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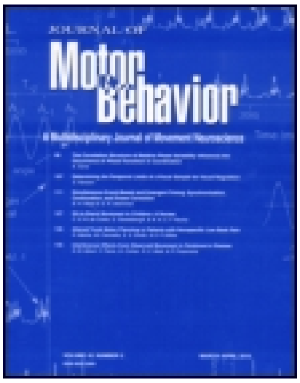
published in
Journal of Motor Behavior
1998

DOI (link to publisher)
10.1080/00222899809601337

document version
Publisher's PDF, also known as Version of record



Journal of Motor Behavior
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/vjmb20

Breaking the Reflectional Symmetry of Interlimb Coordination Dynamics

Polemnia G. Amazeen [a], Eric L. Amazeen [a] & M. T. Turvey [b]

[a] Faculty of Human Movement Sciences, Vrije Universiteit, Amsterdam Center for the Ecological Study of Perception and Action University of Connecticut, Storrs

[b] Center for the Ecological Study of Perception and Action, University of Connecticut, Storrs Haskins Laboratories, New Haven

Version of record first published: 02 Apr 2010.

To cite this article: Polemnia G. Amazeen, Eric L. Amazeen & M. T. Turvey (1998): Breaking the Reflectional Symmetry of Interlimb Coordination Dynamics, Journal of Motor Behavior, 30:3, 199-216

To link to this article: http://dx.doi.org/10.1080/00222899809601337

Journal of Motor Behavior, 1998, Vol. 30, No. 3, 199–216

Breaking the Reflectional Symmetry of Interlimb Coordination Dynamics

Polemnia G. Amazeen
Eric L. Amazeen
Faculty of Human Movement Sciences
Vrije Universiteit, Amsterdam
Center for the Ecological Study
of Perception and Action
University of Connecticut, Storrs

M. T. Turvey
Center for the Ecological Study
of Perception and Action
University of Connecticut, Storrs
Haskins Laboratories, New Haven

ABSTRACT. Interlimb rhythmic coordination is reflectionally symmetric when the left and right limb segments are identical in uncoupled frequencies and spatial orientation. In the present studies (4 experiments, with a total of 31 participants), when reflectional symmetry was broken through differences in timing (frequency), the resulting stable states were related by reflection and were identical for paired identically oriented limb segments behaving either as inverted or as ordinary pendulums. When reflectional symmetry was broken both temporally and spatially (coordinating inverted and ordinary pendular motions), the resulting stable states were different from those produced by identically oriented pendulums but nevertheless were related by reflection. In the Discussion, the authors focus on (a) symmetry breaking as leading to one of a number of symmetrically related states and (b) extending coordination dynamics with reflectional symmetry so that temporal and spatial asymmetries can both be accommodated.

Key words: dynamics, group theory, interlimb coordination, inverted pendulum, symmetry

A common but nonetheless remarkable feature of the human movement system is its ability to synchronize the rhythmic motions of any one body segment with any other. For example, pianists are adept at coordinating the movements of their fingers upon a keyboard but may also incorporate in their overall body motion a tapping foot, a nodding head, and a swaying torso. Of significance to the present article is the observation that coordination occurs quite easily despite differences in the sizes and orientations of the body segments; in fact, coordination is more likely to be between dissimilar body segments, whether by design (e.g., an arm and a leg) or by occasion (e.g., two arms, one of which is weighted by a bag). In research on two-limb 1:1 frequency-locked coordinations, the focus has been predominantly the impact of differently sized body segments on the resulting coordination (see summaries in Amazeen, Amazeen, & Turvey, in press; Kelso, 1994a; Schmidt & Turvey, 1995; Turvey, 1994). Our focus in the present article is on incorporating the influence of differently oriented body segments through the use of mathematical (symmetry) group theory for 1:1 frequency-locked coordinations.[1]

In Figure 1a, a prototypical synchronization is depicted, namely, the synchronization of the upper limbs that occurs during walking. Notice that the left and right arms are identical to each other both in size and in orientation. The limbs are *reflectionally symmetric*; that is, a *reflectional transformation* or exchange of the left and right limbs—as if the individual were viewed either from behind or in a mirror—leaves the appearance of that two-limb system, for our purposes (i.e., overlooking the orientation of the palms and the position of the individual fingers), unchanged. Physically identical arms prefer the same frequency of movement and therefore contribute equally to the coordination during the basic rhythmic act of walking. Two prototypical patterns that are observed for two-limb monofrequency (1:1 frequency-locked) coordination are in-phase, in which the

Correspondence address: P. G. Amazeen, Faculty of Human Movement Sciences, Vrije Universiteit, Van der Boechorststraat 9, 1081 BT Amsterdam, the Netherlands. E-mail address: P_Amazeen@fbw.vu.nl

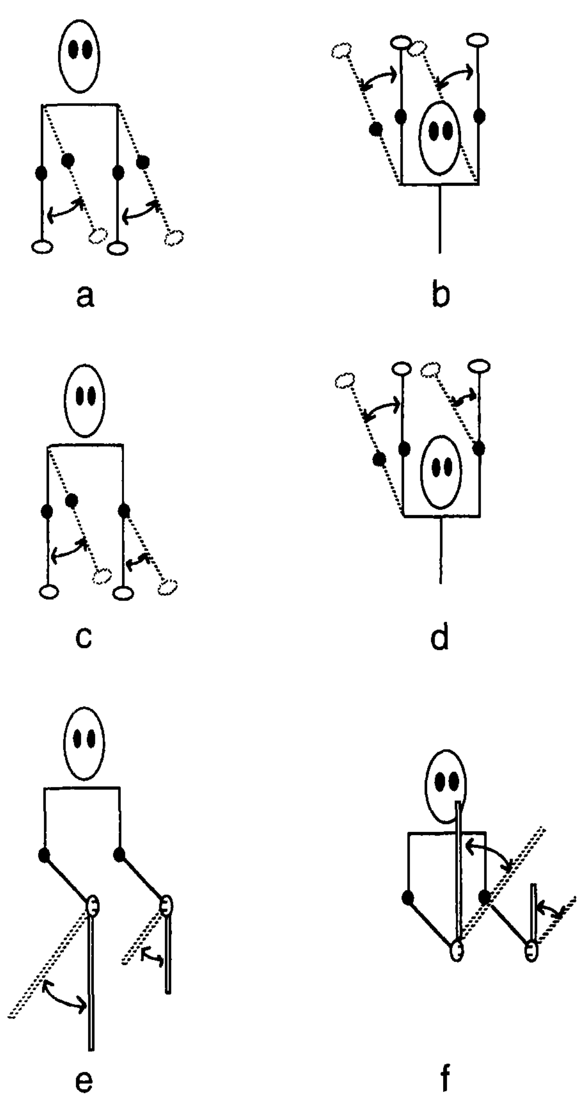


FIGURE 1. Reflectional symmetry is preserved when the limbs or limb segments of a coordination are physically identical and identically oriented, such as two arms either (**a**) oriented in the direction of gravity or (**b**) raised. Whenever reflectional symmetry is preserved, symmetric solutions (that is, perfect in-phase 0 and perfect antiphase π) should be produced. That hypothesis was tested in Experiments 1 and 2. Reflectional symmetry is broken when the limb segments are physically (and, therefore, temporally) different ($\Delta\omega \neq 0$), as in the coordination of an arm and a forearm that are either (**c**) oriented in the direction of gravity or (**d**) raised. (**e**) Kugler and Turvey's (1987) system of hand-held pendulums can be used to control the differential loading on the limbs so that one can experimentally model (a) and (c). (**f**) When inverted, the hand-held pendulums are experimental models of (b) and (d). The solutions produced by those two systems, in which $\Delta\omega \neq 0$, were compared in Experiment 2.

limb segments are always at the same position in their movement cycles, and antiphase, in which the two limbs are always exactly opposite to each other in their movement cycles (e.g., Collins & Stewart, 1993a, 1993b; Kelso, 1984). When the two-limb system is set into motion, its behavior remains categorically unchanged by a reflectional transformation; that is, in-phase remains in-phase and antiphase remains antiphase.[2] Therefore, both in appearance and in behavior, the two-limb system of Figure 1a belongs to the reflectional symmetry group.

A two-limb system in which the two arms are oriented upward is depicted in Figure 1b. Although the two arms are oriented differently than the arms in Figure 1a, the systems depicted in Figures 1a and 1b both remain unchanged across a reflectional transformation. Therefore, both two-limb systems are reflectionally symmetric. In the majority of research on rhythmic coordination in two-limb systems, the focus has been the behavior of either downward-oriented limb segments (depicted in Figure 1a) or horizontally oriented limb segments (e.g., Byblow, Carson, & Goodman, 1994; Lee, Swinnen, & Vershueren, 1995; Wuyts, Summers, Carson, Byblow, & Semjen, 1996). Nevertheless, if the underlying principle guiding the coordination is symmetry group membership, the findings should hold for the upward-oriented (that is, inverted) limb segments of Figure 1b as well. At issue is the phenomenon of motor equivalence, that is, the ability of two seemingly different motor systems to behave the same functionally. To the extent that different two-limb configurations are symmetrically equivalent, they should produce the same solution.

The reflectional symmetry of Figures 1a and 1b is easily broken, as depicted in Figures 1c and 1d. Restricting the oscillations of one arm to rotation about the elbow changes the size of the limb segment and, therefore, its preferred frequency of movement (Kugler & Turvey, 1987). The coupled full arm–forearm system is no longer reflectionally symmetric, because dissimilar limb segments cannot be interchanged without changing the physical and behavioral characteristics of the system. The importance of group theory lies in the way in which seemingly different objects and events are shown to relate to each other, that is, are shown to belong to the same symmetry group. That finding is best summarized in the *Extended Curie Principle*, as expressed by Stewart and Golubitsky (1992): "physically realizable states of a symmetric system come in bunches, related to each other by a symmetry" (p. 58).[3] To put that statement in the context of interlimb coordinations, let us consider any two-limb coordination (i.e., categorically, reflectionally symmetric, as in Figures 1a and 1b, or reflectionally asymmetric, as in Figures 1c and 1d). For the purposes of argument, apply the transformation that defines the symmetry group—reflection—by looking at the same two-limb coordination from behind. The Extended Curie Principle states that the first configuration (frontal view) will appear either identical to or as a perfect mirror image of the second configuration (rear view). The argument from group theory is that the same rule should hold behaviorally. Rephrasing the Extended Curie Principle: Symmetry is never lost, but redistributed (Stewart & Golubitsky, 1992); with reflectionally asymmetric configurations, such as in Figures 1c and 1d, reflectional symmetry is shared between two different states of the same system.

Coordination Dynamics

With the principles of group theory in mind, let us now consider in more detail the predictions that may be made. Monofrequency (1:1 frequency-locked) coordinations are accommodated by the following motion equation, written over the order parameter relative phase: $\phi = \theta_{left} - \theta_{right}$, where θ is the phase angle of the individual oscillator (Haken, Kelso, & Bunz, 1985; Kelso, Delcolle, & Schöner, 1990; Schöner, Haken, & Kelso, 1986):

$$\dot{\phi} = \Delta\omega - a \sin\phi - 2b \sin 2\phi + \sqrt{Q}\xi_t . \tag{1}$$

Reflectional asymmetry is captured by the parameter $\Delta\omega$ ($= \omega_{left} - \omega_{right}$, where ω is the preferred movement frequency of the individual oscillator), and the coefficients a and b are the strength of the coupling between the limbs. The last right-hand term is a Gaussian white noise term of strength Q that arises from the interactions of the very many (neural, muscular, and vascular) subsystems. Equation 1 is reflectionally symmetric when there are no timing differences between the limbs ($\Delta\omega = 0$); that condition has been referred to as the *elementary coordination dynamics* (Kelso, 1994a).

Predictions regarding the location and relative stability of the stable phase relations, or fixed points, of Equation 1 are presented in Figure 2. In the laboratory, the location of the fixed points is indexed as the deviation or shift of mean relative phase ϕ_{ave} from an intended phase ϕ_ψ of either 0 or π (Figure 2a), and their stability is inversely related to the variability with which ϕ_{ave} is produced, as indexed by the standard deviation of relative phase $SD\phi$ (Figure 2b). Although the principles of group theory apply to patterns of both fixed point shift and variability, presentation of the former serves to illustrate. The predictions of both Figures 2a and 2b are presented more fully when they are tested in Experiment 2.

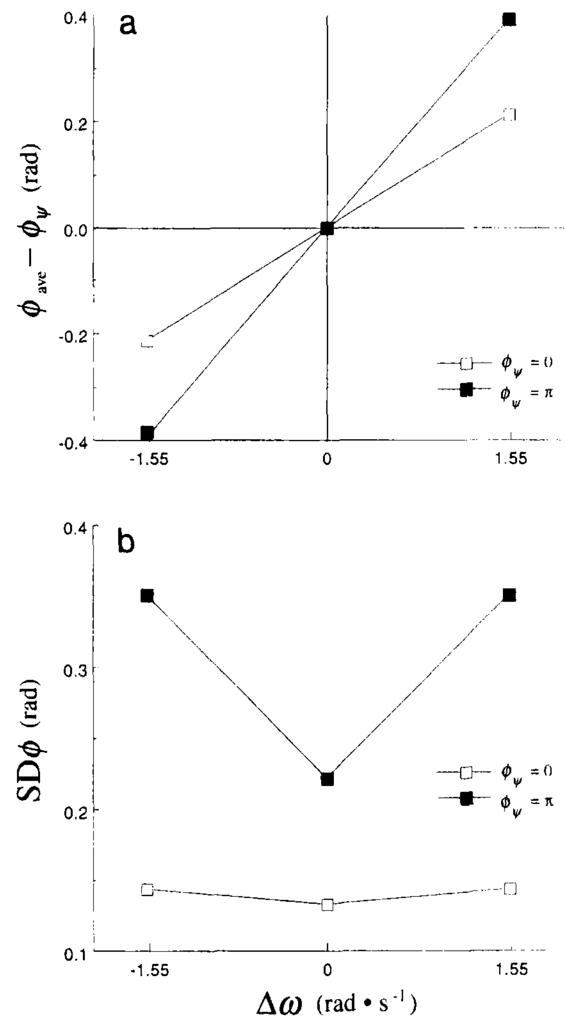


FIGURE 2. Predictions from Equation 1 of (**a**) fixed point shift and (**b**) variability, as a function of frequency competition, $\Delta\omega$, for an intended phase, ϕ_ψ, of both 0 and π. Fixed point shift was measured as the deviation of average relative phase, ϕ_{ave}, from intended phase. Variability was measured as the standard deviation of relative phase, $SD\phi$. Coupling strength was held constant (b/a = 1) for all predictions.

As noted, according to the Extended Curie Principle, whenever symmetry is broken, it is not lost entirely but, rather, is redistributed or shared among multiple system states (Stewart & Golubitsky, 1992). The manner of redistribution is such that those states will be related to each other via the transformation that defines the symmetry group. When $\Delta\omega = 0$, the two-limb system is reflectionally symmetric. When reflectional symmetry is broken (as in the mental exercise presented earlier), that is, $\Delta\omega \neq 0$, there are two related states, say, (a) $\Delta\omega = -d$ and (b) $\Delta\omega = +d$ rad s^{-1}. In words, they can be described as (a) left is x larger than right and (b) right is x larger than left, where x is some size difference that causes the difference d in preferred frequencies of the two limbs. Individually, those systems do not remain invariant under a reflectional transformation, but together they are related to each other via reflection; that is, when (a) left is x larger than right is left–right reflected, it turns into (b) right is x larger than left. Mathematically, the reflection can be represented as multiplication by -1; the present example is an instance of reflectional symmetry sharing because $-1(a) = (b)$. Reflectional symmetry is not lost when $\Delta\omega \neq 0$ but is shared between the two states of the system, (a) $\Delta\omega = -d$ and (b) $\Delta\omega = +d$.

The significance of that observation lies not in identifying the symmetry status of the coordination condition but, more importantly, in predicting the symmetry of the coordinative solution. Reflectionally symmetric ($\Delta\omega = 0$) limbs produce reflectionally symmetric solutions ($\phi_{ave} = 0$, $\phi_{ave} = \pi$). Conversely, reflectionally asymmetric limbs produce reflectionally asymmetric solutions ($\phi_{ave} - \phi_{\psi} \neq 0$). For a given value of $\Delta\omega \neq 0$, ϕ_{ave} is displaced a given amount, say, f rad, from perfect in-phase and perfect antiphase. That is, when (a) $\Delta\omega = -d$, $\phi_{ave} - \phi_{\psi} = -f$, and when (b) $\Delta\omega = +d$, $\phi_{ave} - \phi_{\psi} = +f$ (see Figure 2a). Both the coordination conditions and their solutions are related to each other by reflection, that is, $-1(a) = (b)$. The pattern of fixed point shift is qualitatively the same for in-phase and antiphase, although it is amplified at antiphase. Although fixed point shift has been discussed repeatedly in the literature, it has not, thus far, been addressed in terms of group theory. A major promise of the group theoretic perspective is that it may provide a tool for understanding how an organism demonstrates generativity, that is, the ability to produce a skill in novel circumstances (see Fodor, 1975, and Fodor & Pylyshyn, 1988, for development of this term in the field of language); any organism that is capable of producing one asymmetric state of coordination may have at its disposal the ability to demonstrate transfer—without prior experience—to the other related, or "shared," states as well. The most straightforward illustration of that is the ability of a learner to acquire the $\pi/2$ phase relation and to demonstrate, without additional practice, $-\pi/2$ (e.g., Zanone & Kelso, 1992). Although both $\pi/2$ and $-\pi/2$ may be produced with symmetric limb segments, their relation to each other holds in the form of the Extended Curie Principle.

Support for the predictions in Figure 2 has been well documented in the literature for downward-oriented limbs (e.g., Figure 1c) whose preferred frequencies researchers have manipulated by varying the physical characteristics of hand-held pendulums (see Figure 1e and Amazeen, Sternad, & Turvey, 1996; Kugler & Turvey, 1987; Schmidt, Shaw, & Turvey, 1993; Sternad, Amazeen, & Turvey, 1996). If Equation 1 is a generalized form of the coordination dynamics for any two-limb system belonging to the reflectional symmetry group, then it should apply equally to inverted systems in which reflectional symmetry has been broken through an asymmetry in timing of the limbs (e.g., Figure 1d); we tested that theory empirically with inverted pendulums (e.g., Figure 1f) in Experiment 2.

Breaking Reflectional Symmetry Through Differences in Spatial Orientation

Researchers have broken reflectional symmetry by imposing differences in the timing of the coordination components in experiments in which $\Delta\omega$ has been manipulated, although those experiments have not been labeled as such. There are considerably fewer studies of reflectional symmetry breaking through the imposition of differences in the spatial orientation of the coordination components (e.g., Jeka & Kelso, 1995; Kelso, Buchanan, & Wallace, 1991; Kelso & Jeka, 1992). In Figures 3a and 3b, instances of coordination between one inverted limb and one downward-oriented limb (in its ordinary position) are depicted. Viewed simply as a geometric arrangement, Figures 3a and 3b are both reflectionally asymmetric, because they are changed by a reflection transformation. However, because reflection transforms Figure 3a into Figure 3b, and vice versa, they are related states. The symmetry that is broken when the two limbs are oriented differently is redistributed among those two configurations.

One can use group theory to predict that the solution produced by Figure 3a should be reflectionally related to the solution produced by Figure 3b. For example, if Figure 3a produces $\phi_{ave} - \phi_{\psi} = -f$, then Figure 3b should produce $\phi_{ave} - \phi_{\psi} = +f$. In its current form, Equation 1 has no means for accommodating spatial symmetry breaking. Therefore, it offers the alternate prediction that two limbs that are equivalent in terms of their preferred timing ($\Delta\omega = 0$) should produce perfect in-phase and antiphase ($\phi_{ave} - \phi_{\psi} = 0$). We tested those predictions in Experiments 3 and 4.

Breaking Reflectional Symmetry Through Differences in Both Timing and Spatial Orientation

Reflectional symmetry is broken through differences in either timing or spatial orientation, but it is also broken when the limbs being coordinated are different in both timing and orientation. A coordination in which the limb segments (experimentally controlled by hand-held pendulums) being coordinated are of different sizes ($\Delta\omega < 0$) and different spatial orientations is depicted in Figure 3c. The system is clearly reflectionally asymmetric, because reflection drastically alters its physical appearance. That fact alone allows for the prediction from group theory that, behaviorally, the system will not produce perfect in-phase or antiphase coordinations (i.e., $\phi_{ave} - \phi_{\psi} \neq 0$). Currently, there is no basis for predicting whether the deviation of ϕ_{ave} from 0 and π produced by differences in limb orientation alone will be greater or less than the deviation produced by differences in both timing and limb orientation.

The limb configuration that is, by group theory, the reflectional partner of the configuration in Figure 3c is shown in Figure 3d. Left–right exchange of the coordination components in Figure 3c yields the configuration depicted in Figure 3d, and vice versa. For contrast, note that the configuration depicted in Figure 3e is identical to the one in Figure 3c in terms of its timing differences ($\Delta\omega = -d$) but is reflectionally related to neither Figure 3c nor Figure 3d. The resulting prediction is that behaviorally, Figures 3c and 3d (but not Figure 3e) should be related. Specifically, if Figure 3c produces $\phi_{ave} - \phi_{\psi} = -f$, then Figure 3d should produce $\phi_{ave} - \phi_{\psi} = +f$. We tested that prediction in Experiment 4. The alternate hypothesis comes from Equation 1: Because it lacks a manner in which to specify symmetry breaking through differences in spatial orientation, Equation 1 predicts that because the configurations depicted in Figures 3c and 3e are identical in $\Delta\omega$, they should produce the same coordinative solution. Importantly, then, the finding that differences in spatial orientation alter the coordination dynamics of Equation 1 calls for the accommodation

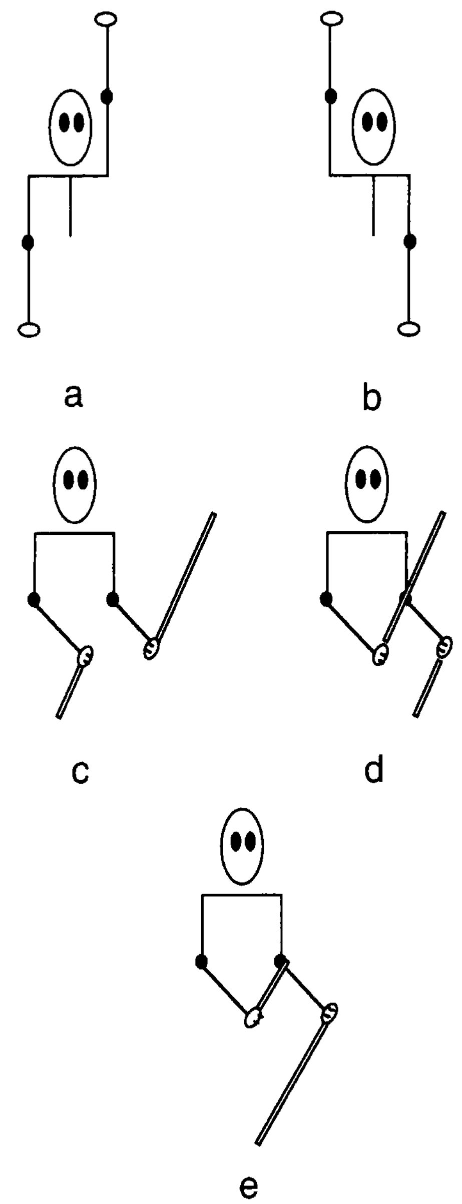


FIGURE 3. (**a**) and (**b**) Reflectional symmetry is broken when the limb segments are differently oriented, such as in the coordination of an arm that is oriented in the direction of gravity with an arm that is raised. (**c**) to (**e**) We asked participants to coordinate the movements of an ordinary pendulum that was held in one hand with the movements of an inverted pendulum that was held in the other hand in order to experimentally control differences in timing of two differently oriented limb segments. Any two coordinative systems that are related by reflection should demonstrate solutions that are likewise related by reflection. Therefore, it is expected that (a) and (b) will demonstrate equal but mirror-image coordinative patterns, as will (c) and (d), but not (c) and (e) or (d) and (e). The alternate hypothesis, from Equation 1, is that because (c) and (e)—but not (c) and (d)—are identical in terms of their differences in timing $\Delta\omega$, they will produce identical solutions. That hypothesis was tested in Experiment 4.

by Equation 1 of additional manners of reflectional symmetry breaking.

EXPERIMENT 1

The ratio of parameters b/a in Equation 1 is inversely related to the movement frequency of the coupled limb segments, as claimed in the original development of the equation (Haken et al., 1985) and shown experimentally for downward-oriented limb segments (e.g., Amazeen et al., 1996; Schmidt et al., 1993). The undamped, undriven frequency (the frequency of a gravity pendulum) of any hand-plus-pendulum system can be calculated from the simple pendulum length L (dependent on both the length and mass of the specific pendulum); specifically,

$$\omega = (g/L)^{1/2}, \tag{2}$$

where g is the constant acceleration caused by gravity (Kugler & Turvey, 1987). The output of Equation 2 is an estimation of the frequency of oscillation of a pendulum resulting from gravity alone. Practically, however, hand-held pendulums are both damped and driven, so additional forces subtract from and contribute to the pendulum's preferred frequency (see Beek, Schmidt, Morris, Sim, & Turvey, 1995, for a detailed analysis of the contributing linear and nonlinear stiffness and friction functions). The result is an observed pendular frequency that is estimated by, but not identical to, the output of Equation 2 (see Amazeen, Schmidt, & Turvey, 1995, for a demonstration of individual differences). Theoretically, calculation of the preferred movement frequency of ordinary (downward-oriented) pendulums is the same for inverted (upward-oriented) pendulums (e.g., Smith & Blackburn, 1992). We evaluated the validity of that claim in Experiment 1 by changing the physical properties and, therefore, the movement frequencies of the hand-held inverted pendulums.

The influence of movement frequency on location and stability of the fixed points is well documented for the coordination of reflectionally symmetric ordinary pendulums (e.g., Amazeen et al., 1996; Amazeen et al., 1995; Schmidt et al., 1993; Sternad et al., 1996; Sternad, Turvey, & Schmidt, 1992). When the timing of the limb segments is identical ($\Delta\omega = \omega_{left} - \omega_{right} = 0$ rad s^{-1}), there is no shift of the fixed points at $\phi_\psi = 0$ and $\phi_\psi = \pi$, but the production of antiphase is more variable than the production of inphase. In Experiment 1, that prediction was evaluated across a range of movement frequencies for the reflectionally symmetric limb configuration depicted in Figure 1b and was modeled experimentally as the synchronization of hand-held inverted pendulums of equal preferred movement frequencies. If the critical comparison is the reflectional symmetry similarity rather than the orientation difference, then the results for the inverted pendulums of the present experiment should replicate previously obtained results of experiments in which ordinary pendulums were used.

Method

Participants

Five men and 2 women, all graduate students at the University of Connecticut, participated in the experiment. Six of the 7 had previously participated in hand-held pendulum studies (although none had previously participated in inverted pendulum experiments), and 1 was left-handed. Data from both the naïve and left-handed participants were compared with data from the remaining participants. Because no statistical difference was found, experience and handedness were not considered as factors in the analysis.

Design

The data collected in this study were the movement trajectories of the two hand-held pendulums. Measures included ω_{ave}, the frequency of oscillation averaged over the two pendulums; ϕ_{ave}, the relative phase (the estimate of the stable fixed point) averaged over each trial; and $SD\phi$ (the estimate of fluctuations about the stable fixed point) per trial.

Participants were instructed to maintain a relative phase, ϕ_ψ, of either 0 (in-phase) or π (antiphase) rad. There were five symmetric pairs of pendulums ($\Delta\omega = 0$ rad s^{-1}) of varying movement frequencies, resulting in 10 conditions (2 ϕ_ψ × 5 pendulum pairs) with two trials per condition.

Apparatus

The five right hand and the five left hand pendulums were wooden rods (85 g, 1 m in length, 0.012 m in diameter); each was held in the center of the hand so that the pendulums were vertical and the hand was positioned 0.6 m from the top (see Figure 4). It should be noted that the extension of the pendulums both below and above the hand signifies a departure from the standard hand-held pendulums procedure (see Kugler & Turvey, 1987). Extension of the pendulum above the hand was required so that an inverted pendulum could be created. Data collection required a portion of the pendulum to extend below the hand, but the length and weight of that portion and, therefore, its impact on the pendulum's spatial orientation were minimized. Calculation of movement frequency using Equation 2 took those changes into account, but the influence of that change in method on obtained movement frequency and extent of coordination was left to empirical determination.

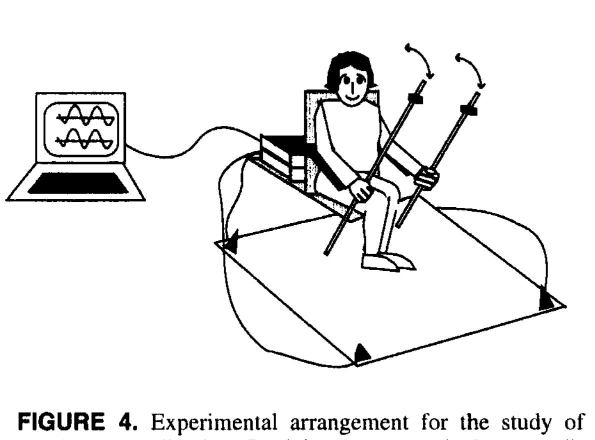


FIGURE 4. Experimental arrangement for the study of interlimb coordination. Participants were asked to coordinate the movements of their pendulums so that an intended relative phase, ϕ_ψ, of either 0 or π would be obtained. Characteristics of the pendulums were varied so that timing differences, $\Delta\omega$, could be introduced. Pendulums can be weighted above the hand (inverted) or below the hand (ordinary); we weighted the pendulums to manipulate their spatial orientation. The participant depicted in the figure is producing in-phase $\phi_\psi = 0$ with inverted pendulums.

A 200-g metal ring was positioned on each rod at one of five positions (0.2, 0.3, 0.4, 0.5, and 0.58 m) above the hand; ω was greatest when the metal ring was closest to the hand (0.89 Hz) and least (0.64 Hz) when the metal ring was farthest from the hand. More fully, the five movement frequencies corresponding to the five metal ring positions of 0.2, 0.3, 0.4, 0.5, and 0.58 m above the hand were 0.89, 0.83, 0.76, 0.69, and 0.64 Hz, respectively. On any given trial, the position of the metal ring was the same for both the left-hand and right-hand rods so that $\Delta\omega = 0$.

Because the movement registration device was sonic, participants sat within a 1-m^2-base experimental cube lined with foam to minimize reflections (see Figure 4). A specially designed chair elevated their legs to allow for unobstructed data collection. A Sonic 3-Space Digitizer (SAC Corporation, Stratford, CT) collected movement trajectories of each pendulum. A sonic emitter attached to the bottom of each pendulum emitted sparks at the rate of 90 Hz. Microphones positioned in the four corners of the experimental cube registered the position of the emitter by computing the distance of the emitter from the three of the four microphones that registered the least number of errors during that trial. Motion analysis digitizer software (MASS; ESI Technologies, Columbus, OH) stored the slant range time series for use on a 80486-based microcomputer. We then used MASS to calculate the mean frequency of oscillation of each of the pendulums, their primary angle of excursion, and ϕ. The three measures, ω_{ave}, ϕ_{ave}, and $SD\phi$ were calculated for each individual trial. Because there were no order effects (that is, there was no statistical difference between measures obtained for the first and second repetitions of a given trial), we averaged all three measures across replications to obtain a single data point for each experimental condition.

Procedure

Participants held each pendulum vertically, with the center of the palm positioned 0.6 m from the top of each pendulum. They were instructed to position their wrists at the end of the armrests and to create as smooth and as continuous a trajectory as possible, holding the pendulum firmly in the hand so that rotation about the wrist rather than rotation about the finger joints would be assured. Their gaze was focused straight ahead during the course of a trial, and, in keeping with standard hand-held pendulums procedure (Treffner & Turvey, 1996), they were instructed to avoid visually guiding the movements of the pendulums. Pendular motion was restricted to the plane parallel to the participant's sagittal plane. On any given trial, participants were instructed that they should coordinate the hand-held pendulums to establish either in-phase ($\phi_\psi = 0$) or antiphase ($\phi_\psi = \pi$) 1:1 frequency locking. They were permitted to elect a comfortable frequency and to control the beginning of each trial. We expected that the chosen movement frequency ω_{ave} would vary as a function of the calculated movement frequency ω. Each trial was 30 s; data collection lasted approximately 45 min. All experimental procedures reported in the present experiments adhered to the ethical guidelines of the American Psychological Association.

Results and Discussion

Frequency

We analyzed cycle by cycle (i.e., peak to peak) the frequency of oscillations of the left and the right inverted pendulums to determine whether 1:1 frequency locking had occurred. In all trials, the number of cycles exhibited by the left and right pendulums either was identical or differed by only one cycle. The frequency of each cycle was then calculated for left and right pendulums separately; on average, the difference in frequency per cycle was always less than .01 Hz. The conclusion that 1:1 frequency locking occurred is significant for the purposes of relative phase calculations; it indicates that ϕ_{ave} could be calculated straightforwardly for coupled inverted pendulums. A 2 × 5 analysis of variance (ANOVA) revealed that across the five pendulum pairs, ω_{ave} differed significantly, $F(4, 24) = 17.05$, $p < .0001$, in accordance with the differences among the pairs in ω (ω_{ave} = 0.85, 0.88, 1.00, 1.08, and 1.21 Hz, from the slowest to the fastest pendulum pair). Note that participants elected a movement frequency that was slightly but consistently higher than the calculated value; that trend has been witnessed for ordinary hand-held pendulums (e.g., ω_{ave} = 1.27 Hz to 1.65 Hz when ω = 1.13 Hz; Amazeen et al., 1995) and is suggested to be a function of neuromuscular contributions to the preferred frequency of the hand-and-pendulum system (Beek et al., 1995). The important finding is that extension of pendulums both above and below the hand did not influence the elected frequency of oscillation. Rather, manipulation of the pendulum's undamped, undriven movement frequency alone affected the elected frequency of oscillation. The implication is that one can use Equation 2 to calculate the movement frequency for both ordinary and inverted pendulums.

Although the frequency elected by participants was slightly higher for in-phase (ω_{ave} = 1.02 Hz) than for antiphase (ω_{ave} = 0.99 Hz), there was no significant difference between the two phase relations, $F(1, 6) = 1.39$, $p > .05$, nor was there an interaction between ϕ_ψ and pendulum pair, $F(4, 24) < 1$. In previous research on the interlimb coordination of ordinary pendular motions, no difference has been found between in-phase and antiphase in the magnitude of the freely elected ω_{ave} (Sternad et al., 1996; Turvey, Rosenblum, Schmidt, & Kugler, 1986).

Mean and Standard Deviation of Relative Phase

An ANOVA revealed that the differences between the pairs of pendulums and, therefore, the differences in movement frequency, ω, did not affect $\phi_{ave} - \phi_\psi$, $F(4, 24) < 1$. Intended phase ϕ_ψ similarly had no effect, $F(1, 6) < 1$, nor did it interact with the pendulum pairs, $F(4, 24) = 1.00$, $p > .05$. The absence of any significant effects of ω on $\phi_{ave} - \phi_\psi$ is consistent with Equation 1 and replicates previous findings (e.g., Amazeen et al., 1996; Sternad et al., 1996).

In general, $SD\phi$ was slightly higher than, but within the ballpark of, those values obtained for the coordination of ordinary pendulums (e.g., 0.28 rad in the present study; 0.23 rad in Amazeen et al., 1996). That finding indicates that participants were able to stably produce a required phase relation with inverted pendulums. Both Equation 1 and previous research on the synchronization of symmetric ordinary pendulums revealed that $SD\phi$ is significantly greater at π than at 0 (e.g., Amazeen et al., 1995; Treffner & Turvey, 1996). That contrast was found in the present experiment ($SD\phi$ = 0.23 rad at $\phi_\psi = 0$; $SD\phi$ = 0.33 rad at $\phi_\psi = \pi$), $F(1, 6) = 64.03$, $p < .001$. There was no significant effect of movement frequency on $SD\phi$, $F(4, 24) = 2.24$, $p > .05$, and pendulum pair and ϕ_ψ did not interact significantly, $F(4, 24) = 2.15$, $p > .05$.

In Experiment 1, we demonstrated that the same methods used previously to investigate synchronization of ordinary pendular motions can be applied to the investigation of inverted pendular motions. In agreement with expectation, we found that 1:1 frequency locking of inverted pendular motions follows the same coordination dynamics as 1:1 frequency locking of ordinary pendular motions. In Experiment 2, the synchronizations of inverted and ordinary pendular motions were compared directly under the broken symmetry conditions expressed by $\Delta\omega \neq 0$.

EXPERIMENT 2

In Figure 2 are shown the predictions from Equation 1 regarding the stable coordinations for the broken symmetry conditions depicted in Figures 1c and 1d (and their experimental models, depicted in Figures 1e and 1f). Predictions for fixed point shift (Figure 2a) were outlined in the introduction. Patterns of variability likewise demonstrate membership in the reflectional symmetry group. Variability of relative phase ($SD\phi \geq 0$) is minimal for reflectionally symmetric systems ($\Delta\omega = 0$) and increases for reflectionally asymmetric systems ($\Delta\omega \neq 0$; see Figure 2b). In keeping with the Extended Curie Principle , the broken symmetry of $\Delta\omega = +1.55$ rad s^{-1} leads to a state (e.g., $SD\phi$ = 0.35 rad for $\phi_\psi = \pi$) that is reflectionally related to the state resulting from the broken symmetry of $\Delta\omega = -1.55$ rad s^{-1} ($SD\phi$ = 0.35 rad for $\phi_\psi = \pi$); in that particular instance, reflection requires only a sign change for

$\Delta\omega$, because $SD\phi \geq 0$. Although the pattern of variability is the same for in-phase and antiphase, $SD\phi$ is uniformly greater for antiphase. The implication is that two phase relations (0 and π) can be members of the same symmetry group and yet be differentially stable according to the coordination dynamics (Schöner, Jiang, & Kelso, 1990). The rule that symmetry breaking of Equation 1 leads to one of a number of symmetrically related states was expected to hold equally for the coordinations of coupled ordinary and coupled inverted pendular motions. We expected that a given value of $\Delta\omega$ would result in the same ϕ_{ave} and $SD\phi$ regardless of whether ordinary or inverted pendular motions were being synchronized. Confirmation of those predictions will support the claim that the consequences of Equation 1's symmetry are system independent.

Method

Participants

Four men and 4 women, all undergraduate students at the University of Connecticut, participated in the experiment in exchange for credit toward their introductory psychology course. One of the 8 participants was left-handed, but no difference between her performance and that of the right-handed participants was observed. None of the participants had previously participated in hand-held pendulum tasks.

Design

Participants were instructed to maintain $\phi_\psi = 0$ or $\phi_\psi = \pi$ with pendulums that were either both inverted (metal ring above the hand) or both ordinary (metal ring below the hand). We manipulated (in a manner similar to Experiment 1) positions of the metal rings to produce one zero and two nonzero values of $\Delta\omega = \omega_{left} - \omega_{right}$. Accordingly, participants were run under 12 conditions (2 orientations × 2 ϕ_ψ × 3 $\Delta\omega$), with two trials per condition.

Apparatus

Pendulums were assembled from the same wooden rods and 200-g metal rings used in Experiment 1. There were three pendulum pairs, distinguished by the value of $\Delta\omega$. Those were $\Delta\omega = 0$ when the metal rings on both pendulums were positioned 0.5 m from the hand and $\Delta\omega = \pm1.55$ when the metal rings were positioned 0.20 m from the hand for one pendulum and 0.58 m from the hand for the other pendulum. In the latter case, $\Delta\omega$ was negative when the pendulum with the metal ring at 0.20 m was held in the right hand and positive when it was held in the left hand.

A simple pendulum length L can be calculated for any coupled system of two hand-held pendulums (see Kugler & Turvey, 1987). Equation 2 was used so that the undamped, undriven movement frequency of the two-pendulum system could be determined. That frequency, referred to as the *virtual* frequency ω_v, is the undamped, undriven frequency a pair of pendulums would exhibit if the pendulums were rigidly coupled so that a perfect in-phase or perfect antiphase relation was maintained at each point in their individual cycles (see Sternad et al., 1996, for details). To reiterate an earlier point, although hand-held pendulums are both damped and driven, Equation 2 provides an estimation of the preferred movement frequency. The metal rings were positioned so that the same virtual frequency was obtained for all three pairs of pendulums: $\omega_v = 0.69$ Hz. An electronic metronome paced the pendulums at 0.69 Hz so that the same frequency would be produced for all three pairs of pendulums. Data collection for Experiment 2 was identical to that used in Experiment 1.

Procedure

Participants held the pendulums vertically (see Figure 4). The center of the palm of each hand was positioned either 0.6 m from the top of each pendulum, with the metal rings located above both hands (inverted pendulums), or 0.6 m from the bottom of each pendulum, with the metal rings located below both hands (ordinary pendulums). We manipulated orientation in blocks of six trials in order to minimize the number of pendulum exchanges during the course of the experiment. Participants were instructed to coordinate the end point of the pendular movement with the beep of the metronome. Instructions to participants were otherwise identical to those used in Experiment 1. Data collection lasted approximately 35 min.

Results and Discussion

Frequency

A cycle-by-cycle comparison of the frequency of oscillations of the left and right pendulums indicated that the required 1:1 frequency locking was achieved. Because the same frequency was required for all pendulum conditions, an assessment of the effect of orientation on obtained movement frequency was not possible.

Phase Portraits

The regularity of both (a) an inverted and (b) an ordinary pendulum in the context of their normalized phase portraits are shown in Figure 5. Note that the phase plane orbit for the inverted pendulum is larger than the orbit for the ordinary pendulum and slightly off-centered (–0.40 rad). That indicates that the inverted pendular motion was quite large and centered slightly off of the vertical. Despite those differences, the phase plane orbits for inverted and ordinary pendulums were qualitatively similar in terms of their variability and the degree to which they were circular, or harmonic. The implication is that relative phase may be computed in the same manner for systems of coupled ordinary pendulums, systems of coupled inverted pendulums, and systems in which an inverted pendulum is coupled with an ordinary pendulum (Experiments 3 and 4).

Mean Relative Phase

In Figure 6, $\phi_{ave} - \phi_\psi$ for the Orientation × ϕ_ψ × $\Delta\omega$ interaction is depicted; data for (a) inverted and (b) ordinary pendulums are displayed separately. Notice that the pattern of results for inverted and ordinary pendulums is qualitatively similar. Comparison of Figure 6 with Figure 2a indicates that the expected main effect of $\Delta\omega$ was achieved for both inverted and ordinary pendulums: $\phi_{ave} - \phi_{\psi}$ was minimal (0.04 rad) when $\Delta\omega = 0$, increased in the negative direction (−0.21 rad) when $\Delta\omega = -1.55$, and increased in the positive direction (0.29 rad) when $\Delta\omega = +1.55$. An ANOVA revealed that observed trend to be significant, $F(2, 14) = 121.43$, $p < .0001$; all three levels of $\Delta\omega$ were significantly different from each other (Tukey, $p < .01$). In support of group theory, $\phi_{ave} - \phi_{\psi}$ was nearly equivalent in magnitude for equal but opposite values of $\Delta\omega$. The Orientation × $\Delta\omega$ interaction was nonsignificant, $F(2, 14) = 1.59$, $p > .05$, indicating that the effect of $\Delta\omega$ was identical for both inverted and ordinary pendulum systems.

When reflectional symmetry is broken by $\Delta\omega \neq 0$, fixed point shift is predicted by Equation 1 to be greater for $\phi_{\psi} = \pi$ than for $\phi_{\psi} = 0$, with the magnitude of the difference dependent on the choice of parameters a and b. Greater fixed point shift for antiphase has been found in some studies (e.g., Sternad et al., 1996; Treffner & Turvey, 1995) but not in others (e.g., Amazeen et al., 1995; Schmidt et al., 1993; Sternad et al., 1992). The ϕ_{ψ} × $\Delta\omega$ interaction was nonsignificant, $F(2, 14) = 1.58$, $p > .05$, supporting the finding that fixed point shift did not differ across intended phase. Although the Orientation × ϕ_{ψ} × $\Delta\omega$ interaction was marginally significant, $F(2, 14) = 3.74$, $p = .05$, post hoc ANOVAs revealed that the ϕ_{ψ} × $\Delta\omega$ interaction was significant for neither ordinary, $F(2, 14) = 2.67$, $p > .05$, nor inverted, $F(2, 14) = 2.92$, $p > .05$, pendulums. Therefore,

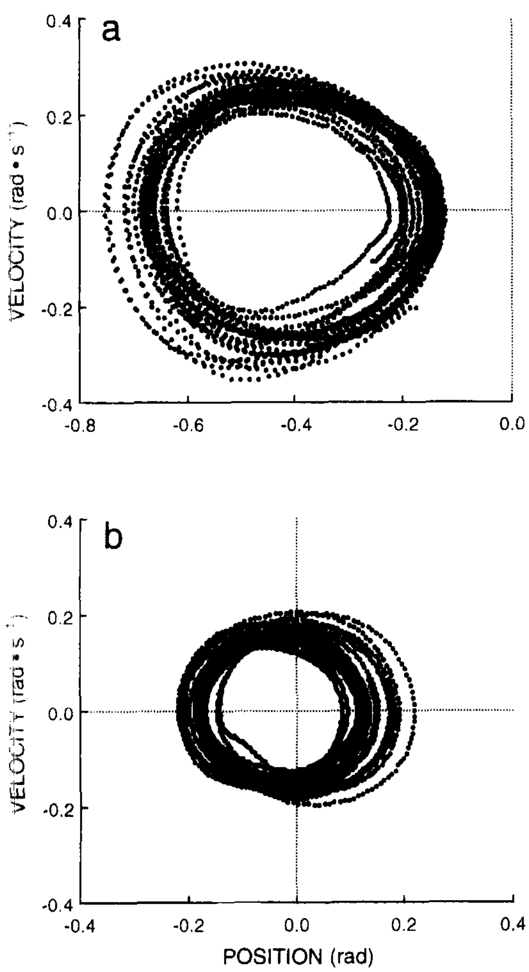


FIGURE 5. Normalized phase portraits for (**a**) an inverted and (**b**) an ordinary pendulum, each held in the right hand of a representative participant for two different trials ($\Delta\omega = 0$; $\phi_{\psi} = \pi$) in Experiment 2. The proximity of both phase plane orbits to a perfect limit cycle indicates that relative phase may be computed in the same manner for both inverted and ordinary pendulums.

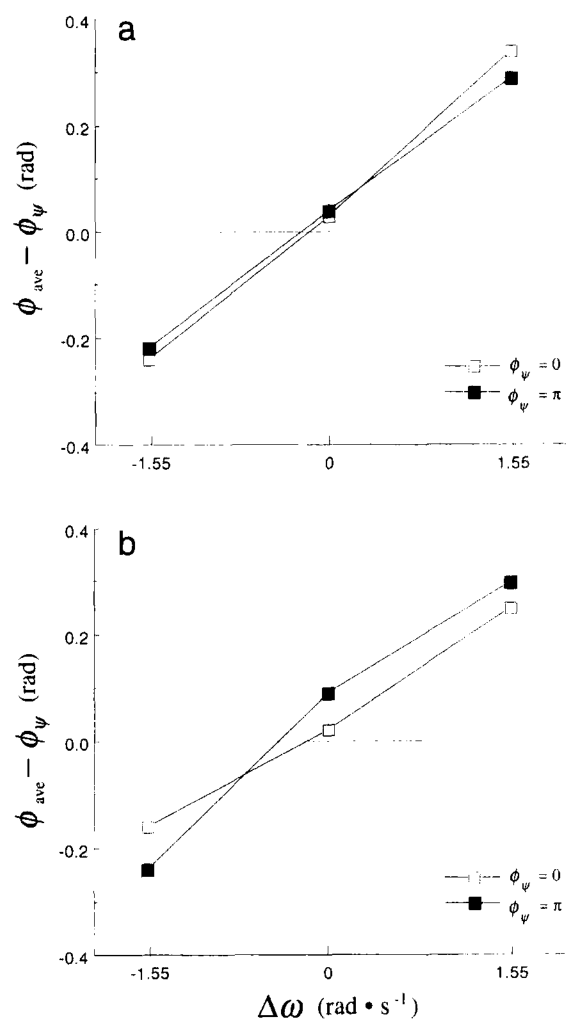


FIGURE 6. Deviation of mean relative phase from intended phase, $\phi_{ave} - \phi_{\psi}$, for the Orientation × ϕ_{ψ} × $\Delta\omega$ interaction in Experiment 2. Results for (**a**) inverted pendulums and (**b**) ordinary pendulums are displayed separately, although the pattern of fixed point shift was not differentially affected by pendulum orientation.

differences in the orientation of the two pendulum systems (inverted or ordinary) did not differentially affect the standard pattern of fixed point shift.

Standard Deviation of Relative Phase

$SD\phi$ as a function of orientation and $\Delta\omega$ is depicted in Figure 7. Comparison of Figure 7 with Figure 2b indicates that the standard pattern of variability was achieved: $SD\phi$ was minimal (0.18 rad) when $\Delta\omega = 0$ and increased at both $\Delta\omega = -1.55$ ($SD\phi = 0.26$ rad) and $\Delta\omega = +1.55$ ($SD\phi = 0.27$ rad). The $\Delta\omega$ trend was significant, $F(2, 14) = 15.85$, $p < .0005$. In agreement with Equation 1 and the Extended Curie Principle, equal deviations from $\Delta\omega = 0$, that is, $\Delta\omega = \pm 1.55$, produced identical $SD\phi$. The $\Delta\omega$ effect was not identical for ordinary and inverted pendulums, $F(2, 14) = 17.21$, $p < .0005$. Although simple effects analyses for the Orientation × $\Delta\omega$ interaction revealed the $\Delta\omega$ trend to be significant for both inverted, $F(2, 14) = 4.39$, $p < .05$, and ordinary pendulums, $F(2, 14) = 23.81$, $p < .001$, the trend was considerably weaker for inverted pendulums. The source of the difference was at $\Delta\omega = 0$: $SD\phi$ was significantly greater for inverted pendulums than for ordinary pendulums at $\Delta\omega = 0$, $F(1, 7) = 276.41$, $p < .001$, but not at $\Delta\omega = -1.55$, $F(1, 7) = 3.13$, $p > .05$, or $\Delta\omega = +1.55$, $F(1, 7) = 5.04$, $p > .05$. Therefore, although both inverted and ordinary pendulums revealed the same effect of $\Delta\omega$ on stability, it is obvious that inverted pendulums were less stable than ordinary pendulums, at least at $\Delta\omega = 0$.

Variability differences are often but not always found between in-phase and antiphase in fixed point data; antiphase usually proves to be less stable (e.g., Amazeen et al., 1995; Sternad et al., 1996). In the present experiment, $SD\phi$ was significantly higher for $\phi_\psi = \pi$ (0.26 rad) than for $\phi_\psi = 0$ (0.22 rad), $F(1, 7) = 43.68$, $p < .0005$, supporting the notion that two equally symmetric phase relations can be differentially stable in the coordination dynamics. It is interesting that the differential stability of coupled inverted and coupled ordinary pendulums (at $\Delta\omega = 0$) places them in the same category. The lack of an Orientation × ϕ_ψ interaction, $F(1, 7) = 1.53$, $p > .05$, indicates that the in-phase–antiphase difference held for both inverted and ordinary pendulums. Therefore, in agreement with Equation 1, both inverted and ordinary pendulum systems showed less variability in the vicinity of 0 than in the vicinity of π.

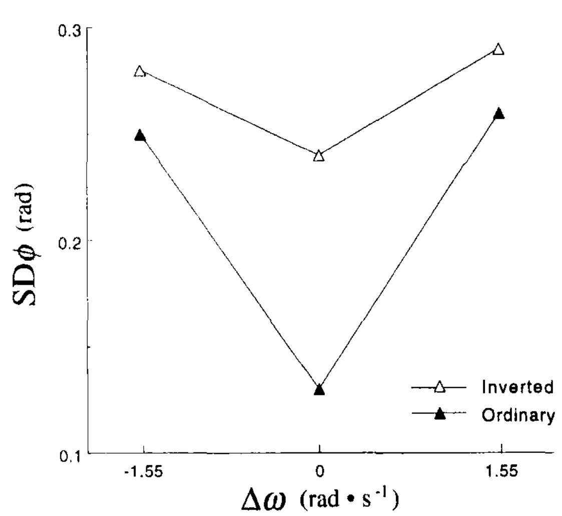


FIGURE 7. Standard deviation of relative phase, $SD\phi$, as a function of orientation and $\Delta\omega$ in Experiment 2. Although the same pattern of variability was witnessed for both ordinary and inverted pendulums, the inverted pendulums were considerably less stable at $\Delta\omega = 0$.

In Experiment 2, we tested the following two hypotheses: First, for both kinds of coupled systems, breaking the reflectional symmetry of Equation 1 in reflectionally related ways, that is, $\Delta\omega = \pm 1.55$, leads to one of a number of reflectionally related states. That hypothesis was confirmed for both ordinary and inverted pendulum systems. Second, breaking the reflectional symmetry of Equation 1 affects the equilibria of a system of coupled inverted pendulums (mimicking the two arms raised, as in Figure 1d) in precisely the same way that it affects the equilibria of a system of coupled ordinary pendulums (mimicking the two arms hanging by the side, as in Figure 1c). Although the location of the stable states of the inverted and ordinary pendulum systems was shifted identically, at $\Delta\omega = 0$, the equilibria of coupled inverted pendulums were less stable—that is, more variable—than the equilibria of coupled ordinary pendulums. That latter outcome points to the significance of physical differences between inverted and ordinary pendular motions.

At the level of the individual pendulum—that is, at the local level—inversion is an unstable state. An inverted pendulum that is not externally supported does not remain inverted; even when it is supported, gravity serves to displace it from the vertical position. Therefore, at the local level, an inverted pendulum (a raised arm) is physically different from an ordinary pendulum (a hanging arm). The argument put forth in the present article, however, is that a system of two inverted pendulums abides by the same coordination dynamics—at the global level—and therefore is dynamically the same as a system of two ordinary pendulums. The topological similarity of phase portraits for coupled ordinary pendulums and coupled inverted pendulums supports that claim. That systems of inverted pendulums are less stable than systems of ordinary pendulums does not negate the fact that their coordination dynamics is qualitatively the same. The global dynamics appear to be indifferent to the orientation of the particular two-limb system but sensitive to its reflectional symmetry.[4]

EXPERIMENT 3

It is not trivial to find, as was the case in Experiment 2, that reflectional symmetry and its breaking have qualitatively the same consequences for systems of coupled inverted or coupled ordinary rhythmic movements. That finding emphasizes the fact that the coordination dynamics of Equation 1 is governed by symmetry-based properties that are defined over the coordinative system as a whole rather than simply by the characteristics of the individual subsystems. The finding of Experiment 2 that a system of inverted pendulums nevertheless retains the stamp of the individual subsystems (in the form of greater variability) implies that coordinating the movements of physically identical but differently oriented limb segments (e.g., Figures 3a and 3b) should break the reflectional symmetry of the elementary coordination dynamics (Equation 1, when $\Delta\omega = 0$) in much the same manner as do differences in timing ($\Delta\omega \neq 0$). Specifically, to the extent that the two limb segments cannot contribute equally to the coordination, breaking reflectional symmetry through differences in spatial orientation will produce asymmetrical solutions (i.e., $\phi_{ave} - \phi_{\psi} \neq 0$).

In Experiment 3, one-to-one frequency locking of an inverted and an ordinary pendulum was compared with the 1:1 frequency locking of two ordinary pendulums for three values of $\Delta\omega$—specifically, 0 and ± 1.55 rad s^{-1}. At $\Delta\omega = 0$, we expected that coordination of differently oriented pendulums—but not identically oriented pendulums—would be shifted from the archetypal patterns of $\phi = 0$ and $\phi = \pi$ because of reflectional symmetry breaking. For the same reason, shift of the fixed points should be witnessed at $\Delta\omega = \pm 1.55$ for both differently oriented and identically oriented pendulum pairs. There is no theoretical basis for predicting whether the shift of ϕ from 0 and π produced by differences in spatial orientation alone will be greater or less than the deviation produced by differences in both timing (that is, $\Delta\omega = \pm 1.55$) and spatial orientation; the minimal expectation is that the factors of orientation and $\Delta\omega$ will interact.

Method

Participants

Four men and 4 women, all undergraduate students at the University of Connecticut, participated in the experiment in exchange for credit toward their introductory psychology course. One of the 8 participants was left-handed, but no difference between her performance and that of the right-handed participants was observed. None of the participants had previously participated in hand-held pendulum tasks.

Design

Participants were instructed to maintain $\phi_{\psi} = 0$ or $\phi_{\psi} = \pi$ with either two ordinary pendulums (both weighted below the hand) or two pendulums of opposite orientation (one weighted above the hand, one weighted below the hand). Because the rods extended both below and above the hand, we chose the convention of defining ϕ according to the spatial relation between the two upper segments of the rods (equivalent to ϕ defined for the lower segments of the rods), which also corresponded to the anatomical configuration of the flexor–extensor muscles in the wrists. Antiphase is depicted in Figure 8 for differently oriented pendulums. Note that although there was an antiphase relation both spatially—the upper segments of the pendulums were opposite in cycle—and anatomically—the left wrist was flexed while the right was extended—the weighted portion of the differently oriented pendulums was pointed in the same direction, that is, forward. In contrast, when the pendulums were identically oriented, the definition of ϕ extended to the relation between the weighted portions of the pendulums: In-phase corresponded to weighted portions that pointed in the same direction, and antiphase corresponded to weighted portions that pointed in opposite directions.

Manipulations of $\Delta\omega = \omega_{left} - \omega_{right}$ were identical to those in Experiment 2. That is, for both differently oriented and identically oriented pendulums, when the pendulum that was held in the left hand was faster (weighted closer to the hand) than the pendulum that was held in the right hand, $\Delta\omega > 0$, and when the pendulum that was held in the left hand was slower (weighted farther from the hand) than the pendulum that was held in the right hand, $\Delta\omega < 0$. Accordingly, participants were run under 12 conditions (2 orientations × 2 ϕ_{ψ} × 3 $\Delta\omega$), with two trials per condition. Data collection was otherwise identical to the procedure used in Experiment 2.

Procedure

Participants held the rods vertically. In the identically oriented pendulums condition, the center of the palm was positioned 0.6 m from the bottom of each rod, with the metal ring located below the hand. In the differently oriented pen-

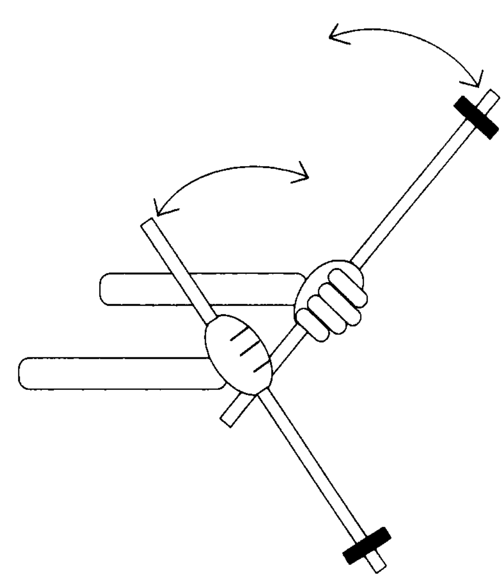


FIGURE 8. Reflectional symmetry was broken when participants coordinated the movements of an inverted pendulum that was held in one hand with those of an ordinary pendulum that was held in the other hand. Relative phase was defined according to the spatial relationship between the upper segments of the pendulums. Antiphase is depicted.

dulums condition, the left palm was positioned 0.6 m from the top of the rod, with the metal ring located above the hand, and the right palm was positioned 0.6 m from the bottom of the rod, with the metal ring located below the hand. The experimental procedure was identical to that used in Experiment 2.

Results and Discussion

Frequency

A cycle-by-cycle comparison of the frequency of oscillations of the left and right pendulums indicated that the required 1:1 frequency locking was achieved.

Mean Relative Phase

In Figure 9a, $\phi_{ave} - \phi_\psi$ as a function of $\Delta\omega$ is depicted for both differently oriented and identically oriented pendulums. Although there was a significant $\Delta\omega$ trend, $F(2, 14) = 105.03$, $p < .0001$, with all three levels of $\Delta\omega$ significantly different from each other (Tukey, $p < .01$), there was a clear difference between the differently oriented pendulums and the identically oriented pendulums. Replicating the results of Experiment 2, $\phi_{ave} - \phi_\psi$ for the identically oriented (coupled ordinary) pendulums was minimal at $\Delta\omega = 0$ (0.09 rad) and deviated from zero in a direction specified by the sign of $\Delta\omega$ ($\phi_{ave} - \phi_\psi = -0.25$ rad at $\Delta\omega = -1.55$; $\phi_{ave} - \phi_\psi = 0.29$ rad at $\Delta\omega = +1.55$). In contrast, for differently oriented pendulums, $\phi_{ave} - \phi_\psi$ was minimal at $\Delta\omega = +1.55$ (0.10 rad) and negative for both $\Delta\omega = -1.55$ (−0.30 rad) and $\Delta\omega = 0$ (−0.26 rad). When the pendulums satisfied $\Delta\omega = 0$, but were opposite in orientation, the ordinary pendulum (held in the right hand) was phase advanced of the inverted pendulum (held in the left hand). The main effect of orientation was significant, $F(1, 7) = 62.50$, $p < .0001$; fixed point shift was significantly more negative for differently oriented pendulums than for identically oriented pendulums. The Orientation × $\Delta\omega$ interaction was also significant, $F(2, 14) = 23.22$, $p < .0001$; simple effects analyses revealed that whereas the two pendulum orientations were statistically equal at $\Delta\omega = -1.55$, $F(1, 7) = 1.99$, $p > .05$, fixed point shift was significantly more negative for differently oriented pendulums at both $\Delta\omega = 0$, $F(1, 7) = 97.99$, $p < .001$, and $\Delta\omega = +1.55$, $F(1, 7) = 27.48$, $p < .001$.

There was a main effect of intended phase, $F(1, 7) = 62.50$, $p < .0001$; $\phi_{ave} - \phi_\psi$ was significantly more negative for in-phase than for antiphase. There was a marginal interaction of ϕ_ψ with $\Delta\omega$, $F(2, 14) = 3.66$, $p = .05$; simple effects analyses pointed to no difference between $\phi_\psi = 0$ and $\phi_\psi = \pi$ at $\Delta\omega = -1.55$ ($\phi_{ave} - \phi_\psi = -0.27$ rad for both 0 and π) but in-phase was significantly less than antiphase at both $\Delta\omega = 0$ ($\phi_{ave} - \phi_\psi = -0.13$ rad for 0; $\phi_{ave} - \phi_\psi = -0.04$ rad for π), $F(1, 7) = 14.79$, $p < .01$, and $\Delta\omega = +1.55$ ($\phi_{ave} - \phi_\psi = 0.15$ rad for 0; $\phi_{ave} - \phi_\psi = 0.24$ rad for π), $F(1, 7) = 6.21$, $p < .05$.

An anomalous finding was the significant deviation of ϕ_{ave} from π (0.12 rad), $t(7) = 3.61$, $p < .01$, but not from 0 (0.06 rad), $t(7) = 1.98$, $p > .05$, for the identically oriented, ordinary pendulums. A similar, but nonsignificant, trend was evident also in Experiment 2 (see Figure 6b). For both in-phase and antiphase coordinations under $\Delta\omega = 0$, Equation 1 predicts that $\phi_{ave} - \phi_\psi$ will not be significantly different from 0, and typically the prediction has been upheld unless there were handedness effects (Treffner & Turvey, 1995). Participants in the present experiment were mostly right-handed (just one exception), and the tendency is for right-handed participants to show a right hand lead, that is, $\phi_{ave} - \phi_\psi < 0$ (Amazeen, Amazeen, Treffner, & Turvey, 1997; Riley, Amazeen, Amazeen, Treffner, & Turvey, 1997; Treffner & Turvey, 1995, 1996). In the present experiment, antiphase coordination resulted in $\phi_{ave} - \phi_\psi > 0$, contrary to what would be expected on the basis of a right hand preference.

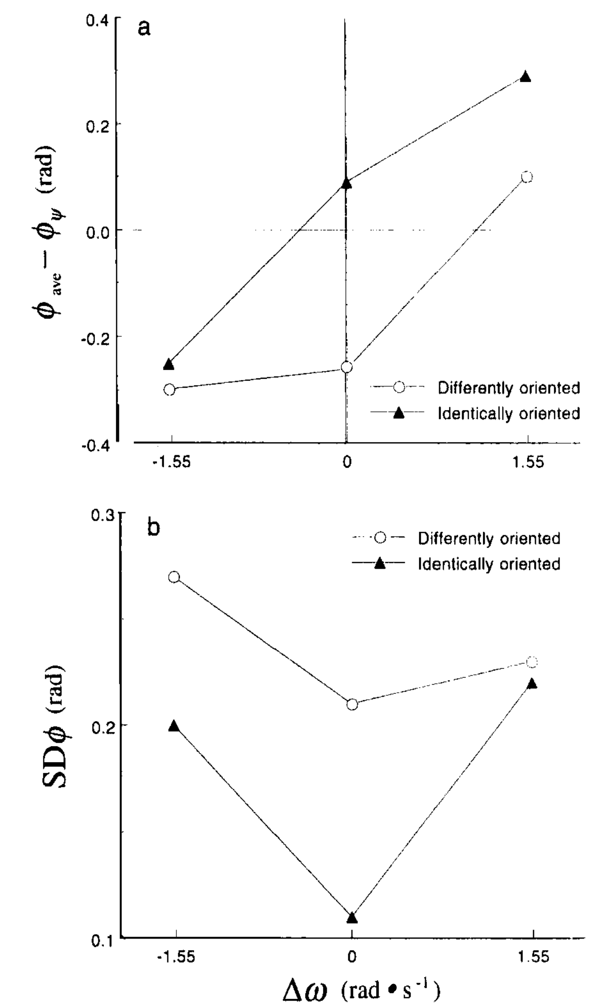


FIGURE 9. Patterns of (**a**) fixed point shift, as indexed by $\phi_{ave} - \phi_\psi$, and (**b**) variability, as indexed by $SD\phi$, as a function of $\Delta\omega$ and pendulum orientation (differently oriented or identically oriented), in Experiment 3. Comparison of Figure 9 with Figure 2 indicates that the results for identically oriented pendulums replicate standard findings, whereas the pattern for differently oriented pendulums is significantly skewed.

Standard Deviation of Relative Phase

$SD\phi$ as a function of $\Delta\omega$ and orientation is depicted in Figure 9b. Comparison of Figure 9b with Figure 2b reveals the expected symmetric pattern of variability for identically oriented pendulums and an asymmetric pattern of variability for differently oriented pendulums. Indeed, the Orientation × $\Delta\omega$ interaction was significant, $F(2, 14) = 73.04$, $p < .0001$. Although simple effects analyses revealed a significant $\Delta\omega$ trend for both differently oriented pendulums, $F(2, 14) = 28.36$, $p < .001$, and identically oriented pendulums, $F(2, 14) = 89.75$, $p < .001$, the trend for differently oriented pendulums was skewed. Tukey pairwise comparisons ($p < .01$) revealed that, for differently oriented pendulums, $SD\phi$ at both $\Delta\omega = 0$ and $\Delta\omega = +1.55$ was statistically equal and significantly less than $SD\phi$ at $\Delta\omega = -1.55$. Comparison of Figures 11a and 11b for differently oriented pendulums reveals the implication of that finding: that both near-perfect production of a phase relation (at $\Delta\omega = +1.55$) and imperfect production of a phase relation (at $\Delta\omega = 0$) can be equally stable.

The two pendulum configurations (identically oriented and differently oriented) were differentially stable, $F(1, 7) = 90.53$, $p < .0001$. Simple effects analyses from the Orientation × $\Delta\omega$ interaction indicated that $SD\phi$ was significantly greater for differently oriented pendulums at both $\Delta\omega = -1.55$, $F(1, 7) = 77.10$, $p < .001$, and $\Delta\omega = 0$, $F(1, 7) = 158.26$, $p < .001$, but that the two orientation conditions were statistically equal at $\Delta\omega = +1.55$, $F(1, 7) < 1$. As expected, antiphase was significantly more variable than in-phase, $F(1, 7) = 99.84$, $p < .0001$. More important, the fact that ϕ_ψ interacted with neither orientation nor $\Delta\omega$ indicates that in-phase was uniformly more stable for both orientation conditions. To reiterate an earlier point, if both identically oriented pendulums and differently oriented pendulums are members of the reflectional symmetry group, then they may be accommodated by the same coordination dynamics, and yet—like in-phase and antiphase—exhibit differential stability.

Our focus in Experiment 3 was on the correspondences between equilibria of differently oriented pendulums and identically oriented pendulums. From the perspective of Equation 1, the fact that differently oriented pendulums at $\Delta\omega = 0$ did not coordinate at $\phi_{ave} - \phi_\psi = 0$ implies a breaking of reflectional symmetry. Given the identical results for coupled ordinary and coupled inverted pendulums found in Experiment 2, it is reasonable to conclude that the uncoupled frequencies of an ordinary and an inverted pendulum of the same L are identical and, therefore, that $\Delta\omega$ was, in fact, zero. Consequently, the deviation of $\phi_{ave} - \phi_\psi$ from 0 must be attributed to a breaking of reflectional symmetry different in kind from that captured by $\Delta\omega$.

EXPERIMENT 4

In Experiment 3, differently oriented pendulums were constituted by an ordinary right pendulum and an inverted left pendulum (a left [L] -up configuration). They could just as well have been constituted in the reverse manner—by an inverted right pendulum and an ordinary left pendulum (a right [R] -up configuration). A prediction following from group theory is that two reflectionally related states of the system will produce solutions that are related by the reflectional transformation. More concretely, the asymmetric solution produced by the configuration depicted in Figure 3a (e.g., $\phi_{ave} - \phi_\psi = -f$), for example, should be reflectionally related to the solution produced by the configuration depicted in Figure 3b (e.g., $\phi_{ave} - \phi_\psi = +f$). In Experiment 4, those two configurations were compared in in-phase and antiphase coordination at three $\Delta\omega$ values: 0 and ± 1.55 rad s^{-1}. On the understanding that symmetry breaking is more accurately considered to be symmetry sharing (Stewart & Golubitsky, 1992), we expected that if the coordination dynamics of differently oriented pendulums abide by reflectional symmetry, then the three fixed points of the R-up configuration and the three fixed points of the L-up configuration should be related by reflection. Specifically, if both $\Delta\omega$ and $\phi_{ave} - \phi_\psi$ are multiplied by −1 for the R-up pendulums, then (other things being equal) they should yield the corresponding fixed points for L-up.

Method

Participants

Four men and 4 women, all undergraduate students at the University of Connecticut, participated in the experiment in exchange for credit toward their introductory psychology course. All 8 participants were right-handed and naïve with respect to the hand-held pendulum task.

Design

Participants were instructed to maintain $\phi_\psi = 0$ or $\phi_\psi = \pi$ with pendulum pairs that were configured so that either the pendulum held in the left hand was inverted (L-up) or the pendulum held in the right hand was inverted (R-up). Manipulations of $\Delta\omega$ were identical to those in Experiments 2 and 3. Therefore, participants were run under 12 conditions (2 configurations × 2 ϕ_ψ × 3 $\Delta\omega$), with two trials per condition. Data collection was identical to the procedures used in Experiments 2 and 3.

Procedure

Participants held the rods vertically. In the L-up condition, the left palm was positioned 0.6 m from the top of the rod, with the metal ring located above the hand, and the right palm was positioned 0.6 m from the bottom of the rod, with the metal ring located below the hand. In the R-up condition, the left palm was positioned 0.6 m from the bottom of the rod, with the metal ring located below the hand, and the right palm was positioned 0.6 m from the top of the rod, with the metal ring located above the hand. The experimental procedure was identical to that used in Experiments 2 and 3.

Results and Discussion

Frequency

A cycle-by-cycle comparison of the frequency of oscillations of the left and right pendulums indicated that the required 1:1 frequency locking was achieved.

Mean Relative Phase

In Figure 10a, $\phi_{ave} - \phi_{\psi}$ as a function of $\Delta\omega$ is depicted for both L-up and R-up pendulum configurations. Notice that $\phi_{ave} - \phi_{\psi}$ was negative at both $\Delta\omega = -1.55$ and $\Delta\omega = 0$ for L-up and positive at both $\Delta\omega = 0$ and $\Delta\omega = +1.55$ for R-up, indicating that the inverted pendulum tended to lag the ordinary pendulum in both configurations. It can also be seen that the $\Delta\omega$ trend for L-up was identical to that for L-up in Experiment 3, whereas, for R-up, the $\Delta\omega$ trend was an exact mirror image through the origin (0, 0). Reflection of the results for R-up took the form of exchanging the left and right pendulums both for $\Delta\omega$—that is, $-1(\Delta\omega) = (\omega_{right} - \omega_{left})$—and for $\phi_{ave} - \phi_{\psi}$, where $-1(\phi) = (\theta_{right} - \theta_{left})$. The result of reflecting the R-up fixed points is shown in Figure 10b.

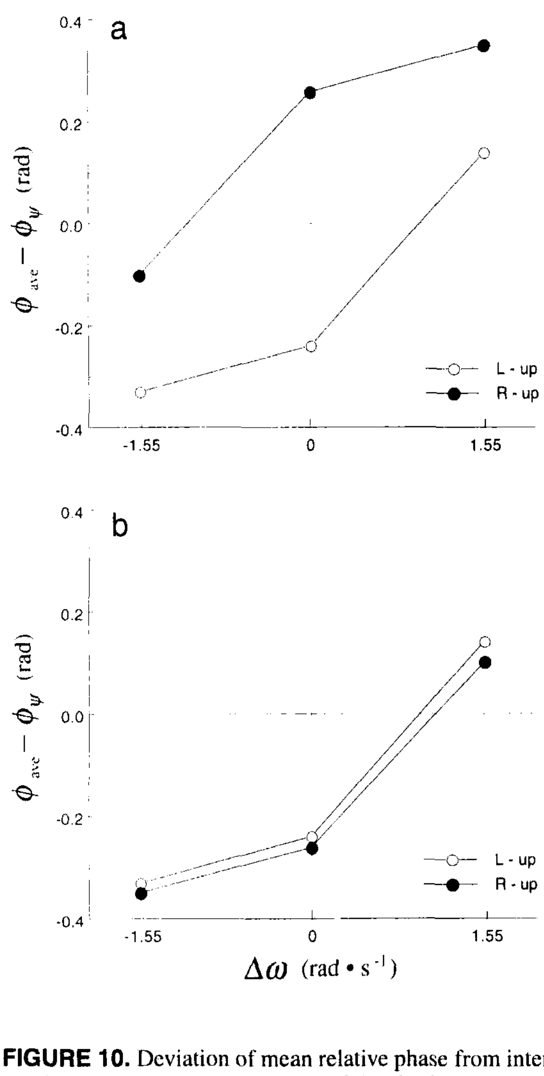


FIGURE 10. Deviation of mean relative phase from intended phase, $\phi_{ave} - \phi_{\psi}$, as a function of $\Delta\omega$ for L-up and R-up pendulum configurations in Experiment 4, both (**a**) before and (**b**) after transformation. We transformed data by multiplying both $\Delta\omega$ and $\phi_{ave} - \phi_{\psi}$ by -1. If differently oriented pendulums abide by reflectional symmetry, then fixed points for L-up and R-up should be identical following a reflectional transformation. L = left, and R = right.

As is suggested by Figure 10b, the $\Delta\omega$ trends for the two configurations (L-up and R-up) were identical; there was no main effect of configuration, $F(1, 7) < 1$, and there was no interaction between configuration and $\Delta\omega$, $F(1, 7) < 1$. In replication of the results for differently oriented pendulums in Experiment 3, there was the significant $\Delta\omega$ trend, $F(2, 14) = 117.10$, $p < .0001$, with minimal fixed point shift at $\Delta\omega = +1.55$ (0.12 rad) and negative and statistically equal shift for both $\Delta\omega = -1.55$ (−0.34 rad) and $\Delta\omega = 0$ (−0.25 rad; Tukey, $p < .01$). The $\phi_{\psi} \times \Delta\omega$ interaction was significant, $F(2, 14) = 5.04$, $p < .05$; in partial replication of the results of Experiment 3, simple effects analyses revealed that the production of in-phase was significantly more negative than the production of antiphase at $\Delta\omega = +1.55$, $F(1, 7) = 31.62$, $p < .001$, but neither at $\Delta\omega = -1.55$, $F(1, 7) = 9.50$, $p > .01$, nor at $\Delta\omega = 0$, $F(1, 7) = 7.79$, $p > .01$.

Standard Deviation of Relative Phase

$SD\phi$ as a function of $\Delta\omega$ for both L-up and R-up pendulum configurations is shown in Figure 11a. Across both configurations, $SD\phi$ was minimal (0.24 rad) when $\Delta\omega = 0$ and increased equally for both $\Delta\omega = -1.55$ (0.31 rad) and $\Delta\omega = +1.55$ (0.31 rad). That $\Delta\omega$ trend was significant, $F(2, 14) = 15.18$, $p < .0005$. Although the $\Delta\omega$ effect was not numerically identical for L-up and R-up, with $SD\phi$ at $\Delta\omega = -1.55$ greatest for L-up and $SD\phi$ at $\Delta\omega = +1.55$ greatest for R-up, the Orientation × $\Delta\omega$ interaction was not significant, $F(2, 14) = 1.55$, $p > .05$. The results of transforming the R-up data points in the manner described earlier (with reference to the mean relative phase data in Figure 10) suggested that any asymmetry of the $\Delta\omega$ trend was statistically identical for both configurations, $F(2, 14) < 1$ (see Figure 11b). Analyses performed on the transformed data revealed statistical significance only between $SD\phi$ at $\Delta\omega = -1.55$ and $SD\phi$ at $\Delta\omega = 0$. That result replicates the finding of Experiment 3 that both symmetric and asymmetric phase relations, as produced by $\Delta\omega = +1.55$ and $\Delta\omega = 0$, respectively, can be equally stable.

As expected, in-phase ($SD\phi = 0.27$ rad) was significantly more stable than antiphase ($SD\phi = 0.30$ rad), $F(1, 7) = 99.84$, $p < .0001$; however, a significant interaction of ϕ_{ψ} with pendulum configuration, $F(1, 7) = 5.91$, $p < .05$, revealed the differential stability of in-phase and antiphase for the L-up configuration only, $F(1, 7) = 23.52$, $p < .01$ (see also the results of Experiment 3).

Experiment 4 was primarily directed at the expectation that if the coordination dynamics of differently oriented pendular motions were reflectionally asymmetric, then the

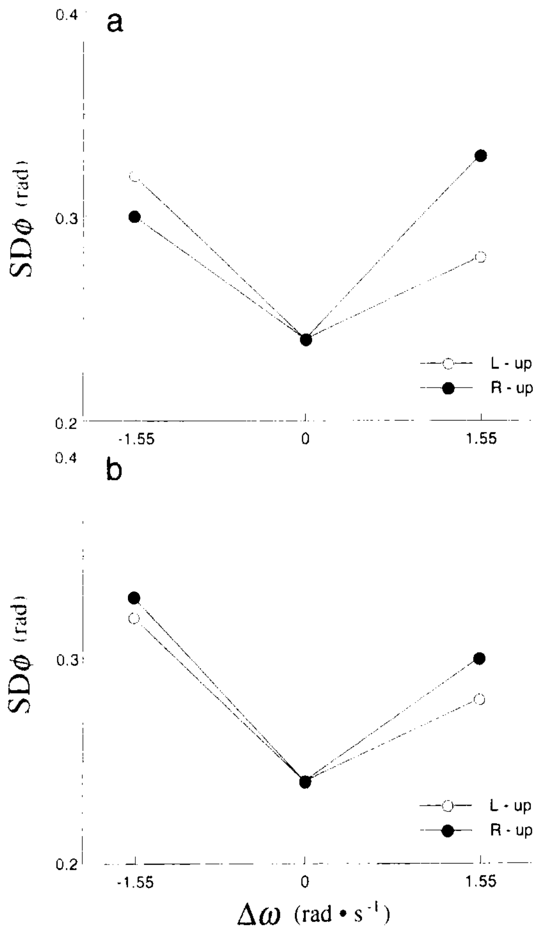


FIGURE 11. Standard deviation of relative phase, $SD\phi$, as a function of $\Delta\omega$ for L-up and R-up pendulum configurations in Experiment 4, both (**a**) before and (**b**) after transformation (described in Figure 10). If differently oriented pendulums abide by reflectional symmetry, then $SD\phi$ for L-up and R-up should be identical following a reflectional transformation. L = left, and R = right.

fixed points for R-up and L-up induced by manipulations of $\Delta\omega$ would be related by a reflectional transformation. That expectation was confirmed (see Figure 10). As a secondary feature, Experiment 4 together with Experiment 3 provided results with bearing on experiments that have been directed at intersegmental coordinations of homologous muscle groups (two arms or two legs) and nonhomologous muscle groups (one arm and one leg) (Jeka & Kelso, 1995; Kelso & Jeka, 1992). Using a spatial definition of ϕ in which flexion–flexion pairings were considered in-phase for homologous limb segments and flexion–extension pairings were considered in-phase for nonhomologous limb segments, Kelso and Jeka replicated the standard findings regarding the differential stability of the phase modes 0 and π (e.g., Kelso, 1984). Comparison of homologous with nonhomologous muscle group pairings revealed higher $SD\phi$ for nonhomologous limb pairings, a result that is consistent with the findings of Experiment 3 that differently oriented limbs were less stable than identically oriented limbs.

GENERAL DISCUSSION

In the present series of experiments, we addressed the ability to coordinate differently oriented and differently sized limb segments. The experimental procedures mimicked the coordination of the arms hanging by the sides (frequency locking of ordinary pendulums), the arms raised above the head (frequency locking of inverted pendulums), and the coordination of a hanging and a raised arm (frequency locking of an ordinary and an inverted pendulum). Defining a coordination pattern by ϕ, that is, the phase difference between the two rhythmically moving segments, our focus in the experiments was on the stable coordination patterns (fixed points or equilibria) that characterize the three different spatial arrangements of limb segments. In Experiments 1 and 2, we studied the coordinations of two arms oriented in the same way with respect to gravity. Expectations were shaped by the coordination dynamics of Equation 1, in which reflectional symmetry is broken through differences in the timing $\Delta\omega$ of the limb segments. It is important to note that the results of the experiments showed that two coordinated limb segments behaving as inverted pendulums were identical in their equilibria values to two coordinated limb segments behaving as ordinary pendulums. For those two identically oriented systems, the coordination dynamics at the global level were the same despite differences in the physical dynamics at the local level (see Kelso, 1994b; Schöner, 1994).

In Experiments 3 and 4, we investigated the coordination of two arms that were oriented in opposite ways with respect to gravity. The results of Experiment 3 showed that the coordination dynamics of differently oriented limb segments were characterized by different equilibria than the coordination dynamics of identically oriented limb segments. The implication of that finding is that the coordination dynamics of Equation 1 are incomplete with respect to reflectional symmetry breaking through differences in spatial orientation. The fact that a combination of temporal and spatial differences produced coordinative solutions that were closer to symmetry (i.e., perfect in-phase 0 and antiphase π coordination) than did spatial differences alone indicates the presence of an interaction between the two types of symmetry breaking that needs to be accommodated by the coordination dynamics. Finally, in Experiment 4, we showed that the equilibria of differently oriented limb segments depicted in Figures 3c (L-up) and 3d (R-up) were related by a reflectional transformation. That the coordination of differently oriented limb segments belongs to the same reflectional symmetry group as does the coordination of temporally different limb segments implies that the accommodation of differently oriented limb segments may be accomplished through the expansion, rather than replacement, of Equation 1.

The reflectional relatedness of L-up and R-up coordination systems in Experiment 4 bears a striking similarity to Treffner and Turvey's (1995) finding that identical solutions were produced when the right hand versus left hand distinction was transformed into a distinction between the preferred and nonpreferred hand. Treffner and Turvey (1995, 1996) showed that handedness effects on coordination result from an asymmetric coupling between the preferred and nonpreferred hand, with the preferred hand having a stronger influence over the nonpreferred hand than the nonpreferred hand has over it. That finding was demonstrated by a preferred hand lead in relative phase. In Experiments 3 and 4 of the present study, the manner in which asymmetric solutions were produced seemed to indicate an asymmetric coupling between the pendulums in which the ordinary pendulum was phase advanced of the inverted pendulum; that is, the ordinary pendulum appeared to act as a type of preferred hand. That conclusion is not terribly surprising, because ordinary pendulums are oriented in the direction of gravity and their coordination was shown in Experiment 2 to be more stable than the coordination of inverted pendulums.

If Treffner and Turvey's (1995, 1996) work on bilateral asymmetries were tapping into the more general issue of breaking reflectional symmetry by spatial differences, then their expansion of Equation 1 through the addition of two cosine terms should accommodate both the asymmetric patterns of fixed point shift and (questionably) the imperfect correspondence between fixed point shift and variability. The latter imperfection was noteworthy both in the determination of the differential stability of in-phase and antiphase (see Experiment 4) and in the lack of a conclusive statement regarding the asymmetries at $\Delta\omega = 0$. In both Experiments 3 and 4, temporally identical pendulums were found to produce greater fixed point shift and less variability than temporally different pendulums. When the components of coordination are spatially identical, as in Experiment 2, fixed point and variability data correspond—the greater the shift, the greater the variability.

The literature on movement disorders seems to indicate that the finding that asymmetric solutions can be more (or equally) stable is not entirely unexpected. Although the stability of the asymmetric gaits of stroke and Parkinson's patients has yet to be empirically demonstrated, it has been argued that because those gaits are regularly and repeatedly produced, they are, in fact, more stable in those populations than symmetric gaits (e.g., Wagenaar, 1990; Wagenaar & van Emmerik, 1994). It is not unreasonable to believe that the change in gait that is witnessed in Parkinson's and stroke patients might be mediated through a spatial asymmetry that is produced by the disorder. If manipulation of the added parameters in Treffner and Turvey's (1995, 1996) model adequately describes the general category of spatial asymmetries, then the model may be able to address gait asymmetries produced by movement disorders. Unfortunately, Treffner and Turvey's model does not account for the spatial asymmetries observed in the present study.

The second option, then, is to consider an expansion of $\Delta\omega$ (the detuning term or imperfection parameter, as characterized by Collins, Sternad, & Turvey, 1996; Sternad, Collins, & Turvey, 1995)—introduced in Equation 1 to represent temporal differences in the component oscillators—so that it includes their spatial differences as well. Consideration of the imperfection parameter as a general symmetry breaking term would allow for representation of both the temporal and spatial composition of a coordinative pattern, as well as their interactive qualities, as observed in the present research. Further empirical investigation of the temporal–spatial interaction, as well as the influence of the coupling strength between the component oscillators, b/a, is required before serious consideration is given to the precise form of the expansion.

The present study was an empirical demonstration of the Extended Curie Principle that whenever asymmetric solutions are witnessed, they are balanced by asymmetric solutions that are qualitatively the same but opposite in direction (Stewart & Golubitsky, 1992). Stewart and Golubitsky applied the concept of symmetry sharing to symmetric equations that have asymmetric solutions. To date, however, the asymmetries seen in two-limb coordination have been accommodated only by asymmetric equations (e.g., Equation 1). Nevertheless, the Extended Curie Principle appears to predict the form of the present results. Therefore, a third option exists in which reflectional symmetry breaking is accommodated by a symmetric equation similar in kind to the elementary coordination dynamics (i.e., $\Delta\omega = 0$), in which the symmetry has not been broken but, rather, reduced (see Schöner et al., 1990, for instruction on the reduction of an equation's symmetry).

The symmetric relatedness of solutions permits nature to maximize its resources by allowing systems that are equally asymmetric to produce qualitatively the same solution. As Stewart and Golubitsky (1992) remarked, "[*M*]*athematically* the laws that apply to symmetric systems can sometimes predict not just a single effect, but a whole set of symmetrically related effects" (p. 15). The challenge now is to find the coordination dynamics that underlie the symmetric structure of interlimb coordination.

ACKNOWLEDGMENT

This research was supported by National Science Foundation Grant SBR 94-22650, awarded to M. T. Turvey. Additional support for the preparation of this manuscript was provided by grant USF'96 awarded to P. J. Beek by the Vrije Universiteit in Amsterdam. Preliminary findings were reported in Amazeen et al. (in press). The authors would like to acknowledge Geoffrey P. Bingham for his comments on an earlier version of the manuscript and Andreas Daffertshofer for his technical advice.

NOTES

1. Group theory has been applied successfully to gait transitions in quadrupeds (e.g., Collins & Richmond, 1994; Collins & Stew-

art, 1993a; Haken, 1996; Schöner, Jiang, & Kelso, 1990). Three aspects of its previous application remain categorically distinct from the present application and, therefore, prohibit direct comparison: symmetry breaking that is induced (asymmetric conditions produce asymmetric solutions) rather than spontaneous (symmetric conditions produce asymmetric solutions—currently that is possible only with four-limb coordinations); the use of the Extended Curie Principle in group theory for generating hypotheses (see text for details); and a focus on stable state behavior rather than on phase transitions. Relatedly, the quadrupedal gaits analyzed were all among limbs that differed in neither timing nor spatial orientation. Therefore, it would be inappropriate to go into further details in the context of the present article.

2. An important distinction must be made between the symmetry group (in this case, reflectional) and the members of the symmetry group (in-phase and antiphase) that are accommodated by a coordination dynamics (e.g., Equation 1). In both group theory and the coordination dynamics, in-phase and antiphase are both termed *symmetric phase relations*; an example of an asymmetric phase relation is 90° (π/2), a phase relation that is roughly equivalent to the bipedal gallop (e.g., Peck & Turvey, 1997). The difference between the two symmetric phase relations is that in-phase is temporally preserved over a reflectional transformation, whereas antiphase, albeit unchanged categorically, is phase advanced by ½ cycle. The left limb forward and the right limb forward antiphase postures are different only in terms of their (static) initial condition; that condition is lost as soon as the system is set into motion and, therefore, has no effect on its dynamics. Because the dynamics of in-phase and antiphase are unaffected by reflection, no predictions follow from group theory regarding differences between them. The coordination dynamics alone make a distinction between the in-phase and antiphase members of the reflectional symmetry group: In-phase is more stable than antiphase and, therefore, more persistent over frequency scaling (see phase transition experiments of Kelso, 1984).

3. Stewart and Golubitsky (1992) applied the Extended Curie Principle to systems in which symmetry is spontaneously broken, that is, to symmetric systems—describable by symmetric equations—that produce asymmetric solutions. Although spontaneous symmetry breaking has been used to describe gait transitions in quadrupeds (e.g., Collins & Richmond, 1994; Collins & Stewart, 1993a; Haken, 1996; Schöner et al., 1990), it is currently not applicable to stable state two-limb coordinative patterns because, to date, only asymmetric equations have been able to produce the witnessed asymmetric patterns (e.g., Amazeen et al., 1996; Kelso et al., 1990; Rand, Cohen, & Holmes, 1988). Stewart and Golubitsky labeled that latter instance of symmetry breaking *induced* and refrained from discussing the applicability of the Extended Curie Principle. In the empirical work presented in this article, we tested the extent to which Stewart and Golubitsky's Extended Curie Principle can be applied to systems in which symmetry breaking is induced.

4. Implicit in Experiment 2 was one additional hypothesis with methodological significance. The method of determining uncoupled frequencies appropriate for the computation of $\Delta\omega$ takes the undamped, undriven frequency (see Equation 2 and den Hartog, 1985/1934) as the relevant quantity. The results of Experiment 1 indicated that when an ordinary, hanging pendulum is turned upside down, the movement frequency remains the same (see Smith & Blackburn, 1992, for application to purely physical systems). However, the neuromuscular organization for oscillating an inverted pendulum cannot be identical to that for oscillating a hanging pendulum. The neuromuscular difference is based on the fact that gravity tends to remove the inverted pendulum from the vertical, in contrast to its effect in the ordinary hanging situation; then, gravity tends to return the pendulum to the vertical. The evidence from Experiment 2, that the equilibria of coupled inverted and coupled hanging pendulums were the same for $\Delta\omega = \pm1.55$, reinforces the hypothesis that it is the undamped, undriven (unforced) frequencies that enter into the calculation of $\Delta\omega$ rather than the damped, driven (forced) frequencies defined by the neuromuscular driving of the hand-held pendulums.

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Submitted July 7, 1997
Revised November 18, 1997
Second revision January 20, 1998