

Boundary Layer Resistance and Temperature Distribution on Still and Flapping Leaves

I. THEORY AND LABORATORY EXPERIMENTS¹

Received for publication March 29, 1971

JEAN-YVES PARLANGE,² PAUL E. WAGGONER, AND GARY H. HEICHEL
The Connecticut Agricultural Experiment Station, New Haven, Connecticut 06504

ABSTRACT

If the evaporation is uniform on a flat exposed leaf, forced convection will also be nearly uniform, and the leaf temperature will vary with the square root of the distance from the leading edge. Then the resistance expressed in terms of the proper, *i.e.*, average, temperature has the same value as the resistance of a leaf at uniform temperature. Compared to a steady laminar flow, the turbulence of a realistic wind decreases the resistance by a constant factor of about 2.5. The same constant factor was observed whether the leaf was flapping or not, when the wind velocity was not too low.

The boundary layer resistance of the air around a leaf determines how fast the energy gained from radiation will be lost by forced convection and evaporation. Combined with the stomatal resistance (2, 16), the boundary layer resistance determines the partitioning of the loss caused by evaporation and forced convection. Recently, the suitability of the conventional boundary layer resistance has been questioned (6, 7, 12), and here we examine to what extent a nonuniform temperature over the leaf and turbulence or flapping of the leaf could invalidate the conventional calculation of the boundary layer resistance.

Pohlhausen (17) has derived an equation for the heat exchange between a flat plate at uniform temperature and the bulk air when the flow over the plate is steady and laminar. His result for the resistance to heat exchange at a point x , measured from the leading edge of the leaf, is

$$r_T = (\text{Pr}^{2/3}/0.332)(x/U\nu)^{1/2} \quad (1)$$

where the boundary layer resistance r_T is defined by

$$r_T = \rho C_p \theta / H \quad (2)$$

Here θ is the difference between the temperatures of the plate and bulk air and is assumed constant in the present case. U is the wind velocity; ν is the kinematic viscosity; Pr is the Prandtl number; ρ and C_p are the air density and specific heat at constant pressure, respectively; and H is the heat removed by forced convection per unit of time and area.

¹ This work was supported by Connecticut and McIntire-Stennis funds.

² Permanent address: Engineering and Applied Science, Yale University, New Haven, Conn.

Since most observers have employed a single resistance for the entire leaf, we now derive an average resistance, \bar{r}_T , by

$$\bar{r}_T = \rho C_p \theta / \bar{H} \quad (3)$$

where \bar{H} is the average over the leaf of the quantity H , or, from equation 1,

$$\bar{r}_T = (\text{Pr}^{2/3}/[2 \times 0.332])(l/U\nu)^{1/2} \quad (4)$$

The average width of the leaf, l , is defined precisely in the "Appendix." We may notice that the resistance to the diffusion of water vapor through the boundary layer is commonly obtained by multiplying \bar{r}_T by the two-thirds power of the Lewis number, which is about 1.3, or sometimes is simply assumed to be equal to \bar{r}_T , with a small error.

There are obvious unrealities in applying equation 4 to a real crop. Real leaves are not flat, but the effect of curvature is slight (15). On the other hand, the effect upon r_T of nonuniform temperature, extreme turbulence, and flapping or tilting of the leaves cannot be easily discarded. We now examine whether these latter unrealities could cause \bar{r}_T to be half of that given in equation 4 as Monteith's formula (12) suggests or even much less as others (6, 7) suggest. These results (6, 7, 12) can be represented by a factor β ,

$$\bar{r}_T = (\text{Pr}^{2/3}/[2 \times 0.332])(l/U\nu)^{1/2}/\beta \quad (5)$$

where Monteith's results imply that β is 2.5 and the others (6, 7) imply that β is 10 or more. We shall examine the consequence of nonuniform temperature theoretically and show that it has little effect upon \bar{r}_T . This theoretical analysis also predicts the variation of temperature over the leaf, which in turn can be used to measure the effect of turbulence and flapping. The effect of extreme turbulence will be measured by observing both the rate of cooling of a leaf after a change in radiation and the temperature variation over a leaf. Finally, the effect of flapping upon the cooling and temperature variation is observed.

Theoretical Resistance of a Leaf with Nonuniform Temperature. Since convection, H , is the difference between radiation and evaporation, H will be nearly constant over an exposed flat leaf if the variation in evaporation is small. This will certainly be true if evaporation is slight relative to radiation, but it may be approximately true in other cases. Therefore, an assumption of uniform H is an alternative to, and perhaps a more realistic one than, the assumption of uniform θ that underlies the Pohlhausen equation, equation 1.

For uniform H , one can derive (15) the temperature variation along the leaf:

$$0.453 \beta k \theta \text{Pr}^{1/2} = H(x\nu/U)^{1/2} \quad (6)$$

where k is the conductivity of the air. The factor, β , represents the

effect of turbulence and rises from 1.0 for laminar steady winds (9). From the definition of r_T in equation 2, equation 6 yields

$$r_T = (Pr^{2/3}/0.453 \beta)(x/U\nu)^{1/2} \quad (7)$$

Equation 6 shows that the temperature θ varies as $x^{1/2}$, a prediction that we shall test when we come to experiments. We notice that equations 1 and 7 are of the same form, except for a numerical coefficient, since the factor β could have been introduced into equation 1 as well. We now define an average resistance, \bar{r}_T , in terms of the average temperature of the leaf, $\bar{\theta}$, or

$$\bar{r}_T = \rho C_p \bar{\theta} / H \quad (8)$$

Equations 3 and 8 are slightly different since the heat flux, H , rather than the temperature, θ , is now constant on the leaf. Integration of equation 6 over the average width, \bar{l} , of the leaf together with equation 8 yields

$$\bar{r}_T = (2 Pr^{2/3}/[3 \times 0.453 \times \beta])(\bar{l}/U\nu)^{1/2} \quad (9)$$

By coincidence 2×0.332 is about $3 \times 0.453/2$, so that equations 5 and 9 are numerically identical. Consequently the same numerical answer is obtained when either the temperature or the heat flux is uniform. This result is not indicated by equations 6 and 7 of Reference 13 and equations 8 and 9 of Reference 14. They replaced, mistakenly, the factor $\frac{2}{3}$ in our equation 9 by $\frac{1}{2}$.

Monteith's (12) survey of observations was fitted by

$$r_T = 1.3 (\bar{l}/U)^{1/2} \quad (10)$$

where \bar{l} is in centimeters, U is in centimeters, sec, and \bar{r}_T pertains to a single surface. In air, equation 9 becomes

$$\bar{r}_T = (3.2/\beta)(\bar{l}/U)^{1/2} \quad (11)$$

Thus equation 11 can be reconciled with the observations collected by Monteith if β has a constant value of 2.5. Thus in the varying conditions surveyed by Monteith, the heat transfer was about 2.5 times larger than the heat transfer that would be obtained for a steady flow at speed U .

We must now examine whether it is reasonable to expect the turbulence of the wind to be responsible for a value of $\beta \approx 2.5$. Natural winds are always turbulent with eddies often as large as the leaf. If only a few eddies are smaller than the boundary layer thickness, the boundary layer is unsteady but has a laminar structure (17) as implied by the analytic form of equation 11. Due to turbulence the main stream velocity varies both in amplitude and direction. It has been shown by careful experimental (1, 9, 17) and some theoretical studies (8, 17) that indeed exchanges are increased when the turbulence and unsteadiness of the main stream increase. These results apply only to very slight turbulence and, hence, they cannot be extrapolated safely to the great turbulence

encountered in natural winds. Nevertheless, they show qualitatively if not quantitatively that a factor, $\beta \approx 2.5$, is perfectly acceptable but not the factor 10 as recently suggested (6, 7). A most important consequence of Monteith's empirical equation is the implication that the level of the main stream turbulence does not enter the equation explicitly. This is different from what happens for a low level of turbulence (1, 9, 17). Obviously, concluding that atmospheric turbulence is uniform in space and constant in time would be incorrect. Rather, the implication is that at some level of turbulence saturation takes place, and the heat transfer reaches a maximum. Possibly this event may be connected with the separation of the boundary layer when the angle of attack of the leaf reaches a critical value. If saturation takes place, it makes a very high value of β (e.g., 10) even more improbable.

If no saturation occurs, a fluttering leaf must have a lower resistance, i.e., a larger β , than a leaf held steady in the wind. For this reason we experimented with both fixed and fluttering leaves.

This remark does not imply that flutter and turbulence are equivalent. Rather, we suggest the possibility that, once β has reached some maximum, it is impossible to increase its value further, either through flutter or an increase in turbulence. Notice also that if β has a value less than the maximum, for instance, if the wind had little turbulence, then flutter should increase the value of β until the maximal value is reached.

For greater accuracy, experiments were first conducted in a laboratory under controlled conditions. Such experiments always have the risk of being unrealistic if outdoor conditions are improperly modeled. Consequently in the next paper we repeat these experiments in the field. Even then, the laboratory experiments are invaluable as a guide in the choice of the proper experimental methods.

MATERIALS AND METHODS

Leaf Ventilation. It is particularly important to obtain as good a simulation of natural wind as possible. A fan was placed in front of the leaf. Three velocities were used: 60, 120, and 160 cm, sec. A hot wire anemometer, type 641N thermal anemometer manufactured by Wilhelm Lambrecht K. G., Göttingen, Germany, showed that the amplitude of the fluctuations in velocity was about 40 cm/sec in all cases. The needle of the anemometer paused about 50 to 80 times/min. The size of the eddies varied between 1 cm (smallest opening in the screen in front of the fan) to 50 cm (size of the fan). These conditions represented fairly well the natural wind conditions that we later measured outdoors.

To avoid the use of "equivalent" leaf (see "Appendix") as an additional source of error, a 20- × 20-cm square of tobacco leaf (*Nicotiana tabacum* L.) was stretched on a wire frame (Fig. 1). The frame was about 2.75 mm in diameter, which is of the order of the boundary layer thickness over the leaf. Consequently it must be expected that the leading edge of the frame has some effect in obstructing the air flow, hence in increasing the leaf temperature. For evaluation of this effect, a less sturdy frame was constructed, where the leading edge was a 0.1-mm wire. This leading edge had no effect on heat exchange since it was thinner than the leaf. As expected, measurements indicated that, in this case, the boundary layer began at the front edge of the leaf, i.e., the front edge of the leaf was at the air temperature. On the other hand, with the thicker wire, the boundary layer seems to begin at the wire itself, rather than at the front edge of the leaf. In that case the experimental results are interpreted by taking the effective origin of the coordinate x at the wire (which was typically placed 1 cm in front of the leaf). The thicker frame was used extensively as it held the leaf more firmly, making the experiments easily reproducible, with less scatter in the results. Flutter was produced by oscillating the leading edge of the frame at a frequency of 4 cycles/sec and an amplitude of 12 cm. The resulting motion

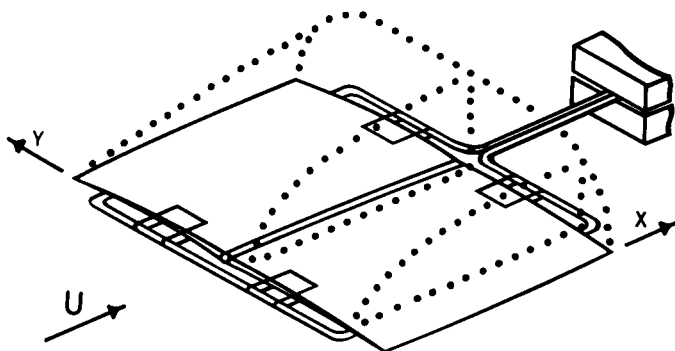


FIG. 1. Leaf on metallic frame and sketch of temperature distribution. The leading edge could be oscillated while the bracket at the rear was stationary.

Table I. Measured Value of $(d\theta/dt)_0$ When the Lights Are First Turned Off

The cooling rates, in C/sec, are measured at five points with $U \approx 120$ cm/sec and $\bar{\theta} \approx 7$ C.

No flutter	0.44	0.47	0.48	0.47	0.44
With flutter	0.45	0.46	0.49	0.46	0.44

is clearly more rapid than under natural conditions, providing an estimate of the maximal effect of flutter.

Radiation. The leaf was irradiated by incandescent lamps submerged in water and delivering a total irradiance of 1.40 calories $\text{cm}^{-2} \text{min}^{-1}$. For determination of the absorptivity, half of the leaf was painted with optical black. The ratio of the warming of green and black portions gives the absorptivity. The average of measurements with five different tobacco leaves gave an absorptivity of 0.44 ± 0.02 . The same experiment was performed in sunlight, where the absorptivity was 0.68 ± 0.03 as expected (4). The low absorptivity with the lamps is simply due to the fact that most of the radiation (0.9 calorie $\text{cm}^{-2} \text{min}^{-1}$) is at wavelengths longer than 0.7 , where the absorptivity of leaves is low (4). The absorbed energy per square centimeter of leaf surface is then $1.4 \times 0.44/2 = 0.31$ calorie $\text{cm}^{-2} \text{min}^{-1}$.

Two other energy fluxes must also be taken into account: radiation from the leaf to the walls and evaporation from the leaf. In our experiments the leaf is 5 to 10 C warmer than the surroundings. A warming of the leaf of 7 C causes a loss of 0.061 calorie $\text{cm}^{-2} \text{min}^{-1}$, which is small but significant.

Experiments were performed about 1 hr after the leaf was cut to permit the stomata to close. Porometric (18) observations showed that the change in stomatal resistance was insignificant during the experiments. Weighing the leaf before and after the

experiments showed that evaporation was always close to 0.01 calorie $\text{cm}^{-2} \text{min}^{-1}$.

The difference between the gain of radiation and the losses of radiation and evaporation when $\bar{\theta} \approx 7$ C must be carried out by convection and is $H = 0.237 \pm 0.015$ calorie $\text{cm}^{-2} \text{min}^{-1}$. This amount will vary slightly when $\bar{\theta}$ changes, due to the radiation loss from the leaf. Only average corrections for evaporation and radiation were made although these corrections are a function of θ and hence of position on the leaf. Nevertheless this variation is only a correction on a correction, which is much less than the error introduced by the uncertainty of measurement, and consequently can be safely ignored.

The determination of H in the foregoing is rather indirect and depends on the estimates of many terms. It is interesting that a direct measurement of H is possible (10). With the thermocouple placed in any position, the lamps are switched off, and the rate of cooling is recorded. The energy equation becomes

$$H + sm(d\theta/dt)_0 = 0 \quad (12)$$

where $(d\theta/dt)_0$ is the rate of cooling when the lamps are first turned off, s is the specific heat of the leaf, and m is the mass of leaf per unit area. The value of $2m$ varied from 0.0185 to 0.0195 g/cm^2 in a series of eight measurements and s was about 0.88 calorie $\text{g}^{-1} \text{C}^{-1}$ (10). Table I gives the cooling rates at five points of a leaf with an average temperature of about 7 C above the air temperature when the lamps were on. The average of the five measurements without flapping in Table I gives $H = 0.231 \pm 0.012$ calorie $\text{cm}^{-2} \text{min}^{-1}$, in excellent agreement with the result obtained above from the measurement of leaf absorptivity. The experiment was repeated with a flapping leaf. The results, also reported in Table I, indicate that the amount of heat to be carried out by convection remains the same, whether the leaf flaps or not, as should be expected. Figure 2 indicates the actual recording of a measurement at $x \approx 10.5$ cm, showing the temperature

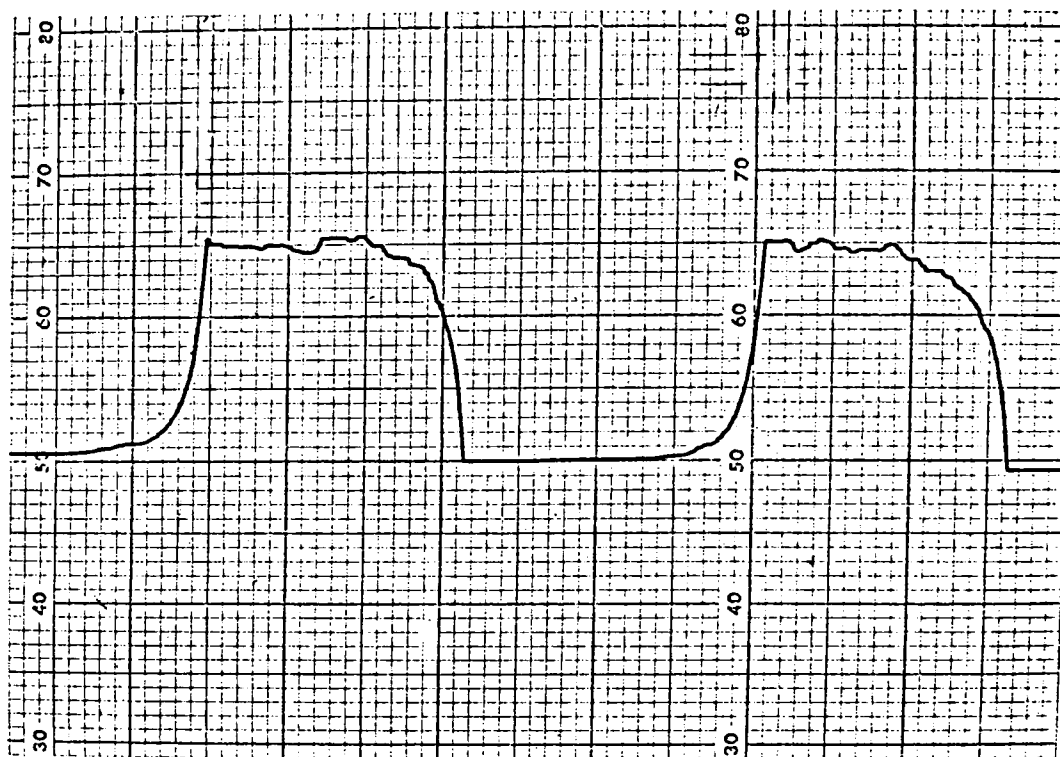


FIG. 2. Recording of temperature (vertical scale: one graduation equals 0.5 C) as a function of time (horizontal scale: one graduation equals 12 sec). Starting from the right the recordings show warming and cooling as the lamps are turned on and off. The first recording on the right is for a still leaf, the second is for a flapping leaf.

as a function of time with and without flutter. The lamps are first turned on and θ rises until equilibrium is reached for $\theta \approx 7.5$ C. The lamps are then turned off and the temperature shows an exponential decay until $\theta \sim 0$. The results are identical with and without flutter.

Temperature. Three different techniques were used to measure temperature.

Thermocouples. The main difficulty with thermocouples is to maintain a proper contact between the leaf and the junction. Soft leaves, e.g., tobacco, permitted the best contacts while harder leaves, e.g., corn, permitted only poor contact. It is also necessary to have junctions sufficiently smaller than the boundary layer thickness. This is made easier when the junctions are more or less pressed into the soft epidermis of tobacco. To check the effect of the thermocouple size, 0.25- and 0.10-mm-diameter copper-constantan thermocouples were compared. As expected, the larger thermocouple indicated temperature slightly cooler than the smaller thermocouple. The average difference of 0.2 C was much smaller than the scatter in the data, as will be seen later. Consequently, observations with both thermocouples were used without corrections.

Radiometers. They avoid the difficulty of contact with the leaf although they sense only an average over an area. Observations with a Stoll-Hardy radiometer were compared with those with the thermocouple. Even when thermocouples indicated fully 8 C temperature variation along the leaf, the radiometer showed only a 2 C variation. On the other hand, the radiometer always indicated the correct average leaf temperature. Since radiometers have been used to measure leaf temperatures by many researchers, this may possibly be a reason why large temperature gradients are rarely observed. The difficulty with radiometers is that if placed too close to the leaf they interfere with the flow field and if placed sufficiently far they then measure the average temperature of a large portion of the leaf. Consequently we only used the radiometer to obtain easily and quickly the average leaf temperature, to determine, for instance, the absorptivity of the leaf.

Liquid Crystals. A third alternative must be devised for eventual use in the field. The device must be easy to use, show spatial temperature differences, and be intimately in contact with the leaf. For this purpose we employed liquid crystals, with color that indicates the temperature with great precision. The method demands some care and will be described in detail in the next paper, since liquid crystals were particularly useful in the field. At present it is sufficient to say that observations in the laboratory with liquid crystals were consistent with those of the thermocouples.

RESULTS

Observed Resistance of a Still Leaf in Extremely Turbulent Air. From the point of view of applications, it is most important to know the average resistance to heat exchange. It is then necessary to estimate the value of β in either equation 9 or equation 5. Equation 6 provides us with the means to measure β . We take $k = 5.8 \times 10^{-8}$ calories $\text{cm}^{-1} \text{sec}^{-1} \text{C}^{-1}$, $\text{Pr} = 0.72$, and $\nu = 0.16$ cm^2/sec . The value of H is effectively fixed by the lamps, but the two parameters x and U can be changed at will, providing a variety of different conditions to estimate β from the measurement of θ .

The expected temperature distribution over a 20- × 20-cm tobacco leaf is sketched in Figure 1. The temperature, θ , is zero near the leading edge and increases with x . Close enough to the two sides the temperature drops rapidly; this result is not included in equation 6 since the latter ignores the three-dimensional effects of the sides. To estimate β and check the validity of equation 6 precisely, we recorded temperatures at various distances from the leading edge and on four lines roughly parallel at approximately 5 and 8 cm from each side.

Figure 3 shows the results obtained at $U \approx 160$ cm/sec for a still leaf, with $H \approx 0.255$ calorie $\text{cm}^{-2} \text{min}^{-1}$. The solid line corresponds to equation 6 with $\beta = 2.7$. The agreement with the experimental results for this value of β is reasonable. The experimental points show some scatter. The data were gathered over a 6-month period with different leaves and some scatter is unavoidable. The leaves are never identical nor absolutely flat; ribs and other geometrical characteristics have an obvious influence on heat transfer which affects the value of θ at a given point. Notice also that the temperature near the trailing edge of the leaf seems to be systematically lower than predicted by equation 6 with $\beta = 2.7$. This is most likely due to the finite size of the leaf: the effects of the sides and the end of the leaf are felt more strongly at the trailing edge of the leaf. It would be meaningless to correct equation 6 empirically to take this effect into account, since the correction would not be generally valid and could not be used anyway for real leaves outdoors. Furthermore, this flattening out of the temperature near the trailing edge does not affect the value of β significantly, compared to the scatter already present, and only that value of β is of primary importance. Altogether the data points give $\beta = 2.7 \pm 0.4$.

Higher velocities should not affect the results. On the other hand, for low values of U , the fluctuations in velocity become relatively more important. In particular the fluctuations in the direction of the wind become larger. As a result the effects of the sides and the end of the leaf should be more pronounced and as a consequence the temperature distribution on the leaf should be flatter. To investigate this effect, we decreased the velocity first to 120 cm/sec and then to 60 cm/sec.

Results for 120 cm/sec with a still leaf and $H \approx 0.237$ calorie $\text{cm}^{-2} \text{min}^{-1}$ are indicated in Figure 4. The solid line corresponds to equation 6 with $\beta = 2.4$. Everything mentioned for the case $U \approx 160$ cm/sec still applies when $U \approx 120$ cm/sec. For that speed there is still no excessive flattening of the temperature distribution. Consideration of all experimental points gives $\beta \approx 2.4 \pm 0.4$ in agreement with the previous case.

The last case is for $U \approx 60$ cm/sec and $H \approx 0.216$ calorie $\text{cm}^{-2} \text{min}^{-1}$. For this case the velocity fluctuations of 40 cm/sec are comparable to U itself. The temperature distribution in Figure 5 is quite flat. Obviously it would be meaningless to try to fit equation 6 to such results. Nevertheless one can still compute an over-all β from equations 8 and 9 or from equations 3 and 5. The result, taking $\bar{\theta} \approx 10$ C, is $\beta = 2.6 \pm 0.4$.

In the present experiments the average wind direction is constant and the flow is unidirectional. In the field, however, the average wind direction often varies and even complete reversals are occasionally observed. If such changes in direction occur rapidly, temperature gradients are likely to be much less important over most of the leaf than for a unidirectional wind. This in fact is what happened in our last experiment (Fig. 5). The fluctuations in velocity due to the eddies are comparable to the average wind velocity. Consequently the wind direction is also changing greatly and rapidly. In that case three-dimensional effects become important, and except very near the edges the temperature gradients are less important, as shown in Figure 5. Thus we expect that changes in wind direction will not greatly affect the value of β in the field since β remains 2.6 in our last laboratory experiments.

As the free stream velocity decreases, natural convection becomes progressively more important. This effect can be easily estimated with known empirical relations for the natural convection from square horizontal plates (11). The result is that, even with an average θ of 10 C as in the last experiment, the heat carried by natural convection is about 0.02 calorie $\text{cm}^{-2} \text{min}^{-1}$. Since H was more than 10 times this value, the heat is indeed mostly carried by forced convection even for velocities as slow as 60 cm/sec. The error which is then introduced in the

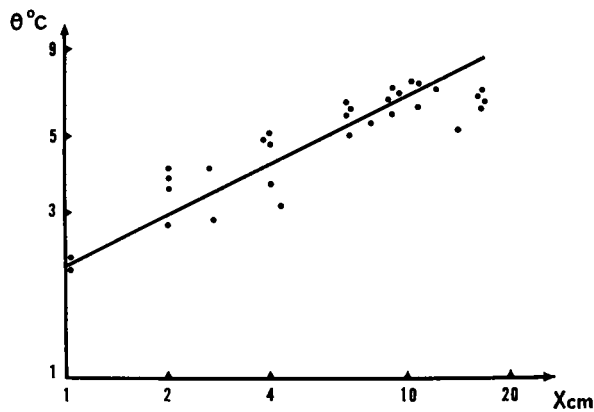


FIG. 3. Temperature θ C as a function of distance X cm from the leading edge for $U = 160$ cm/sec and $H = 0.255$ calorie $\text{cm}^{-2} \text{min}^{-1}$. The solid line corresponds to equation 6 with $\beta = 2.7$.

value of β by neglecting natural convection is less than the error already introduced by the uncertainty in H and the scatter of the results.

All previous results are in essential agreement with Monteith's formula. We are now going to examine whether flutter has any effect on the temperature distribution and the value of β .

Observed Resistance of a Flapping Leaf in Extremely Turbulent Air. It is clear that, at low enough U , flutter must have an effect, *e.g.*, when $U = 0$ flutter must cool the leaf. Of course, in the field, flutter is caused by the wind itself so that the velocity of flapping could not exceed the velocity U and will usually be much less. The average speed associated with our oscillating frame is about 100 cm/sec (4 cycles/sec and 12-cm amplitude). Consequently it would be unreal to use such flutter when $U \approx 60$ cm/sec. The effect for 160 cm/sec must also be less than for 120 cm/sec. In Figure 4 the results of flutter on temperature distribution are represented when $U \approx 120$ cm/sec. Apparently flutter has no effect, in the sense that the experimental points are mixed with those obtained for a still leaf. (We already mentioned that H was unaffected by flutter.)

On the basis of our laboratory experiments several conclusions can be made. For a highly turbulent wind and if the average wind velocity is not too low, equation 6 is valid with $\beta \approx 2.5 \pm 0.4$. When the wind is too low the temperature gradient along the leaf is small, but the average resistance for heat transfer is still associated with a β of the same order. Whether these conclusions still apply in the field will be checked in the next paper. We also expect that the mild flutter of leaves in natural wind has no effect on heat transfer, since it had no effect in the laboratory under rather extreme conditions.

Deviations of Resistances from Conventional Estimates. Three sorts of resistances have been observed: large resistances as predicted by equation 4 for laminar steady flow, those that agree with Monteith and us, and very small resistances.

The large resistances have usually been observed in wind tunnel experiments with low turbulence although Gates (3) has also measured large resistances, apparently in the field. Notice also that in wind tunnel experiments the leaf is sometimes replaced by a metallic model (13, 14). Compared to a leaf, the very high heat conduction of the metal model tends to make its temperature uniform.

We must also suggest why very small resistances have twice been calculated (6, 7). Measuring leaf temperatures is difficult and a large relative error is likely when θ is small, *e.g.*, less than 1 C (5, 7). In this connection we noticed that when Kanemasu *et al.* (7) observed substantial θ of 3.6 and 4 C, their \bar{r}_T was only seven to eight times smaller than the resistance for $\beta = 1$, *i.e.*,

only three times smaller than the resistance obtained from equation 10.

Since observers in the field have often assumed that leaves have a nearly uniform temperature, there is always a possibility, particularly when the measured temperature difference is very small, that the result was effectively obtained near the edge of the leaf. In that case the measured θ would not be representative of $\bar{\theta}$ and would yield an abnormally low \bar{r}_T .

The remarks above apply also to those observers whose results agree with Monteith's equation. Through chance or insight they measured the "right" temperature even when very small (5). These observations require little further comment except to repeat that a β around 2.5 is required for the turbulence of the main flow, and the same factor applies whether the leaf is still or fluttering, as long as the wind velocity is not too low.

The possibility that \bar{r}_T outdoors may somehow be different from the one we observed is examined in the next paper.

APPENDIX

The average width, \bar{l} , is defined as the width of a rectangle that is equivalent to a real leaf. The real leaf and the equivalent rectangle both have the same longer dimension or length, L , and the same total heat flux carried out by forced convection.

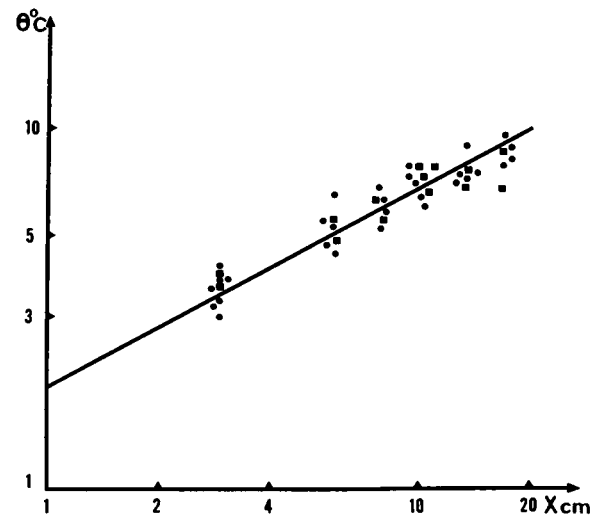


FIG. 4. Temperature θ C as a function of distance X cm from the leading edge for $U = 120$ cm/sec and $H = 0.237$ calorie $\text{cm}^{-2} \text{min}^{-1}$. The solid line corresponds to equation 6 with $\beta = 2.4$. Dots refer to still leaves and squares to flapping leaves.

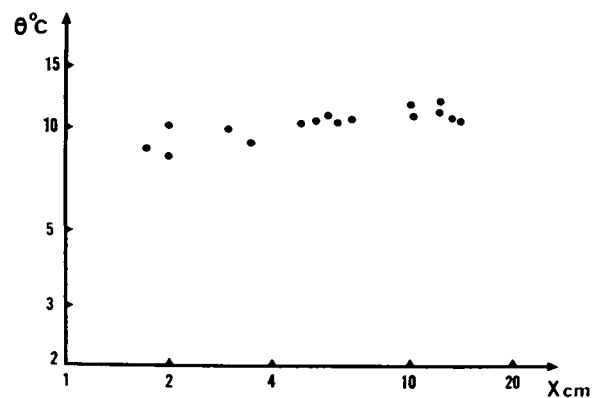


FIG. 5. Temperature θ C as a function of distance X cm from the leading edge for $U = 60$ cm/sec and $H = 0.216$ calorie $\text{cm}^{-2} \text{min}^{-1}$.

We denote by (x, y) the coordinates in the plane of the leaf (Fig. 1), y being in the direction of the longest leaf dimension (L) and x being normal to it. The width of the leaf, $w(y)$, is then a function of y . We consider a wind velocity U in the x direction. In the actual case of a turbulent wind, the instantaneous wind direction fluctuates. Since forced convection is most efficient when the wind blows in the x direction, it is logical to take such a direction as reference to define the average width \bar{l} . We assume that at any y it is possible to apply locally two-dimensional boundary layer results. This approach is correct as long as the length L is at least comparable to the width \bar{l} .

For the important case when the heat flux carried out by forced convection is practically uniform, the leaf and its equivalent must clearly have the same area, or

$$l = \int_0^L w(y) dy / L \quad (13)$$

For the Pohlhausen case of uniform and imposed temperature θ the local heat flux varies like $x^{-1/2}$ according to equations 1 and 2. Hence if the total heat flux is the same for the leaf and its equivalent we must have

$$\int_0^L \int_0^W x^{-1/2} dx dy = \int_0^L \int_0^{\bar{l}} x^{-1/2} dx dy \quad (14)$$

or

$$\bar{l} = [\int_0^L w^{1/2} dy]^2 / L^2 \quad (15)$$

The two average widths given by equations 13 and 14 are in general different. Hence \bar{l} in equations 4 and 9 are also different, unless of course the leaf is rectangular. Actually both definitions give results that are numerically close unless the leaf has a very irregular shape. Notice that equations 9 and 10 of Reference 13 and equations 11 and 12 of Reference 14 are in error. All the denominators should refer to the equivalent, rectangular shape and not to the actual shape. With this correction all those equations reduce at once to our equation 15. This error was corrected later by Parkhurst (private communication) for uniform temperature.

Acknowledgment—The authors are pleased to acknowledge very useful correspondence with Professor C. B. Tanner of the University of Wisconsin.

LITERATURE CITED

- BÜYÜKTÜR, A. R., J. KESTIN, AND P. F. MAEDER. 1964. Influence of combined pressure gradient and turbulence on the transfer of heat from a plate. *Int. J. Heat Mass Transfer* 7: 1175-1186.
- COOK, G. D., J. R. DIXON, AND A. C. LEOPOLD. 1964. Transpiration: its effect on plant leaf temperature. *Science* 144: 546-547.
- GATES, D. M. 1964. Leaf temperature and transpiration. *Agron. J.* 56: 273-277.
- GATES, D. M., H. J. KEEGAN, J. C. SCHLETER, AND V. R. WEIDNER. 1965. Spectral properties of plants. *Appl. Optics* 4: 11-20.
- HUNT, L. A. AND I. I. IMPENS. 1968. Use of antitranspirants in studies of the external diffusion resistance of leaves. *Oecol. Plant.* 3: 1-6.
- HUNT, L. A., I. I. IMPENS, AND E. R. LEMON. 1968. Estimates of the diffusion resistance of some large sunflower leaves in the field. *Plant Physiol.* 43: 522-526.
- KANEMASU, E. T., G. W. THURTELL, AND C. B. TANNER. 1969. Design, calibration and field use of a stomatal diffusion porometer. *Plant Physiol.* 44: 881-885.
- KESTIN, J., P. F. MAEDER, AND H. E. WANG. 1961. On boundary layers associated with oscillating streams. *Appl. Sci. Res.* 10: 1-22.
- KESTIN, J., P. F. MAEDER, AND H. E. WANG. 1961. Influence of turbulence on the transfer of heat from plates with and without a pressure gradient. *Int. J. Heat Mass Transfer* 3: 133-154.
- LINACRE, E. T. 1967. Further studies of the heat transfer from a leaf. *Plant Physiol.* 42: 651-658.
- MCADAMS, W. H. 1954. *Heat Transmission*, Ed. 3. McGraw-Hill Book Co., New York.
- MONTETH, J. L. 1964. Evaporation and environment. *Proc. Symp. Soc. Exp. Biol.* 19: 205-234.
- PARKHURST, D. F., P. R. DUNCAN, D. M. GATES, AND F. KREITH. 1968. Convection heat transfer from broad leaves of plants. *J. Heat Transfer.* 90: 71-76.
- PARKHURST, D. F., P. R. DUNCAN, D. M. GATES, AND F. KREITH. 1968. Wind-tunnel modelling of convection of heat between air and broad leaves of plants. *Agr. Meteorol.* 5: 33-47.
- PARLANGE, J.-Y. 1970. Thermal boundary layer similarity at limiting Prandtl numbers. *Amer. Inst. Aero. Astro. J.* 8: 574-576.
- PARLANGE, J.-Y. AND P. E. WAGGONER. 1970. Stomatal dimensions and resistance to diffusion. *Plant Physiol.* 46: 337-342.
- SCHLICHTING, H. 1960. *Boundary Layer Theory*, Ed. 4. McGraw-Hill Book Co., New York.
- TURNER, N. C. AND J.-Y. PARLANGE. 1970. Analysis of operation and calibration of a ventilated diffusion porometer. *Plant Physiol.* 46: 175-177.