

## BOUNDS ON THE COMPACTNESS OF NEUTRON STARS FROM BRIGHTNESS OSCILLATIONS DURING X-RAY BURSTS

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### ABSTRACT

The discovery of high-amplitude brightness oscillations at the spin frequency or its first overtone in six neutron stars in low-mass X-ray binaries during type I X-ray bursts provides a powerful new way to constrain the compactness of these stars and hence to constrain the equation of state of the dense matter in all neutron stars. Here we report general relativistic calculations of the maximum fractional rms amplitudes that can be observed during bursts, as a function of stellar compactness. We compute the dependence of the oscillation amplitude on the compactness of the star, on the angular dependence of the emission from the surface, on the rotational velocity at the stellar surface, and on whether there are one or two emitting spots. We show that color oscillations caused by the spectral variation with the angle of emission, the rotation of the star, and the limited bandwidth of the detector all tend to increase the observed amplitude of the oscillation. Nevertheless, if two spots are emitting, as appears to be the case in 4U 1636–536 and KS 1731–26, very restrictive bounds on the compactness of the star can be derived.

*Subject headings:* equation of state — gravitation — relativity — stars: neutron — X-rays: bursts

### 1. INTRODUCTION

The determination of the equation of state of neutron star matter has been an important goal of nuclear physics for more than two decades. Progress toward this goal can be made by establishing the properties of neutron stars astrophysically (see, e.g., Lindblom 1992) as well as by improving our understanding of nuclear forces.

Many ways of constraining the properties of neutron stars have been explored. One of the best known is pulse timing of pulsars in binary systems. Although binary pulsar timing has made possible stringent tests of general relativity (see, e.g., Taylor 1992), the  $\approx 1.4 M_{\odot}$  masses derived from timing (see Thorsett et al. 1993) are allowed by all equations of state based on realistic nuclear physics, and hence these observations have not eliminated any of the equations of state currently being considered. The highest known neutron star spin frequency, the 643 Hz frequency of PSR 1937+21, is also allowed by all equations of state currently under consideration. Radius estimates based on the energy spectra of type I (thermonuclear) X-ray bursts and on observations of thermal emission from the surfaces of neutron stars are more restrictive in principle but currently have large systematic uncertainties (see Lewin, van Paradijs, & Taam 1993 and Ögelman 1995).

The discovery of high-frequency brightness oscillations from 16 neutron stars in low-mass X-ray binaries with the *Rossini X-Ray Timing Explorer (RXTE)* holds great promise for providing important new constraints. Oscillations are observed both in the persistent X-ray emission and during type I X-ray bursts. The kilohertz quasi-periodic oscillations (QPOs) observed in the persistent emission have high amplitudes and relatively high coherences (see van der Klis 1998). Two kilohertz QPOs are commonly observed simultaneously in a given source. Although the frequencies of these two QPOs vary by as much as a factor of  $\sim 2$ , the frequency separation  $\Delta\nu$  between them appears to be almost constant in all cases. In both the sonic-

point (Miller, Lamb, & Psaltis 1998) and magnetospheric (Strohmayer et al. 1996) beat-frequency interpretations, the higher frequency in a pair is the orbital frequency at the inner edge of the Keplerian flow, whereas the lower frequency is the beat of the stellar spin frequency with a frequency close to the higher frequency. Such high orbital frequencies yield interesting bounds on the masses and radii of these neutron stars and significant constraints on the equation of state of neutron star matter (Miller et al. 1998).

The brightness oscillations observed during type I X-ray bursts are different in character from the QPOs observed in the persistent emission (see Strohmayer, Zhang, & Swank 1997). Only a single oscillation has been observed from each source during a type I X-ray burst, and the oscillations in the tails of bursts appear to be highly coherent (see, e.g., Smith, Morgan, & Bradt 1997), with frequencies that are always the same for a given source (a comparison of burst oscillations from 4U 1728–34 over about a year shows that the timescale for any variation in the oscillation frequency is  $\geq 3000$  yr; Strohmayer 1997). The burst oscillations in 4U 1728–34 (Strohmayer et al. 1996, 1997) and 4U 1702–42 (Swank 1997) have frequencies that are consistent with the separation frequencies of their kilohertz QPO pairs. The burst oscillations in 4U 1636–536 (Zhang et al. 1997) and KS 1731–260 (Smith et al. 1997; Wijnands & van der Klis 1997) have frequencies that are consistent with twice the separation frequencies of their kilohertz QPO pairs. The burst oscillations are thought to be caused by emission from one or two regions of brighter X-ray emission on the stellar surface, producing oscillations at the stellar spin frequency or its first overtone, respectively (see, e.g., Strohmayer et al. 1997 for compelling evidence in favor of this interpretation). If so, the anisotropy, observed at infinity, of the radiation emitted from the surface, and hence the amplitude of the burst oscillations, typically decreases with increasing gravitational light deflection. The burst oscillations

can therefore be used to constrain the compactness of neutron stars and the equation of state of neutron star matter (Strohmayer 1997).

In this Letter, we report general relativistic calculations of the maximum relative amplitudes of burst oscillations, as a function of stellar compactness, following the procedure described by Pechenick, Ftaclas, & Cohen (1983; see also Chen & Shaham 1989 and Strohmayer 1992). We go beyond these previous treatments by using a more realistic angular intensity distribution at the stellar surface, by including the angle dependence of the energy spectrum, by considering the effects of X-ray color oscillations, and by computing the amplitudes at overtones. We show that gravitational light deflection typically reduces the relative amplitude of the oscillations produced by two spots compared with one spot and that effects other than Doppler shifts and aberration are second order in the rotation rate. We describe our assumptions and method in § 2. In § 3, we present our results and discuss the observational signatures that would provide confidence in the limits on the compactness derived from burst oscillations.

## 2. ASSUMPTIONS AND METHOD

### 2.1. Assumptions

The main purpose of our calculations is to derive constraints on the compactness of the neutron stars that display oscillations during type I X-ray bursts. To do this, we must determine the largest possible amplitude of an oscillation for a given compactness. We therefore make the following assumptions, which maximize the oscillation amplitude (these are discussed further in § 3):

1. The radiation that we see comes directly from the stellar surface.
2. The star radiates from one or two emitting spots; the rest of the surface is dark.
3. The spot or spots are pointlike and located in the rotational equator.
4. If there are two spots, they are identical and antipodal.
5. The observer's line of sight is in the rotational equator.

The effects on the spacetime of the star's rotation change the oscillation amplitude only to second order (see Lamb & Miller 1995 and Miller & Lamb 1996 for details) and are much smaller than first- and second-order Doppler effects for the relatively slow rotation rates of interest. We therefore neglect them, performing all calculations in the Schwarzschild spacetime.

### 2.2. Calculational Method

We compute the intensity seen by an observer when an emitting spot is at a given rotational phase, using the procedure discussed by Pechenick et al. (1983). Let us suppose that the emitting spot is at azimuthal angle  $\phi = 0$  and that the azimuthal angle of the photon when it reaches infinity is  $\phi_{\text{obs}}$ ;  $|\phi_{\text{obs}}|$  is a maximum when the photon is emitted tangentially to the stellar surface in the rotation equator. For a star of mass  $M$  and radius  $R$ , this maximum angle is (Pechenick et al. 1983, eq. [2.13])

$$\phi_{\text{max}} = \int_0^{M/R} \left[ \left(1 - \frac{2M}{R}\right) \left(\frac{M}{R}\right)^2 - (1 - 2u)u^2 \right]^{-1/2} du. \quad (1)$$

Here and below, we use Schwarzschild coordinates and geometrized units in which  $G = c \equiv 1$ .

Let us suppose that the observer is at infinity at azimuthal angle  $\phi_{\text{obs}}$ . If  $\phi_{\text{obs}} > \phi_{\text{max}}$ , the observer cannot see the spot. If  $\phi_{\text{obs}} < \phi_{\text{max}}$ , we solve for  $u_b \equiv M/b$ , the reciprocal of the impact parameter to the spot, using (Pechenick et al. 1983, eq. [2.12])

$$\phi_{\text{obs}} = \int_0^{M/R} [u_b^2 - (1 - 2u)u^2]^{-1/2} du. \quad (2)$$

In the Schwarzschild spacetime, the angle  $\psi$  to the normal at the stellar surface of the light ray that has impact parameter  $b$  at infinity is given implicitly by  $\cos \psi = (1 - b^2/b_{\text{max}}^2)^{1/2}$ , where the maximum impact parameter is  $b_{\text{max}} = R(1 - 2M/R)^{-1/2}$  (see, e.g., Abramowicz, Ellis, & Lanza 1990 and Miller & Lamb 1996). The flux measured by the observer is proportional to  $I_{\nu}[\psi(\phi_{\text{obs}})] \cos \psi(\phi_{\text{obs}})$ , where  $I_{\nu}(\psi)$  is the specific intensity at photon frequency  $\nu$  and angle  $\psi$  from the normal to the spot. As usual,  $I_{\nu}/\nu^3$  and  $I/\nu^4$ , where  $I$  is the frequency-integrated specific intensity, are constant in the absence of absorption or scattering. Hence, changes in  $I_{\nu}$  and  $I$  caused by Doppler shifts and redshifts are easily computed by calculating the change in the frequency of the photons.

If the star were not rotating and the emission from the spot were isotropic, then  $I(\psi)$  would be constant. However, because the star is rotating, the intensity distribution is aberrated, and the photon frequency is Doppler-shifted by the factor of  $1/[\gamma(1 - v \cos \zeta)]$ , where  $v$  is the velocity at the stellar equator,  $\gamma = (1 - v^2)^{-1/2}$ , and  $\zeta$  is the angle between the local direction of motion and the direction of propagation of the photon. Also, the specific intensity is not expected to be isotropic even in the rest frame of the stellar surface, because radiation that diffuses outward through an optically thick scattering atmosphere emerges with a specific intensity distribution  $I_{\text{sc}}(\psi)$  that is nearly 3 times as bright along the normal as in directions tangential to the surface (see Chandrasekhar 1960, chap. 3). Moreover, the energy spectrum will also, in general, depend on the angle from the normal. The angle dependence of the actual spectrum is likely to be complicated (see, e.g., London, Howard, & Taam 1986). However, the qualitative effects of angle dependence are illustrated by a simplified emission model in which the spectrum emerging at angle  $\psi$  is a blackbody spectrum with a temperature  $T$  given by  $T^4 = \frac{3}{4} T_{\text{eff}}^4 (\frac{2}{3} \cos \psi + 0.7)$ , where  $T_{\text{eff}}$  is the effective temperature; this approximates the angle dependence of the energy spectrum from a gray atmosphere.

Using this approach, we compute the brightness seen by an observer at infinity as a function of rotational phase, for a given specific intensity distribution and rotation rate. Intensities can be added linearly, so the addition of a second spot is straightforward. We Fourier-analyze the resulting light curve to determine the oscillation amplitudes at different harmonics of the spin frequency.

## 3. RESULTS AND DISCUSSION

The results of our calculations are summarized in Figure 1, which shows the fractional rms amplitudes of photon number flux oscillations in various situations. Following standard practice (see, e.g., van der Klis 1989, § 4), we always quote the *rms* amplitude; note, however, that a few authors (e.g., Strohmayer et al. 1997) quote half the *peak-to-peak* amplitude, which

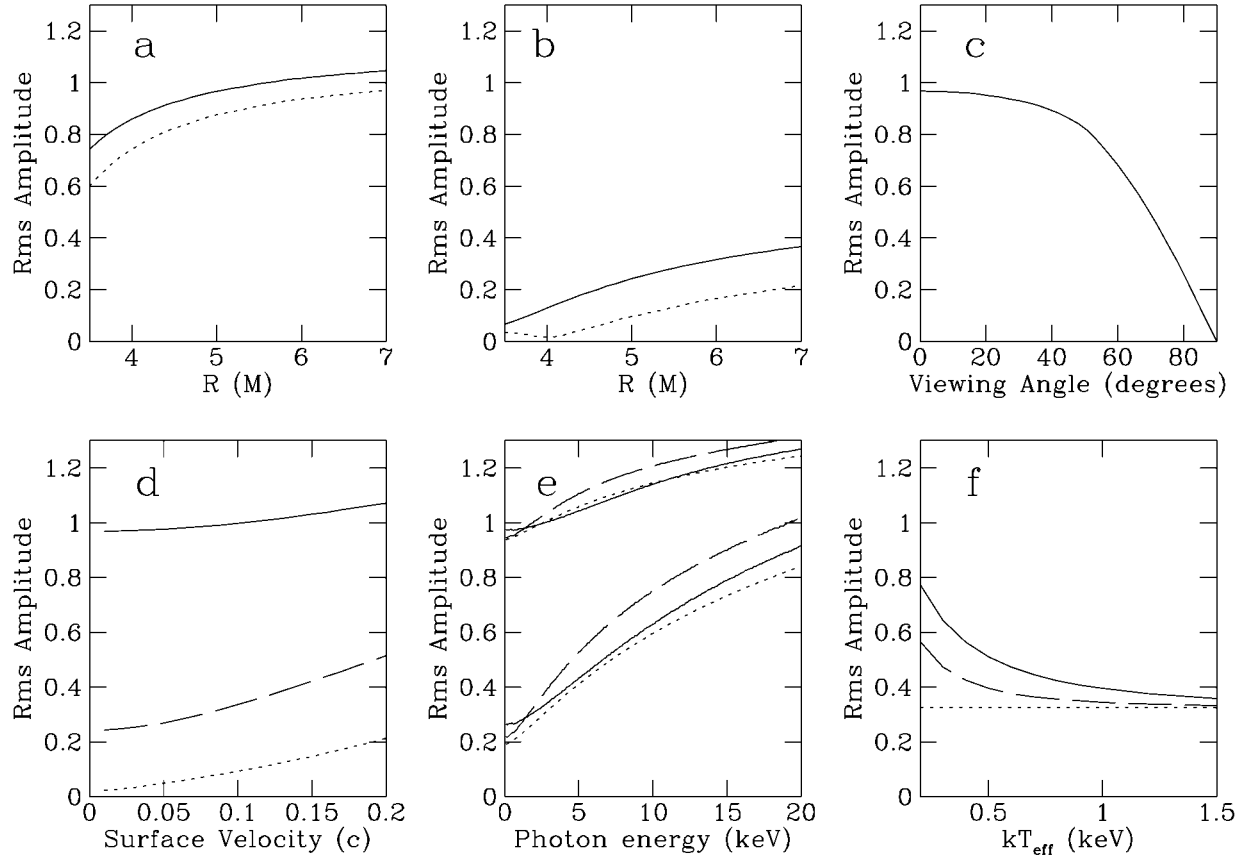


FIG. 1.—Fractional rms amplitude of number flux oscillations for various situations. The observer’s line of sight is assumed to be in the rotational equator, except in (c). The aberration and the Doppler shifts caused by rotation are neglected in (a)–(c). (a) Amplitude as a function of neutron star radius for isotropic emission (dotted line) or the peaked emission  $I_{\text{sc}}(\psi)$  (solid line) from a single spot. (b) Amplitude as a function of radius for the same intensity distributions as in (a), but for two identical, antipodal emitting spots. (c) Amplitude as a function of the angle between the observer’s line of sight and the rotation equator for a single emitting spot,  $R = 5M$ , and  $I(\psi) = I_{\text{sc}}(\psi)$ . (d) Amplitude as a function of surface velocity for one emitting spot,  $R = 5M$ , and  $I(\psi) = I_{\text{sc}}(\psi)$ , at the fundamental of the spin frequency (solid line), the first overtone (dashed line), and the second overtone (dotted line). (e) Amplitude as a function of photon energy for one emitting spot (upper three curves) and two emitting spots (lower three curves), assuming  $R = 5M$  and  $I(\psi) = I_{\text{sc}}(\psi)$  for a surface velocity  $v = 0.1$  and an energy spectrum that is angle independent in the rest frame of the stellar surface (solid lines); for  $v = 0$  and the angle-dependent spectrum described in the text (dotted lines); and for  $v = 0.1$  and the angle-dependent spectrum (dashed lines). (f) Amplitude as a function of effective temperature for two emitting spots,  $R = 5M$ ,  $I(\psi) = I_{\text{sc}}(\psi)$ ,  $v = 0.1$ , and the angle-dependent energy spectrum. The three curves show the amplitude measured by a bolometer (dotted line) and by instruments with two different boxcar energy responses: 4–20 keV (solid line) and 2–20 keV (dashed line). Each detected photon is assumed to produce exactly one count.

is a factor of 1.4 larger for a purely sinusoidal waveform. The amplitudes of oscillations in the photon number and total energy fluxes are equal for nonrotating stars but are generally unequal for rotating stars. Figure 1a compares the oscillation amplitude for isotropic emission and for the peaked emission  $I_{\text{sc}}(\psi)$  from a single spot. The rms amplitude of a nonsinusoidal waveform can be greater than unity, as shown. Figure 1b shows the fractional rms amplitude at the first overtone under the same assumptions, for two identical, antipodal emitting spots. Figure 1c shows the amplitude as a function of the angle between the observer’s line of sight and the rotation equator for a single emitting spot. Figure 1d illustrates the generation of harmonics by Doppler shifts and aberration. Figure 1e shows the amplitude as a function of photon energy for different assumptions about the surface velocity and the rest-frame angle dependence of the energy spectrum. Figure 1f plots the amplitude from two emitting spots as detected by instruments with different bandpasses, as a function of the effective temperature of the surface.

These results illustrate several points: (1) The maximum relative amplitude is significantly greater when one spot is emitting than when two spots are emitting. (2) The amplitude tends to be higher when the neutron star is less compact, i.e.,

when  $R/M$  is larger. The exception occurs for isotropic emission from two spots and very compact neutron stars,  $R < 4M$ , in which case gravitational focusing of emission from the spot on the opposite side of the star from the observer can increase the amplitude (this effect was discussed by Pechenick et al. 1983). (3) The angular distribution of the specific intensity makes a large difference. For example, for  $M = 1.4 M_{\odot}$ ,  $R = 10 \text{ km} = 4.8M$ , and two emitting spots, the maximum amplitude for  $I(\psi) = I_{\text{sc}}(\psi)$  is nearly 3 times greater than the maximum amplitude for isotropic emission. (4) Stellar rotation affects the bolometric amplitude only to second order. However, the rotation shifts the energy spectrum by an amount that is first order in the rotational velocity. As a result, the measured relative count rate amplitude depends strongly on photon energy if the energy spectrum within the bandpass of the instrument is steep, and it may be large even for two emitting spots. As illustrated in Figure 1f, the measured rms amplitude can exceed  $\sim 70\%$  for two emitting spots. The amplitude also depends strongly on the photon energy if, as expected, the dependence of the rest-frame energy spectrum on the angle from the normal is steeper at higher energies. In this case, the amplitude will rise with increasing photon energy, even if Doppler

shifts and aberration are unimportant. (5) Rotation generates significant power at overtones of the spin frequency. (6) The oscillation amplitude is proportional to the cosine of the angle  $\alpha$  between the line of sight of the observer and the rotation equator, and hence the observed amplitude is near its maximum value over a large solid angle: for example, the observed amplitude for  $\alpha$  between  $0^\circ$  and  $30^\circ$ , which occurs 50% of the time for randomly distributed observers, is within 5% of the maximum.

Our results are relevant for determining whether the  $\sim 600$  Hz burst oscillations seen in 4U 1636–536 and KS 1731–26 are at the fundamental or the first overtone of the spin frequency. For example, if a burst oscillation with an rms amplitude greater than 40% is observed and color oscillations are shown to be insignificant, this would strongly favor emission from a single spot and hence would imply that the oscillation is at the star's spin frequency. The reason is that a bolometric oscillation with such a large amplitude is not possible for two antipodal spots (see Fig. 1*b*). If, on the other hand, the amplitude of any oscillation at 1200 Hz is shown to be significantly less than 0.3 times the amplitude of the oscillation at 600 Hz, this would tend to favor a spin frequency of 300 Hz and hence would imply emission from two antipodal spots. The reason is that for a star with a small emitting spot and a radius  $R \approx 10$  km, a spin frequency of 600 Hz produces a surface velocity  $v \approx 0.13c$ , which would generate an oscillation at the first overtone with an amplitude of at least 30% of the amplitude at the fundamental (see Fig. 1*d*).

These results have been computed by assuming that the only emission from the surface is from pointlike spots, that if there are two spots then they are identical and antipodal, and that the emitting spot or spots are in the spin equator. These assumptions produce the largest possible amplitude. Small deviations from them affect the amplitude of the oscillation only to second order in the deviation, as is evident from the results of Pechenick et al. (1983). For example, if emission at the spot comes from a cap of angular radius  $\delta \ll 1$  instead of from a point, then the amplitude is decreased by an amount  $\sim \delta^2$ . If there are two spots and they are offset from antipodal by a small angle  $\beta$ , then the amplitude at twice the spin frequency drops by  $\sim \beta^2$ . If the axis of the spot or spots is offset from the spin equator by a small angle  $\epsilon$ , the amplitude is diminished by  $\sim \epsilon^2$ .

In deriving bounds on the stellar compactness, we have as-

sumed that we are seeing emission directly from the stellar surface. The stellar surface certainly is not observed directly during any phase of photospheric expansion, which is detectable by a variety of effects (see Lewin et al. 1993). Absorption and reradiation by such an expanded atmosphere will diminish the oscillation amplitude. We also may not be seeing the surface directly near the onset or the end of the burst, if the star is surrounded by accreting gas that scatters the radiation from the surface. Scattering by a spherically symmetric cloud tends to isotropize the radiation field and hence to diminish the observed oscillation amplitude (see, e.g., Brainerd & Lamb 1987; Kylafis & Phinney 1989; Miller 1997). Both absorption and scattering by the surrounding gas would therefore strengthen the constraints on the stellar compactness. A significantly anisotropic cloud could, in principle, increase the oscillation amplitude but would almost certainly produce other detectable signatures, such as a change in the relative amplitudes of different harmonics of the spin frequency.

Comptonization by gas surrounding the star could, under some circumstances, increase the measured amplitude of brightness oscillations. The reason is that even relatively small changes in the optical depth or temperature of accreting gas that is corotating with the star could produce large variations with rotational phase in the flux at high energies, and these changes could thereby increase the observed amplitude of oscillations (see Miller et al. 1998 and Miller & Lamb 1992). If the radiation from the surface is significantly Comptonized by the surrounding gas, the spectrum will generally be distorted and will develop an extended high-energy tail (see, e.g., Psaltis, Lamb, & Miller 1995). Therefore, the measurement of a momentary energy spectrum that is close to a modified blackbody and does not have a high-energy tail is evidence that the spectrum from the surface is not significantly distorted by Comptonization and that the bounds we have derived apply.

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