

BOUNDS OF THE PERIMETER OF AN ELLIPSE USING ARITHMETIC, GEOMETRIC AND HARMONIC MEANS

MIAO-KUN WANG, YU-MING CHU, YUE-PING JIANG AND SONG-LIANG QIU

Abstract. In this paper, we present several bounds for the perimeter of an ellipse in terms of arithmetic, geometric, and harmonic means, which improve some known results.

Mathematics subject classification (2010): 41A10, 33E05, 33C05, 26E60.

Keywords and phrases: Perimeter of an ellipse, arithmetic mean, geometric mean, harmonic mean, Toader mean.

REFERENCES

- [1] M. ABRAMOWITZ AND I. A. STEGUN, EDS., *Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables*, Dover, New York, 1992.
- [2] G. ALMKVIST AND B. BERNDT, *Gauss, Landen, Ramanujan, the arithmetic-geometric mean, ellipse, π , and the Ladies Diary*, Amer. Math. Monthly **95** (1988), 585–608.
- [3] H. ALZER AND S.-L. QIU, *Monotonicity theorems and inequalities for the complete elliptic integrals*, J. Comput. Appl. Math. **172** (2004), 289–312.
- [4] G. D. ANDERSON, S.-L. QIU, M. K. VAMANAMURTHY AND M. VUORINEN, *Generalized elliptic integrals and modular equations*, Pacific J. Math. **192** (2000), 1–37.
- [5] G. D. ANDERSON, M. K. VAMANAMURTHY AND M. VUORINEN, *Conformal Invariants, Inequalities, and Quasiconformal Maps*, John Wiley & Sons, New York, 1997.
- [6] R. W. BARNARD, K. PEARCE AND K. C. RICHARDS, *An inequality involving the generalized hypergeometric function and the arc length of an ellipse*, SIAM J. Math. Anal. **31** (2000), 693–699.
- [7] R. W. BARNARD, K. PEARCE AND K. C. RICHARDS, *A monotonicity property involving ${}_3F_2$ and comparisons of the classical approximations of elliptical arc length*, SIAM J. Math. Anal. **32** (2000), 403–419.
- [8] R. W. BARNARD, K. PEARCE AND L. SCHOVANEC, *Inequalities for the perimeter of an ellipse*, J. Math. Anal. Appl. **260** (2001), 295–306.
- [9] B. C. BERNDT, *Ramanujan's Notebooks*, Part III, Springer-Verlag, New York, 1991.
- [10] J. M. BORWEIN AND P. B. BORWEIN, *Pi and the AGM*, John Wiley & Sons, New York, 1987.
- [11] B. C. CARLSON, *Special Functions of Applied Mathematics*, Academic Press, New York, 1977.
- [12] T. R. CHANDRUPATLA AND T. J. OSLER, *The perimeter of an ellipse*, Math. Sci. **35** (2010), 122–131.
- [13] C.-P. CHEN AND F. QI, *The best bounds in Wallis' inequality*, Proc. Amer. Math. Soc. **133** (2005), 397–401.
- [14] Y.-M. CHU AND M.-K. WANG, *Optimal Lehmer mean bounds for the Toader mean*, Result. Math., **61** (2012), 223–229.
- [15] S. PONNUSAMY AND M. VUORINEN, *Univalence and convexity properties of Gaussian hypergeometric functions*, Rocky Mountain J. Math. **31** (2001), 327–353.
- [16] S.-L. QIU AND M. VUORINEN, *Special functions in geometric function theory*, in: Handbook of Complex Analysis: Geometric Function Theory, Vol. 2, Elsevier Sci. B. V., Amsterdam, 2005, pp. 621–659.
- [17] E. D. RAINVILLE, *Special Functions*, MacMillan, New York, 1960.
- [18] GH. TOADER, *Some mean values related to the arithmetic-geometric mean*, J. Math. Anal. Appl. **218** (1998), 358–368.

- [19] M. VUORINEN, *Hypergeometric functions in geometric function theory*, Special functions and differential equations (Madras, 1997), 119–126, Allied Publ., New Delhi, 1998.
- [20] E. T. WHITTAKER AND G. N. WATSON, *A Course of Modern Analysis*, 4th ed., Cambridge Univ. Press, New York, 1962.