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BOUNDED HOLOMORPHIC FUNCTIONS OF SEVERAL COMPLEX VARIABLES. I

BY
DONG SIE KIM[1]

Abstract. A domain of bounded holomorphy in a complex analytic manifold is a maximal domain for which every bounded holomorphic function has a bounded analytic continuation. In this paper, we show that this is a local property: if, for each boundary point of a domain, there exists a bounded holomorphic function which cannot be continued to any neighborhood of the point, then there exists a single bounded holomorphic function which cannot be continued to any neighborhood of the boundary points.

Introduction.. Let X be a topological space. A subset D of X is said to be a *region* if it is open and it is said to be a *domain* if it is open and connected. We denote by $N(p)$ a fundamental system of open neighborhoods of p, where $p \in X$.

1. DEFINITION. Let X be a topological space and U be an open subset of X. Let $C(U)$ be the family of all continuous complex-valued functions on U, then $C(U)$ is an algebra with 1, and it is equipped with the topology of uniform convergence on compact subsets of U. For a pair of open subsets U and V in X such that $V \subset U$ we define $\pi_{UV}: C(U) \to C(V)$ by $\pi_{UV}f = f|V$. Let $A(U)$ be a subalgebra of $C(U)$ with 1 and we assume that $\pi_{UV}A(U) \subset A(V)$; then we call $A = \{A(U), \pi_{UV}\}$ a *presheaf* of *algebras* of *functions*. A presheaf A has the *local belonging property* if, for all open sets U of X and f in $C(U)$, for every $p \in U$ there is $V \in N(p)$, $V \subset U$, such that $f|V \in A(v)$; then $f \in A(U)$.

A *sheaf* A of algebras of functions is a presheaf of algebras of functions with the local belonging property. A is said to be a *ringed structure* on X and the pair (X, A) is said to be a *ringed space*. The functions in $A(U)$ are A-holomorphic functions. We note that $A(U)$ has the relative topology induced by the topology on $C(U)$.

A ringed structure A on X is an n-dimensional *complex analytic structure* on X if for all $x \in X$ there are $U \in N(x)$ and $f_1, \ldots, f_n \in A(U)$ such that

$$F = (f_1, \ldots, f_n): U \to \boldsymbol{C}^n$$

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is a homeomorphism of U onto $F(U)$ with the properties: $F(U)$ is open in $\boldsymbol{C}^n$ and for all W open $\subset U$, $\{f\circ(F|W)^{-1} : f\in A(W)\}=\mathcal{O}(F(W))$, where $\mathcal{O}$ is a complex analytic structure on $\boldsymbol{C}^n$. If X is a hausdorff space we call this pair (X, A) a *complex analytic manifold.*

For a subset U of X, $A(U)$ is *quasi-analytic* if for all nonempty open subsets V of U and for f, g in $A(U)$ such that $f=g$ on V then $f=g$ on U.

We give a characterization of quasi-analyticity in terms of the hausdorffness of the topology on A in the following proposition. The proof may be found in (3).

2. PROPOSITION. *Let (X, A) be a ringed space with X a locally connected hausdorff space. Then A is hausdorff if and only if $A(U)$ is quasi-analytic for all connected subsets U of X.*

Regions of bounded holomorphy.

3. DEFINITION. Let (X, A) be a ringed space and D be a region. We define $B(D)=\{f\in A(D) : f$ is bounded on $D\}$. For a point $p\in\bar{D}-D$ (boundary of D) and $U\in N(p)$, a function $f\in B(D)$ is said to be *extendable* to U if there is a function $g\in B(U)$ such that $f=g$ on $D\cap U$. D is said to be a *weak region* of *bounded holomorphy* if there exists a function $f\in B(D)$ which cannot be extendable beyond the boundary of D.

A is said to be *montel* if for an open set U in X and $F\subset A(U)$ there is $M_K>0$ such that $\|f\|_K<M_K$ for all $f\in F$ and for all compact subsets K of U; then F is relatively compact in $A(U)$.

A is *c.o. complete* if for all open subsets U in X, $A(U)$ is complete in the topology of uniform convergence on compact subsets of U.

We note that an analytic structure A in a complex analytic manifold (X, A) has the montel property, and it is hausdorff and c.o. complete.

We show that the weak bounded holomorphy is a local property in the following theorem.

4. LEMMA. *Let (X, A) be a ringed space. We assume that X is a locally compact and locally connected hausdorff space, and A is hausdorff, c.o. complete and montel. Let D be a region in X and $p\in\bar{D}-D$ such that X is first countable at p. Let B be a closed (relative to the topology of uniform convergence on D) subalgebra of $B(D)$. Then these are equivalent:*

(1°) *For every $U_\alpha\in N(p)$ there is a function $f_\alpha\in B$ which cannot be extended to U.*

(2°) *There is a function $f\in B$ which cannot be extended to any neighborhood of P.*

Proof. It is sufficient to show that (1°) implies (2°). Let $\{U_m : m\in Z_+\}$ be a countable nested basis of open neighborhood of p. Let $B_1(U_m, n)=\{f\in B : f=g|D$ where $g\in B(D\cup U_m)$ and $\|g\|_{U_m}\leqq n\}$, $n\in Z_+$. We claim that $B_1(U_m, n)$ is a closed nowhere dense subset of B. For closedness, let $\{f_k\}$ be any net in $B_1(U_m, n)$ converging uniformly on D to f. We note that $\{f_k\}$ is c.o. convergent to f. Let $\{g_k\}\subset B(D\cup U_m)$ such that $g_k|D=f_k$, $\|g_k\|_{U_m}\leqq n$, $k\in Z_+$. $\{g_k\}$ is uniformly bounded on compact subsets of $D \cup U_m$. Since A is montel $\{g_k\}$ is relatively compact in $A(D \cup U_m)$. Thus there is a subnet $\{g_j\} \subset \{g_k\}$ which converges to $g \in A(D \cup U_m)$. Now $\lim_{\text{c.o.}} g_j | D = \lim_{\text{c.o.}} f_j = f$, so $g | D = f$ and since $\|g_j\|_{U_m} \leqq n$ for $j \in Z_+$, $\|g\|_{U_m} \leqq n$, which concludes that $f \in B_1(U_m, n)$. For nowhere denseness, let $B_1(U_m, n) = \bigcup_n B_1(U_m, n)$. Take $f \in B - B_1(U_m)$ and define $g_j = j^{-1}f + h$ for $h \in B_1(U_m, n)$, $j \in Z_+$. Then $g_j \notin B_1(U_m) \supset B_1(U_m, n)$ and $\lim_j g_j = h$. Since h is an arbitrary element of $B_1(U_m, n)$, int $B_1(U_m, n) = \varnothing$.

Let $B_1 = \bigcup \{B_1(U_m) : m \in Z_+\}$ and $B_2 = \{f \in B : f$ can be extended to some neighborhood of $p\}$. Then $B_1 = B_2$. Now since B has the baire property, $B_1 \subsetneqq B$. Hence there is $f \in B - B_1$, so $f \notin B_2$, f cannot be extended to any neighborhood of p.

5. Theorem. *Let (X, A) be a ringed space. We assume that X is a locally compact locally connected hausdorff space, and A is hausdorff, c.o. complete and montel. Let D be a region in X such that $\bar{D} - D$ is separable and X is first countable on $\bar{D} - D$. Let B be a closed subalgebra of $B(D)$ as in the lemma. Then these are equivalent*:

(1°) *For every $p \in \bar{D} - D$ there is a function $f_p \in B$ which cannot be extended to any $U \in N_{(p)}$.*

(2°) *There is a function $f \in B$ which cannot be extended beyond the boundary of D.*

Proof. Let $\{U_m : m \in Z_+\}$ be a countable basis of nested open neighborhoods of $p \in \bar{D} - D$. Let $B_1(p, U_m, n) = \{f \in B : f = g | D, g \in B(D \cup U_m), \|g\|_{U_m} \leqq n\}$, n, $m \in Z_+$. Then $B_1(p, U_m, n)$ is a closed nowhere dense subset of B as in the proof of the lemma. Let $\{p_i : i \in Z_+\}$ be a countable dense subset of $\bar{D} - D$ and $\{U_m^{(i)}\}$ be a countable basis of nested open neighborhoods of p_i. Let

$$B_2 = \bigcup \{B_1(p_i, U^{(i)}, n) : i, m, n \in Z_+\}$$

and

$$B_3 = \{f \in B : f \text{ can be extended beyond } \bar{D} - D\}.$$

Then $B_2 = B_3$. Since B is baire, $B_2 \subsetneqq B$. Hence there is $f \in B - B_2 = B - B_3$, which asserts (2°).

6. Corollary. *Let (X, A) be a complex analytic manifold and D be a region in X such that $\bar{D} - D$ is separable and X is first countable on $\bar{D} - D$. Let $B = B(D)$. Then these are equivalent*:

(1°) *For every $p \in \bar{D} - D$ there is an $f \in B$ which cannot be extended to any $U \in N(p)$.*

(2°) *D is a weak region of bounded holomorphy.*

7. Definition. Let (X, A) be a ringed space and D be a region in X. Let V be an open subset of X such that $D \cap V \neq \varnothing$ and $V \not\subset D$. $f \in B(D)$ is said to be *continued* to V if there is a connected component Ω of $D \cap V$ and $g \in B(V)$ such that $f = g$ on Ω. We say that g is a *continuation* of f to V. A boundary point p of D is said to be a *boundary singularity* for $f \in B(D)$ if f cannot be continued to any open

neighborhood of p. A region is called a *region* of *bounded holomorphy* if there is an $f \in B(D)$ for which every boundary point of D is a boundary singularity.

We give a characterization of a region of bounded holomorphy by a local property in the next theorem.

8. Lemma. *Let (X, A) be a ringed space. We assume that X is a locally compact and locally connected hausdorff space and A is hausdorff, c.o. complete, and montel. Let D be a region in X and $p \in \bar{D} - D$ such that X is first countable at p. Let B be a closed (relative to the topology of uniform convergence on D) subalgebra of $B(D)$. Then these are equivalent:*

(1°) *For every $U_\alpha \in N(p)$ and every connected component $\Omega_{\alpha\beta}$ of $U_\alpha \cap D$ there is $f_{\alpha\beta} \in B$ such that $f_{\alpha\beta}$ has no continuation to U_α.*

(2°) *There is $f \in B$ such that for all $U \in N(p)$ and for all connected components Ω of $U \cap D$, f has no continuation to U_α, i.e. p is a boundary singularity for f.*

Proof. It suffices to show that (1°) implies (2°). Let $\{U_\alpha : \alpha \in Z_+\}$ be a countable nested basis of open neighborhoods of p and let $\{\Omega_{\alpha\beta} : \beta \in Z_+\}$ be a countable family of connected components of $U_\alpha \cap D$. Let $B_1(\Omega_{\alpha\beta}, n) = \{f \in B$: there is $g \in B(U_\alpha)$ such that $f = g$ on $\Omega_{\alpha\beta}$ and $\|g\|_{U_\alpha} \leqq n\}$, $n \in Z_+$. Then as in the proof of Lemma 4, $B_1(\Omega_{\alpha\beta}, n)$ is a closed nowhere dense subset of B. Let $B_1 = \bigcup_{\alpha,\beta,n} B_1(\Omega_{\alpha\beta}, n)$ and let $B_2 = \{f \in B : f$ can be continued to some neighborhood of $p\}$. Then $B_1 = B_2$, and since $B_1 \subsetneqq B$ there is an $f \in B - B_2$.

9. Theorem. *Let (X, A) be a ringed space. We assume that X is a locally compact, locally connected hausdorff space and A is hausdorff, c.o. complete and montel. Let D be a region in X such that $\bar{D} - D$ is separable and X is first countable on $\bar{D} - D$. Let B be a closed subalgebra of $B(D)$. Then these are equivalent:*

(1°) *For every $p \in \bar{D} - D$ there is a function $f_p \in B$ for which p is a boundary singularity.*

(2°) *There is a function $f \in B$ for which every boundary point is a boundary singularity.*

Proof follows by the lemma and in a similar way as the proof of Theorem 5.

10. Corollary. *Let (X, A) be a complex analytic manifold and D be a region in X such that $\bar{D} - D$ is separable. Let $B = B(D)$. Then these are equivalent:*

(1°) *For every $p \in \bar{D} - D$ there is $f_p \in B$ for which p is a boundary singularity.*

(2°) *D is a region of bounded holomorphy.*

In the following, we show that a weak region of bounded holomorphy is a region of bounded holomorphy when the region is locally connected on the boundary.

11. Definition. Let X be a topological space and D be a region in X. We say that D is *locally connected* at $p \in \bar{D} - D$ if p has a base of open neighborhoods whose intersections with D are connected. D is *locally connected* on the *boundary* of D if D is locally connected at every point of the boundary.

The following lemma will give the proof of Theorem 13.

12. Lemma. *Let X be a locally connected hausdorff space and let D be a region in X which is locally connected on the boundary. Let $V \in N(p)$, $p \in \bar{D}-D$ and U be an open subset of $V \cap D$. Then there is an open set $V_1 \subset U$ such that $V_1 \cap (\bar{D}-D) \neq \varnothing$, $V_1 \cap D$ is connected and $V_1 \cap U \neq \varnothing$.*

Proof. We assume that V is a connected neighborhood of p.

(i) We show that for every connected component Ω of $V \cap D$, $V \cap (\bar{\Omega}-\Omega) \subset \bar{D}-D$. Note that $V \cap (\bar{\Omega}-\Omega) \neq \varnothing$, for otherwise we have $V=(V-\bar{\Omega}) \cup \Omega$ which contradicts its connectedness. Now $\bar{\Omega} \subset \bar{D}$ so that $V \cap (\bar{\Omega}-\Omega) \subset V \cap \bar{D}$. If $V \cap (\bar{\Omega}-\Omega) \cap D \neq \varnothing$, take $p \in V \cap (\bar{\Omega}-\Omega) \cap D$ then there is a connected open set $U' \in N(p)$ such that $U' \subset V \cap D$ and $U' \cap \Omega \neq \varnothing$. Thus $U' \cup \Omega \subset V \cap D$ is connected. But then $U' \cup \Omega = \Omega$ and $p \in \Omega$, which is a contradiction. It follows that $V \cap (\bar{\Omega}-\Omega) \cap D = \varnothing$ so that $V \cap (\bar{\Omega}-\Omega) \subset \bar{D}-D$.

(ii) Choose a connected component Ω of $V \cap D$ such that $\Omega \cap U \neq \varnothing$. Take $q \in V \cap (\bar{\Omega}-\Omega) \subset \bar{D}-D$ and choose a neighborhood V' of q such that $V' \subset V$ and $V' \cap D$ is connected. Let $V_1 = \Omega \cup V'$. Since $\Omega \cap V' \neq \varnothing$, V_1 has the required property.

13. Theorem. *Let (X, A) be a ringed space. We assume that X is a locally compact, locally connected hausdorff space, and A is hausdorff, c.o. complete and montel. Let D be a region in X which is locally connected on the boundary. Let B be a closed subalgebra of $B(D)$. Then these are equivalent:*

(1°) *There is a function $f \in B$ which cannot be extended beyond D.*

(2°) *There is a function $f \in B$ which cannot be continued beyond D.*

Proof. It is immediate from the lemma.

14. Corollary. *Let (X, A) be a complex analytic manifold. Let D be a region which is locally connected on the boundary. Then these are equivalent:*

(1°) *D is a weak region of bounded holomorphy.*

(2°) *D is a region of bounded holomorphy.*

We investigate regions of bounded holomorphy in $(\boldsymbol{C}^n, \mathcal{O})$. First, we have a useful lemma for searching domains of bounded holomorphy.

15. Lemma. *Let (X, A) be a complex analytic manifold and D be a region in X. Let U be a domain such that $D \cap U \neq \varnothing$ and $U \not\subset D$. If every function $f \in B(D)$ can be continued to U and $\tilde{f}$ denotes the continuation of f to U, then $\tilde{f}(U) \subset \mathrm{cl}\,(f(D))$ for all $f \in B(D)$.*

Proof. Let $\alpha \notin \mathrm{cl}\,(f(D))$, then $g=(f-\alpha)^{-1} \in B(D)$, and so has a continuation $\tilde{g} \in B(U)$. Now $g \cdot (f-\alpha) \equiv 1$ on D, and $g \cdot (f-\alpha) = \tilde{g} \cdot (\tilde{f}-\alpha) = 1$ on a connected component Ω of $D \cap U$. So by analytic continuation, $\tilde{g} \cdot (\tilde{f}-\alpha) \equiv 1$ on U. Hence $\alpha \notin \tilde{f}(U)$. So $\tilde{f}(U) \subset \mathrm{cl}\,(f(D))$.

16. *Simple examples of domains of bounded holomorphy in $(\boldsymbol{C}^n, \mathcal{O})$.*

(1°) An open polydisc

$$\boldsymbol{P}(w:r) = \boldsymbol{P}(w_1, \ldots, w_n : r_1, \ldots, r_n) = \{s \in \boldsymbol{C}^n : |s_i - w_i| < r_i,\ 1 \leqq i \leqq n\} \subset \boldsymbol{C}^n$$

is a domain of bounded holomorphy. For, take a boundary point $s \in \bar{\boldsymbol{P}}(w:r)$; then $|s_j| = r_j$ for some j. Now for any polydisc $\boldsymbol{P}_1(s:\varepsilon)$, $\|z_j\|_{\boldsymbol{P}_1} > r_j$. Hence $z_j(\boldsymbol{P}) \not\subset \mathrm{cl}\,(Z_j(\boldsymbol{P}))$. By Lemma 15, $\boldsymbol{P}$ is a domain of bounded holomorphy. Moreover, an analytic polyhedron and a bounded complete Reinhardt domain are domains of bounded holomorphy.

(2°) A simply connected domain D in $\boldsymbol{C}$ which is locally connected on the boundary of D is a domain of bounded holomorphy.

17. PROPOSITION. *Let $\{D_j : j \in Z_+\}$ be an indexed set of regions of bounded holomorphy in $\boldsymbol{C}^n$. Let $D = \bigcap_{j=1}^{\infty} D_j$ and assume that D is open. Then D is a region of bounded holomorphy in $\boldsymbol{C}^n$.*

Proof. For a point $p \in \bar{D} - D$ there exists $m \in Z_+$ such that $p \notin D_m$. Then there exists $f \in B(D_m)$ which is a singular function at p. Thus $f \,|\, D \in B(D)$ is singular at p.

18. PROPOSITION. *A finite cartesian product of regions of bounded holomorphy is a region of bounded holomorphy.*

Proof. We shall prove this for the case of a product of two regions. Let D_1 and D_2 be regions of bounded holomorphy in $\boldsymbol{C}^n$ and let $f_i \in B(D_i)$, $i = 1, 2$, be singular functions. Define $F_1 \in B(D \times \boldsymbol{C}^n)$ by $F_1(s, t) = f_1(s)$ and $F_2 \in B(\boldsymbol{C}^n \times D_2)$ by $F_2(s, t) = f_2(t)$. Then F_1 is a singular function at every point of (bdry D_1) $\times \boldsymbol{C}^n$ and so is F_2 for $\boldsymbol{C}^n \times$ (bdry D_2). For, if F_1 is not, then there is $V \in N(p)$, $p \in$ (bdry D_1) $\times \boldsymbol{C}^n$ such that F_1 can be continued to V. Let W be the image of V into $\boldsymbol{C}^n \supset D_1$ then $F_1 \,|\, W = f_1$ can be continued to W. But W is a neighborhood of a boundary point of D_1. This is absurd (similarly for F_2). Now bdry $(D_1 \times D_2) =$ (bdry D_1) $\times \bar{D}_2 \cup \bar{D}_1 \times$ (bdry D_2). Thus if $p \in$ bdry $(D_1 \times D_2)$, then F_1 or F_2 is a singular function at p. Hence $D_1 \times D_2$ is a domain of bounded holomorphy.

19. PROPOSITION. *Every convex (in the geometric sense) domain D in $\boldsymbol{C}^n$ is a domain of bounded holomorphy.*

Proof. Since such a domain D is the intersection of the open halfspaces in $\boldsymbol{C}^n$ (as a real vector space R^{2n}) which contain it, by Proposition 17 it suffices to show that every open halfspace in $\boldsymbol{C}^n$ is a domain of bounded holomorphy. Let $S = \{(z_1, \ldots, z_n) \in \boldsymbol{C}^n : \mathrm{Re}\, z_i > 0, i = 1, \ldots, n\}$. Then any open halfspace in $\boldsymbol{C}^n$ can be identified as S by a translation and a complex linear transformation. Hence again it suffices to show that S is a domain of bounded holomorphy. But this is so; for, let $H = \{z \in \boldsymbol{C} : \mathrm{Re}\, z > 0\}$, then since H can be mapped onto the open unit disc by a Riemann map, H is a domain of bounded holomorphy. Now $S = \prod^n H$, a finite cartesian product. Hence S is a domain of bounded holomorphy by Proposition 18.

20. PROPOSITION. *Let D be a region in $\boldsymbol{C}^n$, $n > 1$, and let K be a compact subset of D such that $D - K$ is connected. Then for every $f \in B(D - K)$ there exists $\tilde{f} \in B(D)$ such that $f = \tilde{f}$ on $D - K$.*

Proof. Since $B(D-K)\subset\mathcal{O}(D-K)$, for every function $f\in B(D-K)$ there is $\tilde{f}\in\mathcal{O}(D)$ such that $f=\tilde{f}$ on $D-K$ by a theorem of Hartog's. So it suffices to show that those extensions are still bounded on D. But this is clear from Lemma 15.

21. *Let D be a region in C^n and let $B=B(D)$. Then B is a Banach algebra with the supremum norm on D.* The *spectrum of B*, denoted by $S(B)$, is the set of nonzero complex homomorphisms of B. For $z\in D$ if we define $h_z(f)=f(z)$, $f\in B$, then $h_z\in S(B)$. Hence we obtain a mapping $\rho: D\to S(B)$, $\rho(z)=h_z$. To each $f\in B$ we associate a function $\hat{f}$ defined on $S(B)$ by defining $\hat{f}(h)=h(f)$. Since $\hat{f}\circ\rho=f$, the mapping $f\mapsto\hat{f}$ is one-to-one. We endow $S(B)$ with the weakest topology which makes $\hat{f}$ continuous. Then $S(B)$ is compact and the mapping $f\mapsto\hat{f}$ is an isometry of B onto $\hat{B}=\{\hat{f}:f\in B\}$. Hence we may assume that B is defined on $S(B)$. Let $f_1,\ldots,f_n\in B$. The *joint spectrum* of $f_1,\ldots,f_n$ is the set; $\sigma(f_1,\ldots,f_n)=\{(f_1(h),\ldots,f_n(h)):h\in S(B)\}$. For given $f_1,\ldots,f_n\in B$ we define $\pi: S(B)\to C^n$ by $\pi(h)=(f_1(h),\ldots,f_n(h))$, then π is a continuous map. If D is relatively compact in C^n then the coordinate functions $z_1,\ldots,z_n$ belong to B and $\pi S(B)\supset D$ since the point evaluation maps are in $S(B)$. Furthermore, since $S(B)$ is compact $\pi S(B)\supset\bar{D}$. Now we have the following theorem:

22. THEOREM. *Let D be a relatively compact region in C^n with* int $\bar{D}=D$. *If $\pi S(B)=\bar{D}$ then D is a region of bounded holomorphy.*

Proof. If we assume that D is not a region of bounded holomorphy, then every function $f\in B$ has an extension $\tilde{f}$ to a neighborhood V of a boundary point p of D. By Lemma 15, $\tilde{f}(V)\subset\mathrm{cl}\,(f(D))$. Hence the extensions $\tilde{f}$, $f\in B$ are continuous with respect to the supnorm on D. Now take a point $z\in V-\bar{D}$, consider the point evaluation map h_z, $h_z(\tilde{f})=\tilde{f}(z)$ for all $f\in B$, then $h_z\in S(B)$ and $\pi(h_z)=z\in V-\bar{D}$, which is absurd.

We note that if int $\bar{D}\neq D$ then the theorem is false; consider

$$D=\{z\in C: 0<|z|<1\}$$

then $B(D)=B(D\cup\{0\})$ and $S(B)=\bar{D}$. But D is not a domain of bounded holomorphy.

REFERENCES

1. R. Gunning and H. Rossi, *Analytic functions of several complex variables*, Prentice-Hall, Englewood Cliffs, N. J., 1965. MR **31** #4927.

2. L. Hörmander, *An introduction to complex analysis in several variables*, Van Nostrand, Princeton, N. J., 1966. MR **34** #2933.

3. F. Quigley, *Lectures on several complex variables*, Tulane University, New Orleans, La., 1964/65, 1965/66.

4. W. Rudin, *Some theorems on bounded analytic functions*, Trans. Amer. Math. Soc. **78** (1955), 333–342. MR **16**, 685.

5. M. Shauck, *Algebras of holomorphic functions in ringed spaces*, Dissertation, Tulane University, New Orleans, La., 1966.

UNIVERSITY OF FLORIDA,
GAINESVILLE, FLORIDA 32601