BRANCH-AND-BOUND SCHEDULING FOR THERMAL GENERATING UNITS

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ABSTRACT - Scheduling thermal generation units plays an important role in power system economic operations. Each day power generating units have to be selected to realize a reliable production of electric energy with the fewest fuel costs. This paper presents a new branch-and-bound algorithm for the unit scheduling problem. An efficient branching method based on the 'heap' data structure and a simple intuitive bounding rule are proposed. Computational results indicate that the presented approach locates the optimum schedule in less time than many existing techniques.

INTRODUCTION

To economically commit available thermal generating units under constraints is still the subject under intensive research[1-15]. Costs of unit commitment are incurred both from starting up generating units and from dispatching them. The overall cost over the study period is to be minimized and the variables to be determined are the hourly statuses of generating units, namely ON(committed) or OFF(uncommitted).

Constraints in the unit commitment problem can naturally be divided into two categories : the "coupling" constraints and the "local" constraints. The "coupling" constraints reflect the sum of the power generated by all units. This whole generation must meet the system demand, including network transmission losses and spinning reserves. The "local" constraints deal with each thermal unit individually. The first is the upper and lower limits on generated power, if the unit is ON. And there are technical constraints, together with the requirement of limiting equipment fatigue, leading to the imposition of minimum up times and minimum down times.

Optimal unit commitment is the cheapest production policy that selects the most economical start-up and shut-down times for each unit such that all constraints are satisfied for the study period. Many attempts have been made to solve this problem. References [1-4] propose solution methods based on unit priority lists and heuristic rules to improve a given feasible solution. In references [5, 6] a solution methodology based on duality, Lagrangian relaxation, and nondifferentiable optimization is described. Merlin and Sandrin [7] propose another Lagrangian relaxation method: this is a decomposition method using Lagrange multipliers which provides a new solution to the conventional problem of thermal unit commitment. Dillion et al. [8] and Pang et al. [9] formulate the unit commitment problem as a linear mixed-integer programming problem and then use standard integer programming algorithms to solve for the commitment schedule. Dynamic programming has long been used by many authors to solve the unit commitment problem [10-15]. In order to keep the problem tractable, these methods assume some partial ordering which specifies which set of units can be used for dispatch at a given load level.

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Among all, the branch-and-bound approach [16] is of particular interest because of its ability to handle operating constraints. Fox and Bond [17] present a method to allow incorporating an existing dispatch algorithm into a structured binary search for the unit commitment solution. In [18], Ohuchi and Kaji propose a branch-and-bound algorithm which is more efficient than the dynamic programming method by their comparison results. Like dynamic programming, branch-and-bound is an intelligently structured search of the space of all feasible solutions. Most commonly, the space of all feasible solutions is repeatedly partitioned into smaller and smaller subsets, and a lower bound (in the case of minimization) is calculated for the cost of the solutions within each subset. After each partitioning, those subsets with a bound that exceeds the cost of a known feasible solution are excluded from all further partitionings. The partitioning continues until a feasible solution is found such that its cost is no greater than the bound for any subset.

In this paper, a new branch-and-bound algorithm for the unit scheduling problem is presented. The mathematical formulation of the scheduling problem is depicted in Section 2. In Section 3, an efficient branching method based on the 'heap' data structure and a simple intuitive bounding rule are proposed. Preliminary computational results of two utility systems are shown in Section 4. Section 5 gives the conclusions.

PROBLEM FORMULATION

In mathematical terms, the unit commitment problem is described as

$$\text{Min} \sum_{k=1}^{M}\sum_{i=1}^{N} C_i[P_i(k)]$$

with constraints

$$\sum_{i=1}^{N} P_i(k) = GR(k) \qquad k=1,...,M$$

$$I_{i,k}*\underline{P}_i < P_i(k) < I_{i,k}*\overline{P}_i$$

$$[t_{on}(i,k-1) - T_{on}(i)] * (I_{i,k-1} - I_{i,k}) > 0$$

$$[t_{off}(i,k-1) - T_{off}(i)] * (I_{i,k-1} - I_{i,k}) > 0$$

where the notations used are

$P_i(k)$: generation of the ith unit at stage k

$C_i[\ ]$: cost function of unit i

N : number of units to be scheduled

M : number of stages

GR(k) : generation requirement at stage k, which includes load demand, spinning reserve, and network losses

$\underline{P}_i$, $\overline{P}_i$: lower and upper generation limits of unit i

$T_{on}(i)$, $T_{off}(i)$: minimum up and down time of unit i

$t_{on}(i,k)$, $t_{off}(i,k)$: the time period for unit i having been continuously up or down till stage k

THE ALGORITHM

In this section, the proposed branch-and-bound algorithm and the operations of embedded data structures are described.

Data Representations

In a convenient way, the commitment schedule of generating units is usually represented by a matrix, called the commitment matrix. Every row of this matrix represents the schedule of a generating unit and every column denotes the unit schedule for each hour. It records the commitment statuses ("0" for OFF, "1" for ON, and "x" for undetermined) of all generating units hour by hour from the beginning to the end. It is noted that this kind of data representation is clear and straightforward, but it consumes a lot of memory space. Storing each partially fulfilled scheme for interim computation requires an NxM matrix. Thus while a larger scaled system is under study, this situation would be intolerable.

A new way to store data based on the concept of file compression [19] is proposed for the unit commitment schedule. It is primarily designed to reduce space consumption without using additional computing time. The idea is simply to store the run lengths of 0/1 strings, taking advantage of the fact that the runs alternate between 0 and 1 to avoid storing the 0's and 1's themselves. For this idea to work satisfactorily there must be few short runs and this is indeed the situation in the unit commitment problem. Because of the operating constraints, the shortest run lengths of '1' and '0' are not less than the hours of minimum up times and minimum down times, respectively. Also it is observed that for most practical utility systems under normal operations, their daily load cycles have one major peak or two peaks relatively close to each other. Unit start-up costs and minimum up and down time constraints preclude multiple start-ups. Thus, it is assumed in this paper that generating units with nonzero start-up cost will cycle at most once in a 24-hour period. This cycling assumption allows us to replace the commitment decision variable of a unit is at each hour with the indication of the start and stop time of the unit. Thus the problem's decision space is considerably reduced. The compressed representation is an Nx3 matrix. The first column of the compressed commitment matrix denotes the hours already scheduled. The second and third columns signify the start-up and the shut-down times, respectively. As illustration, the complete commitment matrix and its compressed counterpart for an 8-hour unit commitment schedule of a 4-unit system are given in Fig. 1.

1 1 1 1 x x x x	4 1 0
1 1 0 0 0 x x x	5 1 3
1 1 1 x x x x x	3 1 0
0 0 0 0 x x x x	4 0 1
(a) the complete matrix	(b) compressed form

Fig.1 the commitment matrix and its compressed representation

Lower Bound Computation

To compute the lower bound for each partially fulfilled commitment schedule, we define a simple and efficient method : assuming available for generation, removing the lower generation limits, and ignoring the start-up costs for all undetermined states in the commitment matrix. This method comes from a practical concept that adding generating units without start-up costs and lower generation limits for contributing to the same amount of load requirement will not incur higher dispatching cost. The worst case is at most as high as when all added units contribute nothing to the total generation. Since the best schedule is among all possible combinations of 'ON' and 'OFF', the cost computed by this method must be not higher than the optimum value of this set of all commitment schedules. The obtained value will be a lower bound of the objective function. This lower bound is acquired by violating the constraint of the lower generation limit and ignoring the start-up cost. It gives a rough measure of the possibility of finding the optimum schedule.

The lower bounding procedure involves the economic dispatch problem as a subproblem. That is, for any selected subset of the total number of units to provide the generation requirement they should be operated in the optimum economic fashion. The λ-iteration method [20] is the most popular approach for computing the dispatch cost. For comparison, the approximation method of base point and participation factors [20] is also used in the examples.

Heapsort for Lowest-first Branching

We always focus our attention on the set of combinations which has the largest possibility of containing the optimum solution. It is obvious that the partial commitment matrix with the lowest bound has the potential to generate the lowest-cost commitment schedule. So the branch-and-bound processes proceed with the set of the lowest bound.

The processes of branching with the set of which lower bound is the lowest and computing lower bounds for the created subsets are continued. With this branch-and-bound process proceeds, the lowest bound monotonically grows because more unit statuses are specified and fewer units contribute to the load requirement in a specific time interval. This process stops when the lowest bound belongs to a completed commitment schedule. This completed commitment schedule is thus the optimum solution, because other solutions will incur costs higher than the lowest bound.

Among the data manipulating algorithms, heapsort[21] is especially appropriate for the branch-and-bound process described above. Heapsort permits one to insert elements into a set and also to find the best element efficiently. The embedded data structure is a heap, which is a complete binary tree with the property that the value associated with each element is at least as large as the values with its children (if they exist).

To our purpose, the set with the lowest bound is placed at the root of the heap and the lower bound associated with each set is at least as small as the values with its children. We always branch with the set at the root and each branching will generate two subsets. After computing lower bounds for these two newly generated set, the one with the smaller value is placed at the root and the other one is appended at the bottom of the heap to retain the shape of a complete binary tree. In order not to violate the ordering property of a heap, certain operations may be necessary to exchange the positions of the set at the root with its offspring and the one appended at the bottom with its ancestors . The branch-and-bound process continues by splitting the root node and rearranging data sequences in the heap. Upon completing every branching and bounding cycle the root node is the set with the lowest bound.

A simplified flow chart of the proposed branch-and-bound algorithm is given in Fig. 2.

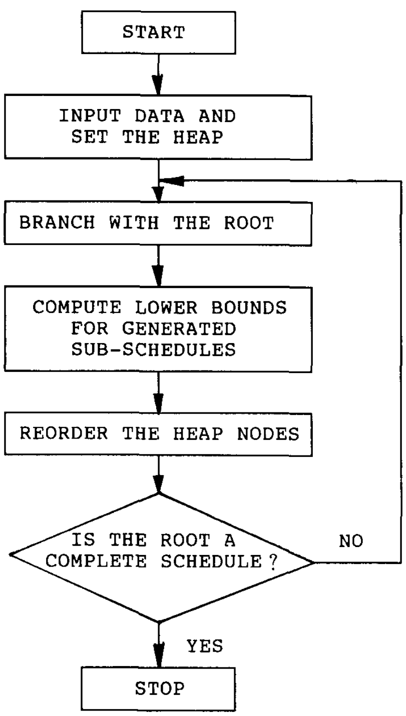

Fig. 2 The simplified flow chart of the proposed algorithm

Initially the unit data, including the minimum up/down times, the start-up cost, the upper and lower generation limits, and the initial status, etc. and the system data such as the hourly total generation requirement are read into the program. A heap with only the root node is formed at this stage. The root is branched into two sub-schedules. Lower bounds for each newly born node is then computed and these two nodes are inserted into the heap. The heap nodes have to be reordered to retain its property. If the new root node is a complete commitment schedule after one cycle of the branch-bound-reorder process, then the whole procedure stops with the root as the solution. Otherwise, the whole branch-bound-reorder process repeats.

NUMERICAL EXAMPLES

The objective of this section is to illustrate the capability of the proposed branch-and-bound algorithm in terms of its solution quality and computational requirements. Two example cases are studied, one with 10 units and 24 hours and the other with 20 units and 36 hours. For comparison, the λ-iteration method and the approximation method of base point and participation factors are both used for computing the dispatch cost. All the computations are performed on PC 386-33.

Case I: A 10-Unit System

In this example, a system with 10 generating units is studied. The system unit data and the generation requirements for each stage are shown in Table A-1 and Table A-2 of Appendix A, respectively. The consumed computing time and the total cost are shown in Table 1 and the determined schedule is given in Table 2.

Table 1 The computing time and the total cost

	Time(sec)	Cost($)
λ - iteration	10.93	78907
base point & participation factors	6.92	79169

Table 2 The determined commitment schedule

(a) λ-iteration

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1
1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1
1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

(b)base point & participation factors

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1
1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Case II: A 20-Unit System

A 20-unit midwestern system is studied. The unit data and the generation requirements for a 36-hour commitment horizon are given in Appendix B. The total cost and the computing time is given in Table 3 and the commitment schedule determined by the proposed method is shown in Table 4.

Table 3 The computing time and the total cost

	Time(sec)	Cost($)
λ - iteration	215.38	1006875
base point & participation factors	52.03	1065436

Table 4 The determined commitment schedule

(a) λ-iteration

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

(b)base point & participation factors

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	1	1
0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	1	1	1
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	1	1	1
0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

CONCLUSIONS

This paper presents a new branch-and-bound method for scheduling thermal generating units. The decision variables are the start and stop times and the generation levels of the units. A simple rule is defined to compute the lower bound of each candidate schedule for interim computation usage and the branching process takes place on the sub-schedule with the lowest lower bound. The heap data-storage structure and space-saving encoded data representations for partially-fulfilled commitment schedules are utilized to facilitate the branch-and-bound procedure. By successive branching and bounding, the unit commitment schedule with the minimum cost can be obtained. Two examples, a 10-unit 24-hour and a 20-unit 36-hour case, are shown to illustrate the effectiveness of the proposed algorithm.

It is observed that obtaining the optimum solution is possible by using the proposed approach for reasonable-sized system. Yet the consumed computing time may be intolerable for large-scale systems. Three future directions for overcoming this obstacle are under investigation. The first is trying to find a tighter lower bound without significantly increasing the computing time, such as the successive Lagrangian decomposition [22]. The second one is to improve the efficiency of the economic dispatch, which consumes a large part of the total computing time in unit commitment. The need for a faster dispatching method is evident if the number of economic dispatches that have to be performed to produce a unit commitment schedule is examined. The last resort is pruning more sub-schedules which are less likely to be optimum. Then the determined schedule would be suboptimal, the same as what the truncated dynamic programming gets.

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APPENDICES

A. Data For The 10-Unit System

Table A-2 The 24 hourly generation requirements

Hour	Gen. Req.	Hour	Gen. Req.
1	2000	13	1200
2	1980	14	1160
3	1940	15	1140
4	1900	16	1160
5	1840	17	1260
6	1870	18	1380
7	1820	19	1560
8	1700	20	1700
9	1510	21	1820
10	1410	22	1900
11	1320	23	1950
12	1260	24	1990

Table A-1 The system unit data

UNIT NO.	Max o/p	Min o/p	A($)	B($/MW)	C($/MW^2)	STC($)	Min ON	Min OFF	IT(HR)
1	60	10	15	2.2034	0.00510	10	3	2	-20
2	80	20	25	1.9161	0.00396	12	3	5	-20
3	100	30	40	1.8518	0.00393	12	2	2	-10
4	120	25	32	1.6966	0.00382	13	3	2	10
5	150	50	29	1.8015	0.00212	11	3	2	10
6	280	75	72	1.5354	0.00261	18	6	6	10
7	320	120	49	1.2643	0.00289	13	8	2	10
8	445	125	82	1.2163	0.00148	15	10	5	20
9	520	250	105	1.1954	0.00127	14	12	7	20
10	550	250	100	1.1285	0.00135	20	12	3	20

B. Data For The 20-Unit System

Table B-1 The system unit data

UNIT NO.	Max o/p	Min o/p	A($)	B($/MW)	C($/MW^2)	STC($)	Min ON	Min OFF	IT(HR)
1	55	55	650.7	28.12	0.00209	144	2	3	-40
2	55	55	661.2	27.91	0.00214	144	2	3	-40
3	55	55	644.5	27.79	0.00173	144	2	3	-40
4	85	25	476.6	27.74	0.00079	1072	4	6	-20
5	55	55	665.8	27.27	0.00222	144	2	3	-40
6	55	55	660.8	25.92	0.00413	144	2	3	-40
7	55	55	692.4	25.54	0.00951	144	2	3	-40
8	300	60	471.6	23.90	0.00070	13133	48	48	-50
9	162	25	367.5	23.71	0.00171	3960	48	48	-20
10	160	20	372.2	22.68	0.00254	3960	48	48	-45
11	80	20	371.0	22.26	0.00712	703	4	6	-5
12	470	150	958.2	21.60	0.00043	23490	48	48	50
13	460	135	1313.6	21.05	0.00063	23577	48	48	50
14	465	135	1168.1	21.04	0.00078	24099	48	48	0
15	162	25	445.4	19.70	0.00398	3888	48	48	-50
16	80	20	455.6	19.58	0.10908	767	4	6	-5
17	455	150	969.8	17.26	0.00031	20129	48	48	100
18	130	20	702.7	16.51	0.00211	2274	48	48	50
19	130	20	702.7	16.51	0.00211	2274	48	48	50
20	455	150	1078.8	16.19	0.00048	19939	48	48	100

Table B-2 The 36 hourly generation requirements

Hour	Gen. Req.	Hour	Gen. Req.
1	950	19	1500
2	900	20	1480
3	850	21	1380
4	900	22	1210
5	1000	23	1100
6	1210	24	1100
7	1370	25	1000
8	1380	26	900
9	1500	27	880
10	1600	28	850
11	1650	29	1100
12	1550	30	1100
13	1600	31	1200
14	1600	32	1350
15	1540	33	1380
16	1500	34	1500
17	1380	35	1600
18	1350	36	1650

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