# Branching Fractions and $C P$ Asymmetries in $B^{0} \rightarrow \pi^{0} \pi^{0}, B^{+} \rightarrow \pi^{+} \pi^{0}$, and $B^{+} \rightarrow K^{+} \pi^{0}$ Decays and Isospin Analysis of the $B \rightarrow \pi \boldsymbol{\pi}$ System 

B. Aubert, ${ }^{1}$ R. Barate, ${ }^{1}$ D. Boutigny, ${ }^{1}$ F. Couderc, ${ }^{1}$ Y. Karyotakis, ${ }^{1}$ J. P. Lees, ${ }^{1}$ V. Poireau, ${ }^{1}$ V. Tisserand, ${ }^{1}$ A. Zghiche, ${ }^{1}$ E. Grauges-Pous, ${ }^{2}$ A. Palano, ${ }^{3}$ A. Pompili, ${ }^{3}$ J. C. Chen, ${ }^{4}$ N. D. Qi, ${ }^{4}$ G. Rong, ${ }^{4}$ P. Wang, ${ }^{4}$ Y. S. Zhu, ${ }^{4}$ G. Eigen, ${ }^{5}$ I. Ofte, ${ }^{5}$ B. Stugu, ${ }^{5}$ G. S. Abrams, ${ }^{6}$ A. W. Borgland, ${ }^{6}$ A. B. Breon, ${ }^{6}$ D. N. Brown, ${ }^{6}$ J. Button-Shafer, ${ }^{6}$ R. N. Cahn, ${ }^{6}$ E. Charles, ${ }^{6}$
C. T. Day, ${ }^{6}$ M. S. Gill, ${ }^{6}$ A. V. Gritsan, ${ }^{6}$ Y. Groysman, ${ }^{6}$ R. G. Jacobsen, ${ }^{6}$ R. W. Kadel, ${ }^{6}$ J. Kadyk, ${ }^{6}$ L. T. Kerth, ${ }^{6}$

Yu. G. Kolomensky, ${ }^{6}$ G. Kukartsev, ${ }^{6}$ G. Lynch, ${ }^{6}$ L. M. Mir, ${ }^{6}$ P. J. Oddone, ${ }^{6}$ T. J. Orimoto, ${ }^{6}$ M. Pripstein, ${ }^{6}$ N. A. Roe, ${ }^{6}$
M. T. Ronan, ${ }^{6}$ W. A. Wenzel, ${ }^{6}$ M. Barrett, ${ }^{7}$ K. E. Ford, ${ }^{7}$ T. J. Harrison, ${ }^{7}$ A. J. Hart, ${ }^{7}$ C. M. Hawkes, ${ }^{7}$ S. E. Morgan, ${ }^{7}$ A. T. Watson, ${ }^{7}$ M. Fritsch, ${ }^{8}$ K. Goetzen, ${ }^{8}$ T. Held, ${ }^{8}$ H. Koch, ${ }^{8}$ B. Lewandowski, ${ }^{8}$ M. Pelizaeus, ${ }^{8}$ T. Schroeder, ${ }^{8}$ M. Steinke, ${ }^{8}$ J. T. Boyd, ${ }^{9}$ N. Chevalier, ${ }^{9}$ W. N. Cottingham, ${ }^{9}$ M. P. Kelly, ${ }^{9}$ T. E. Latham, ${ }^{9}$ F. F. Wilson, ${ }^{9}$ T. Cuhadar-Donszelmann, ${ }^{10}$ C. Hearty, ${ }^{10}$ N.S. Knecht, ${ }^{10}$ T. S. Mattison, ${ }^{10}$ J. A. McKenna, ${ }^{10}$ D. Thiessen, ${ }^{10}$ A. Khan, ${ }^{11}$ P. Kyberd, ${ }^{11}$ L. Teodorescu, ${ }^{11}$ A. E. Blinov, ${ }^{12}$ V. E. Blinov, ${ }^{12}$ V. P. Druzhinin, ${ }^{12}$ V. B. Golubev, ${ }^{12}$ V. N. Ivanchenko, ${ }^{12}$ E. A. Kravchenko, ${ }^{12}$ A. P. Onuchin, ${ }^{12}$ S. I. Serednyakov, ${ }^{12}$ Yu. I. Skovpen, ${ }^{12}$ E. P. Solodov, ${ }^{12}$ A. N. Yushkov, ${ }^{12}$ D. Best, ${ }^{13}$ M. Bruinsma, ${ }^{13}$ M. Chao, ${ }^{13}$
I. Eschrich, ${ }^{13}$ D. Kirkby, ${ }^{13}$ A. J. Lankford, ${ }^{13}$ M. Mandelkern, ${ }^{13}$ R. K. Mommsen, ${ }^{13}$ W. Roethel, ${ }^{13}$ D. P. Stoker, ${ }^{13}$ C. Buchanan, ${ }^{14}$ B. L. Hartfiel, ${ }^{14}$ A. J. R. Weinstein, ${ }^{14}$ S. D. Foulkes, ${ }^{15}$ J. W. Gary, ${ }^{15}$ O. Long, ${ }^{15}$ B. C. Shen, ${ }^{15}$ K. Wang, ${ }^{15}$ D. del Re, ${ }^{16}$ H. K. Hadavand, ${ }^{16}$ E. J. Hill, ${ }^{16}$ D. B. MacFarlane, ${ }^{16}$ H. P. Paar, ${ }^{16}$ Sh. Rahatlou, ${ }^{16}$ V. Sharma, ${ }^{16}$ J. W. Berryhill, ${ }^{17}$ C. Campagnari, ${ }^{17}$ A. Cunha, ${ }^{17}$ B. Dahmes, ${ }^{17}$ T. M. Hong, ${ }^{17}$ A. Lu,,$^{17}$ M. A. Mazur, ${ }^{17}$ J. D. Richman, ${ }^{17}$ W. Verkerke, ${ }^{17}$ T. W. Beck,,$^{18}$ A. M. Eisner, ${ }^{18}$ C. A. Heusch, ${ }^{18}$ J. Kroseberg, ${ }^{18}$ W. S. Lockman, ${ }^{18}$ G. Nesom, ${ }^{18}$ T. Schalk, ${ }^{18}$ B. A. Schumm, ${ }^{18}$ A. Seiden,,$^{18}$ P. Spradlin, ${ }^{18}$ D. C. Williams, ${ }^{18}$ M. G. Wilson, ${ }^{18}$ J. Albert, ${ }^{19}$ E. Chen, ${ }^{19}$ G. P. Dubois-Felsmann, ${ }^{19}$ A. Dvoretskii, ${ }^{19}$ D. G. Hitlin,,${ }^{19}$ I. Narsky, ${ }^{19}$ T. Piatenko, ${ }^{19}$ F. C. Porter, ${ }^{19}$ A. Ryd, ${ }^{19}$ A. Samuel, ${ }^{19}$ S. Yang, ${ }^{19}$ S. Jayatilleke, ${ }^{20}$ G. Mancinelli, ${ }^{20}$ B. T. Meadows, ${ }^{20}$ M. D. Sokoloff, ${ }^{20}$ F. Blanc, ${ }^{21}$ P. Bloom, ${ }^{21}$ S. Chen, ${ }^{21}$ W. T. Ford, ${ }^{21}$ U. Nauenberg, ${ }^{21}$ A. Olivas, ${ }^{21}$ P. Rankin, ${ }^{21}$ W. O. Ruddick, ${ }^{21}$ J. G. Smith, ${ }^{21}$ K. A. Ulmer, ${ }^{21}$ J. Zhang, ${ }^{21}$ L. Zhang, ${ }^{21}$ A. Chen, ${ }^{22}$
E. A. Eckhart, ${ }^{22}$ J. L. Harton, ${ }^{22}$ A. Soffer, ${ }^{22}$ W. H. Toki, ${ }^{22}$ R. J. Wilson, ${ }^{22}$ Q. Zeng, ${ }^{22}$ B. Spaan, ${ }^{23}$ D. Altenburg, ${ }^{24}$ T. Brandt, ${ }^{24}$ J. Brose, ${ }^{24}$ M. Dickopp, ${ }^{24}$ E. Feltresi, ${ }^{24}$ A. Hauke, ${ }^{24}$ H. M. Lacker, ${ }^{24}$ R. Nogowski, ${ }^{24}$ S. Otto, ${ }^{24}$ A. Petzold, ${ }^{24}$ J. Schubert, ${ }^{24}$ K. R. Schubert, ${ }^{24}$ R. Schwierz, ${ }^{24}$ J. E. Sundermann, ${ }^{24}$ D. Bernard, ${ }^{25}$ G. R. Bonneaud, ${ }^{25}$ P. Grenier, ${ }^{25}$ S. Schrenk, ${ }^{25}$ Ch. Thiebaux, ${ }^{25}$ G. Vasileiadis, ${ }^{25}$ M. Verderi, ${ }^{25}$ D. J. Bard, ${ }^{26}$ P. J. Clark, ${ }^{26}$ F. Muheim, ${ }^{26}$ S. Playfer, ${ }^{26}$ Y. Xie, ${ }^{26}$ M. Andreotti, ${ }^{27}$ V. Azzolini, ${ }^{27}$ D. Bettoni, ${ }^{27}$ C. Bozzi, ${ }^{27}$ R. Calabrese, ${ }^{27}$ G. Cibinetto, ${ }^{27}$ E. Luppi, ${ }^{27}$ M. Negrini, ${ }^{27}$ L. Piemontese, ${ }^{27}$ A. Sarti, ${ }^{27}$ F. Anulli, ${ }^{28}$ R. Baldini-Ferroli, ${ }^{28}$ A. Calcaterra, ${ }^{28}$ R. de Sangro, ${ }^{28}$ G. Finocchiaro, ${ }^{28}$ P. Patteri, ${ }^{28}$ I. M. Peruzzi, ${ }^{28}$ M. Piccolo,,${ }^{28}$ A. Zallo, ${ }^{28}$ A. Buzzo, ${ }^{29}$ R. Capra, ${ }^{29}$ R. Contri,,${ }^{29}$ G. Crosetti, ${ }^{29}$ M. Lo Vetere, ${ }^{29}$ M. Macri, ${ }^{29}$ M. R. Monge, ${ }^{29}$ S. Passaggio, ${ }^{29}$ C. Patrignani, ${ }^{29}$ E. Robutti, ${ }^{29}$ A. Santroni, ${ }^{29}$ S. Tosi, ${ }^{29}$ S. Bailey, ${ }^{30}$ G. Brandenburg, ${ }^{30}$ K. S. Chaisanguanthum, ${ }^{30}$ M. Morii, ${ }^{30}$ E. Won, ${ }^{30}$ R. S. Dubitzky, ${ }^{31}$ U. Langenegger, ${ }^{31}$ J. Marks, ${ }^{31}$ U. Uwer, ${ }^{31}$ W. Bhimji, ${ }^{32}$ D. A. Bowerman, ${ }^{32}$ P. D. Dauncey, ${ }^{32}$ U. Egede, ${ }^{32}$ J. R. Gaillard, ${ }^{32}$ G. W. Morton, ${ }^{32}$ J. A. Nash, ${ }^{32}$ M. B. Nikolich, ${ }^{32}$ G. P. Taylor, ${ }^{32}$ M. J. Charles, ${ }^{33}$ G. J. Grenier, ${ }^{33}$ U. Mallik, ${ }^{33}$ J. Cochran, ${ }^{34}$ H. B. Crawley, ${ }^{34}$ J. Lamsa, ${ }^{34}$ W. T. Meyer, ${ }^{34}$ S. Prell, ${ }^{34}$ E. I. Rosenberg, ${ }^{34}$ A. E. Rubin, ${ }^{34}$ J. Yi, ${ }^{34}$ N. Arnaud, ${ }^{35}$ M. Davier, ${ }^{35}$ X. Giroux, ${ }^{35}$ G. Grosdidier, ${ }^{35}$ A. Höcker, ${ }^{35}$ F. Le Diberder, ${ }^{35}$ V. Lepeltier, ${ }^{35}$ A. M. Lutz, ${ }^{35}$ T. C. Petersen, ${ }^{35}$ S. Plaszczynski, ${ }^{35}$ M. H. Schune, ${ }^{35}$ G. Wormser, ${ }^{35}$ C. H. Cheng, ${ }^{36}$ D. J. Lange, ${ }^{36}$ M. C. Simani, ${ }^{36}$ D. M. Wright, ${ }^{36}$ A. J. Bevan, ${ }^{37}$ C. A. Chavez, ${ }^{37}$ J. P. Coleman, ${ }^{37}$ I. J. Forster, ${ }^{37}$ J. R. Fry, ${ }^{37}$ E. Gabathuler, ${ }^{37}$ R. Gamet, ${ }^{37}$ D. E. Hutchcroft, ${ }^{37}$ R. J. Parry, ${ }^{37}$ D. J. Payne, ${ }^{37}$ C. Touramanis, ${ }^{37}$ C. M. Cormack, ${ }^{38}$ F. Di Lodovico, ${ }^{38}$ C. L. Brown, ${ }^{39}$ G. Cowan, ${ }^{39}$ R. L. Flack, ${ }^{39}$ H. U. Flaecher, ${ }^{39}$ M. G. Green, ${ }^{39}$ P. S. Jackson, ${ }^{39}$ T. R. McMahon, ${ }^{39}$ S. Ricciardi, ${ }^{39}$ F. Salvatore, ${ }^{39}$ M. A. Winter, ${ }^{39}$ D. Brown, ${ }^{40}$ C. L. Davis, ${ }^{40}$ J. Allison, ${ }^{41}$ N. R. Barlow, ${ }^{41}$ R. J. Barlow, ${ }^{41}$ M. C. Hodgkinson, ${ }^{41}$ G. D. Lafferty, ${ }^{41}$ J. C. Williams, ${ }^{41}$ C. Chen, ${ }^{42}$ A. Farbin, ${ }^{42}$ W. D. Hulsbergen, ${ }^{42}$ A. Jawahery, ${ }^{42}$ D. Kovalskyi, ${ }^{42}$ C. K. Lae, ${ }^{42}$ V. Lillard, ${ }^{42}$ D. A. Roberts, ${ }^{42}$ G. Blaylock, ${ }^{43}$ C. Dallapiccola, ${ }^{43}$ S. S. Hertzbach, ${ }^{43}$ R. Kofler, ${ }^{43}$ V. B. Koptchev, ${ }^{43}$ T. B. Moore, ${ }^{43}$ S. Saremi, ${ }^{43}$ H. Staengle, ${ }^{43}$ S. Willocq, ${ }^{43}$ R. Cowan, ${ }^{44}$ K. Koeneke, ${ }^{44}$ G. Sciolla, ${ }^{44}$ S. J. Sekula, ${ }^{44}$ F. Taylor, ${ }^{44}$ R. K. Yamamoto, ${ }^{44}$ P. M. Patel, ${ }^{45}$ S. H. Robertson, ${ }^{45}$ A. Lazzaro, ${ }^{46}$ V. Lombardo, ${ }^{46}$ F. Palombo, ${ }^{46}$ J. M. Bauer, ${ }^{47}$ L. Cremaldi, ${ }^{47}$ V. Eschenburg, ${ }^{47}$ R. Godang, ${ }^{47}$ R. Kroeger, ${ }^{47}$ J. Reidy, ${ }^{47}$ D. A. Sanders, ${ }^{47}$ D. J. Summers, ${ }^{47}$ H. W. Zhao, ${ }^{47}$ S. Brunet, ${ }^{48}$ D. Côté, ${ }^{48}$ P. Taras, ${ }^{48}$ H. Nicholson, ${ }^{49}$ N. Cavallo, ${ }^{50, *}$ F. Fabozzi, ${ }^{50, *}$ C. Gatto, ${ }^{50}$ L. Lista, ${ }^{50}$ D. Monorchio, ${ }^{50}$ P. Paolucci, ${ }^{50}$ D. Piccolo, ${ }^{50}$ C. Sciacca, ${ }^{50}$ M. Baak, ${ }^{51}$ H. Bulten, ${ }^{51}$ G. Raven, ${ }^{51}$ H. L. Snoek, ${ }^{51}$ L. Wilden, ${ }^{51}$ C. P. Jessop, ${ }^{52}$ J. M. LoSecco, ${ }^{52}$ T. Allmendinger, ${ }^{53}$ G. Benelli, ${ }^{53}$ K. K. Gan, ${ }^{53}$ K. Honscheid, ${ }^{53}$ D. Hufnagel, ${ }^{53}$ H. Kagan, ${ }^{53}$ R. Kass, ${ }^{53}$ T. Pulliam, ${ }^{53}$ A. M. Rahimi, ${ }^{53}$ R. Ter-Antonyan, ${ }^{53}$ Q. K. Wong, ${ }^{53}$ J. Brau, ${ }^{54}$ R. Frey, ${ }^{54}$ O. Igonkina, ${ }^{54}$ M. Lu, ${ }^{54}$ C. T. Potter, ${ }^{54}$ N. B. Sinev, ${ }^{54}$ D. Strom, ${ }^{54}$ E. Torrence, ${ }^{54}$
F. Colecchia, ${ }^{55}$ A. Dorigo, ${ }^{55}$ F. Galeazzi, ${ }^{55}$ M. Margoni, ${ }^{55}$ M. Morandin, ${ }^{55}$ M. Posocco, ${ }^{55}$ M. Rotondo, ${ }^{55}$ F. Simonetto, ${ }^{55}$ R. Stroili, ${ }^{55}$ C. Voci, ${ }^{55}$ M. Benayoun, ${ }^{56}$ H. Briand, ${ }^{56}$ J. Chauveau, ${ }^{56}$ P. David, ${ }^{56}$ Ch. de la Vaissière, ${ }^{56}$ L. Del Buono, ${ }^{56}$ O. Hamon, ${ }^{56}$ M. J. J. John, ${ }^{56}$ Ph. Leruste, ${ }^{56}$ J. Malcles, ${ }^{56}$ J. Ocariz, ${ }^{56}$ L. Roos, ${ }^{56}$ G. Therin, ${ }^{56}$ P. K. Behera, ${ }^{57}$ L. Gladney, ${ }^{57}$ Q. H. Guo, ${ }^{57}$ J. Panetta, ${ }^{57}$ M. Biasini, ${ }^{58}$ R. Covarelli, ${ }^{58}$ M. Pioppi, ${ }^{58}$ C. Angelini, ${ }^{59}$ G. Batignani, ${ }^{59}$ S. Bettarini, ${ }^{59}$ M. Bondioli, ${ }^{59}$ F. Bucci, ${ }^{59}$ G. Calderini, ${ }^{59}$ M. Carpinelli, ${ }^{59}$ F. Forti, ${ }^{59}$ M. A. Giorgi, ${ }^{59}$ A. Lusiani, ${ }^{59}$ G. Marchiori, ${ }^{59}$ M. Morganti, ${ }^{59}$ N. Neri, ${ }^{59}$ E. Paoloni, ${ }^{59}$ M. Rama, ${ }^{59}$ G. Rizzo, ${ }^{59}$ G. Simi, ${ }^{59}$ J. Walsh,,${ }^{59}$ M. Haire, ${ }^{60}$ D. Judd, ${ }^{60}$ K. Paick, ${ }^{60}$ D. E. Wagoner, ${ }^{60}$ N. Danielson, ${ }^{61}$ P. Elmer, ${ }^{61}$ Y. P. Lau, ${ }^{61}$ C. Lu, ${ }^{61}$ V. Miftakov, ${ }^{61}$ J. Olsen, ${ }^{61}$ A. J. S. Smith, ${ }^{61}$ A. V. Telnov, ${ }^{61}$ F. Bellini, ${ }^{62}$ G. Cavoto, ${ }^{61,62}$ A. D’Orazio, ${ }^{62}$ E. Di Marco, ${ }^{62}$ R. Faccini, ${ }^{62}$ F. Ferrarotto, ${ }^{62}$ F. Ferroni, ${ }^{62}$ M. Gaspero, ${ }^{62}$ L. Li Gioi, ${ }^{62}$ M. A. Mazzoni, ${ }^{62}$ S. Morganti, ${ }^{62}$ M. Pierini, ${ }^{62}$ G. Piredda, ${ }^{62}$ F. Polci, ${ }^{62}$ F. Safai Tehrani, ${ }^{62}$ C. Voena, ${ }^{62}$ S. Christ, ${ }^{63}$ H. Schröder, ${ }^{63}$ G. Wagner, ${ }^{63}$ R. Waldi, ${ }^{63}$ T. Adye, ${ }^{64}$ N. De Groot, ${ }^{64}$ B. Franek, ${ }^{64}$ G. P. Gopal, ${ }^{64}$ E. O. Olaiya, ${ }^{64}$ R. Aleksan, ${ }^{65}$ S. Emery, ${ }^{65}$ A. Gaidot, ${ }^{65}$ S. F. Ganzhur, ${ }^{65}$ P.-F. Giraud, ${ }^{65}$ G. Hamel de Monchenault, ${ }^{65}$ W. Kozanecki, ${ }^{65}$ M. Legendre, ${ }^{65}$ G. W. London, ${ }^{65}$ B. Mayer, ${ }^{65}$ G. Schott, ${ }^{65}$ G. Vasseur, ${ }^{65}$ Ch. Yèche, ${ }^{65}$ M. Zito, ${ }^{65}$ M. V. Purohit, ${ }^{66}$ A. W. Weidemann, ${ }^{66}$ J. R. Wilson, ${ }^{66}$ F. X. Yumiceva, ${ }^{66}$ T. Abe, ${ }^{67}$ M. Allen, ${ }^{67}$ D. Aston, ${ }^{67}$ R. Bartoldus, ${ }^{67}$ N. Berger, ${ }^{67}$ A. M. Boyarski, ${ }^{67}$ O. L. Buchmueller, ${ }^{67}$ R. Claus, ${ }^{67}$ M. R. Convery, ${ }^{67}$ M. Cristinziani, ${ }^{67}$ G. De Nardo, ${ }^{67}$ J. C. Dingfelder, ${ }^{67}$ D. Dong, ${ }^{67}$ J. Dorfan, ${ }^{67}$ D. Dujmic, ${ }^{67}$ W. Dunwoodie, ${ }^{67}$ S. Fan, ${ }^{67}$ R. C. Field, ${ }^{67}$ T. Glanzman, ${ }^{67}$ S. J. Gowdy, ${ }^{67}$ T. Hadig, ${ }^{67}$ V. Halyo, ${ }^{67}$ C. Hast, ${ }^{67}$ T. Hryn'ova, ${ }^{67}$ W. R. Innes, ${ }^{67}$ M. H. Kelsey, ${ }^{67}$ P. Kim, ${ }^{67}$ M. L. Kocian, ${ }^{67}$ D. W. G. S. Leith, ${ }^{67}$ J. Libby, ${ }^{67}$ S. Luitz, ${ }^{67}$ V. Luth, ${ }^{67}$ H. L. Lynch, ${ }^{67}$ H. Marsiske, ${ }^{67}$ R. Messner, ${ }^{67}$ D. R. Muller, ${ }^{67}$ C.P. O’Grady, ${ }^{67}$ V.E. Ozcan, ${ }^{67}$ A. Perazzo, ${ }^{67}$ M. Perl, ${ }^{67}$ B. N. Ratcliff, ${ }^{67}$ A. Roodman, ${ }^{67}$ A. A. Salnikov, ${ }^{67}$ R. H. Schindler, ${ }^{67}$ J. Schwiening, ${ }^{67}$ A. Snyder, ${ }^{67}$ A. Soha, ${ }^{67}$ J. Stelzer, ${ }^{67}$ J. Strube, ${ }^{54,67}$ D. Su, ${ }^{67}$ M. K. Sullivan, ${ }^{67}$ J. Thompson, ${ }^{67}$ J. Va'vra, ${ }^{67}$ S. R. Wagner, ${ }^{67}$ M. Weaver, ${ }^{67}$ W. J. Wisniewski, ${ }^{67}$ M. Wittgen, ${ }^{67}$ D. H. Wright, ${ }^{67}$ A. K. Yarritu, ${ }^{67}$ C. C. Young, ${ }^{67}$ P. R. Burchat, ${ }^{68}$ A. J. Edwards, ${ }^{68}$ S. A. Majewski, ${ }^{68}$ B. A. Petersen, ${ }^{68}$ C. Roat, ${ }^{68}$ M. Ahmed, ${ }^{69}$ S. Ahmed, ${ }^{69}$ M. S. Alam, ${ }^{69}$ J. A. Ernst, ${ }^{69}$ M. A. Saeed, ${ }^{69}$ M. Saleem, ${ }^{69}$ F. R. Wappler, ${ }^{69}$ W. Bugg, ${ }^{70}$ M. Krishnamurthy, ${ }^{70}$ S. M. Spanier, ${ }^{70}$ R. Eckmann, ${ }^{71}$ H. Kim, ${ }^{71}$ J. L. Ritchie, ${ }^{71}$ A. Satpathy, ${ }^{71}$ R. F. Schwitters, ${ }^{71}$ J. M. Izen, ${ }^{72}$ I. Kitayama, ${ }^{72}$ X. C. Lou, ${ }^{72}$ S. Ye, ${ }^{72}$ F. Bianchi, ${ }^{73}$ M. Bona, ${ }^{73}$ F. Gallo, ${ }^{73}$ D. Gamba, ${ }^{73}$ L. Bosisio, ${ }^{74}$ C. Cartaro, ${ }^{74}$ F. Cossutti, ${ }^{74}$ G. Della Ricca, ${ }^{74}$ S. Dittongo, ${ }^{74}$ S. Grancagnolo, ${ }^{74}$ L. Lanceri, ${ }^{74}$ P. Poropat, ${ }^{74, \dagger}$ L. Vitale, ${ }^{74}$ G. Vuagnin, ${ }^{74}$ F. Martinez-Vidal, ${ }^{2,75}$ R. S. Panvini, ${ }^{76}$ Sw. Banerjee, ${ }^{77}$ B. Bhuyan, ${ }^{77}$ C. M. Brown, ${ }^{77}$ D. Fortin, ${ }^{77}$ P. D. Jackson, ${ }^{77}$ R. Kowalewski, ${ }^{77}$ J. M. Roney, ${ }^{77}$ R. J. Sobie, ${ }^{77}$ J. J. Back, ${ }^{78}$ P. F. Harrison, ${ }^{78}$ G. B. Mohanty, ${ }^{78}$ H. R. Band, ${ }^{79}$ X. Chen, ${ }^{79}$ B. Cheng, ${ }^{79}$ S. Dasu, ${ }^{79}$ M. Datta, ${ }^{79}$ A. M. Eichenbaum, ${ }^{79}$ K. T. Flood, ${ }^{79}$ M. Graham, ${ }^{79}$ J. J. Hollar, ${ }^{79}$ J. R. Johnson, ${ }^{79}$ P. E. Kutter, ${ }^{79}$ H. Li, ${ }^{79}$ R. Liu, ${ }^{79}$ A. Mihalyi, ${ }^{79}$ Y. Pan, ${ }^{79}$ R. Prepost, ${ }^{79}$ P. Tan, ${ }^{79}$ J. H. von Wimmersperg-Toeller, ${ }^{79}$ J. Wu, ${ }^{79}$ S.L. Wu, ${ }^{79} \mathrm{Z}$. Yu, ${ }^{79}$ M. G. Greene, ${ }^{80}$ and H. Neal ${ }^{80}$
(BABAR Collaboration)

[^0][^1]Based on a sample of $227 \times 10^{6} B \bar{B}$ pairs collected by the $B A B A R$ detector at the PEP-II asymmetricenergy $B$ Factory at SLAC, we measure the branching fraction $\mathcal{B}\left(B^{0} \rightarrow \pi^{0} \pi^{0}\right)=(1.17 \pm 0.32 \pm 0.10) \times$ $10^{-6}$, and the asymmetry $C_{\pi^{0} \pi^{0}}=-0.12 \pm 0.56 \pm 0.06$. The $B^{0} \rightarrow \pi^{0} \pi^{0}$ signal has a significance of 5.0 $\sigma$. We also measure $\mathcal{B}\left(B^{+} \rightarrow \pi^{+} \pi^{0}\right)=(5.8 \pm 0.6 \pm 0.4) \times 10^{-6}, \mathcal{B}\left(B^{+} \rightarrow K^{+} \pi^{0}\right)=(12.0 \pm 0.7 \pm$ $0.6) \times 10^{-6}$, and the charge asymmetries $\mathcal{A}_{\pi^{+} \pi^{0}}=-0.01 \pm 0.10 \pm 0.02$ and $\mathcal{A}_{K^{+} \pi^{0}}=0.06 \pm 0.06 \pm$ 0.01 . Using isospin relations, we find an upper bound on the angle difference $\left|\alpha-\alpha_{\text {eff }}\right|$ of $35^{\circ}$ at the $90 \%$ C.L.

DOI: 10.1103/PhysRevLett. 94.181802
PACS numbers: $13.25 . \mathrm{Hw}, 11.30 . \mathrm{Er}, 12.15 . \mathrm{Hh}$

In the standard model (SM), the Cabibbo-KobayashiMaskawa (CKM) matrix $V_{q q^{\prime}}$ [1] describes the chargedcurrent couplings in the quark sector. The unitarity triangle is a useful representation of relations between CKM matrix elements, and measurements of its sides and angles provide a stringent test of the SM. Following the success in measuring the CKM angle $\beta$ [2], an important challenge for the $B$ factories is the determination of the remaining angles. The extraction of the CKM angle $\alpha \equiv \arg \left[-V_{\mathrm{td}} V_{\mathrm{tb}}^{*} / V_{\mathrm{ud}} V_{\mathrm{ub}}^{*}\right]$ from the time-dependent $C P$-violating asymmetry in the $B^{0} \rightarrow \pi^{+} \pi^{-}$decay mode [3] is complicated by the interference of competing amplitudes ("tree" and "penguin") with different weak phases. The difference between $\alpha$ and $\alpha_{\text {eff }}$, where $\alpha_{\text {eff }}$ is derived from the time-dependent $B^{0} \rightarrow \pi^{+} \pi^{-} C P$ asymmetry, may be evaluated using the isospin-related decays $B^{0} \rightarrow$ $\pi^{0} \pi^{0}$ and $B^{+} \rightarrow \pi^{+} \pi^{0}$ [4]. Here and throughout this Letter, charge conjugate reactions are included implicitly. For $B^{0} \rightarrow \pi^{0} \pi^{0}$ the asymmetry may deviate from zero if the tree and penguin amplitudes have different weak and strong phases. In the SM the decay $B^{+} \rightarrow \pi^{+} \pi^{0}$ is governed by a pure tree amplitude since penguin diagrams cannot contribute to the $I=2$ final state; as a result, no charge asymmetry is expected. The $B \rightarrow K \pi$ system is a rich source of information on the understanding of $C P$ violation, as has been illustrated by the recent observation of direct $C P$ asymmetry in $B^{0} \rightarrow K^{+} \pi^{-}$decays [5]. Both the rate and asymmetry of the $B^{+} \rightarrow K^{+} \pi^{0}$ decay may be used to extract constraints on penguin contributions to the $B \rightarrow K \pi$ amplitudes [6].

In this Letter, we report a constraint on $\Delta \alpha_{\pi \pi} \equiv \alpha-$ $\alpha_{\text {eff }}$, using the measurement of the asymmetry $C_{\pi^{0} \pi^{0}}$ and updated measurements of the branching fractions for $B^{0} \rightarrow$ $\pi^{0} \pi^{0}$ and $B^{+} \rightarrow \pi^{+} \pi^{0}$ and the charge asymmetry $\mathcal{A}_{\pi^{+} \pi^{0}}$. We also measure the branching fraction for the $B^{+} \rightarrow$ $K^{+} \pi^{0}$ decay and its charge asymmetry $\mathcal{A}_{\underline{K}^{+} \pi^{0}}$. The asymmetry $C_{\pi^{0} \pi^{0}}$ is defined as $\left(\left|A_{00}\right|^{2}-\left|\bar{A}_{00}\right|^{2}\right) /\left(\left|A_{00}\right|^{2}+\right.$ $\left|\bar{A}_{00}\right|^{2}$ ), where $A_{00}\left(\bar{A}_{00}\right)$ is the $B^{0}\left(\bar{B}^{0}\right) \rightarrow \pi^{0} \pi^{0}$ decay amplitude. For $B^{ \pm}$modes, the $C P$-violating charge asymmetry is defined as $\mathcal{A}=\left(|\bar{A}|^{2}-|A|^{2}\right) /\left(|\bar{A}|^{2}+|A|^{2}\right)$, where $A(\bar{A})$ is the $B^{+}\left(B^{-}\right)$decay amplitude. This study is based on $227 \times 10^{6} \Upsilon(4 S) \rightarrow B \bar{B}$ decays (on-resonance), collected with the $B A B A R$ detector. We also use $16 \mathrm{fb}^{-1}$ of data recorded 40 MeV below the $B \bar{B}$ production threshold (off-resonance).

The BABAR detector is described in Ref. [7]. The primary components used in this analysis are a tracking system consisting of a five-layer silicon vertex tracker and a 40-layer drift chamber (DCH) surrounded by a 1.5 T solenoidal magnet, an electromagnetic calorimeter comprising $6580 \mathrm{CsI}(\mathrm{Tl})$ crystals, and a ring imaging Cherenkov counter (DIRC).

The reconstruction and selection of $\pi^{0}$ mesons is described in Ref. [8]. Candidate tracks are required to be within the tracking fiducial volume, to originate from the interaction point, to consist of at least 12 DCH hits, and to be associated with at least 6 Cherenkov photons in the DIRC.
$B$ meson candidates are reconstructed by combining a $\pi^{0}$ with a charged pion or kaon $\left(h^{+}\right)$or by combining two $\pi^{0}$ mesons. Two variables, used to isolate the $B^{0} \rightarrow \pi^{0} \pi^{0}$ and $B^{+} \rightarrow h^{+} \pi^{0}$ signal events, take advantage of the kinematic constraints of $B$ mesons produced at the $\Upsilon(4 S)$. The first is the beam-energy-substituted mass $m_{\mathrm{ES}}=$ $\sqrt{\left(s / 2+\mathbf{p}_{i} \cdot \mathbf{p}_{B}\right)^{2} / E_{i}^{2}-\mathbf{p}_{B}^{2}}$, where $\left(E_{i}, \mathbf{p}_{i}\right)$ is the fourmomentum of the initial $e^{+} e^{-}$system, $\mathbf{p}_{B}$ is the $B$ candidate momentum, both measured in the laboratory frame, and $\sqrt{s}$ is the $e^{+} e^{-}$center-of-mass (c.m.) energy. The second variable is $\Delta E=E_{B}-\sqrt{s} / 2$, where $E_{B}$ is the $B$ candidate energy in the c.m. frame. The $\Delta E$ resolution for the signal is approximately 80 MeV for $B^{0} \rightarrow \pi^{0} \pi^{0}$ and 40 MeV for $B^{+} \rightarrow h^{+} \pi^{0}$.

The primary source of background is $e^{+} e^{-} \rightarrow q \bar{q}(q=$ $u, d, s, c$ ) events where a $\pi^{0}$ or $h^{+}$from each jet randomly combine to mimic a $B$ decay. This jetlike $q \bar{q}$ background is suppressed by requiring that the angle $\theta_{S}$ between the sphericity axis of the $B$ candidate and that of the remaining tracks and photons in the event, in the c.m. frame, satisfy $\left|\cos \theta_{S}\right|<0.7(0.8)$ for $B^{0} \rightarrow \pi^{0} \pi^{0} \quad\left(B^{+} \rightarrow h^{+} \pi^{0}\right)$. The other sources of background are $B$ decays to final states containing one vector meson and one pseudoscalar meson, where one pion is produced almost at rest in the $B$ rest frame and the remaining decay products match the kinematics of a $B^{0} \rightarrow \pi^{0} \pi^{0}$ or $B^{+} \rightarrow h^{+} \pi^{0}$ decay.

For the $B^{0} \rightarrow \pi^{0} \pi^{0}$ analysis we restrict the $m_{\mathrm{ES}}-\Delta E$ plane to the region with $m_{\mathrm{ES}}>5.2 \mathrm{GeV} / c^{2}$ and $|\Delta E|<$ 0.4 GeV . For the on-resonance sample we define the signal region as the band in the plane with $|\Delta E|<0.2 \mathrm{GeV}$ and the sideband region as the rest of the plane excluding the
region which is also populated with $B^{+} \rightarrow \rho^{+} \pi^{0}$ events. The entire plane for the off-resonance data and the sideband region for the on-resonance data are kept in the fit in order to constrain the $q \bar{q}$ background parameters. $B^{+} \rightarrow$ $h^{+} \pi^{0}$ candidates are selected in the region with $m_{\mathrm{ES}}>$ $5.22 \mathrm{GeV} / c^{2}$ and $-0.11<\Delta E<0.15 \mathrm{GeV}$.
For $B^{0} \rightarrow \pi^{0} \pi^{0}$ candidates, the other tracks and clusters in the event are used to determine whether the other $B$ meson ( $B_{\text {tag }}$ ) decays as a $B^{0}$ or $\bar{B}^{0}$ (flavor tag). We use a multivariate technique [9] to determine the flavor of the $B_{\text {tag }}$ meson. Events are assigned to one of several mutually exclusive categories based on the estimated mistag probability and on the source of tagging information.

The number of signal $B$ candidates is determined with an extended, unbinned maximum-likelihood fit. The probability density function (PDF) $\mathcal{P}_{i}\left(\vec{x}_{j} ; \vec{\alpha}_{i}\right)$ for a signal or background hypothesis is the product of PDFs for the variables $\vec{x}_{j}$ given the set of parameters $\vec{\alpha}_{i}$. The likelihood function is a product over the $N$ events of the $M$ signal and background hypotheses:

$$
\begin{equation*}
\mathcal{L}=\exp \left(-\sum_{k=1}^{M} n_{k}\right) \prod_{j=1}^{N}\left[\sum_{i=1}^{M} c_{i j} \mathcal{P}_{i}\left(\vec{x}_{j} ; \vec{\alpha}_{i}\right)\right] . \tag{1}
\end{equation*}
$$

For $B^{0} \rightarrow \pi^{0} \pi^{0}$ the coefficients $c_{i j}$ are defined as $c_{i j}=$ $\frac{1}{2}\left(1-s_{j} A_{i}\right) n_{i}$, where $s_{j}$ refers to the sign of the flavor tag of the other $B$ in the event $j$ and is zero for untagged events. The fit parameters $n_{i}$ and $A_{i}$ are the number of events and raw asymmetry for $B^{0} \rightarrow \pi^{0} \pi^{0}$ signal, $B^{+} \rightarrow \rho^{+} \pi^{0}$ background, and continuum background components. The average of branching fraction measurements [10] is used to fix $n\left(B^{+} \rightarrow \rho^{+} \pi^{0}\right)$ to $32 \pm 6$. The raw asymmetry for the signal is $(1-2 \chi)(1-2 \omega) C_{\pi^{0} \pi^{0}}$, where $\chi=0.186 \pm$ 0.004 [11] is the neutral $B$ mixing probability, and $\omega$ is the mistag probability.

For $B^{+} \rightarrow h^{+} \pi^{0}$ the probability coefficients are $c_{i j}=$ $\frac{1}{2}\left(1-q_{j} \mathcal{A}_{i}\right) n_{i}$, where $q_{j}$ is the charge of the track $h$ in the event $j$. The fit parameters $n_{i}$ and $\mathcal{A}_{i}$ are the number of events and asymmetry for $B^{+} \rightarrow \pi^{+} \pi^{0}$ and $B^{+} \rightarrow K^{+} \pi^{0}$ signal, continuum, and $B$ background components. The $B$ background yields are fixed to the expected number of events using the current world averages of branching ratios [12], which are $18 \pm 4$ for $B^{0} \rightarrow \rho^{+} \pi^{-}$and $B^{+} \rightarrow \rho^{+} \pi^{0}$ combined, and $3 \pm 1$ events for $B^{0} \rightarrow \rho^{-} K^{+}$. Uncertainties on these numbers are dominated by the uncertainty on selection efficiencies, due to the sensitivity to the tight requirement in $\Delta E$.

The variables $\vec{x}_{j}$ used for $B^{0} \rightarrow \pi^{0} \pi^{0}$ are $m_{\mathrm{ES}}, \Delta E$, and a Fisher discriminant $F$. The Fisher discriminant is an optimized linear combination of $\sum_{i} p_{i}$ and $\sum_{i} p_{i} \cos ^{2} \theta_{i}$, where $p_{i}$ is the momentum and $\theta_{i}$ is the angle with respect to the thrust axis of the $B$ candidate, both in the c.m. frame, for all tracks and neutral clusters not used to reconstruct the $B$ meson. For both the $B^{0} \rightarrow \pi^{0} \pi^{0}$ signal and the $B^{+} \rightarrow$
$\rho^{+} \pi^{0}$ background the $m_{\text {ES }}$ and $\Delta E$ variables are correlated, and therefore a two-dimensional PDF from a smoothed, simulated distribution is used. For the continuum background, the $m_{\mathrm{ES}}$ distribution is modeled as a threshold function [13] and the $\Delta E$ distribution as a second-order polynomial. The PDF for the $F$ variable is modeled as a parametric step function (PSF) [8] for all event components. A PSF is a variable width binned distribution whose parameters are the heights of each bin. The limits of the ten bins $F$ PSF are chosen so that each bin contains $10 \%$ of the signal sample. For $B^{0} \rightarrow \pi^{0} \pi^{0}$ and $B^{+} \rightarrow \rho^{+} \pi^{0}$ the $F$ PSF parameters are correlated with the flavor tagging, and the PSF parameters are different for each tagging category. Simulated events are used to determine the PSF distributions for both $B^{0} \rightarrow \pi^{0} \pi^{0}$ and $B^{+} \rightarrow$ $\rho^{+} \pi^{0}$. For $q \bar{q}$ background, the $F$ PSF parameters are free in the fit.

An additional discriminating variable for $B^{+} \rightarrow h^{+} \pi^{0}$ is the Cherenkov angle $\theta_{c}$ of the $h^{+}$track. The PDF parameters for $m_{\mathrm{ES}}, \Delta E, \theta_{c}$, and $F$ for the background are determined using the data, while the PDFs for the signal are found from a combination of simulated events and data. The $m_{\mathrm{ES}}$ and $\Delta E$ distributions for $q \bar{q}$ events are treated as in the $B^{0} \rightarrow \pi^{0} \pi^{0}$ case, with parameters allowed to vary freely in the fit. For the signal, the $m_{\mathrm{ES}}$ and $\Delta E$ distributions are both modeled as a Gaussian distribution with a low-side power law tail whose parameters are determined from the simulation. The means of the Gaussian components are determined from the fit to the $B^{+} \rightarrow h^{+} \pi^{0}$ sample, and their values are used to set the neutral energy scale in the $B^{0} \rightarrow \pi^{0} \pi^{0}$ analysis. The neutral energy resolution is studied using photons from $e^{+} e^{-} \rightarrow \mu^{+} \mu^{-} \gamma$ events and from $B$ decays such as $B \rightarrow K^{*} \gamma$. The mean of $\Delta E$ for the $B^{+} \rightarrow K^{+} \pi^{0}$ mode is a function of the kaon laboratory momentum, since a pion mass hypothesis is used. The distribution of $F$ is modeled as a Gaussian function with an asymmetric variance for the signal, whose parameters are obtained from simulation, and as a double Gaussian for the continuum background, whose parameters are determined in the likelihood fit. The difference of the measured and expected values of $\theta_{c}$ for the pion or kaon hypothesis, divided by the uncertainty on $\theta_{c}$, is modeled as a double Gaussian function, whose parameters are obtained from a control sample of kaons and pions, from $D^{*+} \rightarrow D^{0}\left(K^{-} \pi^{+}\right) \pi^{+}$decays.

The result of the maximum-likelihood fit for $B^{0} \rightarrow$ $\pi^{0} \pi^{0}$ is $n\left(B^{0} \rightarrow \pi^{0} \pi^{0}\right)=61 \pm 17$ (see Table I), with a corresponding statistical significance of $5.2 \sigma$. The asymmetry is $C_{\pi^{0} \pi^{0}}=-0.12 \pm 0.56$. Shown in Fig. 1 are distributions of $m_{\mathrm{ES}}, F$, and $\Delta E$, for signal-enriched samples of $B^{0} \rightarrow \pi^{0} \pi^{0}$ candidates.

With changes in the analysis technique to measure the $C P$ asymmetry, we now find $44 \pm 13$ signal events in the first $123 \times 10^{6} B \bar{B}$ events, compared to $46 \pm 13$ found in Ref. [8]. The additional $104 \times 10^{6} B \bar{B}$ events data set has a

TABLE I. The results for the modes $B^{0} \rightarrow \pi^{0} \pi^{0}$ and $B^{+} \rightarrow h^{+} \pi^{0}$ are summarized. For each mode, the sample size $N$, number of signal events $N_{S}$, total detection efficiency $\varepsilon$, branching fraction $\mathcal{B}$, asymmetry $\mathcal{A}$ or $C_{\pi^{0} \pi^{0}}$, and the $90 \%$ confidence interval for the asymmetry are shown. For $C_{\pi^{0} \pi^{0}}$ the confidence interval is obtained inferring minimum coverage inside the physical region [ $-1,1$. The first errors are statistical, the second systematic, with the exception of $\varepsilon$ whose error is purely systematic.

| Mode | $N$ | $N_{S}$ | $\varepsilon(\%)$ | $\mathcal{B}\left(10^{-6}\right)$ | Asymmetry | (90\% C.L.) |
| :---: | ---: | ---: | :---: | :---: | ---: | :---: |
| $B^{0} \rightarrow \pi^{0} \pi^{0}$ | 8153 | $61 \pm 17$ | $23.5 \pm 1.4$ | $1.17 \pm 0.32 \pm 0.10$ | $-0.12 \pm 0.56 \pm 0.06$ | $[-0.88,0.64]$ |
| $B^{+} \rightarrow \pi^{+} \pi^{0}$ | 29950 | $379 \pm 41$ | $28.7 \pm 1.1$ | $5.8 \pm 0.6 \pm 0.4$ | $-0.01 \pm 0.10 \pm 0.02$ | $[-0.19,0.21]$ |
| $B^{+} \rightarrow K^{+} \pi^{0}$ | 13165 | $682 \pm 39$ | $25.0 \pm 1.0$ | $12.0 \pm 0.7 \pm 0.6$ | $0.06 \pm 0.06 \pm 0.01$ | $[-0.06,0.18]$ |

signal of $17 \pm 11$. The signal rates in these two subsets agree at the $1.3 \sigma$ level. This result also reflects an improved understanding of high energy $\pi^{0}$ detection efficiency. Using a sample of $\pi^{0}$ mesons from $\tau^{+} \rightarrow$ $\pi^{+} \pi^{0} \nu_{\tau}$ decays, we apply a $\pi^{0}$ efficiency correction of $0.99 \pm 0.03$ to our GEANT simulation, compared to a correction of $0.88 \pm 0.08$ applied in Ref. [8].

For $B^{+} \rightarrow h^{+} \pi^{0}$ the likelihood fit results are summarized in Table I. Using the event-weighting technique described in Ref. [14], we show signal and background projections in Fig. 2. For each event, a weight to be signal or background is assigned based on a fit performed without the specific variable that is plotted. The resulting distributions are normalized to the event yields, and are compared to the PDFs used in the full fit.

Systematic uncertainties on the event yields and $C P$ asymmetries are evaluated on data control samples, or


FIG. 1 (color online). Distributions and PDF projections for $B^{0} \rightarrow \pi^{0} \pi^{0}$. Shown are (a) $m_{\mathrm{ES}}$, (b) $F$, and (c) $\Delta E$ for candidates that satisfy an optimized requirement on the signal probability, based on all variables except the one being plotted. The three projections contain, respectively, $25 \%, 68 \%, 45 \%$ of the signal, $14 \%, 17 \%, 31 \%$ of the $\rho^{+} \pi^{0}$ background, and $2.2 \%, 4.4 \%, 1.3 \%$ of the continuum background. For (d) the requirement is loosened to include $80 \%$ of the signal. PDF projections are shown as a dashed line for $q \bar{q}$ background, a dotted line for $B$ background, and a dash-dotted line for signal.
by varying the fixed parameters and refitting the data. In order of decreasing importance, the dominant systematics on the $B^{0} \rightarrow \pi^{0} \pi^{0}$ branching fraction arise from the uncertainty on the $\Delta E$ resolution, the efficiency of the $\pi^{0}$ reconstruction, and the uncertainty on $B$ background event yields. The significance of the $B^{0} \rightarrow \pi^{0} \pi^{0}$ signal yield, taking systematic effects into account, is $5.0 \sigma$. The systematic uncertainty on $C_{\pi^{0} \pi^{0}}$ is dominated by the uncertainties on the $B$ background asymmetry and tagging efficiency.

For $B^{+} \rightarrow h^{+} \pi^{0}$ the dominant systematic uncertainties arise from the $F$ signal PDF parameters, selection efficiencies, and the $\Delta E$ resolution. Additional systematics arise from uncertainties on the $B$ background event yields and particle identification. The systematic uncertainty on the charge asymmetries is dominated by the $1 \%$ upper limit on the charge bias in the detector [15].

To extract information on $\Delta \alpha_{\pi \pi}$ we use the isospin relations [4] in conjunction with $B A B A R$ measurements of $C_{\pi^{+} \pi^{-}}=-0.09 \pm 0.15 \pm 0.04$ [3], the branching fraction $\mathcal{B}\left(B^{0} \rightarrow \pi^{+} \pi^{-}\right)=(4.7 \pm 0.6 \pm 0.2) \times 10^{-6}$ [16], the $B^{0} \rightarrow \pi^{0} \pi^{0}$ and $B^{+} \rightarrow \pi^{+} \pi^{0}$ decay rates, and the $C_{\pi^{0} \pi^{0}}$ values reported here. We scan over all values of $\left|\Delta \alpha_{\pi \pi}\right|$ and calculate a $\chi^{2}$ for the decay amplitudes using the method described in Ref. [17]. The $\chi^{2}$ is converted into a confidence level shown in Fig. 3, from which we derive an upper bound on $\left|\Delta \alpha_{\pi \pi}\right|$ of $35^{\circ}$ at the $90 \%$ C.L.


FIG. 2 (color online). Distributions and PDF projections for $B^{+} \rightarrow h^{+} \pi^{0}$, using the method described in the text. For $m_{\mathrm{ES}}$ (a) the signal distributions are combined, while for $\Delta E$ (b) the signal $B^{+} \rightarrow \pi^{+} \pi^{0}$ (open circles and dash-dotted curve) and $B^{+} \rightarrow K^{+} \pi^{0}$ (solid circles and curve) are shown separately. The insets show the combined background components.


FIG. 3 (color online). Constraints on the $\left|\Delta \alpha_{\pi \pi}\right|$ in terms of confidence level. We find an upper bound on $\left|\Delta \alpha_{\pi \pi}\right|$ of $35^{\circ}$ at the $90 \%$ C.L.

In summary, we observe $61 \pm 17 \pm 5 \quad B^{0} \rightarrow \pi^{0} \pi^{0}$ events with a significance of $5.0 \sigma$ including systematic uncertainties. This corresponds to a branching fraction of $\mathcal{B}\left(B^{0} \rightarrow \pi^{0} \pi^{0}\right)=(1.17 \pm 0.32 \pm 0.10) \times 10^{-6}$, where the first error is statistical and the second is systematic. We measure the asymmetry $C_{\pi^{0} \pi^{0}}=-0.12 \pm 0.56 \pm 0.06$. We report branching fractions $\mathcal{B}\left(B^{+} \rightarrow \pi^{+} \pi^{0}\right)=(5.8 \pm$ $0.6 \pm 0.4) \times 10^{-6}$ and $\mathcal{B}\left(B^{+} \rightarrow K^{+} \pi^{0}\right)=(12.0 \pm 0.7 \pm$ $0.6) \times 10^{-6}$. The charge asymmetries are $\mathcal{A}_{\pi^{+} \pi^{0}}=$ $-0.01 \pm 0.10 \pm 0.02$ and $\mathcal{A}_{K^{+} \pi^{0}}=0.06 \pm 0.06 \pm 0.01$; we find no evidence for $C P$ violation. In contrast to the recent measurements of charge asymmetry in $B^{0} \rightarrow K^{+} \pi^{-}$ decays [5], the $\mathcal{A}_{K^{+} \pi^{0}}$ value reported here is compatible with zero. We use isospin relations on $B \rightarrow \pi \pi$ decay rates and asymmetries to find an upper bound of $\left|\Delta \alpha_{\pi \pi}\right|<35^{\circ}$ at the $90 \%$ C.L.

We are grateful for the excellent luminosity and machine conditions provided by our PEP-II colleagues, and for the substantial dedicated effort from the computing organizations that support $B A B A R$. The collaborating institutions wish to thank SLAC for its support and kind hospitality. This work is supported by DOE and NSF (USA), NSERC (Canada), IHEP (China), CEA and CNRS-IN2P3 (France), BMBF and DFG (Germany), INFN (Italy), FOM (The Netherlands), NFR (Norway), MIST (Russia), and PPARC (United Kingdom). Individuals have received support from CONACyT (Mexico), A.P. Sloan Foundation, Research Corporation, and Alexander von Humboldt Foundation.

Note added.-After submission of the present manuscript the Belle Collaboration submitted an updated measurement of the branching fraction for $B^{0} \rightarrow \pi^{0} \pi^{0}$ and its $C P$ asymmetry [18].
*Also with Università della Basilicata, Potenza, Italy. ${ }^{\dagger}$ Deceased.
[1] N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).
[2] B. Aubert et al. (BABAR Collaboration), Phys. Rev. Lett. 89, 201802 (2002); K. Abe et al. (Belle Collaboration), Phys. Rev. D 66, 071102 (2002).
[3] B. Aubert et al. (BABAR Collaboration), hep-ex/0408089; K. Abe et al. (Belle Collaboration), Phys. Rev. Lett. 93, 021601 (2004).
[4] M. Gronau and D. London, Phys. Rev. Lett. 65, 3381 (1990).
[5] B. Aubert et al. (BABAR Collaboration), Phys. Rev. Lett. 93, 131801 (2004); Y. Chao et al. (Belle Collaboration), Phys. Rev. Lett. 93, 191802 (2004).
[6] M. Gronau and J.L. Rosner, Phys. Rev. D 59, 113002 (1999); H. J. Lipkin, Phys. Lett. B 445, 403 (1999); J. Matías, Phys. Lett. B 520, 131 (2001).
[7] B. Aubert et al. (BABAR Collaboration), Nucl. Instrum. Methods Phys. Res., Sect. A 479, 1 (2002).
[8] B. Aubert et al. (BABAR Collaboration), Phys. Rev. Lett. 91, 241801 (2003); B. Aubert et al. (BABAR Collaboration), Phys. Rev. Lett. 91, 021801 (2003).
[9] B. Aubert et al. (BABAR Collaboration), hep-ex/0408127 [Phys. Rev. Lett. (to be published)].
[10] B. Aubert et al. (BABAR Collaboration, Phys. Rev. Lett. 93, 051802 (2004); J. Zhang et al. (Belle Collaboration), Phys. Rev. Lett. 94, 031801 (2005).
[11] S. Eidelman et al. (Particle Data Group), Phys. Lett. B 592, 1 (2004).
[12] B. Aubert et al. (BABAR Collaboration), Phys. Rev. Lett. 91, 201802 (2003); A. Gordon et al. (Belle Collaboration), Phys. Lett. B 542, 183 (2002); C. P. Jessop et al. (CLEO Collaboration), Phys. Rev. Lett. 85, 2881 (2000).
[13] H. Albrecht et al. (ARGUS Collaboration), Z. Phys. C 48, 543 (1990).
[14] M. Pivk and F. Le Diberder, physics/0402083.
[15] B. Aubert et al. (BABAR Collaboration), Phys. Rev. D 65, 051101 (2002).
[16] B. Aubert et al. (BABAR Collaboration), Phys. Rev. Lett. 89, 281802 (2002).
[17] J. Charles et al., hep-ph/0406184 [Eur. Phys. Jour. C (to be published)].
[18] Y. Chao et al., following Letter, Phys. Rev. Lett. 94, 181803 (2005).


[^0]:    ${ }^{1}$ Laboratoire de Physique des Particules, F-74941 Annecy-le-Vieux, France
    ${ }^{2}$ Universitad Autonoma de Barcelona, E-08193 Bellaterra, Barcelona, Spain
    ${ }^{3}$ Dipartimento di Fisica and INFN, Università di Bari, I-70126 Bari, Italy
    ${ }^{4}$ Institute of High Energy Physics, Beijing 100039, China
    ${ }^{5}$ Institute of Physics, University of Bergen, N-5007 Bergen, Norway
    ${ }^{6}$ Lawrence Berkeley National Laboratory and University of California, Berkeley, California 94720, USA
    ${ }^{7}$ University of Birmingham, Birmingham, B15 2TT, United Kingdom
    ${ }^{8}$ Ruhr Universität Bochum, Institut für Experimentalphysik 1, D-44780 Bochum, Germany
    ${ }^{9}$ University of Bristol, Bristol BS8 1TL, United Kingdom
    ${ }^{10}$ University of British Columbia, Vancouver, British Columbia, Canada V6T $1 Z 1$
    ${ }^{11}$ Brunel University, Uxbridge, Middlesex UB8 3PH, United Kingdom
    ${ }^{12}$ Budker Institute of Nuclear Physics, Novosibirsk 630090, Russia
    ${ }^{13}$ University of California at Irvine, Irvine, California 92697, USA
    ${ }^{14}$ University of California at Los Angeles, Los Angeles, California 90024, USA
    ${ }^{15}$ University of California at Riverside, Riverside, California 92521, USA
    ${ }^{16}$ University of California at San Diego, La Jolla, California 92093, USA
    ${ }^{17}$ University of California at Santa Barbara, Santa Barbara, California 93106, USA
    ${ }^{18}$ University of California at Santa Cruz, Institute for Particle Physics, Santa Cruz, California 95064, USA
    ${ }^{19}$ California Institute of Technology, Pasadena, California 91125, USA
    ${ }^{20}$ University of Cincinnati, Cincinnati, Ohio 45221, USA
    ${ }^{21}$ University of Colorado, Boulder, Colorado 80309, USA
    ${ }^{22}$ Colorado State University, Fort Collins, Colorado 80523, USA

[^1]:    ${ }^{23}$ Institut fur Physik, Universität Dortmund, D-44221 Dortmund, Germany
    ${ }^{24}$ Institut für Kern- und Teilchenphysik, Technische Universität Dresden, D-01062 Dresden, Germany
    ${ }^{25}$ Ecole Polytechnique, LLR, F-91128 Palaiseau, France
    ${ }^{26}$ University of Edinburgh, Edinburgh EH9 3JZ, United Kingdom
    ${ }^{27}$ Dipartimento di Fisica and INFN, Università di Ferrara, I-44100 Ferrara, Italy
    ${ }^{28}$ Laboratori Nazionali di Frascati dell'INFN, I-00044 Frascati, Italy
    ${ }^{29}$ Dipartimento di Fisica and INFN, Università di Genova, I-16146 Genova, Italy
    ${ }^{30}$ Harvard University, Cambridge, Massachusetts 02138, USA
    ${ }^{31}$ Physikalisches Institut, Universität Heidelberg, Philosophenweg 12, D-69120 Heidelberg, Germany
    ${ }^{32}$ Imperial College London, London, SW7 2AZ, United Kingdom
    ${ }^{33}$ University of Iowa, Iowa City, Iowa 52242, USA
    ${ }^{34}$ Iowa State University, Ames, Iowa 50011-3160, USA
    ${ }^{35}$ Laboratoire de l'Accélérateur Linéaire, F-91898 Orsay, France
    ${ }^{36}$ Lawrence Livermore National Laboratory, Livermore, California 94550, USA
    ${ }^{37}$ University of Liverpool, Liverpool L69 72E, United Kingdom
    ${ }^{38}$ Queen Mary, University of London, E1 4NS, United Kingdom
    ${ }^{39}$ University of London, Royal Holloway and Bedford New College, Egham, Surrey TW20 OEX, United Kingdom
    ${ }^{40}$ University of Louisville, Louisville, Kentucky 40292, USA
    ${ }^{41}$ University of Manchester, Manchester M13 9PL, United Kingdom
    ${ }^{42}$ University of Maryland, College Park, Maryland 20742, USA
    ${ }^{43}$ University of Massachusetts, Amherst, Massachusetts 01003, USA
    ${ }^{44}$ Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA
    ${ }^{45}$ McGill University, Montréal, Québec, Canada H3A $2 T 8$
    ${ }^{46}$ Dipartimento di Fisica and INFN, Università di Milano, I-20133 Milano, Italy
    ${ }^{47}$ University of Mississippi, University, Mississippi 38677, USA
    ${ }^{48}$ Laboratoire René J. A. Lévesque, Université de Montréal, Montréal, Québec, Canada H3C 3J7
    ${ }^{49}$ Mount Holyoke College, South Hadley, Massachusetts 01075, USA
    ${ }^{50}$ Dipartimento di Scienze Fisiche and INFN, Università di Napoli Federico II, I-80126, Napoli, Italy
    ${ }^{51}$ NIKHEF, National Institute for Nuclear Physics and High Energy Physics, NL-1009 DB Amsterdam, The Netherlands
    ${ }^{52}$ University of Notre Dame, Notre Dame, Indiana 46556, USA
    ${ }^{53}$ The Ohio State University, Columbus, Ohio 43210, USA
    ${ }^{54}$ University of Oregon, Eugene, Oregon 97403, USA
    ${ }^{55}$ Dipartimento di Fisica and INFN, Università di Padova, I-35131 Padova, Italy
    ${ }^{56}$ Laboratoire de Physique Nucléaire et de Hautes Energies, Universités Paris VI et VII, F-75252 Paris, France
    ${ }^{57}$ University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA
    ${ }^{58}$ Dipartimento di Fisica and INFN, Università di Perugia, I-06100 Perugia, Italy
    ${ }^{59}$ Dipartimento di Fisica, Scuola Normale Superiore and INFN, Università di Pisa, I-56127 Pisa, Italy
    ${ }^{60}$ Prairie View A\&M University, Prairie View, Texas 77446, USA
    ${ }^{61}$ Princeton University, Princeton, New Jersey 08544, USA
    ${ }^{62}$ Dipartimento di Fisica and INFN, Università di Roma La Sapienza, I-00185 Roma, Italy
    ${ }^{63}$ Universität Rostock, D-18051 Rostock, Germany
    ${ }^{64}$ Rutherford Appleton Laboratory, Chilton, Didcot, Oxon, OX1 1 0QX, United Kingdom
    ${ }^{65}$ DSM/Dapnia, CEA/Saclay, F-91191 Gif-sur-Yvette, France
    ${ }^{66}$ University of South Carolina, Columbia, South Carolina 29208, USA
    ${ }^{67}$ Stanford Linear Accelerator Center, Stanford, California 94309, USA
    ${ }^{68}$ Stanford University, Stanford, California 94305-4060, USA
    ${ }^{69}$ State University of New York, Albany, New York 12222, USA
    ${ }^{70}$ University of Tennessee, Knoxville, Tennessee 37996, USA
    ${ }^{71}$ University of Texas at Austin, Austin, Texas 78712, USA
    ${ }^{72}$ University of Texas at Dallas, Richardson, Texas 75083, USA
    ${ }^{73}$ Dipartimento di Fisica Sperimentale and INFN, Università di Torino, I-10125 Torino, Italy
    ${ }^{74}$ Dipartimento di Fisica and INFN, Università di Trieste, I-34127 Trieste, Italy
    ${ }^{75}$ Universitad de Valencia, E-46100 Burjassot, Valencia, Spain
    ${ }^{76}$ Vanderbilt University, Nashville, Tennessee 37235, USA
    ${ }^{77}$ University of Victoria, Victoria, British Columbia, Canada V8W 3P6
    ${ }^{78}$ Department of Physics, University of Warwick, Coventry CV4 7AL, United Kingdom
    ${ }^{79}$ University of Wisconsin, Madison, Wisconsin 53706, USA
    ${ }^{80}$ Yale University, New Haven, Connecticut 06511, USA
    (Received 13 December 2004; published 12 May 2005)

