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Research Article

Bounds of Some Topological Indices of the Cartesian Product of F-sum Graphs

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Abstract. Lower and upper bounds of Randic index(R), sum-connectivity index (SCI), arithmetic geometric index (AG_1), Harmonic index (H) and multiplicative indices of Cartesian productof F-sum of connected graphs are obtained.

Keywords. Randic index; Sum-connectivity index, AG_1-index; Harmonic index; Multiplicative indices; F-sum; Cartesian product

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1. Introduction

Chemical graph theory is one of the tools, which implements graph theory to study mathematical modeling of chemical compounds. A topological index in the chemical graph theory is used to predict bioactivity of the molecular graphs of chemical compounds [8]. A molecular graph models the chemical structures of molecules and molecular compounds by considering atoms as vertices and the chemical bonds between the atoms as edges. Topological indices are widely studied for derived graphs like subdivision graph, line graphs, middle graphs, total graph and many more [9]. Eliasi and Taeri have discussed some of the topological indices for F-sum graphs [3]. Inspired by the above concepts we have discussed some of the other degree based topological indices to F-sum graphs.

Throughout the paper we consider a simple, finite and connected graph G with vertex set V_G and edge set E_G, the order of $G = |V_G| = n$ and the size of $G = |E_G| = m$. Let uv represent an edge between the two vertices u and v and $d_G(v)$ denotes the degree of a vertex v. We denote δ_G be the minimum degree and Δ_G be the maximum degree of the graph G. For undefined terminologies we refer to [5].

Topological indices are numerical values associated with chemical compoundsto study the correlation of chemical structures with various physical properties, chemical reactivity or biological activity. A hugenumber of molecular graphbased topological indices (molecular structure descriptor) are studied depending on vertex degrees. Vertex degree based topological indices have been studied for different graphs [1, 2, 4, 6, 7]. Among which we consider few topological indices which are:

Definition 1. Randic index of $G : R(G) = \sum\limits_{uv\in E_G} \frac{1}{\sqrt{d_G(u)d_G(v)}}$.

Definition 2. Sum-connectivity index of $G : SCI(G) = \sum\limits_{uv\in E_G} \frac{1}{\sqrt{d_G(u)+d_G(v)}}$.

Definition 3. Arithmetic geometric index of $G : AG_1(G) = \sum\limits_{uv\in E_G} \frac{d_G(u)+d_G(v)}{2\sqrt{d_G(u)\ d_G(v)}}$.

Definition 4. Harmonic index of $G : H(G) = \sum\limits_{uv\in E_G} \frac{2}{d_G(u)+d_G(v)}$.

Definition 5. Multiplicative Randic index of $G : \chi(G) = \prod\limits_{uv\in E_G} \frac{1}{d_G(u)d_G(v)}$.

Definition 6. Multiplicative sum-connectivity index of $G : X\pi(G) = \prod\limits_{uv\in E_G} \frac{1}{d_G(u)+d_G(v)}$.

Definition 7. Multiplicative arithmetic geometric index of $G : AG_1\pi(G) = \prod\limits_{uv\in E_G} \frac{d_G(u)+d_G(v)}{2\sqrt{d_G(u)\ d_G(v)}}$.

Definition 8. Multiplicative harmonic index of $G : H\pi(G) = \prod\limits_{uv\in E_G} \frac{2}{d_G(u)+d_G(v)}$.

The Cartesian product of two graphs G and H, denoted by $G\square H$ is a graphwith vertex set $V_{(G\square H)} = V_G \times V_H$ and $(u_1,v_1)(u_2,v_2) \in E_{(G\square H)}$ whenever $[u_1 = u_2$ and $v_1v_2 \in E_H]$ or $u_1u_2 \in E_G$ and $v_1 = v_2$.

Proposition 1. *Let G and H be two graphs, then*

(i) $|V_{(G\square H)}| = |V_G||V_H|$ *and* $|E_{(G\square H)}| = |V_H||E_G| + |V_G||E_H|$,

(ii) $d_{G\square H}(u,v) = d_G(u) + d_H(v)$.

For agiven connected graph $G(V,E)$ the following the four derived graphs are defined as below and vertex set of all the four graphs is $V(G)\cup E(G)$:

(1) To obtain $S(G)$ an additional vertex in each edge of G is inserted, that is every edge of G is replaced by a path of length 2. The inserted vertices correspond to edges of G. The obtained new graph is called a subdivision graph of G.

(2) Graph $R(G)$ contains edges of $S(G)$ as well as edges of G.

(3) Graph $Q(G)$ contains edges of $S(G)$ and also edges of line graph of G.

(4) Graph $T(G)$ contains edges of $S(G)$, edges of line graph of G and also the edges of G. $T(G)$ is called total graph of G.

The following four new operations that are based on $S(G)$, $R(G)$, $Q(G)$, $T(G)$ was given by Eliasi and Taeri [3]:

Let F denotes one of S, R, Q or T. The F-sum, denoted by $G +_F H$, of graphs G and H, is a graph with the set of vertices $V_{(G+_F H)} = (V_G \cup E_G) \times V_H$ and $(u_1,v_1)(u_2,v_2) \in E_{(G+_F H)}$, if and only if $[u_1 = u_2 \in V_G$ and $v_1v_2 \in E_H]$ or $[v_1 = v_2 \in V_H$ and $u_1u_2 \in E_{F(G)}]$.

Proposition 2. *Size of F-sum of graphs is:*

(1) *if* $G = G_1 +_S G_2$, *then the size of* G *is* $n_1m_2 + 2n_2m_1$,

(2) *if* $G = G_1 +_Q G_2$, *then the size of* G *is* $n_1m_2 + \frac{n_2m_1(m_1+3)}{2}$,

(3) *if* $G = G_1 +_R G_2$, *then the size of* G *is* $n_1m_2 + 3n_2m_1$,

(4) *if* $G = G_1 +_T G_2$, *then the size of* G *is* $n_1m_2 + \frac{n_2m_1(m_1+5)}{2}$,

where $|V_{G_1}| = n_1$, $|V_{G_2}| = n_2$, $|E_{G_1}| = m_1$ *and* $|E_{G_2}| = m_2$.

In this paper, the lower and upper bounds of Randic index (R), sum-connectivity index (SCI), arithmetic geometric index (AG_1), Harmonic index (H) and multiplicative indices of Cartesian product of F-sum of connected graphs are obtained.

2. Main Results

Throughout the paper, we consider A_1, A_2, B_1, B_2 to be simple, connected graphs with

$$|V_{A_1}| = a_1,\ |V_{A_2}| = a_2,\ |V_{B_1}| = b_1,\ |V_{B_2}| = b_2,\ |E_{A_1}| = a_1',\ |E_{A_2}| = a_2',\ |E_{B_1}| = b_1',\ |E_{B_2}| = b_2'.$$

We give the lower and upper bounds of Randic index (R), sum-connectivity index (SCI), arithmetic geometric index (AG_1), Harmonic index (H) and also multiplicative indices of these for the Cartesian product of F-sum of connected graphs.

Theorem 1. *Let* $A = A_1 +_S B_1$ *and* $B = A_2 +_S B_2$ *be any two F-sum graphs with the order and size of* A, B *are* r_1, r_2 *and* s_1, s_2, *respectively. Then*

(a) $\frac{\alpha}{\Delta_A+\Delta_B} \le R(A\Box B) \le \frac{\alpha}{\delta_A+\delta_B}$,

(b) $\frac{\alpha}{\sqrt{2(\Delta_A+\Delta_B)}} \le SCI(A\Box B) \le \frac{\alpha}{\sqrt{2(\delta_A+\delta_B)}}$,

(c) $\alpha\left(\frac{\delta_A+\delta_B}{\Delta_A+\Delta_B}\right) \le AG_1(A\Box B) \le \alpha\left(\frac{\Delta_A+\Delta_B}{\delta_A+\delta_B}\right)$,

(d) $\frac{\alpha}{\Delta_A+\Delta_B} \le H(A\Box B) \le \frac{\alpha}{\delta_A+\delta_B}$

where $\alpha = r_1s_2 + s_1r_2$.

Proof. Consider the graphs A and B with vertex sets $\{x_1,x_2,\dots,x_{a_1(b_1+b_1')}\}$ and $\{y_1,y_2,\dots,y_{a_2(b_2+b_2')}\}$ respectively with the order, size, maximum degree and minimum degree as follows and also for convenience we denote $d_{A\square B}(u)$ as $d_C(u)$.

$$|V(A)| = b_1(a_1 + a_1') = r_1, \quad |V(B)| = b_2(a_2 + a_2') = r_2,$$
$$|E(A)| = (b_1'a_1 + 2b_1a_1') = s_1, \quad |E(B)| = (b_2'a_2 + 2b_2a_2') = s_2$$

and

$$\Delta_A = \Delta_{A_1} + \Delta_{B_1}, \quad \Delta_B = \Delta_{A_2} + \Delta_{B_2}, \quad \delta_A = \delta_{A_1} + \delta_{B_1}\delta_B = \delta_{A_2} + \delta_{B_2}.$$

(a) By definition,

$$R(A\square B) = \sum_{(x_i,y_j)(x_k,y_l)\in E_{A\square B}} \frac{1}{\sqrt{d_C(x_i,y_j)\, d_C(x_k,y_l)}}$$
$$= \sum_{x_i\in V_A} \sum_{y_jy_l\in E_B} \frac{1}{\sqrt{d_C(x_i,y_j)\, d_C(x_i,y_l)}} + \sum_{y_j\in V_B} \sum_{x_ix_k\in E_A} \frac{1}{\sqrt{d_C(x_i,y_j)\, d_C(x_k,y_j)}}. \tag{2.1}$$

By using Proposition 1(ii), we obtain

$$d_C(x_i,y_j)d_C(x_k,y_l) = [d_A(x_i) + d_B(y_j)][d_A(x_k) + d_B(y_l)].$$

Since, for any vertex $x \in V_A$, $d_A(x) \le \Delta_A$ and $d_A(x) \ge \delta_A$, therefore, by using these facts, we obtain

$$d_C(x_i,y_j)d_C(x_k,y_l) \le (\Delta_A + \Delta_B)(\Delta_A + \Delta_B) = (\Delta_A + \Delta_B)^2, \tag{2.2}$$
$$\frac{1}{d_C(x_i,y_j)d_C(x_k,y_l)} \ge \frac{1}{(\Delta_A + \Delta_B)^2}. \tag{2.3}$$

By using inequality (2.3) in (2.1), we obtain

$$R(A\square B) \ge |V(A)|\,|E(B)| \frac{1}{\sqrt{(\Delta_A + \Delta_B)^2}} + |E(A)|\,|V(B)| \frac{1}{\sqrt{(\Delta_A + \Delta_B)^2}}.$$

Therefore, we get

$$R(A\square B) \ge [r_1s_2 + s_1r_2] \frac{1}{\Delta_A + \Delta_B}.$$

By using similar arguments with $d_A(x) \ge \delta_A$, we obtain

$$R(A\square B) \le [r_1s_2 + s_1r_2] \frac{1}{\delta_A + \delta_B}.$$

By taking,$r_1s_2 + s_1r_2 = \alpha$ part (a) is proved.

(b) By definition,

$$SCI(A\square B) = \sum_{(x_i,y_j)(x_k,y_l)\in E_{A\square B}} \frac{1}{\sqrt{d_C(x_i,y_j) + d_C(x_k,y_l)}}$$
$$= \sum_{x_i\in V_A} \sum_{y_jy_l\in E_B} \frac{1}{\sqrt{d_C(x_i,y_j) + d_C(x_i,y_l)}}$$
$$+ \sum_{y_j\in V_B} \sum_{x_ix_k\in E_A} \frac{1}{\sqrt{d_C(x_i,y_j) + d_C(x_k,y_j)}}. \tag{2.4}$$

By using Proposition 1(ii), we obtain

$$d_C(x_i,y_j) + d_C(x_k,y_l) = d_A(x_i) + d_B(y_j) + d_A(x_k) + d_B(y_l).$$

Since, for any vertex $x \in V_A$, $d_A(x) \le \Delta_A$ and $d_A(x) \ge \delta_A$, therefore, by using these facts,

$$d_C(x_i,y_j)+d_C(x_k,y_l) \le \Delta_A+\Delta_B+\Delta_A+\Delta_B = 2(\Delta_A+\Delta_B), \tag{2.5}$$

$$\frac{1}{d_C(x_i,y_j)+d_C(x_k,y_l)} \ge \frac{1}{2(\Delta_A+\Delta_B)}. \tag{2.6}$$

By using inequality (2.6) in (2.4), we obtain

$$SCI(A\square B) \ge |V(A)|\,|E(B)|\frac{1}{\sqrt{2(\Delta_A+\Delta_B)}} + |E(A)|\,|V(B)|\frac{1}{\sqrt{2(\Delta_A+\Delta_B)}}.$$

Therefore, we get

$$SCI(A\square B) \ge [r_1s_2+s_1r_2]\times\frac{1}{\sqrt{2(\Delta_A+\Delta_B)}}.$$

By using similar arguments with $d_A(a) \ge \delta_A$, we obtain

$$SCI(A\square B) \le [r_1s_2+s_1r_2]\times\frac{1}{\sqrt{2(\delta_A+\delta_B)}}.$$

By taking, $r_1s_2+s_1r_2=\alpha$ part (b) is proved.

(c) By definition,

$$\begin{aligned} AG_1(A\square B) &= \sum_{(x_i,y_j)(x_k,y_l)\in E_{A\square B}} \frac{d_C(x_i,y_j)+d_C(x_k,y_l)}{2\sqrt{d_C(x_i,y_j)d_C(x_k,y_l)}} \\ &= \sum_{x_i\in V_A}\sum_{y_jy_l\in E_B} \frac{d_C(x_i,y_j)+d_C(x_k,y_l)}{2\sqrt{d_C(x_i,y_j)d_C(x_k,y_l)}} \\ &\quad + \sum_{y_j\in V_B}\sum_{x_ix_k\in E_A} \frac{d_C(x_i,y_j)+d_C(x_k,y_l)}{2\sqrt{d_C(x_i,y_j)d_C(x_k,y_l)}}. \end{aligned} \tag{2.7}$$

By using inequality (2.2) and (2.5) in (2.7) and using the same procedure as above, we obtain

$$AG_1(A\square B) \ge [r_1s_2+s_1r_2]\times\left(\frac{\delta_A+\delta_B}{\Delta_A+\Delta_B}\right)$$

and

$$AG_1(A\square B) \le [r_1s_2+s_1r_2]\times\left(\frac{\Delta_A+\Delta_B}{\delta_A+\delta_B}\right).$$

By taking, $r_1s_2+s_1r_2=\alpha$ part (b) is proved.

(d) By definition,

$$\begin{aligned} H(A\square B) &= \sum_{(x_i,y_j)(x_k,y_l)\in E_{A\square B}} \frac{2}{d_C(x_i,y_j)+d_C(x_k,y_l)} \\ &= \sum_{x_i\in V_A}\sum_{y_jy_l\in E_B} \frac{2}{d_C(x_i,y_j)+d_C(x_k,y_l)} \\ &\quad + \sum_{y_j\in V_B}\sum_{x_ix_k\in E_A} \frac{2}{d_C(x_i,y_j)+d_C(x_k,y_l)}. \end{aligned} \tag{2.8}$$

By using inequality (2.6) in (2.8) and using the same procedure as above, we obtain

$$H(A\square B) \ge [r_1s_2+s_1r_2]\times\frac{1}{\Delta_A+\Delta_B} \quad\text{and}\quad H(A\square B) \le [r_1s_2+s_1r_2]\times\frac{1}{\delta_A+\delta_B}$$

by substituting $r_1s_2+s_1r_2=\alpha$ part (d) is proved. Hence the theorem. □

Corollary 1. *Let $A=A_1+_S B_1$ and $B=A_2+_S B_2$, then*

(a) $\left(\frac{1}{\Delta_A+\Delta_B}\right)^{\alpha}\le \chi(A\square B)\le \left(\frac{1}{\delta_A+\delta_B}\right)^{\alpha}$,

(b) $\left(\frac{1}{\sqrt{2(\Delta_A+\Delta_B)}}\right)^{\alpha}\le X\pi(A\square B)\le \left(\frac{1}{\sqrt{2(\delta_A+\delta_B)}}\right)^{\alpha}$,

(c) $\left(\frac{\delta_A+\delta_B}{\Delta_A+\Delta_B}\right)^{\alpha}\le AG_1\pi(A\square B)\le \left(\frac{\Delta_A+\Delta_B}{\delta_A+\delta_B}\right)^{\alpha}$,

(d) $\left(\frac{1}{\Delta_A+\Delta_B}\right)^{\alpha}\le H\pi(A\square B)\le \left(\frac{1}{\delta_A+\delta_B}\right)^{\alpha}$.

Proof. Consider the graphs A and B with vertex set $\{x_1,x_2,\dots,x_{a_1(b_1+b_1')}\}$ and $\{y_1,y_2,\dots,y_{a_2(b_2+b_2')}\}$, respectively. Then

(a) By definition,

$$\chi(A\square B)=\prod_{(x_i,y_j)(x_k,y_l)\in E_{A\square B}}\frac{1}{\sqrt{d_{A\square B}(x_i,y_j)d_{A\square B}(x_k,y_l)}}. \tag{2.9}$$

By using inequality (2.3) in (2.9) and using the same procedure as above theorem, we obtain

$$\chi(A\square B)\ge\left(\frac{1}{\Delta_A+\Delta_B}\right)^{r_1s_2+s_1r_2}\quad\text{and}\quad \chi(A\square B)\le\left(\frac{1}{\delta_A+\delta_B}\right)^{r_1s_2+s_1r_2}$$

by substituting $r_1s_2+s_1r_2=\alpha$ part (a) is proved. Hence part (b), (c) and (d) can be obtained by using similar procedure as in (a). □

Similar to the above, we can obtain the following results.

Theorem 2. *Let $A=A_1+_Q B_1$ and $B=A_2+_Q B_2$, be any two F-sum graphs with the order and size of A, B are r_1, r_2 and s_1, s_2, respectively. Then*

(a) $\frac{\beta}{\Delta_A+\Delta_B}\le R(A\square B)\le \frac{\beta}{\delta_A+\delta_B}$,

(b) $\frac{\beta}{\sqrt{2(\Delta_A+\Delta_B)}}\le SCI(A\square B)\le \frac{\beta}{\sqrt{2(\delta_A+\delta_B)}}$,

(c) $\beta\left(\frac{\delta_A+\delta_B}{\Delta_A+\Delta_B}\right)\le AG_1(A\square B)\le \beta\left(\frac{\Delta_A+\Delta_B}{\delta_A+\delta_B}\right)$,

(d) $\frac{\beta}{\Delta_A+\Delta_B}\le H(A\square B)\le \frac{\beta}{\delta_A+\delta_B}$,

where $r_1=\frac{1}{2}\left[b_1\left(a_1+a_1'\right)\right]$, $r_2=\frac{1}{2}\left[b_2\left(a_2+a_2'\right)\right]$, $s_1=\frac{1}{2}\left[2b_1'a_1+b_1a_1'\left(a_1'+3\right)\right]$, $s_2=\frac{1}{2}\left[2b_2'a_2+a_2'b_2\left(a_2'+3\right)\right]$, $\beta=r_1s_2+s_1r_2$, $\delta_A+\delta_B=\delta_{A_1}+\delta_{A_2}+\delta_{B_1}+\delta_{B_2}$ *and* $\Delta_A+\Delta_B=\Delta_{A_1}+\Delta_{A_2}+\Delta_{B_1}+\Delta_{B_2}$.

Corollary 2. *Let $A=A_1+_Q B_1$ and $B=A_2+_Q B_2$, then*

(a) $\left(\frac{1}{\Delta_A+\Delta_B}\right)^{\beta}\le \chi(A\square B)\le \left(\frac{1}{\delta_A+\delta_B}\right)^{\beta}$,

(b) $\left(\frac{1}{\sqrt{2(\Delta_A+\Delta_B)}}\right)^{\beta}\le X\pi(A\square B)\le \left(\frac{1}{\sqrt{2(\delta_A+\delta_B)}}\right)^{\beta}$,

(c) $\left(\frac{\delta_A+\delta_B}{\Delta_A+\Delta_B}\right)^{\beta}\le AG_1\pi(A\square B)\le \left(\frac{\Delta_A+\Delta_B}{\delta_A+\delta_B}\right)^{\beta}$,

(d) $\left(\frac{1}{\Delta_A+\Delta_B}\right)^\beta \le H\pi(A\square B) \le \left(\frac{1}{\delta_A+\delta_B}\right)^\beta$.

Theorem 3. *Let $A = A_1 +_R B_1$ and $B = A_2 +_R B_2$, then*

(a) $\frac{\gamma}{\Delta_A+\Delta_B} \le R(A\square B) \le \frac{\gamma}{\delta_A+\delta_B}$,

(b) $\frac{\gamma}{\sqrt{2(\Delta_A+\Delta_B)}} \le SCI(A\square B) \le \frac{\gamma}{\sqrt{2(\delta_A+\delta_B)}}$,

(c) $\gamma\left(\frac{\delta_A+\delta_B}{\Delta_A+\Delta_B}\right) \le AG_1(A\square B) \le \gamma\left(\frac{\Delta_A+\Delta_B}{\delta_A+\delta_B}\right)$,

(d) $\frac{\gamma}{\Delta_A+\Delta_B} \le H(A\square B) \le \frac{\gamma}{\delta_A+\delta_B}$,

where $r_1 = b_1(a_1+a'_1)$, $r_2 = b_2(a_2+a'_2)$, $s_1 = b'_1a_1+3a'_1b_1$, $s_2 = b'_2a_2+3a'_2b_2$, $\gamma = r_1s_2+s_1r_2$, $\delta_A+\delta_B = \delta_{A_1}+\delta_{A_2}+\delta_{B_1}+\delta_{B_2}$ *and* $\Delta_A+\Delta_B = \Delta_{A_1}+\Delta_{A_2}+\Delta_{B_1}+\Delta_{B_2}$.

Corollary 3. *Let $A = A_1 +_R B_1$ and $B = A_2 +_R B_2$, then*

(a) $\left(\frac{1}{\Delta_A+\Delta_B}\right)^\gamma \le \chi(A\square B) \le \left(\frac{1}{\delta_A+\delta_B}\right)^\gamma$,

(b) $\left(\frac{1}{\sqrt{2(\Delta_A+\Delta_B)}}\right)^\gamma \le X\pi(A\square B) \le \left(\frac{1}{\sqrt{2(\delta_A+\delta_B)}}\right)^\gamma$,

(c) $\left(\frac{\delta_A+\delta_B}{\Delta_A+\Delta_B}\right)^\gamma \le AG_1\pi(A\square B) \le \left(\frac{\Delta_A+\Delta_B}{\delta_A+\delta_B}\right)^\gamma$,

(d) $\left(\frac{1}{\Delta_A+\Delta_B}\right)^\gamma \le H\pi(A\square B) \le \left(\frac{1}{\delta_A+\delta_B}\right)^\gamma$.

Theorem 4. *Let $A = A_1 +_T B_1$ and $B = A_2 +_T B_2$, then*

(a) $\frac{\eta}{\Delta_A+\Delta_B} \le R(A\square B) \le \frac{\eta}{\delta_A+\delta_B}$,

(b) $\frac{\eta}{\sqrt{2(\Delta_A+\Delta_B)}} \le SCI(A\square B) \le \frac{\eta}{\sqrt{2(\delta_A+\delta_B)}}$,

(c) $\eta\left(\frac{\delta_A+\delta_B}{\Delta_A+\Delta_B}\right) \le AG_1(A\square B) \le \eta\left(\frac{\Delta_A+\Delta_B}{\delta_A+\delta_B}\right)$,

(d) $\frac{\eta}{\Delta_A+\Delta_B} \le H(A\square B) \le \frac{\eta}{\delta_A+\delta_B}$,

where $r_1 = \frac{1}{2}\left[b_1\left(a_1+a'_1\right)\right]$, $r_2 = \frac{1}{2}\left[b_2\left(a_2+a'_2\right)\right]$, $s_1 = \frac{1}{2}\left[2b'_1a_1+b_1a'_1\left(a'_1+5\right)\right]$, $s_2 = \frac{1}{2}\left[2b'_2a_2+a'_2b_2\left(a'_2+5\right)\right], \eta = r_1s_2+s_1r_2$, $\delta_A+\delta_B = \delta_{A_1}+\delta_{A_2}+\delta_{B_1}+\delta_{B_2}$ *and* $\Delta_A+\Delta_B = \Delta_{A_1}+\Delta_{A_2}+\Delta_{B_1}+\Delta_{B_2}$.

Corollary 4. *Let $A = A_1 +_T B_1$ and $B = A_2 +_T B_2$, then*

(a) $\left(\frac{1}{\Delta_A+\Delta_B}\right)^\eta \le \chi(A\square B) \le \left(\frac{1}{\delta_A+\delta_B}\right)^\eta$,

(b) $\left(\frac{1}{\sqrt{2(\Delta_A+\Delta_B)}}\right)^\eta \le X\pi(A\square B) \le \left(\frac{1}{\sqrt{2(\delta_A+\delta_B)}}\right)^\eta$,

(c) $\left(\frac{\delta_A+\delta_B}{\Delta_A+\Delta_B}\right)^\eta \le AG_1\pi(A\square B) \le \left(\frac{\Delta_A+\Delta_B}{\delta_A+\delta_B}\right)^\eta$,

(d) $\left(\frac{1}{\Delta_A+\Delta_B}\right)^\eta \le H\pi(A\square B) \le \left(\frac{1}{\delta_A+\delta_B}\right)^\eta$.

3. Conclusion

Topological indices have a significance importance in the study of physicochemical properties of chemical compounds. In this paper lower and upper bounds for some of the degree based topological indices of cartesian product of F-sum graphs are computed. However, these results can be extended to other topological indices for different product of F-sum graphs.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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