BRDF Reconstruction Using Compressive Sensing

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Figure 1: A sample scene has been rendered using alum-bronze and blue-metallic-paint materials from MERL database [9]. Left: Image based on the original data. Right: Image based on the reconstructed data using only 5% of the original (The Peak-to-Signal Ratio is 51.63 dB).

ABSTRACT

Compressive sensing is a technique for efficiently acquiring and reconstructing the data. This technique takes advantage of sparseness or compressibility of the data, allowing the entire measured data to be recovered from relatively few measurements. Considering the fact that the BRDF data often can be highly sparse, we propose to employ the compressive sensing technique for an efficient reconstruction. We demonstrate how to use compressive sensing technique to facilitate a fast procedure for reconstruction of large BRDF data. We have showed that the proposed technique can also be used for the data sets having some missing measurements. Using BRDF measurements of various isotropic materials, we obtained high quality images at very low sampling rates both for diffuse and glossy materials. Similar results also have been obtained for the specular materials at slightly higher sampling rates.

Keywords

BRDF reconstruction, compressive sensing.

1 INTRODUCTION

Real world materials display different reflection characteristics. Accurate representation of the distribution of light reflected from the surface of a material has long been studied in computer graphics. A class of functions called Bidirectional Reflectance Distribution Function (BRDF) defined in terms of incoming and outgoing light directions is commonly used to describe such reflectance properties.

Various models have been proposed for approximating the BRDF. It has been shown that some of these models meet the reciprocity and energy conserving principles.

However they generally fail to capture the reflectance properties of all material types. A natural approach to tackle this problem is to fit the underlying models to measured BRDF data. Since small numbers of parameters are involved in these models, only the corresponding estimates need to be stored for reconstruction of BRDF. Fitting can also be performed on data sets having some missing measurements. Nevertheless, such fitting procedure leads to some approximation errors for certain materials and its implementation is difficult in most cases because of its computational complexity [12].

A general and simple method for approximating the BRDF would be to use directly the measured BRDF data which is obtained on a regular grid using some version of gonioreflectometers. Then the intermediate BRDF values can be estimated by an interpolation. However, the BRDF data obtained in this way is generally noisy and contains some missing observations due to some difficulties in measuring BRDF around grazing angles [14].

A major difficulty of using the BRDF data is its large size. Even if the raw data were correct and complete, its size would be prohibitively large for an efficient storage and for a rendering application. An alternative approach is to compress the measured BRDF data using some well-known compression techniques. These techniques are based on using basis functions (splines, spherical harmonics, wavelets, and Zernike polynomials), dimension reduction techniques (Principal Component Analysis, Independent Component Analysis and Cluster Analysis) and matrix factorization (Non-negative Matrix Factorization and Tensor Products). Empirical results have shown that these compression based techniques can provide an accurate and compact representation of BRDF data but do not offer an efficient importance sampling [8].

Generally, BRDF measuring systems suffer from occlusion problems because of using cameras, projectors, or even mirrors. In such cases, acquiring BRDFs over a full hemisphere may not always be possible. In some other cases measurements taken at certain angles may be prohibitively noisy and cannot be used for rendering. A possible approach for handling this problem is to ignore the missing or highly noisy measurements and fit an analytical model to the remaining part of the data [12]. Cleary the resulting fitting will not be adequate due to increased degree of the lack-of-fit of the model.

In this work we propose to employ an interesting technique namely the compressive sensing that can be used to reconstruct the missing BRDF measurements efficiently. It turns out that the proposed technique also provides an effective way of compressing the BRDF data.

The compressive sensing (which is also referred in the literature as compressed sensing or compressive sampling) has been evolving rapidly [3]. This technique takes advantage of the sparseness or compressibility of the data, allowing the entire measured data to be recovered from relatively few measurements using some optimization techniques. It has been shown that compressive sensing can also be used for data sets that contain some missing measurements [6].

We apply the compressive sensing approach on a large BRDF data for rendering applications. Based on the empirical results, we show that compressive sensing technique can be used effectively for image reconstruction. In Figure 1, we present rendered images based on measured BRDF data sets with 95% of its elements removed randomly and reconstructed on the right, and the original image on the left. This example illustrates the power of the proposed technique for reconstructing measured BRDF data using only a small portion of it. We also demonstrate the effectiveness of compressive

sensing technique for reconstruction of BRDF data having some missing or noisy measurements.

The paper is organized as follows: In Section 2 we explain briefly the compressive sensing technique. In Section 3 we present the problems encountered during BRDF data acquisition. In Section 4 we describe our reconstruction algorithm and in Section 5 we show experimental results. Section 6 is devoted to conclusions and discussions.

2 COMPRESSIVE SENSING

Compressive sensing technique, emerging over the past few years, has attracted considerable attention in digital signal processing. In this section we summarize compressive sensing technique for completeness. A good treatment of the topic may be found in [2, 5].

An n-dimensional signal is called sparse if it can be represented as a linear combination of smaller number of some basis vectors. The key idea behind compressive sensing technique is that sparse signals can be reconstructed perfectly in terms of smaller number of basis vectors.

Suppose that discrete time signals are represented by an $n \times 1$ column vector $\mathbf{x}$. Without loss of generality, higher dimensional data can also be represented by a vector by making an appropriate arrangement of the signal measurements. For example four dimensional BRDF data with a resolution $n = n_1 \times n_2 \times n_3 \times n_4$ where $n_i, (i = 1, 2, 3, 4)$ is the resolution of the ith dimension, can be viewed as an $n \times 1$ vector. It is well known that a vector can be transformed into another $n \times 1$ vector $\mathbf{s}$. through an $n \times n$ orthogonal basis matrix Ψ as

$$\mathbf{x} = \Psi \mathbf{s} \qquad (1)$$

Since Ψ is an orthogonal matrix, this equation can be solved for $\mathbf{s}$ as $\mathbf{s} = \Psi' x$ where Ψ' is the transpose of Ψ. If $\mathbf{s}$ is known or can be estimated from the sample data then $\mathbf{x}$ can be reconstructed easily from the above equation. This representation similar to that of principal components in the sense that the vectors $\mathbf{x}$ and $\mathbf{s}$ are the equivalent representations of the signals. In principal components representation, Ψ is determined adaptively from the sample data and the first $k \leq n$ nonzero entries of $\mathbf{s}$ corresponding to significant row vectors in Ψ are used to recover $\mathbf{x}$.

Compressive sensing technique uses the sparsity property of the signals. If a signal is sparse then some of the entries of $\mathbf{s}$ in Eq. (1) is expected to be zero leading to a representation with a reduced dimensionality. The underlying approach provides a non-adaptive technique where entries of the matrix is fixed and only a small

portion of the sample data is used for reconstruction of the vector $\mathbf{x}$.

Suppose that a signal is k-sparse that is only k entries of $\mathbf{s}$ in Eq. (1) is nonzero. Let $\mathbf{y} = \varphi\mathbf{x}$ where φ is an $m \times n$ sampling matrix ($m \le n$) and $\mathbf{y}$ is an $m \times 1$ vector. From Eq. (1) $\mathbf{y}$ can be expressed as

$$\mathbf{y} = \varphi\mathbf{x} = \varphi\Psi\mathbf{s} = \Theta\mathbf{s} \qquad (2)$$

where Θ is an $m \times n$ transformation matrix, Ψ and $\mathbf{s}$ are defined as in Eq. (1). Clearly, this system of m simultaneous linear equations with n unknowns cannot be solved for $\mathbf{s}$ as the number of independent linear equations is much less than the number of unknowns. In a special case when signals are assumed to be k-sparse then a solution could be possible if the locations of the nonzero coefficients are known and $m \ge k$. A necessary and sufficient condition is that the transformation matrix should not change the lengths of the k-sparse vectors [1]. It has been shown that this condition is satisfied if the sampling matrix φ is chosen to be an identically and independently distributed gaussian matrix. An interesting result with this Gaussian matrix is that k-sparse signals of length n can be reconstructed using only $m \times 1$ vector $\mathbf{y}$ where $m \ge ck\log(n/k) < n$ and c is a small constant random number generated from a Gaussian distribution.

The reconstruction algorithm for a k-sparse sample of size n should be able to determine the k nonzero and $(n-k)$ zero entries in $n \times 1$ vector $\mathbf{s}$. Finding the best combination out of m nonzero and $n-m$ zero combinations of the entries in $\mathbf{s}$ is difficult. However an approximate solution can be obtained by minimizing the quantity

$$\xi(\mathbf{s}) = \sum_{i=1}^{n} |s_i| \qquad (3)$$

with the constraint $\mathbf{y} = \Theta\mathbf{s}$ where $\mathbf{s} = (s_1, s_2, \cdots, s_n)$ is the sparse coefficient vector [1]. This process is known as ℓ_1-norm optimization. An interesting result with ℓ_1-norm optimization is that it tends to concentrate the energy of the signals onto a few nonzero entries of $\mathbf{s}$ as opposed to the least squares which tends to spread the energy around. The reconstruction algorithm then consists of the following steps:

i Determine an $m \times n$ sampling matrix φ
ii Obtain the $m \times 1$ measurement vector as $\mathbf{y} = \varphi\mathbf{x}$
iii Determine an $n \times n$ orthogonal basis matrix Ψ
iv Find the coefficient vector $\mathbf{s}$ using ℓ_1-minimization
v Reconstruct $\mathbf{x}$ using Eq. (1).

3 BRDF DATA WITH MISSING OR NOISY MEASUREMENTS

Commonly, BRDF measurements are obtained using a gonioreflectometer, a computer controlled device which

typically has a photometer and a light source. Often the underlying system requires huge amount of measurements. For example, when an angular resolution of 1 degree is used, with a uniform sampling then the underlying system would require approximately one and a half million of measurements. It has been reported that measurements of reflectance at grazing angles are difficult to obtain accurately. For example, in evaluating several analytical BRDF models, Ngan et. al. [12] have ignored the data within an incoming and outgoing angles greater than 80 degrees considering that they are in general unreliable. In some other cases the optical elements of the system do not allow measurements at all at certain positions resulting considerable amount of missing data [11]. Experimental results have shown that approximately 60-65% of the measurements taken at the grazing angles and 10-15% of the measurements in the remaining region contain some reciprocity related errors [8]. On the other hand, Romerio et. al. [13] have mentioned about the existence of lens flare artifacts in BRDF measurements.

4 RECONSTRUCTION OF BRDF MEASUREMENTS

In this work, the compressive sensing technique is applied on isotropic BRDF data which is assumed to have some missing measurements. For this purpose the three dimensional data is divided into sub-sample blocks of size $15 \times 15 \times 15$. Random samples were generated from these sub-samples at a predefined sampling ratio. Finally, the resulting uniform random samples were used to reconstruct the underlying blocks employing the compressive sensing technique. An ℓ_1-norm optimization algorithm proposed by van den Berg and Friedlander [16] was used for estimating the sparse vector $\mathbf{s}$ as defined in Eq. (1).

As was explained in the preceding section, the compressive sensing technique requires using a sampling matrix φ, and an orthogonal basis matrix which is incoherent with this sampling matrix. A number of sampling methods have been proposed for reconstructing signal data [3, 4]. Unfortunately, these methods cannot be applied on BRDF data directly unless an appropriate sampling strategy is used for obtaining the BRDF measurement. It is obvious from Eq. (2) that when the vector $\mathbf{x}$ contains some missing data points, that is when some of the entries are missing, then the corresponding dot product between the vector $\mathbf{x}$ and the rows of the Gaussian matrix φ cannot be determined. To overcome this difficulty, we proceed to use a different sampling procedure namely point sampling which is based on using a permutation matrix instead of a random Gaussian matrix. It is shown that the permutation matrices are coherent with the basis matrices which produce highly sparse data like the ones that are based
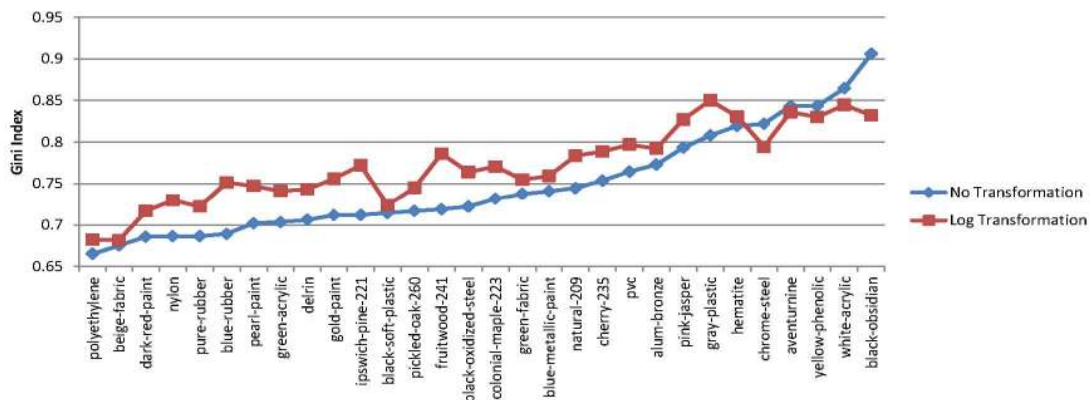
Figure 2: Computed Gini's indices of sparsity coefficients obtained in Fourier domain for BRDF measurements of 30 isotropic materials. Higher index values are found with log transformations.

on wavelets [15]. In our work we created an $n \times n$ basis matrix Ψ whose entries are obtained through Fourier transforms. It is reported that Fourier basis matrices often do not produce sparse coefficients in vector **s** for real-world images [15]. However, our empirical results based on BRDF data have shown that highly sparse coefficients can be obtained with Fourier basis matrices when they are used with permutation matrices (point sampling). This property of using Fourier basis matrices is demonstrated in Figure 2. In this figure, Gini's indices are obtained and plotted for each material. Similar results based on log transformations of BRDF are also obtained and shown on the same figure. Higher values of Gini's index corresponds to higher sparsity of the measured data. It is seen that, the Gini's indices obtained for this case is found in the range (0.68, 0.85).

The permutation matrix which consists of zeros and ones are provided by generating these numbers using a simple random sampling techniqe without replacement. During the sampling process, if an entry of this matrix corresponding to a missing value in the vector **x** is 1 then it is set to 0 and the next available position is set to 1.

It is assumed that the BRDF measurements must be positive [10]. However some data sets contain negative values as a result of certain sampling errors. We used log transform of the BRDF measurements to preserve

the underlying property of BRDF. Our empirical results have shown that such transformation also increases the sparsity of the BRDF data. This situation is illustrated in Figure 3. It is seen in the figure that negative values cause some artifacts under illumination.

5 RESULTS

To demonstrate the efficiency of the compressive sensing approach, we considered a data set based on various isotropic materials acquired by Matusik et.al. [10] from MERL MIT database [9]. In this data set 1458000 measurements are provided for each material. We selected 30 isotropic materials to represent various diffuse and reflection properties. ℓ_1-norm estimates of the coefficient vectors for each material were computed in Fourier domain. Gini's index [7] is used as a measure of sparsity and computed for each case.

It is seen in Figure 2 that in 25 materials out of 30, the sparsity indices based on log transformation are found to be higher than those based on the original data.

To investigate the effect of the sampling ratio on the visual quality of the reconstructed images, random samples with ratios 1%, 2.5%, 5%, 10%, 25%, 50% were generated from six different materials namely dark-red-paint, green-fabric, blue-metallic-paint, gold-paint, fruitwood-241, chrome-steel were chosen. These materials were chosen to reflect the diffuse, glossy and specular properties of reflection. Rendered spheres based on the original data is shown in the first row of the Figure 4 while the reconstructed images are presented in the following rows. The insets for each sphere represents the difference image between the corresponding reconstructed and the original images scaled by 8. The Peak-to-Signal(PSNR) values are also given for each reconstructed image.

It is interesting to see that images with a visually acceptable quality could be obtained by sampling only 1% of the measurements for diffuse and glossy mate-
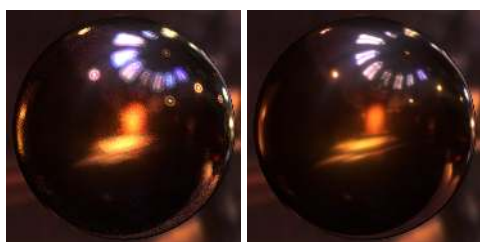

Figure 3: Left: Image based on BRDF measurements. The presence of lighting artifacts due to negative BRDF values. Right: Image obtained with log transformation.
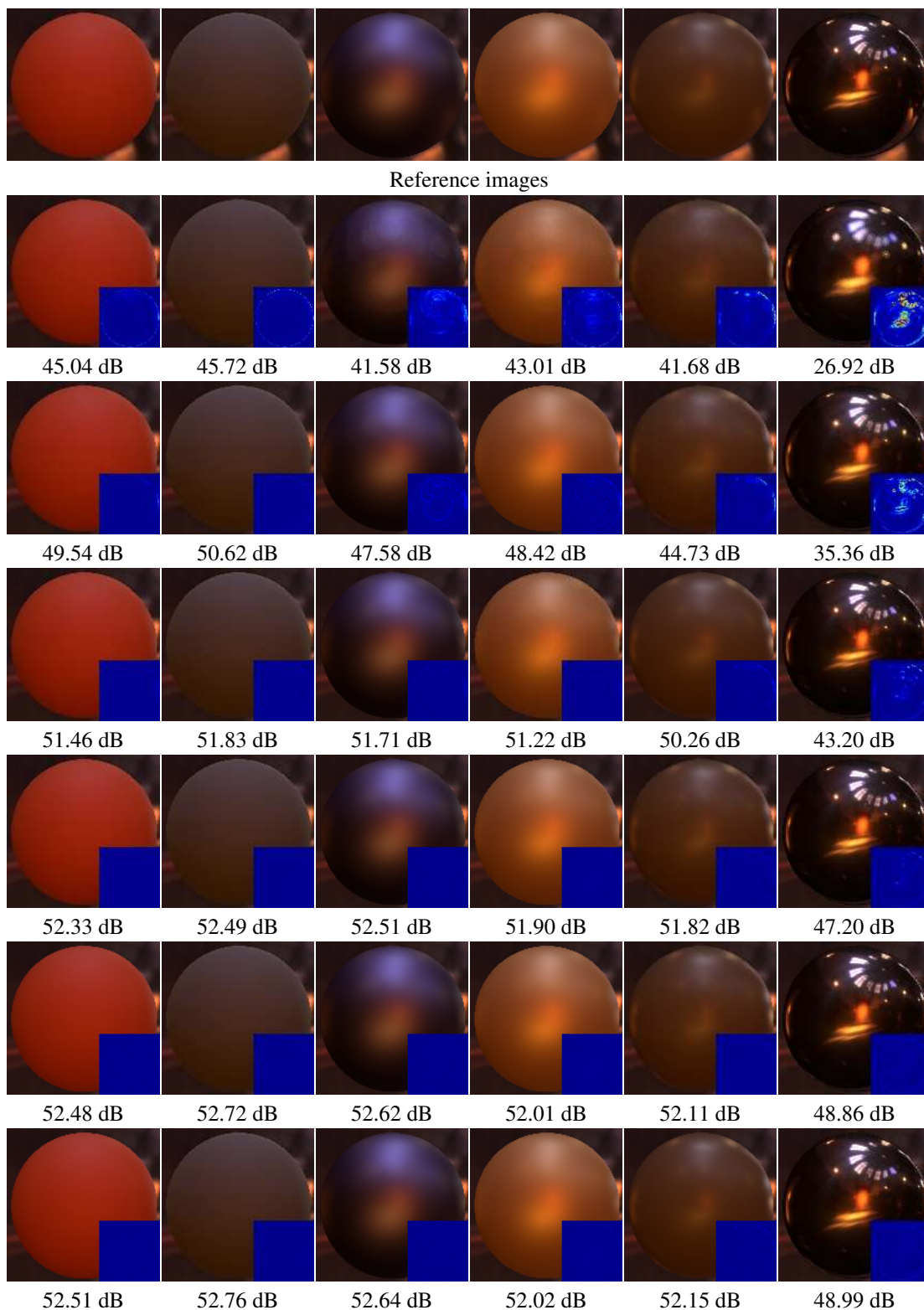
Figure 4: Rendered spheres under global illumination. First two columns: Diffuse materials (*dark-red-paint*, *green-fabric*), Third and fourth columns: Glossy materials (*blue-metallic-paint*, *gold-paint*), Last two columns: Specular materials (*fruitwood-241*, *chrome-steel*) [9]. First row: Original images. Second row through seventh row: Images obtained at 1, 2.5, 5, 10, 25, and 50 percent. Insets indicate the scaled differences between the given image and the corresponding original image.
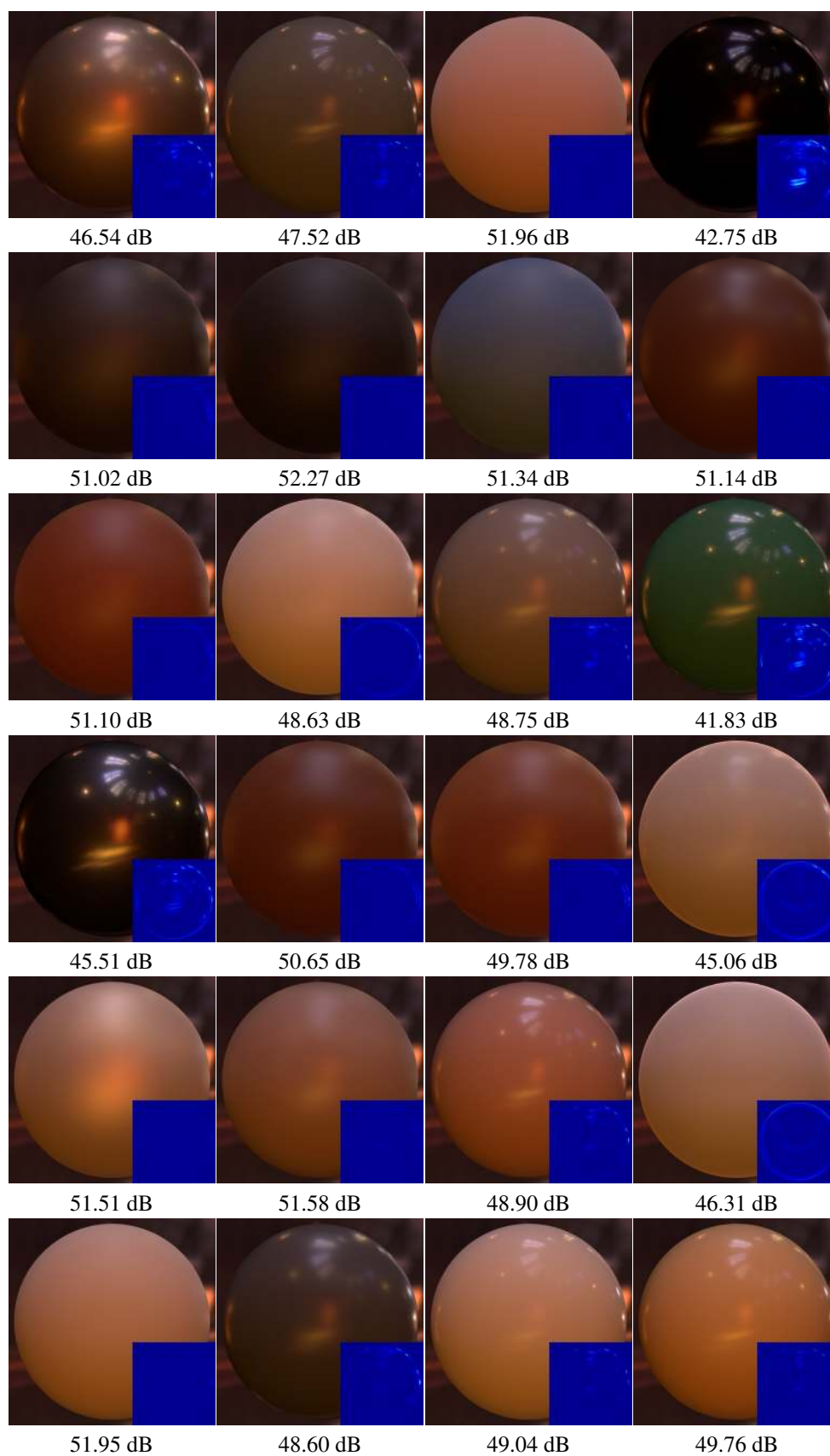
Figure 5: Reconstructed images using 5% of the BRDF measurements of randomly selected 24 isotropic materials from MERL data base. Insets indicate the differences between the given image and the corresponding original image. PSNR values are given for each material.

rials. In all cases except the first two cases for chrome-steel corresponding to 1% and 2.5% sampling ratios, the PSNR values are above 40 db. These results demonstrate the power of the compressive sensing approach when dealing with BRDF data having missing measurements. It can also be seen from Figure 5 that compressive sensing approach produces visually acceptable quality for all material types by sampling only 5% of the original data.

6 CONCLUSIONS AND DISCUSSIONS

In this work we analyzed the potential use of compressive sensing technique to facilitate a fast procedure for processing large BRDF data. In image reconstruction, compressive sensing can be more efficient than traditional sampling when data is sparse. Considering the fact that the BRDF data often can be highly sparse, it can be reconstructed efficiently using compressive sensing technique. We have demonstrated that the proposed technique can also be used for the data sets having some missing or unreliable measurements. Using BRDF measurements of various isotropic materials, we have shown that high quality images can be reconstructed at very low sampling ratios both for diffuse and glossy materials. Similar results also have been obtained for the specular materials at slightly higher sampling ratios.

It is well known that modeling and representation of anisotropic data is difficult. More data acquisition is needed for this case as compared with isotropic materials. We expect that the proposed approach can be extended to BRDF reconstruction for anisotropic materials.

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