOVE PLATE

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I present a joint work with Dominique Blanchard (Université de Rouen, France) and Taras A. Mel'nyk (Kyiv Nat. Taras Shevchenko University, Ukraine)

We investigate the asymptotic behavior, as $\varepsilon \rightarrow 0$, of the Kirchhoff – Love equation satisfied by the transverse displacement U_ε of the middle surface $\Omega_\varepsilon^+ \cup \Omega_\varepsilon^-$ (contained in the (x_1, x_2) -coordinate plane) of a thin three-dimensional plate. The middle surface is composed of two domains. The first one Ω_ε^- is a thin strip with vanishing height h_ε (in direction x_2), as $\varepsilon \rightarrow 0$. The second one Ω_ε^+ is a comb with fine teeth having small cross section $\varepsilon\omega$ and constant height, ε -periodically distributed (in direction x_1) on the upper basis of the thin strip (see Figure 1). The middle surface is assumed clamped on the top of the teeth, with a free boundary elsewhere, and subjected to a transverse load.

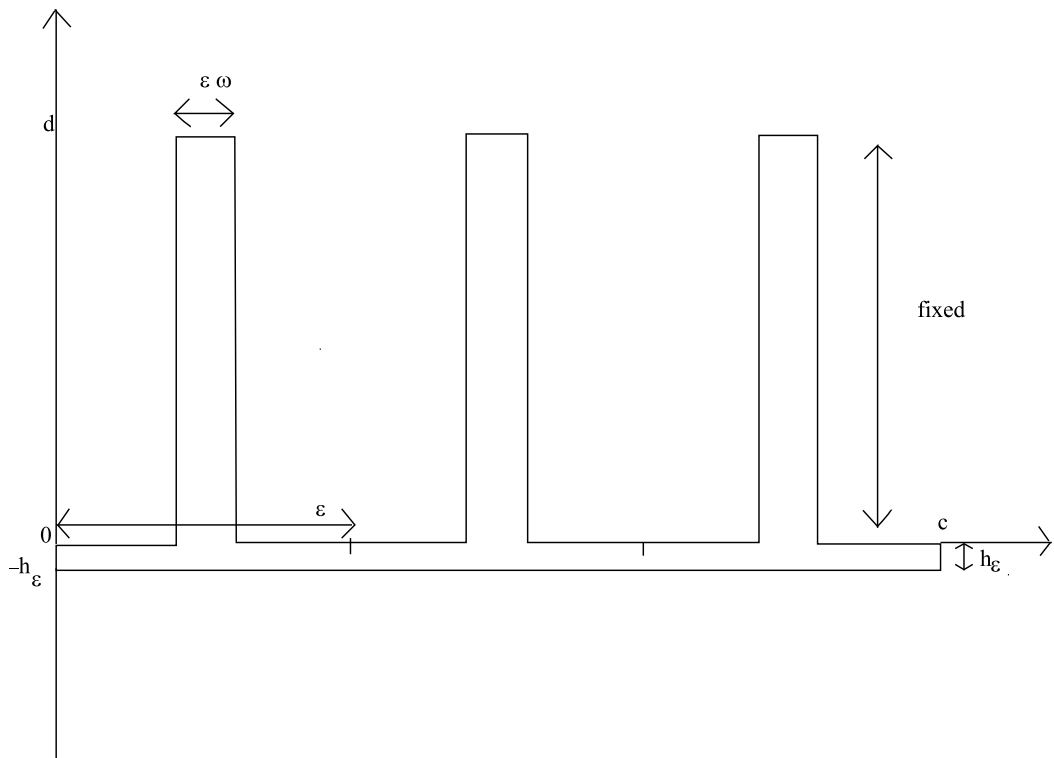


Fig. 1. The middle surface of our three-dimensional plate.

As $\varepsilon \rightarrow 0$, in the limit domain $\Omega^+ =]0, c[\times]0, d[$ of the comb, we obtain a continuum bending model of rods subjected to a force f , clamped on the upper side $\Gamma =]0, c[\times \{d\}$, and subjected on the lower side $\Sigma =]0, c[\times \{0\}$ to applied forces but without applied momentum. The forces on Σ depend on the limit density g of the transverse loads on the thin strip Ω_ε^- , and on the measure of the cross section ω of the reference tooth. The force f depends on the limit of the transverse loads on the teeth.

The limit displacement is independent of x_2 in the rescaled (with respect to h_ε) strip $\Omega^- =]0, c[\times]-1, 0[$. The limit displacement meets a Dirichlet transmission condition between Ω^+ and Ω^- , if $h_\varepsilon \gg \varepsilon^4$, or if $h_\varepsilon \simeq \varepsilon^4$ and $\int_{-1}^0 g(x_1, x_2) dx_2 = 0$ a.e. in $]0, c[$. While, if the strip is thin enough and the transverse loads on the thin strip are strong enough, i.e. $h_\varepsilon \simeq \varepsilon^4$ and $\int_{-1}^0 g(x_1, x_2) dx_2 \neq 0$ in a subset of $]0, c[$ with positive measure, a discontinuity in the Dirichlet transmission condition appears. Roughly speaking, this means that the displacement in the strip oscillates between the teeth of Ω_ε^+ producing a limit average field different from that on the lower extremities of the teeth.

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