helpful experimental advice of Albert Koenig and Tom Davis.

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Breaking the Roton Barrier: An Experimental Study of Motion Faster than the Landau Critical Velocity for Roton Creation in He II†

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We report the first observation of objects moving through He II with equilibrium drift velocities $\bar{v}$ beyond the Landau critical velocity v_c for roton creation. With $T\sim 0.4$ K, $P=25$ bar, $F=2$ kV cm^{-1}, $\bar{v}-v_c$ for negative ions is larger than a theoretical prediction by a factor of $\sim 10^5$. The vortex-ring nucleation rate is found to decrease with F above 300 V cm^{-1}, thus resolving apparent inconsistencies between earlier experiments.

In his celebrated explanation of superfluidity, Landau[1] showed that the kinetic energy of a liquid flowing at velocity v through a tube (or that of a heavy object moving through the liquid) cannot be dissipated through the creation of an excitation of energy ϵ and momentum p in the liquid unless $v \geqslant \epsilon/p$. For He II the minimum value of ϵ/p is nonzero, occurring close to the roton minimum in the elementary excitation spectrum, so that a critical velocity $v_c=(\epsilon/p)_{\min}\approx 50$ m sec^{-1} exists, below which dissipation ought not to occur in the superfluid. Measured critical velocities are usually orders of magnitude smaller, because of the onset of vortex formation at lower velocities, but Rayfield[2] reported that the drift velocity $\bar{v}$ of negative ions in He II under pressure $P>12$ bar below 0.6 K appeared to reach and to be limited by v_c when the applied field was raised to about 70 V cm^{-1}. This has remained the only known situation to which Landau's original criterion for the breakdown of superfluidity appears to be relevant.

Takken[3] has considered roton creation by negative ions moving at velocities slightly greater than v_c on the basis of a wave radiation model in which each ion is assumed to generate a conical wave of coherent roton radiation, much like the disturbance produced by an airplane breaking the sound barrier. By analogy with the aerodynamic case, a rapid increase in drag is expected as the velocity increases past v_c: Takken concluded that an upper bound on v is given by $v_{\text{ub}}=v_c(1+10^{-12}F^2)$, where the electric field F is in V cm^{-1}. A 1% increase of v above v_c would therefore require $F\geqslant 10^5$ V cm^{-1}, implying that any increase of v beyond v_c ought to be almost impossible to observe experimentally.

An attempt by Neeper and Meyer[4] to test this remarkable assertion was thwarted by an unexpected increase in the vortex nucleation rate ν with falling temperature, such that at 0.3 K only vortex rings, and no bare ions, arrived at their collector. Our recent observation[5] that the field emission current at 0.3 K in He II increases dramatically with P above 12 bar, and is temperature independent below 0.4 K, seemed to be inconsistent with Neeper and Meyer's result. Approximate values of ν deduced[6] from the field-emission measurements apparently indicated the feasibility of our present experiment to test Tak-

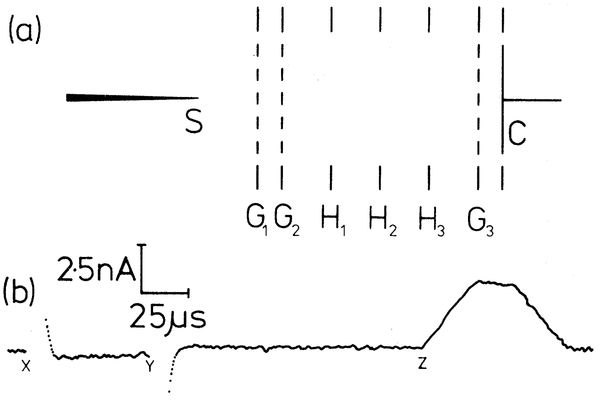

FIG. 1. (a) Electrode structure (diagrammatic) used for the time-of-flight measurements. Pulses of ions from the field emission source S are gated by potentials applied to grids G_1, G_2 into the drift space between grids G_2, G_3, where they propagate in a large electric field maintained at a uniform value by guard rings H_1, H_2, and H_3. The resultant pulses of current induced in the collector C are amplified and signal-averaged prior to being recorded. (b) A typical received signal, after averaging. The transients at x and y indicate, respectively, the opening and closing of the G_1, G_2 gate, and z marks the first arrival of the pulse at G_3.

ken's conclusion by measuring the times of flight of single pulses of ions in large electric fields.

Our experimental cell employed the customary configuration of electrodes [Fig. 1(a)], except in that a field emitter was incorporated in place of the conventional radioactive current source. A typical cycle of operation commenced with the emitter being pulsed to $V_e = -1500$ V with respect to G_1, which was normally held a few volts positive relative to G_2. After the resultant transient had died away, the gate formed by G_1, G_2 was opened for about 50 μsec by applying a negative pulse to G_1. The current pulse arriving at C was taken via a resistor to earth, and the signal appearing across the resistor was amplified and passed to a transient recorder. Finally, after a total emission period of ~600 μsec, V_e was returned to zero. This cycle was repeated at a rate of ~3 Hz, feeding the transient recorder's output into a signal averaging system until the signal-to-noise ratio had reached a satisfactory level. The time calibration of the signal detection system was checked against an independent crystal-controlled standard.

The received pulses [Fig. 1(b)] were of the expected shape,[7] which was invariant over a very wide range of F. Velocities deduced from the pulse arrival times (Fig. 2) were independent of whether the positions of the leading or trailing edges were used, they were independent of T for $0.3\text{ K} \leq T \leq 0.5\text{ K}$, and, contrary to expectations, they were substantially in excess of v_c (dashed line) as computed using accepted values[8] of the Landau parameters.

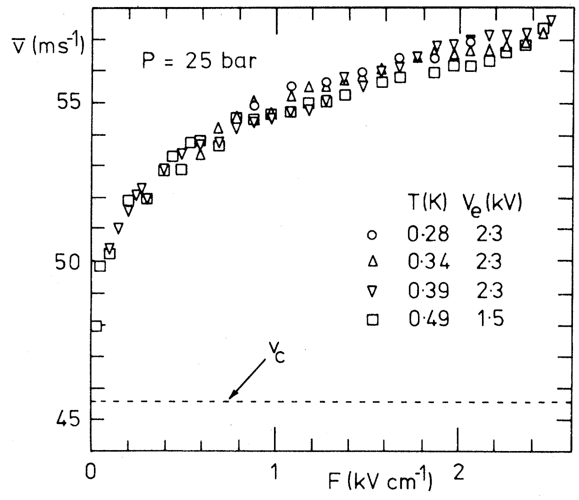

FIG. 2. Measured drift velocity $\bar{v}$ of negative ions as a function of electric field F. The precision of the measurements is indicated by their scatter, and there may, in addition, be a scaling error in $\bar{v}$ of up to ±7%. The dashed line shows v_c.

The systematic error in our values of $\bar{v}$ arises almost entirely from uncertainties in the length G_2G_3, which was determined by measuring individual components at room temperature, and then applying corrections for thermal contraction. This procedure results in an uncertainty of about ±0.3 mm in the total length of brass and nylon components; possible buckling of the grids introduces a further uncertainty which is hard to quantify, but which we believe is less than ±0.4 mm. We conclude that $G_2G_3 = 10.2 \pm 0.7$ mm.

At $F = 2$ kV cm^{-1}, $(\bar{v} - v_c)/v_c = 0.24 \pm 0.09$, compared to $\sim 10^{-6}$ predicted by Takken. Furthermore, the form of our $\bar{v}(F)$ characteristic is qualitatively different from that predicted, which would curve *upwards* with increasing F. We conclude, therefore, that Takken's model is not applicable to negative ions moving in large electric fields through He II under pressure near 0.4 K.

We note that, for an ion of mass m, the instantaneous velocity v required for roton creation is, in fact, not precisely v_c, but[9]

$$v' = [(\epsilon/p) + (p/2m)]_{\min} = (\epsilon'/p') + (p'/2m). \quad (1)$$

Conservation of energy and momentum dictate

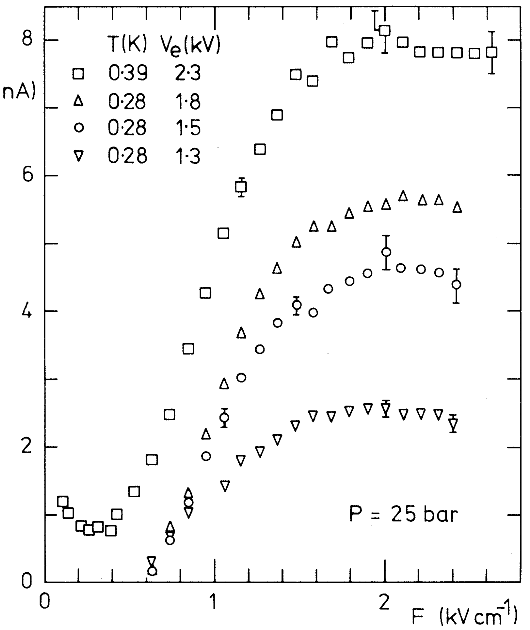

FIG. 3. Pulse height i_p as a function of electric field F.

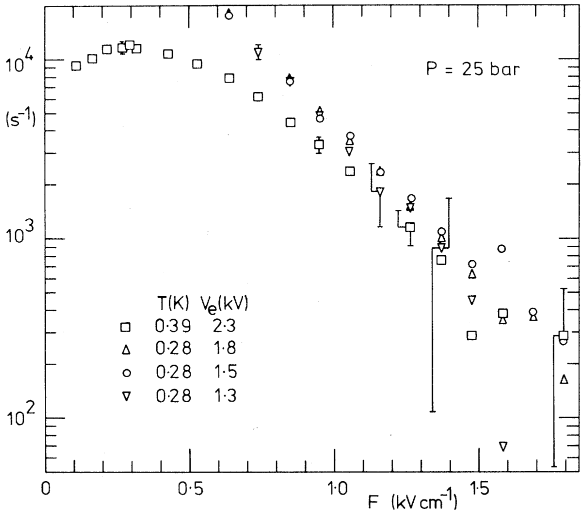

FIG. 4. Charged vortex-ring creation rate ν as a function of electric field F, deduced from the data of Fig. 3 using Eq. (3).

that the instantaneous velocity immediately afterwards be

$$v'' = (\epsilon'/p') - (p'/2m) \tag{2}$$

so that, assuming the ion to accelerate at a constant rate between emission events, $\bar{v} = \epsilon'/p'$. Supposing that the hydrodynamic mass scales approximately with the ionic volume, and using Springett and Donnelly's values[10] of the ionic radius together with Poitrenaud and Williams's measured effective masses at the saturated vapor pressure,[11] we find that, for $P = 25$ atm, $m = 60m_4$.

Using Donnelly's Landau parameters,[8] we calculate from (1) and (2) $v' = 48.3$ m sec^{-1}, $(\epsilon'/p') = 45.6$ m sec^{-1}, $v'' = 42.9$ m sec^{-1}; and $v_c = 45.6$ m sec^{-1}, so that, if all rotons were created at v', we would find $\bar{v} = v_c$ within our experimental error. This is plainly not the case (Fig. 2). We conclude, therefore, that the ion usually accelerates well beyond v' before emitting a roton (or creating a vortex ring). Probably, roton emission and vortex ring creation are competing processes whose relative transition probabilities under any given conditions of T, F, and P will determine the ionic drift velocity which is observed experimentally.

We can deduce the rate ν for creation of charged rings from the variation with F of the height i_p of the pulse at the collector (Fig. 3). For given values of T, P, V_e, and G_1-G_2 potential difference, the pulse magnitude i_0 at G_2 will be independent of F (except, of course, for very small values of F where the effective transparency of G_2 will depart significantly from its geometrical values). We may, therefore, reasonably attribute the variation of i_p with F to the expected vortex nucleation events between G_2 and G_3: Charged vortex rings, because of their relatively very small velocities, represent ions which effectively are lost from the pulse. Taking for i_0 the saturation value of i_p in high field we have calculated ν using the relation

$$\nu = \tau^{-1} \ln(i_0/i_p), \tag{3}$$

where τ is the G_2-G_3 transit time. Values of ν derived in this way (Fig. 4) are independent of emitter, gate, and Frisch grid potentials, but they vary in a complicated manner[12] with F, T, and P; for small F, ν decreases with increasing T, in agreement with the conclusions of Neeper and Meyer.[4] We have, however, verified our earlier inference[6] that for large F, ν *decreases*[13] with increasing F and is apparently independent of T below 0.4 K. This discovery enables the nonappearance of bare ions in Neeper and Meyer's experiment at $F \sim 30$ V cm^{-1} to be reconciled with their apparent presence in a field emission diode[5] for which $F \sim 3000$ V cm^{-1}, and with the present data. Because the probability of thermally activated escapes of ions from vortices under these conditions is quite negligible, the phenomenon probably arises from some modification of the nucleation mechanism itself. This cannot be occurring through the "temperature" of the pulse being above ambient because ν is found to be independent of i_0 although it remains possible, of course, that the nucleation process for an individual ion is perturbed by the bunch of rotons which it has itself created and whose density presumably increases with F. Our results would seem to favor models[14] in which the vortex ring is formed initially in a symmetrical position around the ion which, for small F, will subsequently move sideways and become trapped; and to weigh against models[15] in which a vortex loop is gradually paid out from the ion, which is therefore always trapped.

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Focused-Flow Model of Relativistic Diodes*

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A new model of electron flow in high-voltage diodes and superpinch formation is presented. The relativistic cold-fluid–Maxwell equations are reduced, under certain assumptions, to one equation. The model is based on the solution for the electron flow in large aspect-ratio diodes and the inclusion of cathode and anode plasmas. The anode plasma is assumed to be a field-free region. Results are compared to the parapotential-flow model.

The behavior of relativistic electron flow inside high-voltage (~1 MV) diodes is important for obtaining extremely high current densities[1-3] and for applications to controlled thermonuclear fusion experiments.[4] The strong pinching observed experimentally[1-3] on the anode surface has motivated the theoretical investigation and modeling of the so-called superpinch. The parapotential-flow model[5,6] predicts scaling laws of diode impedance but is unable to treat electron flow near the cathode or the anode. The metapotential-flow model[1] contains the effects of orbit crossings but relies sensitively on anode plasma dynamics and/or strong plasma bias currents for superpinch formation.

In the focused-flow model presented in this work we treat the electrons within the framework of the steady-state ($\partial/\partial t = 0$) relativistic-fluid–