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Brief note

BOUNDARY LAYER ANALYSIS IN NANOFLUID FLOW PAST A PERMEABLE MOVING WEDGE IN PRESENCE OF MAGNETIC FIELD BY USING FALKNER – SKAN MODEL

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In the present work, the effect of various dimensionless parameters on the momentum, thermal and concentration boundary layer are analyzed. In this respect we have considered the MHD boundary layer flow of heat and transfer over a porous wedge surface in a nanofluid. The governing partial differential equations are converted into ordinary differential equations by using the similarity transformation. These ordinary differential equations are numerically solved using fourth order Runge–Kutta method along with shooting technique. The present results have been shown in a graphical and also in tabular form. The results indicate that the momentum boundary layer thickness reduces with increasing values of the pressure gradient parameter β for different situations and also for the magnetic parameter M but increases for the velocity ratio parameter λ and permeability parameter K^*. The heat transfer rate increases for the pressure gradient parameter β, velocity ratio parameter λ, Brownian motion parameter Nb and Prandtl number Pr but opposite result is found for the increasing values of the thermoporesis parameter Nt. The nanoparticle concentration rate increases with an increase in the pressure gradient parameter β, velocity ratio parameter λ, Brownian motion parameter Nb and Lewis number Le, but decreases for the thermoporesis parameter Nt. Finally, the numerical results has compared with previously published studies and found to be in good agreement. So the validity of our results is ensured.

Key words: MHD, nanofluid, wedge flow, porosity, convection.

1. Introduction

The boundary layer theory is important in many engineering fields and real world problems. The main application of this theory is to find the skin friction drag acting on a body moving through a fluid, such as the drag of an airplane wing, a turbine blade, or a complete ship. So on the basis of the Prandtl boundary layer theory, Falkner – Skan developed a model which is known as wedge flow. Therefore a lot of work has been done over the last few years. Among them, Seddeek *et al.* [1] has found a similarity solutions for a steady Falkner- Skan flow and heat transfer over a wedge with variable viscosity and thermal conductivity, Martin [2] discussed the Falkner- Skan flow over a wedge by taking slip boundary conditions, Yacob *et al.* [3] studied the Falker- Skan flow for a static or moving wedge in nanofluids, Hayat *et al.* [4] discussed the Falkner- Skan flow in the case of power – law fluid with mixed convection, and Ashwini *et al.* [5] has discussed the unsteady MHD accelerating flow past a wedge with thermal radiation and internal heat generation. The concept of nanofluid was first introduced by Choi in 1995 which referred to dispersions of nanoparticles in the base fluids such as water, ethylene glycol, and propylene glycol. Later, Buongiorno [6] examined the reasons behind the

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enhancement in heat transfer for a nanofluid and he found that Brownian diffusion and thermophoresis are the main causes, Nield and Kuznetsov [7] and Kuznetsov and Nield [8] investigated the natural convective boundary layer flow of a nanofluid employing the Buongiorno model. Many researchers [9-14] have worked on the boundary layer theory by considering various types of geometry and boundary conditions. By applying the model of Nield and Kuznetsov [7], Khan and Pop [15] were first to study the boundary layer flow of a nanofluid past a linearly stretching sheet. The boundary layer flow and heat transfer over a linearly stretching sheet with a convective boundary condition in a nanofluid were described by Makinde and Aziz [16]. Mutuku [17] discussed the MHD boundary layer flow over a permeable vertical plate with convective heating. Kandasamy *et al*. [18] investigated the MHD boundary layer flow of a nanofluid past a vertical stretching permeable surface with suction/injection. Mustafa *et al.* [19] analyzed the effect of various parameters in a nanofluid near a stagnation point towards a stretching surface. Rana and Bhargava [20] studied the steady, laminar boundary layer flow due to the nonlinear stretching flat surface in a nanofluid. Later, Makinde *et al*. [21] discussed the combined effects of buoyancy force and magnetic field on stagnation-point flow and heat transfer in a nanofluid flow towards a stretching sheet. Hence, the present work is focused on the steady MHD Falkner-Skan flow with magnetic field past a porous wedge in a nanofluid.

2. Governing equations and similarity analysis

Let us consider a two dimensional steady laminar MHD boundary layer flow of a viscous incompressible electrically conducting nanofluid over a non-conducting, non-isothermal stretching porous wedge surface moving with the velocity u_w and the free stream velocity is U. The positive x-coordinate is measured along the surface of the wedge and the positive y-coordinate is measured normal to the x-axis in the outward direction towards the fluid. Taking T_w and T_∞ are the temperature of the wedge wall and free stream of the fluid far away from the wedge, the total angle of the wedge is denoted as $\Omega=\beta\pi$, where β is the Hartree pressure gradient. It is assumed that the base fluid and the nanoparticles are in thermal equilibrium and no slip occurs between them. Also, a uniform magnetic field of strength B_0 is introduced to the normal to the direction of the flow. The governing partial differential equations for the boundary-layer flow of the nanofluid in this problem can be written as follows (using the Buongiorno Model [6]):
Equation of continuity

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0. \tag{2.1}$$

Momentum equation

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{dU}{dx}+\nu_f\frac{\partial^2 u}{\partial y^2}+\frac{\sigma B_0^2}{\rho_f}(U-u)+\frac{\nu}{K}(U-u)\ . \tag{2.2}$$

Energy equation

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_f\frac{\partial^2 T}{\partial y^2}+\tau\left\{D_B\left(\frac{\partial T}{\partial y}\frac{\partial C}{\partial y}\right)+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right\}. \tag{2.3}$$

Concentration equation

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}. \tag{2.4}$$

The above equations are subject to the following boundary conditions

$$u = u_w, \quad v = 0, \quad T = T_w, \quad C = C_w \quad \text{at} \quad y = 0,$$

$$u = U, \quad T \to T_\infty, \quad C \to C_\infty \quad \text{as} \quad y \to \infty$$

where u and v are the velocity components along x and y directions, T_w and C_w are the variable wall temperature and nanoparticle volume fraction of the wall while the uniform temperature and nanoparticle volume fraction far from the wall are T_∞ and C_∞, respectively, ν_f is the kinematic viscosity of the base fluid, ρ_f is the density of the base fluid, σ is the electrical conductivity, B_0 is the magnetic field intensity, g is the acceleration due to gravity, α_f is the thermal diffusivity of the base fluid, D_B is the Brownian diffusion coefficient, D_T is the thermophoresis diffusion coefficient. Here τ is the ratio of the effective heat capacity of the nanoparticle material and the heat capacity of the ordinary fluid, T is the fluid temperature and C is the nanoparticle volume fraction, respectively.

The velocity of the wedge sheet and the free stream velocity are respectively defined as follows

$$u_w = ax^m, \quad V_w = 0, \quad U = bx^m$$

where a, b and d are positive constants with dimension reciprocal time. Among them, b is the initial stretching rate and the exponent m is a function of the wedge angle parameter β where the total apex angle of the wedge is $\beta\pi$ such that $\beta = \dfrac{2m}{1+m}$. To convert the governing equations into a set of ordinary differential equations, we introduce the following similarity transformations

$$\eta = y\sqrt{\frac{U(1+m)}{2x\nu}}, \quad \psi = \sqrt{\frac{2x\nu U}{(1+m)}}\, f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},$$

$$\varphi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \quad u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x}.$$

By applying the above similarity transformations, the partial differential Eqs (2.2), (2.3) and (2.4) are transformed into non-dimensional, nonlinear and coupled ordinary differential equations are as follows

$$f''' + f f'' + \beta\left(1 - f'^2\right) + \left(M + K^*\right)\left(1 - f'\right) = 0, \tag{2.5}$$

$$\theta'' + \Pr\left[f\theta' - \beta f'\theta + Nb\,\theta'\varphi' + Nt\,\theta'^2\right] = 0, \tag{2.6}$$

$$\varphi'' + \frac{Nt}{Nb}\theta'' + \mathrm{Le}\left[f\varphi' - \beta f'\varphi\right] = 0. \tag{2.7}$$

The transform boundary conditions

$$f = 0, \quad f' = \lambda, \quad \theta = 1, \quad \phi = 1 \quad \text{at} \quad \eta = 0,$$

$$f' \to 1, \quad \theta = \phi \to 0 \quad \text{as} \quad \eta \to \infty$$

where

$$M = \frac{\sigma B_0^2 U x}{\rho_f}, \quad \lambda = \frac{a}{b}, \quad \Pr = \frac{\nu_f}{\alpha_f}, \quad Nb = \frac{\tau D_B\left(C_w - C_\infty\right)}{\nu_f},$$

$$Nt = \frac{\tau D_T\left(T_w - T_\infty\right)}{T_\infty \nu_f}, \quad \beta = \frac{2m}{1+m}, \quad \text{Le} = \frac{\nu}{D_B}, \quad K^* = \frac{\nu x}{K\left(1+m\right)U},$$

are the magnetic parameter, velocity ratio, Prandtl number, Brownian motion, thermophoresis, pressure gradient, Lewis number and porosity respectively. The important physical quantities of this problem are the skin friction coefficient C_f, the local Nusselt number Nu and the local Sherwood number Sh which are proportional to the rate of velocity, rate of temperature and rate of nanoparticle volume fraction respectively.

3. Results and discussion

Numerical calculations are carried out by taking $M = 0.1$, $Nb = Nt = 0.1$, $K^* = 0.2 = \beta$, Pr = 6.0, Le = 5.0 and $\lambda = 0.3$. The results are shown in a graphical and also in tabular form. The results are compared with these of others authors and found to be in good agreement as shown in Tab.1 and Tab.2. This ensures the validity and accuracy of the present work. Also, various values of the skin friction, rate of heat transfer and rate of nanoparticle concentration are presented in Tab.3 for different values of λ, Nb, Nt, Le and M by taking $\beta = 1.0$ and $K^* = 0.2$.

Now Fig.1 – Fig.6 we see that the velocity profiles increases with increasing values of the pressure gradient parameter β for different situations. Therefore the velocity profile is increased in the absence of the magnetic field and static wedge and it is seen that the separation occurred at $\beta = -0.198$. But in case of a moving wedge ($M = 0$) the separation occurred at $\beta = -0.35$ but in the presence of the magnetic field ($\lambda = 0.0$) the separation occurred at $\beta = -0.42$. From Fig.4, Fig.5 and Fig.6 it is observed that the velocity profile decreases for increasing values of the porosity parameter, velocity ratio parameter and increases for magnetic parameter. From Fig.7 – Fig.11 it is seen that the temperature increases for the thermophoresis parameter Nt but reverse results arises for other entering parameters. The increment of the Prandtl number results in major effects on the temperature profile which are depicted in Fig.11. The thermal boundary layer thickness reduces with the Prandtl number and it happens due to a decrease of thermal diffusivity for the increment of the Prandtl number.

Fig.12 – Fig.16 depicts for nanoparticle concentration and from these we observed that the nanoparticle concentrations for thermophoresis parameter Nt but reverse results arises for the remaining parameters. The thermophoresis parameter Nt is a key parameter for analyzing the temperature distributions and nanoparticles volume fraction in nanofluid flow. The effect of thermophoresis parameter Nt on the temperature profile and the nanoparticle concentration are presented in Fig.7 and Fig.14. Therefore increase of Nt, the temperature profiles and nanoparticle concentration of the fluid increases. Increase in Nt causes an increment in the thermophoresis force which tends to move nanoparticles from hot to cold areas and consequently it increases the magnitude of temperature profiles and nanoparticle concentration profiles. Ultimately, the thickness of the nanoparticle concentration boundary layer becomes significantly large for a slightly increased value of the thermophoresis parameter. The large values of Le, the nanoparticle concentration significantly decreases and also the nanoparticle concentration boundary layer thickness reduces. But, for a smaller value of Le the overshoot is found near the wedge The Brownian diffusion effect becomes nominal for larger values of the Lewis number for which the nanoparticle concentration boundary layer thickness decreases.

4. Conclusions

According to Falkner – Skan model the velocity profile is exist for $-0.198 \le \beta \le 2$ which is show in Fig.1. Now in the presence of stretching ratio λ, we have seen that the solution exists for $-0.35 < \beta \le 2$ as shown in Fig.2. Again from Fig.3 it is observed that the solution exists for $-0.42 < \beta \le 2$ in the presence of magnetic field but absence of stretching ration. From Fig.4 it is observed that the velocity profile has a point of inflexion for $K^* = 0.5$. Therefore the solution exists for $K^* < 0.5$. The velocity increases for increasing values of magnetic parameter because the free stream velocity U is dominating here but the velocity decreases for stretching ratio parameter as a result the thickness of momentum boundary layer increases.

Again, from Fig.10 – Fig.13, the separation is observed for $Nb = 0.2$, $Nt = 0.14$, $Le = 1.0$ and $\lambda = 0.1$ in the nanoparticle concentration. So for the validity of the present result, the mentioned values should be omitted.

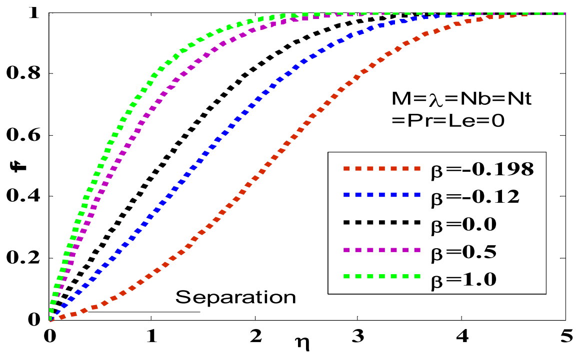

Fig.1. Velocity profile for various values of β.

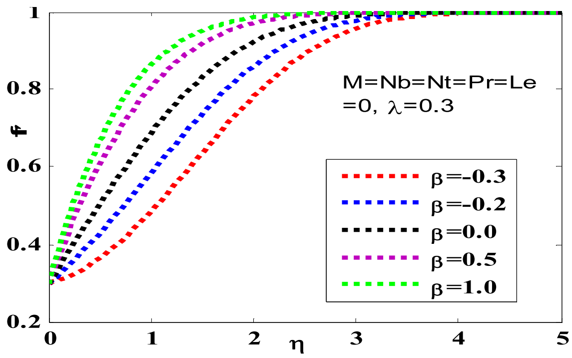

Fig.2. Velocity profile for various values of β.

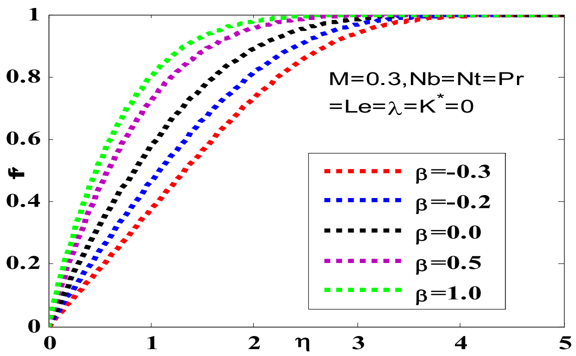

Fig.3. Velocity profile for various values of β.

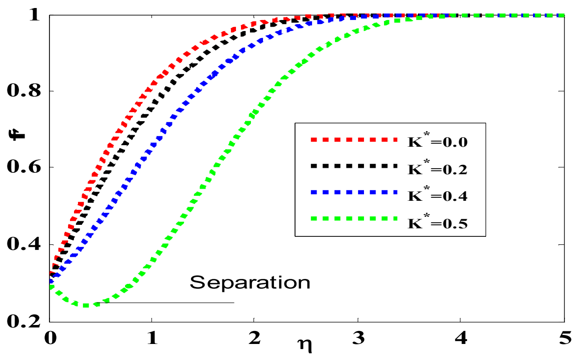

Fig.4. Velocity profile for various values of K^*.

Table 1. Comparison of skin friction $\left[f''(0)\right]$ for different values of β, when $M = Pr = Nb = Nt = K^* = Le = \lambda = 0$.

	Rajagopal *et al.* [23]	White [24]	Mohammadi *et al.* [25]	Khan and Pop [26]	Present results
β	$f''(0)$	$f''(0)$	$f''(0)$	$f''(0)$	$f''(0)$
-0.12	-	-	0.281772	-	0.28211
-0.15	-	-	0.216335	-	0.217153
-0.18	-	-	0.128637	-	0.13138
0.0		0.4696	0.469589	0.4696	0.46964
0.2	0.686708	-	-	-	0.686690
1/6	-	0.6550	-	0.6550	0.6550
1/3	-	0.8021	-	0.8021	0.80212
0.5	0.927680	0.9277	0.927601	0.9277	0.92768
2/3	-	1.0389	-	1.0389	1.0389
1.0	1.232585	1.2326	1.232587	1.2326	1.232587
1.6	1.521514	-	-	-	1.52151399

Table 2. Comparison of local heat transfer rate for different values of Pr when $\beta = 1.0$ and $M = Nb = Nt = K^* = \lambda = \mathrm{Le} = 0$.

Pr	Mohammadi *et al.* [25] $-\theta'(0)$	Present results $-\theta'(0)$
0.72	*0.501508*	*0.50147*
6.0	*1.107140*	*1.1146*
10.0	*1.317881*	*1.3387*

Table 3. Values of skin friction $\left[f''(0)\right]$, local Nusselt number $-\theta'(0)$ and local Sherwood number $-\varphi'(0)$ for different values of A, Nb, Nt, λ when Pr = *1.0* and Le = *2.0*.

λ	*Nb*	*Nt*	Le	*A*	$f''(0)$	$-\theta'(0)$	$-\varphi'(0)$
0.3	*0.4*	*0.4*	*2.0*	*0.2*	*0.928238*	*0.45802*	*0.70207*
0.4	*0.4*	*0.4*	*2.0*	*0.2*	*0.806794*	*0.51330*	*0.78802*
0.6	*0.4*	*0.4*	*2.0*	*0.2*	*0.681510*	*0.59973*	*0.92169*
0.3	***0.6***	*0.4*	*2.0*	*0.2*	-	*0.45802*	*0.75069*
0.3	***0.8***	*0.4*	*2.0*	*0.2*	-	*0.45802*	*0.78123*
0.3	*0.4*	***0.2***	*2.0*	*0.2*	-	*0.50255*	*0.70722*
0.3	*0.4*	***0.6***	*2.0*	*0.2*	-	*0.42096*	*0.74463*
0.3	*0.4*	*0.4*	***1.0***	*0.2*	--	-	*0.45265*
03	*0.4*	*0.4*	***3.0***	*0.2*	-	-	*0.87037*
0.3	*0.4*	*0.4*	*2.0*	***0.3***	*0.961749*	-	-
0.3	*0.4*	*0.4*	*2.0*	***0.4***	*0.994498*	-	-

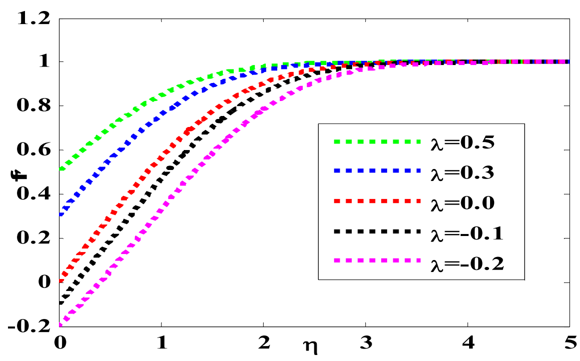

Fig.5. Velocity profile for various values of λ.

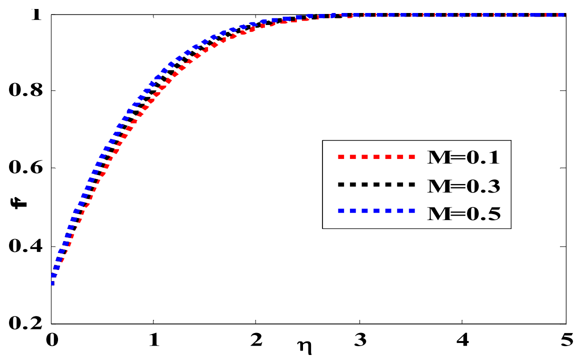

Fig.6. Velocity profile for various values of *M*.

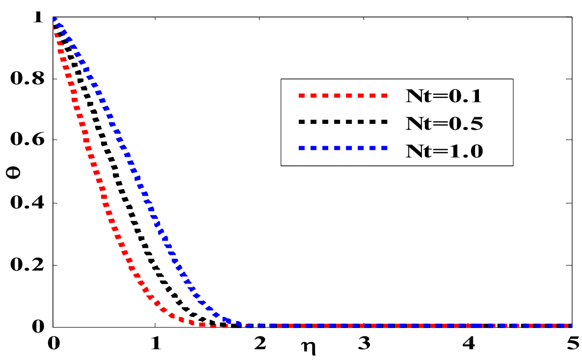

Fig.7. Temperature profile for various values of *Nt*.

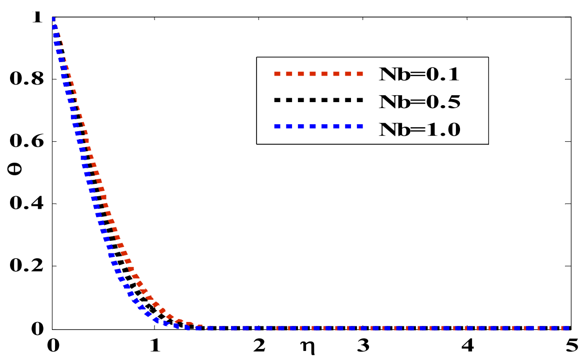

Fig.8. Temperature profile for several values of *Nb*.

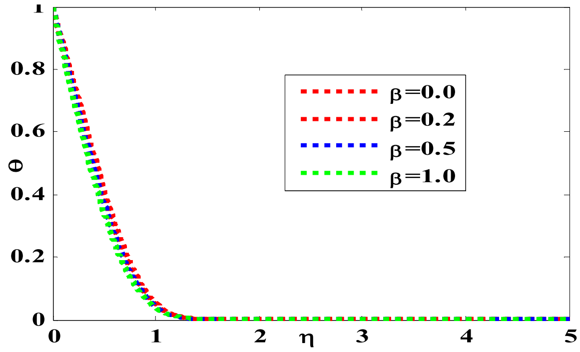

Fig.9. Temperature profile for various values of β.

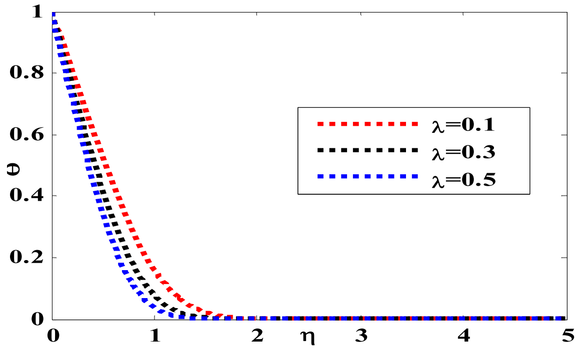

Fig.10. Temperature profile for various values of λ.

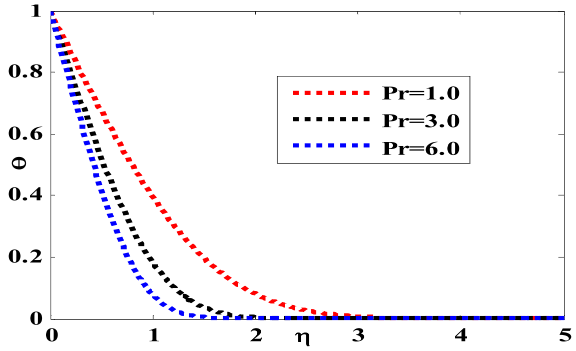

Fig.11. Temperature profile for several values of Pr.

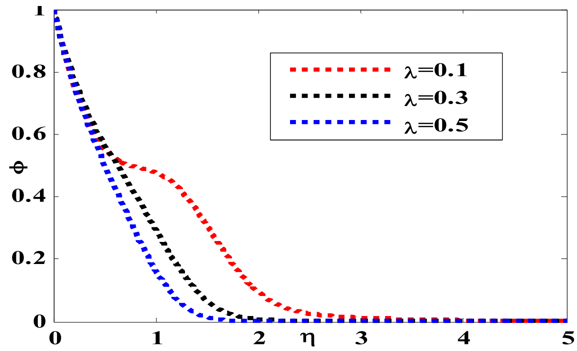

Fig.12. Nanoparticle concentration for λ

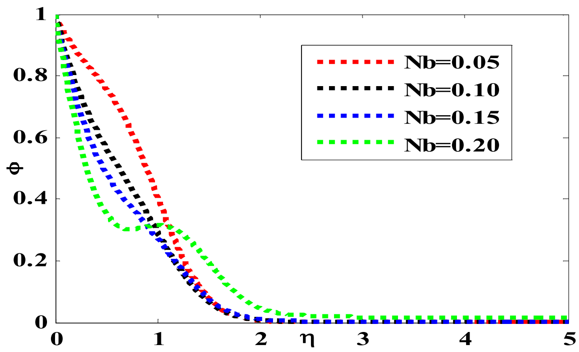

Fig.13. Nanoparticle concentration for various values of *Nb*.

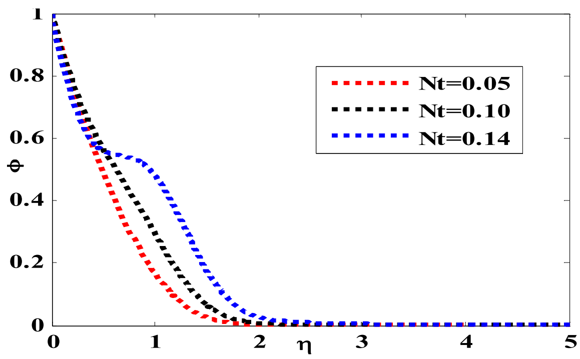

Fig.14. Nanoparticle concentration for various values of *Nt*.

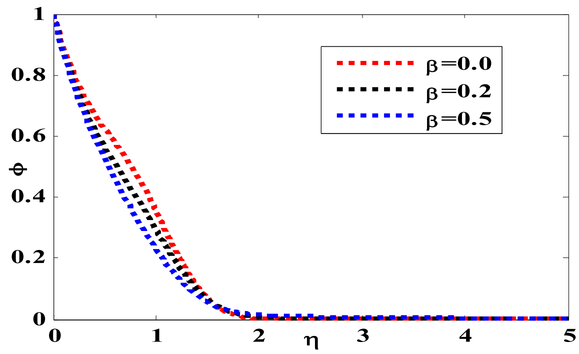

Fig.15. Nanoparticle concentration for several values of β.

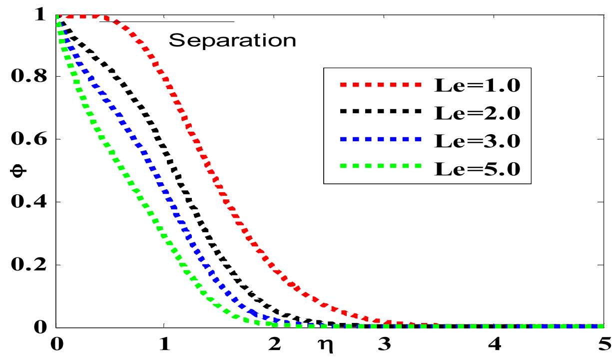

Fig.16. Nanoparticle concentration for various values of Le.

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