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Abstract

The recurrence theory for the breakdown probability in avalanche photodiodes (APDs) is generalized to heterostructure APDs that may have multiple multiplication layers. The generalization addresses layer-boundary effects such as the initial energy of injected carriers as well as the layer-dependent profile of the dead space in the multiplication region. Reducing the width of the multiplication layer serves to both downshift and sharpen the breakdown probability curve as a function of the applied reverse-bias voltage. In structures where the injected carriers have an initial energy that is comparable to the ionization threshold energy, the transition from linear mode to Geiger-mode is more abrupt than in structures in which such initial energy is negligible. The theory is applied to two recently fabricated $Al_{0.6}Ga_{0.4}As$-GaAs heterostructure APDs and to other homostructure thin GaAs APDs and the predictions of the breakdown-voltage thresholds are verified.

SECTION I. Introduction

Avalanche photodiodes (APDs) are highly desirable in fiber-optic communication systems and in many applications that rely on precision radiometric measurements such as photon and photon-coincidence counting. Recently, APDs with thin multiplication layers have been shown to exhibit a significant reduction in the excess noise factor, a feature that is now well known to be attributable to the dead-space effect [1]–[2][3][4][5][6][7][8]. The dead space is the distance a carrier must travel within the APD's multiplication region before acquiring the energy threshold needed for effecting an impact ionization. In essence, dead space results in inhibition in the locations of ionization, which, in turn, brings about orderliness in the avalanche of carrier generation. In APDs with wide multiplication regions (*viz*., > 0.4 μ m), the dead-space distance is negligible relative to the width and its effect on the carrier multiplication is minimal. However, in thin APDs, the dead space can occupy a large fraction of the multiplication region, thereby significantly altering not only the multiplication noise but also the time response, power spectral density, and breakdown [4], [9], [10]. Heterostructure APDs with multiple thin multiplication layers have also been recently fabricated, exhibiting even lower excess noise factors. Such improved performance has been attributable to the combination of the dead-space effect and bandgap-boundary effects, which can serve to further regularize impact ionization through careful bandgap engineering [11]–[12][13].

One aspect of the APD performance whose dependence on the dead space was not analytically investigated is the breakdown probability. Breakdown occurs when the APD's gain becomes infinite. In general, as the applied reverse-bias voltage is raised beyond a threshold, the probability that the gain becomes unstable diverges from zero, and gradually approaches unity as the voltage is further raised. In fact, this threshold voltage is nothing but the breakdown voltage, which is defined as the applied reverse-bias voltage at which the *mean* gain becomes infinite. (Note that as the gain is integer-valued, its mean is finite if and only if the probability of having an infinite gain is zero.) The behavior of the breakdown probability, as a function of the applied revere-bias voltage, is the key indicator of how fast the transition from stable to saturated operation occurs. For example, when an APD is used in the Geiger mode, it is highly desirable that such a transition occur as rapidly as possible so that any incoming photon triggers a measurable response with near certainty. On the other hand, if the transition is not steep, then at any given applied reverse bias, a fraction of the absorbed photons (proportional to the complement of the breakdown probability) will fail to trigger breakdown, which reduces detection efficiency.

In light of the role played by the multiplication-region width in improving the noise and bandwidth characteristics in thin APDs, a natural question that comes to mind is whether thin APDs exhibit improved breakdown characteristics. We have partially answered this question affirmatively in the past by showing that the breakdown voltage decreases as the width decreases [9]. However, the breakdown probabilities for thin

APDs have not been investigated heretofore. In 1999, McIntyre [7] adopted the recurrence principles developed by Hayat et al. [1], [14] and formulated recurrence equations which characterized the breakdown probability for the case of nonuniform fields. Although McIntyre [7] attempted to predict the breakdown probability, he encountered difficulty in numerically solving the recurrence equations near breakdown and beyond. Moreover, no analytical model has yet been developed that can capture the bandgap-boundary effects encountered in heterostructure APDs. Such boundary effects include the initial energy of injected carriers and the layer-dependent profile of the dead space in the multiplication region.

In this paper, we generalize the recurrence equations reported by McIntyre [7] to include boundary effects associated with thin heterostructure APDs. This generalization draws from our recent work on the bandgap-boundary effects on avalanche multiplication noise [11]. The technique developed in this paper provides the means for predicting the probability of breakdown as a function of the applied reverse bias voltage for any heterostructure APD.

SECTION II. Analytical Model

Consider an APD with a multiplication region (possibly consisting of multiple layers) extending from x=0 to x=w, and exhibiting an electric-field profile E(x). Assume that a parent electron is injected at x=0, and that the electron has an initial energy E0 which it acquires, for example, as it travels through a field gradient just before entering the multiplication region. According to basic principles of impact ionization [15], if the electron ionization threshold energy of the material is Eie, then the injected electron must first travel an initial dead-space distance de0, which is the solution to the equation (Eie−E0)=∫de00qE(y)dy, before being able to impact ionize. (de0 is set to zero if E0≥Eie.) Furthermore, according to the hard-threshold dead-space multiplication model, the probability density function (pdf) of the distance ξ to the first impact ionization for the initial carrier is given by [11]

$$h_{e_0}(\xi) = \begin{cases} \alpha(\xi)\exp\left(-\int_{d_{e_0}}^{\xi}(y)dy\right)\alpha, & \xi \geq d_{e_0} \\ 0, & \xi < d_{e_0} \end{cases} \tag{1}$$

where $\alpha(\cdot)$ is the nonlocalized position-dependent ionization coefficient of electrons, which can be calculated from the electric field through a material-dependent parametric model. Upon ionization, two electrons and a hole with zero initial kinetic energy are generated. Moreover, for an offspring electron, born at x with zero initial energy, the pdf of the distance ξ to the first impact ionization is given by [14]

$$h_e(\xi|x) = \begin{cases} \alpha(x+\xi)\exp\left(-\int_{d_e(x)}^{\xi}\alpha(x+y)dy\right), & \xi \geq d_e(x) \\ 0, & \xi < d_e(x) \end{cases} \tag{2}$$

where de(x) is the dead space that it must first travel before being able to impact ionize. A similar argument applies to an offspring hole, in which case the pdf of the distance to the first impact ionization is

$$h_h(\xi|x) = \begin{cases} \beta(x-\xi)\exp\left(-\int_{d_h(x)}^{\xi} \beta(x-y)dy\right), & \xi \geq d_h(x) \\ 0, & \xi < d_h(x) \end{cases} \tag{3}$$

where *dh*(*x*) is dead space associated with a hole born at *x*. In [11], we describe how the dead-space profiles can be computed for a heterostructure. In the above model for ionization, we assumed that the value of the ionization coefficient beyond the dead space is dependent only on the material and the local electric field. A more realistic (but much more complex) model would consider the value to be a function of the history of the carrier. Despite its simplicity, when the above model is used in conjunction with the recurrence technique [1], [14], it has proven to be effective in predicting the low-noise behavior of thin APDs independently of the thickness of the multiplication region.

We now characterize the breakdown probability. Following the notation introduced in [1], let *Z*(*x*) denote the total electron and hole population resulting from a parent electron born at *x* with zero initial energy. Similarly, let *Y*(*x*) denote the total electron and hole population resulting from a parent hole born at *x*. Thus, for the case of electron injection (at *x*=0), the APD gain *G* is given by 0.5(*Z*(0)+1). Let *PZ*(*x*) be defined as the probability that *Z*(*x*) is finite, and similarly, let *PY*(*x*)P{*Y*(*x*)<∞}. McIntyre invoked a recurrence argument and characterized *PZ* and *PY* through the following two nonlinear integral equations [7]:

$$P_Z(x) = \int_{w-x}^{\infty} h_e(\xi|x)d\xi + \int_0^{w-x} P_Z^2(x+\xi)P_Y(x+\xi) \cdot h_e(\xi|x)d\xi \tag{4}$$

$$\text{and } P_Y(x) = \int_x^{\infty} h_h(\xi|x)d\xi + \int_0^{x} P_Y^2(x-\xi)P_Z(x-\xi) \cdot h_h(\xi|x)d\xi. \tag{5}$$

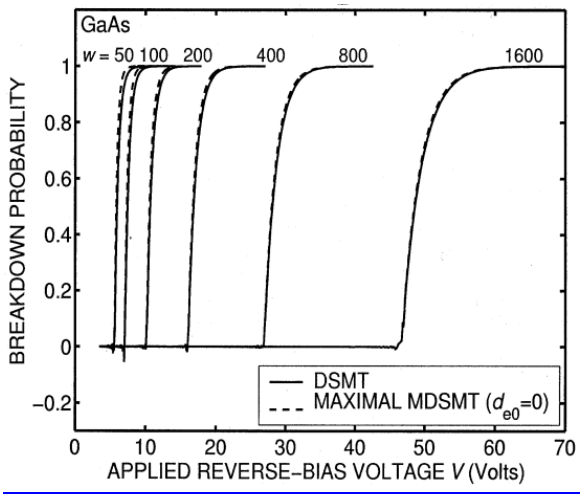

Fig. 1. Breakdown probability for GaAs as a function of the applied reverse-bias voltage for various multiplication-region widths. Solid curves represent the DSMT predictions. Dashed curves represent the maximal-MDSMT predictions.

We now generalize these equations to the case when the parent carrier has nonzero initial energy.

Let $Z0(x)$ be defined as $Z(x)$ with the exception that for the parent electron at x, the distance ξ to the first impact ionization has a pdf $he0(\xi)$ [as defined in (1)]. The key observation here is that upon the first ionization of the injected electron, the two newly-created electrons and hole will have zero initial energy, independently of the initial energy of their parent electron. Consequently, conditional on the initial ionization occurring at ξ, $Z0(x)$ is finite if and only if each one the two offspring electrons and the offspring hole produces a finite number of offsprings. Thus after averaging over all possible ξ, we obtain the following modified recurrence equation for the probability that $Z0(x)$ is finite:

(6)

$$P\{Z_0(x) < \infty\} = \int_{w-x}^{\infty} h_{e_0}(\xi)d\xi + \int_0^{w-x} P_Z^2(x+\xi)P_Y(x+\xi) \cdot h_{e_0}(\xi)d\xi.$$

Hence, to calculate $P\{Z0(x)<\infty\}$, we must first solve for $PZ(\cdot)$ and $PY(\cdot)$ according to (4) and (5), and subsequently use them in calculating the integral given by (6). This is the modified dead-space multiplication theory (MDSMT) for breakdown.

SECTION III. Results

To see the roles of the multiplication-region width and the initial energy of injected carriers on the breakdown characteristics, we numerically solved (4)–(6) and computed the breakdown probability as a function of the applied reverse-bias voltage for GaAs. In these calculations, we assumed a constant electric-field profile within the multiplication region and used the simple approximation $V=Ew$ for the reverse-bias voltage. The parameters for the nonlocalized electron and hole ionization coefficients for GaAs were taken from Saleh et al. [4].

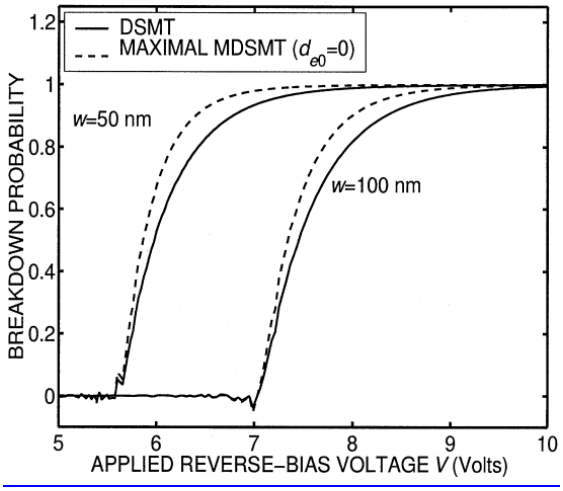

Fig. 2. Magnification of Fig. 1 for w=50 nm and w=100 nm.

Two sets of results were generated for each width. In the first set, we assumed that the injected carrier possessed no initial energy (i.e., $E0=0$), in which case the results were obtained by solving (4) and (5) and the breakdown probability was calculated using $1-PZ(0)$. We refer to this set of results as the DSMT predictions. In the second set of results, we assumed that the initial energy was in excess of the electron ionization energy Eie, in which case the initial dead space $de0$ was set to zero. These results were obtained by inserting $PZ(x)$ and $PY(x)$, which were found for the first set, into (6), and the breakdown probability was calculated using $1-P\{Z0(0)<\infty\}$. We refer to this second set of calculations as the maximal-MDSMT predictions, as they reflect the maximum initial-energy effect. The predicted breakdown probabilities are shown in Fig. 1 for w=50, 100, 200, 400, 800, and 1600 nm. As expected, reducing w serves to cause breakdown to occur at a lower reverse bias. Moreover, for a fixed w, the initial energy of injected carriers causes the breakdown to occur more abruptly, as can be seen from the magnified plots in Fig. 2. This new result can be explained as follows. The initial energy of an injected carrier enhances the probability of the initial impact ionization occurring in the onset of the multiplication process (i.e., near the edge of the multiplication region). This, in turn, will enhance the breakdown probability as each of the offspring electrons will have a higher chance of breakdown as they have a longer distance to travel. Note that the breakdown voltage VB is the voltage corresponding to the point when the breakdown probability begins to exceed zero. We also note that the calculated values of the breakdown probability near breakdown are sensitive to precision error (resulting from discretizing the recurrence equations); however, the calculated values rapidly stabilize beyond the breakdown voltage. We emphasize that in our calculation we used nonlocalized ionization coefficients [4], [5]. The use of the bulk, or so-called localized, ionization coefficients [16] cannot be justified for our technique, as they are not consistent with the dead space theory. It was observed that attempting to use such localized coefficients in the current recurrence technique can lead to unstable solutions.

We also observe from Fig. 2 that both the DSMT and the maximal-MDSMT models predict a more abrupt transition for a thin device than a thick one. However, in order to see the magnitude of this increase in transition abruptness relative to the breakdown voltage, we normalized the transition abruptness by the breakdown voltage. To do so, we calculated the breakdown steepness factor $\Delta V/VB$, where ΔV is the voltage difference corresponding to the transition in breakdown probability from 0.05 to 0.95. (The smaller the steepness factor, the more abrupt the transition is.) Fig. 3 shows the behavior the steepness factor as a function of w. According to DSMT model, as w decreases, the steepness factor increases, and hence the transition from stable to unstable becomes *relatively* less abrupt. However, according to maximal-MDSMT calculations, the steepness factor is almost invariant as w decreases. Hence, the initial energy of injected carriers serves to preserve the stable-to-unstable transition characteristics, especially for low values of w. This is a very desirable feature: it indicates that the reduction in the breakdown voltage with decreasing device thickness can be made available without compromising the relative abruptness of the breakdown transition.

SECTION IV. Comparison With Experiments

The purpose of this section is to validate our predictions of the breakdown voltage for thin APDs. Moreover, since in this paper, the breakdown voltage is extracted directly from the predicted behavior of the breakdown probability as a function of the applied bias, agreement of the breakdown-voltage predictions with experiment will also serve to partially validate the correctness of our model for the breakdown probability. According to our

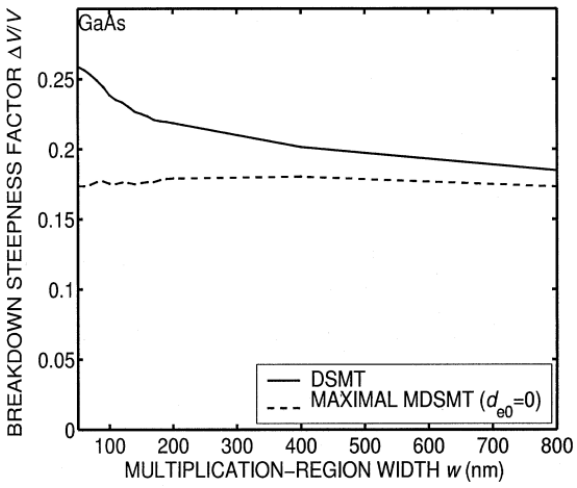

Fig. 3. Characteristics of the transition steepness factor $\Delta V/V$ for GaAs. This factor is zero when the transition is abrupt.

knowledge, there is no relevant experimental data available on the probability of breakdown; thus, our predictions regarding the steepness of the breakdown probability cannot be experimentally verified at this time.

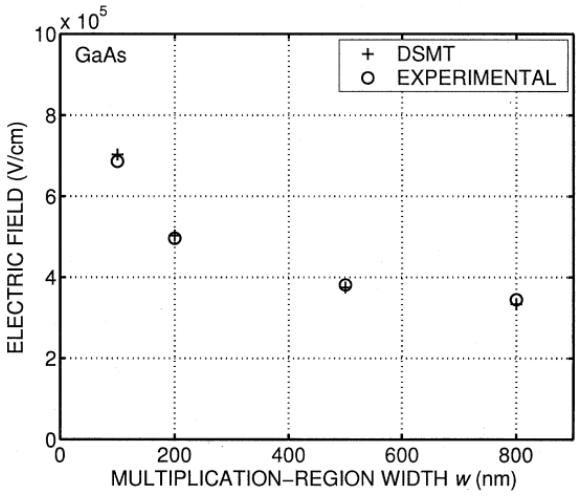

Fig. 4. DSMT predictions and experimental values of the breakdown voltage for various GaAs APDs.

The predicted breakdown voltage *VB* was compared with the measured values obtained for homojunction GaAs APDs. These homojunctions, which were described in detail in [2], have been shown to exhibit negligible initial-energy effect [11]. We, therefore, only considered the DSMT predictions and not the MDSMT in this case. For various multiplication-region widths, the predicted and experimental values for the breakdown voltage were, respectively, 7.03 and 6.86 V (at *w*=100 nm), 10.06 and 9.92 V (at *w*=200 nm), 18.75 and 19.05 V (at *w*=500 nm), and 26.72 and 27.60 V (at *w*=800 nm). The corresponding breakdown electric-field values are shown in Fig. 4, demonstrating the very good agreement between the DSMT predictions and experiment. Such good prediction of the breakdown voltage was also demonstrated earlier using a different technique (which does not yield the breakdown probability) based on impulse-response considerations [9]. We emphasize that the excellent agreement with experiment was obtained as a direct result of using the independently-calculated ionization coefficients and threshold energies [4] in the breakdown recurrence theory without introducing any model adjustments or auxiliary parameters to fit the data.

We also applied the theory to two heterostructure APDs. The first device was a GaAs–$Al_{0.6}Ga_{0.4}As$ heterostructure, where the multiplication is confined to the GaAs layer and *w*=130 nm. (Electrons are injected

from the $Al_{0.6}Ga_{0.4}As$ *p*-layer into the GaAs *i* layer.) It was previously shown that at high electric fields (> 670 kV/cm), the initial-energy effect for this device reached its maximal limit, in which case the initial dead space vanished for the injected electron [11]. The second device was also a GaAs–$Al_{0.6}Ga_{0.4}As$ heterostructure APD, but for which the multiplication takes place in two adjacent $Al_{0.6}Ga_{0.4}As$ (100 nm) and GaAs (30 nm) *i* layers. (Electrons are injected from an $Al_{0.6}Ga_{0.4}As$ *p*-layer into the $Al_{0.6}Ga_{0.4}As$ *i* layer.) The width of the overall multiplication region for the second device was thus 130 nm. (For $Al_{0.6}Ga_{0.4}As$, the parameters for the ionization coefficients were taken from Tan et al. [5].) The effect of initial energy in this device was shown to be very modest, resulting in a dead-space reduction of no more than 10% at E=800 kV/cm [11].[1] We performed three sets of calculations for each device: These included the DSMT and the maximal-MDSMT predictions, as well as the MDSMT predictions, in which case the actual initial energy of injected carriers was used.

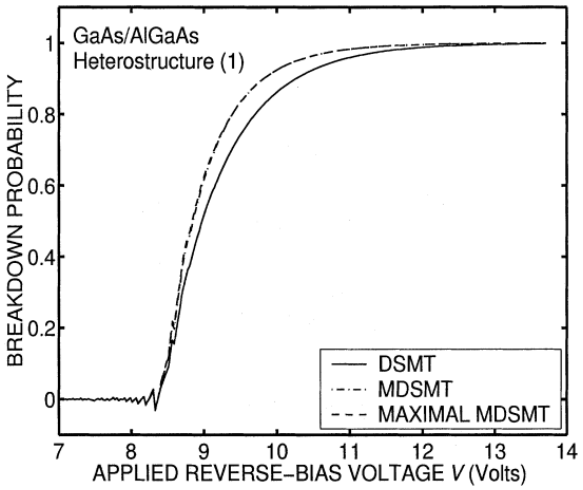

Fig. 5. Breakdown probability versus applied reverse-bias voltage for the first heterostructure GaAs–$Al_{0.6}Ga_{0.4}As$ APD, which has a GaAs multiplication-region width of 130 nm. Three separate curves are shown representing the DSMT-predictions (solid), the MDSMT predictions (dotted-dashed), and the maximal-MDSMT predictions (dashed).

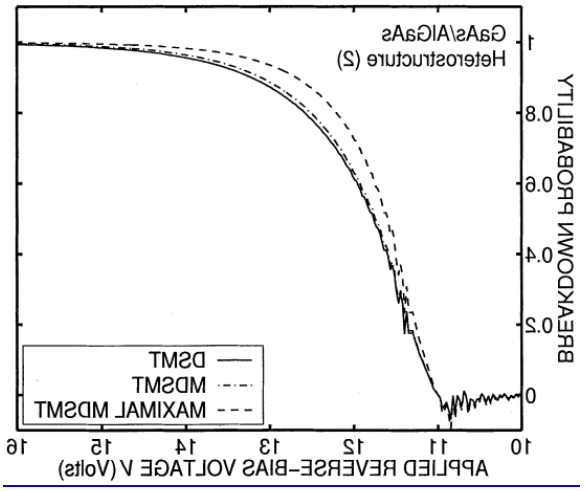

Fig. 6. Same as Fig. 5 but for the second GaAs–$Al_{0.6}Ga_{0.4}As$ heterostructure, which has a two-layer $Al_{0.6}Ga_{0.4}As$–GaAs multiplication region width total width of 130 nm.

These initial energies were previously calculated from the electric-field profile for each device, which were calculated using Medici software according to the doping profiles obtained from secondary ion mass spectroscopy (SIMS) [11]. (A lookup table was generated relating the applied bias voltages to the electric-field profiles.) The predictions for the first heterostructure are shown in Fig. 5. Consistent with the homojunction

GaAs results, the maximal-MDSMT model predicts a more abrupt transition from sub-breakdown to breakdown than the DSMT. Moreover, the maximal-MDSMT and MDSMT curves are almost overlapping, since the initial dead space is almost nonexistent. In contrast, the behavior is different for the second heterostructure, as shown in Fig. 6, where the MDSMT and DSMT predictions are almost indistinguishable. This is because the initial energy in the second device is negligible in comparison to the ionization threshold for $Al_{0.6}Ga_{0.4}As$. As for the breakdown voltage *VB* for these two heterostructures, the MDSMT predictions for the first and second heterostructure APDs were 8.37 and 11.00 V, respectively, whereas the corresponding experimental values were 8.35 and 12.43 V.

We finally make the comment that the initial-energy effect considered in this paper does not affect the breakdown voltage, as can be seen from Figs. 1, 2, 5, and 6. This behavior can be explained from the formulas given in (4) and (6). First, note that $P\{Z0(x)<\infty\}=1$ precisely when the integrands $PZ(x)$ and $PY(x)$ (in the second integral) are unity over the support of $he0(x)$ (i.e., the range over which $he0(x)$ is nonzero). In such a case, the relationship (4) necessarily implies that $PZ(0)=1$ as long as the support of $he0(x)$ includes the support of $he(x)$, which is the case here since $he0(x)$ involves a reduced dead space. Thus, $P\{Z0(x)<\infty\}=1$ implies $PZ(0)=1$, which means that stability in the presence of the injected-carrier's initial energy implies stability in the absence of the initial energy. Conversely, suppose that $P\{Z0(x)<\infty\}<1$, which corresponds to instability in the presence of the initial energy. Then, it follows from (6) that $PZ(x)<1$ or $PY(x)<1$ within the support of $he0(x)$. In such a case, we deduce from the relationship given in (4) that $PZ(0)<1$, since the support of $he0(x)$ includes the support of $he(x)$. This shows that $P\{Z0(x)<\infty\}<1$ implies $PZ(x)<1$, or equivalently, $PZ(x)=1$ implies $P\{Z0(x)<\infty\}=1$, which means that stability in the absence of the injected-carrier's initial energy implies stability in the presence of the initial energy. Thus, we have proved that stability, and thus the breakdown voltage, is independent of the initial energy of injected carriers.

SECTION V. Conclusions

In this paper, we showed that the breakdown characteristics are enhanced in thin APDs. In particular, reducing the thickness of the multiplication region not only serves to reduce the breakdown voltage, as has been observed previously, but it also makes the transition from sub-breakdown to breakdown more abrupt on an absolute scale (the transmission abruptness relative to the breakdown voltage is reduced, however). This feature is particularly desirable for Geiger-mode operation of the APD, as the likelihood of breakdown is enhanced, which leads to enhanced detection and less sensitivity to bias fluctuations. Moreover, the absolute and relative abruptness of the transition can be further enhanced if injected photogenerated carriers have an initial energy comparable to the ionization threshold. Such a phenomenon can occur in heterostructure APDs, and can be manipulated through careful bandgap engineering and doping. APD designs that have the potential for accentuating the initial-energy effect (e.g., certain separate-absorption-charge-multiplication structures) are thus expected to exhibit improved breakdown characteristics.

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