

## BOUNDING EXPECTATIONS OF FUNCTIONS OF RANDOM VECTORS WITH GIVEN MARGINALS AND SOME MOMENTS: APPLICATIONS OF THE MULTIVARIATE DISCRETE MOMENT PROBLEM

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*Abstract.* The paper shows how the bounding technique provided by the multivariate discrete moment problem can be used for bounding expectations of functions of random variables with known univariate marginals and some of the mixed moments. Four examples are presented. In the first one the function is a Monge or related type array, in the second one it is a pseudo-Boolean function. In the further examples bounds are given for values of multivariate generating functions and expectations of special utility functions of random vectors. Numerical results are presented.

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