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Breaking Tri-Bimaximal Mixing and Large θ_{13}

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The long baseline neutrino experiment, T2K, and the reactor experiment, Double Chooz will soon present new data. If we expect $\sin\theta_{13}$ to be $0.1-0.2$, which is close to the present experimental upper bound, we should not persist in the paradigm of the tri-bimaximal mixing. We discuss breaking the tri-bimaximal mixing by adding a simple mass matrix, which could be derived from some non-Abelian discrete symmetries. It is found that $\sin\theta_{13}=0.1-0.2$ is expected in our model independent analysis of the generalized mass matrix for the normal or inverted hierarchical neutrino mass spectrum. On the other hand, $\sin^2\theta_{23}$ and $\sin^2\theta_{12}$ are expected to be not far from 1/2 and 1/3, respectively. As a typical example, we also discuss the A_4 flavor model with the 1 and 1′ flavons, which break the tri-bimaximal mixing considerably. In this modified version of the Altarelli and Feruglio model, $\sin\theta_{13}$ is predicted to be around 0.15 in the case of the normal hierarchical neutrino masses $m_3 \gg m_2, m_1$, and 0.2 in the case of the inverted hierarchy $m_3 \ll m_2, m_1$. The form of the neutrino mass matrix looks rather interesting — it is suggestive of other discrete symmetries as well.

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§1. Introduction

The discovery of the neutrino masses and the lepton mixing has stimulated the work of the flavor symmetries. Recent experiments of the neutrino oscillation go into a new phase of precise determination of the mixing angles and the mass squared differences.[1)–5)] On the basis of these results, the paradigm of the tri-bimaximal mixing for three flavors has been proposed in the lepton sector.[6)–9)] Many of the recent flavor models have aimed at the tri-bimaximal mixing as a leading form of the leptonic mixing.

The flavor symmetry is expected to explain the mass spectrum and the mixing matrix of both quarks and leptons. Especially, the non-Abelian discrete symmetry[10),11)] has been studied intensively in the lepton sectors. Actually, the tri-bimaximal mixing of leptons has been at first understood based on the non-Abelian finite group A_4.[12)–24)]

The tri-bimaximal mixing gives the vanishing θ_{13}, which is the third mixing angle in the conventional parametrization of the lepton flavor mixing matrix. If the tri-bimaximal mixing is guaranteed at the leading order by the underlying theory

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of flavors, a deviation from the tri-bimaximal mixing should be small, and θ_{13} still remains small. On the other hand, the global analyses of the neutrino masses and mixing angles indicate the sizeable θ_{13}. The long baseline neutrino experiment, T2K,[25] and the reactor experiment, Double Chooz[26] will soon present new data. We may expect the rather large $\sin\theta_{13}$, $0.1-0.2$, which is close to the present experimental upper bound. If the true value of $\sin\theta_{13}$ is large, we do not need to persist in the paradigm of the tri-bimaximal mixing. In fact, there are various theoretical proposals which lead to large $\sin\theta_{13}$.[27] The tri-bimaximal structure may be broken considerably.[28)–32]

It should be emphasized that the A_4 flavor symmetry does not necessarily give the tri-bimaximal mixing at the leading order even if the relevant alignments of the vacuum expectation values (VEVs) are realized. Certainly, for the neutrino mass matrix with three flavors, the A_4 symmetry can give the mass matrix with the $(2,3)$ off diagonal matrix due to the A_4 singlet flavon, 1, in addition to the unit matrix and the democratic matrix, which leads to the tri-bimaximal mixing of flavors. However, the $(1,3)$ off diagonal matrix and the $(1,2)$ off diagonal matrix also appear at the leading order if $1'$ and $1''$ flavons exist;[28]

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \text{for } 1', \qquad \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{for } 1''. \tag{1·1}$$

The tri-bimaximal mixing is broken at the leading order in such a case. These additional matrices also appear in the extra-dimensional models with the S_3 and S_4 flavor symmetry.[33),34] The trimaximal mixing model with $\Delta(27)$ also has these matrices effectively.[35]

In this paper, we perform a model independent analysis of the neutrino mass matrix in the presence of the additional terms (1·1). As a concrete realization of such a pattern, we discuss an A_4 flavor model, which is a modified version of the Altarelli and Feruglio model.[15),16] We find that θ_{12} and θ_{23} are not so different compared with the tri-bimaximal mixing, but $\sin\theta_{13}$ is expected to be around 0.2 if the neutrino mass spectrum is hierarchical. Our proposal will be soon tested at the T2K and the Double Chooz experiments in the near future.

In §2, we discuss the neutrino mass matrix breaking the tri-bimaximal mixing in our framework. In §3, we discuss the modified A_4 flavor model and its predictions. Section 4 is devoted to a summary.

§2. Neutrino mass matrix breaking tri-bimaximal mixing

As is well known, the neutrino mass matrix which gives the tri-bimaximal mixing of flavor is given by

$$M_{\rm TBM} = \frac{m_1+m_3}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \frac{m_2-m_1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_1-m_3}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \tag{2·1}$$

Here m_1, m_2, and m_3 are the neutrino masses. The Majorana phases are to be attached to these masses if they exist. Throughout this paper, we use the basis where the charged lepton mass matrix is diagonal. Certainly, the A_4 symmetry can realize the mass matrix in Eq. (2·1). However, the mass matrices in Eq. (1·1) may be added at the leading order in the flavor model with the non-Abelian discrete symmetry. For example, such extra terms appear in the A_4 flavor model if $1'$ and $1''$ flavons couple to the A_4 triplet neutrinos such as $3 \times 3 \times 1'$ and $3 \times 3 \times 1''$ as discussed in the next section.

The two terms in Eq. (1·1) are not independent of each other. It is noted that

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} . \tag{2·2}$$

Thus we may consider the neutrino mass matrix

$$M_\nu = a \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + c \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + d \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} , \tag{2·3}$$

with no loss of generality. Here the parameters a, b, c and d are arbitrary in general. The neutrino masses m_1, m_2 and m_3 are given in terms of these four parameters.

By factoring out the tri-bimaximal mixing matrix $V_{\text{tri-bi}}$

$$V_{\text{tri-bi}} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} , \tag{2·4}$$

the left-handed neutrino mass matrix (2·3) is written as

$$M_\nu = V_{\text{tri-bi}} \begin{pmatrix} a + c - \frac{d}{2} & 0 & \frac{\sqrt{3}}{2} d \\ 0 & a + 3b + c + d & 0 \\ \frac{\sqrt{3}}{2} d & 0 & a - c + \frac{d}{2} \end{pmatrix} V_{\text{tri-bi}}^T . \tag{2·5}$$

At first, suppose the parameters a, b, c, d to be real in order to see the effect of the non-vanishing d clearly. Then, we have the mass eigenvalues of the left-handed neutrinos as

$$a + \sqrt{c^2 + d^2 - cd}, \qquad a + 3b + c + d, \qquad a - \sqrt{c^2 + d^2 - cd}. \tag{2·6}$$

For the normal ordering of the neutrino masses $m_3 > m_2 > m_1$, the neutrino mass squared differences are then given by

$$\Delta m_{31}^2 = -4a\sqrt{c^2 + d^2 - cd} \ , \qquad \Delta m_{21}^2 = (a + 3b + c + d)^2 - (a + \sqrt{c^2 + d^2 - cd})^2, \tag{2·7}$$

where we have chosen the parameter a to be negative. These mass differences are constrained by the observed values Δm_{atm}^2 and Δm_{sol}^2.

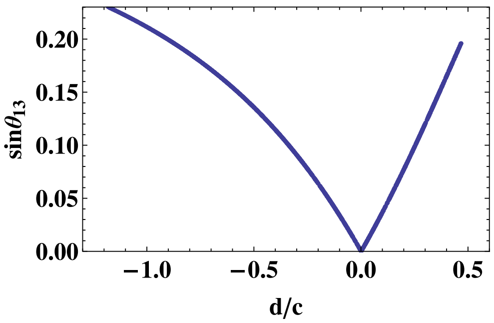

Fig. 1. The d/c dependence of $\sin\theta_{13} \equiv |U_{e3}|$, where c and d are supposed to be real.

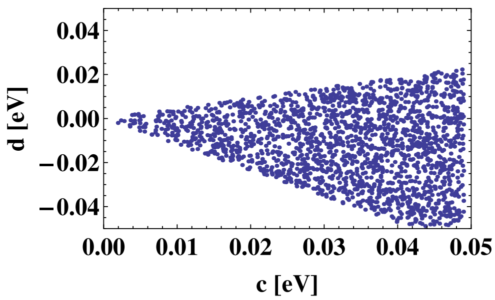

Fig. 2. The scatter plot of the allowed region on the c-d plane, where c and d are supposed to be real.

As the charged lepton mass matrix is diagonal, the mixing matrix $U_{\rm MNS}$ is

$$U_{\rm MNS} = V_{\text{tri-bi}} \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} , \tag{2·8}$$

where

$$\tan 2\theta = \frac{\sqrt{3}d}{-2c+d} \ . \tag{2·9}$$

The relevant mixing matrix elements of $U_{\rm MNS}$ are given as

$$|U_{e2}| = \frac{1}{\sqrt{3}} \ , \quad |U_{e3}| = \frac{2}{\sqrt{6}}\,|\sin\theta| \, , \quad |U_{\mu 3}| = \left| -\frac{1}{\sqrt{6}} \sin\theta - \frac{1}{\sqrt{2}} \cos\theta \right| \ , \tag{2·10}$$

which is the trimaximal lepton mixing.[33)–35)]

We have four free parameters a, b, c, d against two input data $\Delta m^2_{\rm atm}$ and $\Delta m^2_{\rm sol}$. By eliminating a and b with the two mass differences, we can write the observables as the function of c and d. In particular, $|U_{e3}| \equiv \sin\theta_{13}$ is given by the ratio d/c. In Fig. 1, we show the prediction of $\sin\theta_{13}$ versus d/c, where a and b are constrained by the experimental data at 3σ[1)]

$$\Delta m^2_{\rm atm} = (2.07 - 2.75) \times 10^{-3}\ {\rm eV}^2 \ , \qquad \Delta m^2_{\rm sol} = (7.03 - 8.27) \times 10^{-5}\ {\rm eV}^2 \ . \tag{2·11}$$

We get the allowed region $d/c = -1.2 \sim 0.5$, which is obtained by the experimental constraint of the mixing angle at 3σ, $\sin^2\theta_{23} = 0.36 \sim 0.67$.[1)] It is remarked that the magnitude of $\sin\theta_{13}$ is expected to be around 0.2 if c and d are comparable in magnitude. Thus the additional term may not be suppressed compared to the other terms in order to be compatible with the experiments. The input of the experimental data of $\sin^2\theta_{23}$ leads to the allowed region of c and d as shown in Fig. 2. It is found that magnitudes of c and d are comparable to the neutrino mass, and the only positive region of c is allowed for negative a.

As for the inverted mass ordering $m_2 > m_1 > m_3$, which corresponds to the positive a in Eqs. (2·6) and (2·7), we can also predict $\sin\theta_{13}$ versus d/c. The determination of a and b depends on the mass ordering of neutrinos, but c and d are

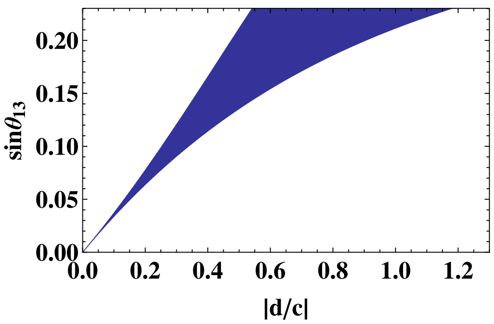

Fig. 3. The $|d/c|$ dependence of $\sin\theta_{13}$, where $\sum m_i \leq 0.07$ eV.

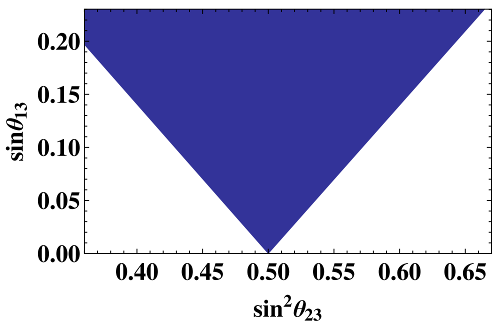

Fig. 4. The allowed region on $\sin^2\theta_{23}$-$\sin\theta_{13}$ plane.

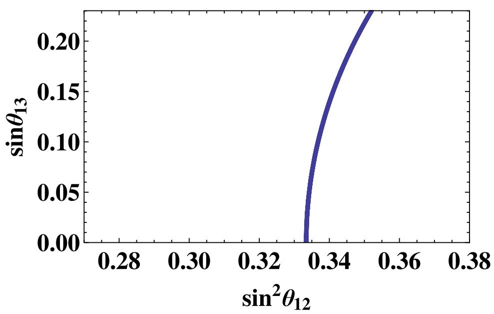

Fig. 5. The allowed region on $\sin^2\theta_{12}$-$\sin\theta_{13}$ plane.

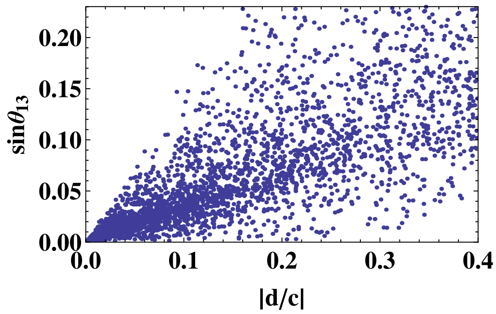

Fig. 6. The scatter plot of the $|d/c|$ dependence of $\sin\theta_{13}$, where $\sum m_i = 0.07 - 0.58$ eV is put.

free from it as seen in Eq. (2·7). The allowed region of c and d is given by the experimental constraint of $\sin^2\theta_{23}$ for the inverted mass hierarchy as well as for the normal one. Therefore, the obtained results are the same as the ones in Figs. 1 and 2.

In these calculations, we have supposed a, b, c, d to be real. However, we should take account of the phases of these parameters in general. Taking a, b, c, d to be complex, we discuss the mixing angles. At first, let us discuss the case of the normal neutrino mass hierarchy. We show our results in Figs. 3, 4 and 5, where the larger $\sum m_i$ is cut at 0.07 eV. Due to phases of a, b, c, d, the predicted $\sin\theta_{13}$ has ambiguity to some extent even if the ratio $|d/c|$ is fixed as seen in Fig. 3. As seen in Fig. 4, $\sin^2\theta_{23} = 0.5$ could be kept thanks to the phases even if $\sin\theta_{13}$ is to be around 0.2. On the other hand, the relation between $\sin\theta_{12}$ and $\sin\theta_{13}$ is independent of the phases as seen in Fig. 5.

Next, we show the $|d/c|$ dependence of $\sin\theta_{13}$ in the case of the quasi-degenerate neutrinos. The expected value of $\sin\theta_{13}$ becomes ambiguous due to phases of parameters. For example, the magnitude of $\sin\theta_{13}$ is allowed to be $0 - 0.2$ for $|d/c| \simeq 0.2$ as seen in Fig. 6, in which $\sum m_i = 0.07 - 0.58$ eV[36)] is taken.

In conclusion, $\sin\theta_{13}$ is expected to be close to the experimental upper bound 0.2 for $|d/c| \simeq \mathcal{O}(1)$ unless the neutrino mass spectrum is quasi-degenerate. In the

Table I. Assignments of $SU(2)$, A_4, and Z_3 representations, where $\omega = e^{\frac{2\pi i}{3}}$.

	(l_e, l_μ, l_τ)	e^c	μ^c	τ^c	$h_{u,d}$	ϕ_l	ϕ_ν	ξ	ξ'
$SU(2)$	2	1	1	1	2	1	1	1	1
A_4	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}''$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}'$
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω	ω

next section, we discuss the models which provide a non-vanishing d.

§3. A_4 model with non-vanishing d

The flavor models with the non-Abelian discrete symmetries can give the non-vanishing d discussed in the previous section. A concrete example of the model is obtained by a slight modification of the A_4 flavor model proposed by Altarelli and Feruglio.[15),16)] We introduce an A_4 singlet ξ', which is a $1'$ flavon, in addition to ϕ_l, ϕ_ν, and ξ as shown in Table I.[*)]

In the lepton sector, the Yukawa interaction which respects the gauge and the flavor symmetry is described by

$$\begin{aligned}\mathcal{L}_\ell &= y^e e^c l\phi_l h_d/\Lambda + y^\mu \mu^c l\phi_l h_d/\Lambda + y^\tau \tau^c l\phi_l h_d/\Lambda \\ &\quad + (y^\nu_{\phi_\nu}\phi_\nu + y^\nu_\xi \xi + y^\nu_{\xi'}\xi') llh_u h_u/\Lambda^2 \ , \end{aligned} \tag{3·1}$$

where y^e, y^μ, y^τ, $y^\nu_{\phi_\nu}$, y^ν_ξ, and $y^\nu_{\xi'}$ are the dimensionless coupling constants, and Λ is the cutoff scale. As is well known, the VEVs $\langle h_{u,d}\rangle = v_{u,d}$, $\langle\xi\rangle = \alpha_\xi\Lambda$, and $\langle\xi'\rangle = \alpha_{\xi'}\Lambda$ and vacuum alignment

$$\langle\phi_l\rangle = \alpha_l\Lambda(1,0,0) \ , \quad \langle\phi_\nu\rangle = \alpha_\nu\Lambda(1,1,1) \tag{3·2}$$

lead to the diagonal charged lepton mass matrix

$$M_l = \alpha_l v_d \begin{pmatrix} y^e & 0 & 0 \\ 0 & y^\mu & 0 \\ 0 & 0 & y^\tau \end{pmatrix} . \tag{3·3}$$

The effective neutrino mass matrix is given as

$$\begin{aligned} M_\nu &= \frac{y^\nu_{\phi_\nu}\alpha_\nu v_u^2}{3\Lambda}\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} + \frac{y^\nu_{\phi_\xi}\alpha_\xi v_u^2}{\Lambda}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + \frac{y^\nu_{\phi_{\xi'}}\alpha_{\xi'} v_u^2}{\Lambda}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \\ &= a\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + b\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + c\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + d\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} , \end{aligned} \tag{3·4}$$

where

$$a = \frac{y^\nu_{\phi_\nu}\alpha_\nu v_u^2}{\Lambda}, \qquad b = -\frac{y^\nu_{\phi_\nu}\alpha_\nu v_u^2}{3\Lambda}, \qquad c = \frac{y^\nu_\xi \alpha_\xi v_u^2}{\Lambda}, \qquad d = \frac{y^\nu_{\xi'}\alpha_{\xi'} v_u^2}{\Lambda}. \tag{3·5}$$

*) By virtue of Eq. (2·2), the following results are valid if we introduce $1''$ flavon instead of $1'$. Brahmachari, Choubey and Mitra[28)] presented a detailed analysis of $\sin\theta_{13}$ only for the case of (ξ', ξ'').

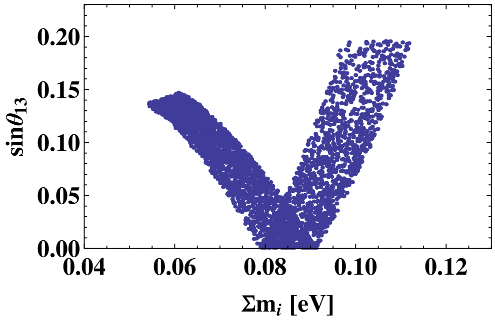

Fig. 7. The $\sum m_i$ dependence of $\sin\theta_{13}$ for normal mass hierarchy.

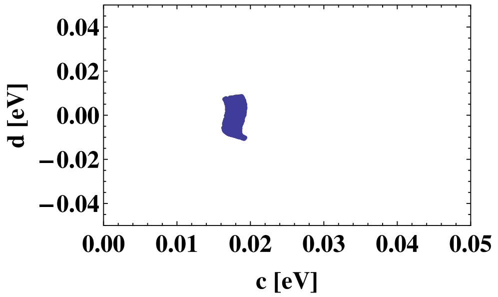

Fig. 8. The allowed region on the c–d plane for normal mass hierarchy.

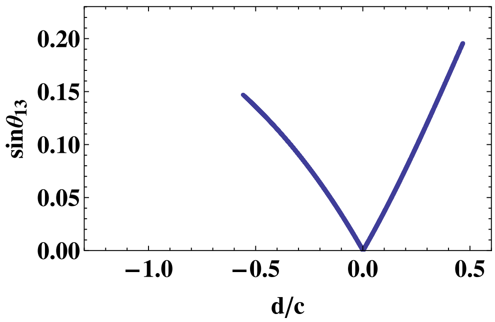

Fig. 9. The d/c dependence of $\sin\theta_{13}$ for normal mass hierarchy.

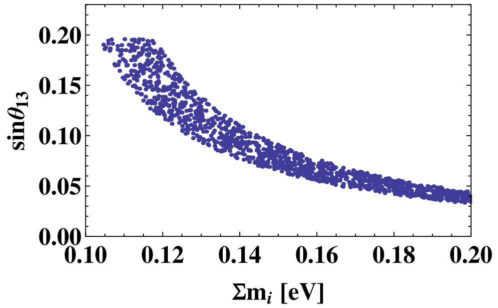

Fig. 10. The $\sum m_i$ dependence of $\sin\theta_{13}$ for inverted mass hierarchy.

As seen in Eqs. (3·4) and (3·5), the non-vanishing d is generated through the coupling $ll\xi' h_u h_u$. Since the relation $a = -3b$ is given in this model, the predicted regions of the lepton mixing angles are reduced compared with the one in the previous section. In the case where the parameters a, c, d are real, they are fixed by the three neutrino masses m_1, m_2 and m_3. That is, $\sin\theta_{13}$ can be plotted as a function of the total mass $\sum m_i$.

In Fig. 7, we show the predicted $\sin\theta_{13}$ versus $\sum m_i$, where the normal hierarchy of the neutrino masses is taken. The leptonic mixing is almost tri-bimaximal, that is $\sin\theta_{13} = 0$, in the regime where $\sum m_i \simeq 0.08 - 0.09$ eV. In the case of $m_3 \gg m_2, m_1$, that is $\sum m_i \simeq 0.05$ eV, $\sin\theta_{13}$ is expected to be around 0.15. Due to the relation $a = -3b$, the allowed region of c and d is restricted as seen in Fig. 8, which should be compared with the result in Fig. 2. It is also noticed that the predicted upper bound of $\sin\theta_{13}$ is 0.2, which comes from the constraint from d/c as seen in Fig. 9. The large region of $|d/c|$ is cut at $d/c \simeq 0.45$ and $d/c \simeq -0.55$ by the input data of $\Delta m^2_{\rm atm}$ and $\Delta m^2_{\rm sol}$.

We can also predict $\sin\theta_{13}$ versus $\sum m_i$ in the case of the inverted hierarchy of neutrino masses. Since we have a constraint of $a = -3b$ in this model, the situation is different from the one in the general analysis of §2, where the predicted $\sin\theta_{13}$ are the same for both cases of the normal and inverted hierarchies of neutrino masses. In this model, the allowed region of c and d is different from that of the normal mass

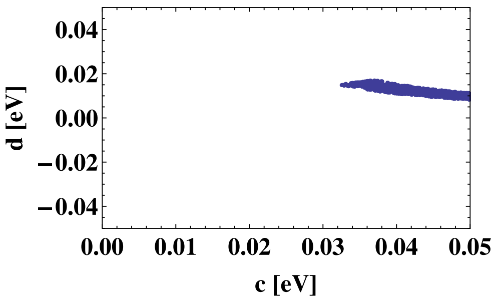

Fig. 11. The allowed region on the c–d plane for inverted mass hierarchy.

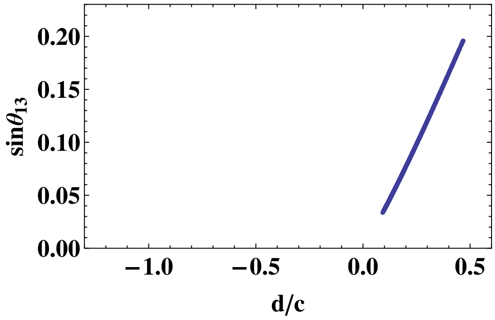

Fig. 12. The d/c dependence of $\sin\theta_{13}$ for inverted mass hierarchy.

hierarchy. Therefore, we get a different prediction of $\sin\theta_{13}$ as seen in Fig. 10. The predicted maximal value of $\sin\theta_{13}$ is 0.2 at $\sum m_i \simeq 0.1$ eV, which corresponds to $m_3 \ll m_2, m_1$. It is noted the tri-bimaximal mixing cannot be realized as seen in Fig. 10. It is easily understood if we consider the $d = 0$ limit in the mass eigenvalues in Eq. (2·6). One finds that $m_2^2 - m_1^2 > 0$ is not realized while keeping $m_1^2 - m_3^2 > 0$. We show the allowed region of c and d in Fig. 11, which is different from the result in Fig. 8. The d/c dependence of $\sin\theta_{13}$ is also shown in Fig. 12.

In the above analysis, we have supposed the parameters a, c, d to be real. We have checked numerically that the predicted $\sin\theta_{13}$ is not so different in the cases of both normal and inverted hierarchies of neutrino masses even if the parameters are taken to be complex.

In conclusion, our modified A_4 model predicts $\sin\theta_{13} = 0.15 - 0.2$ for cases of $m_3 \gg m_2, m_1$ and $m_3 \ll m_2, m_1$.

Finally we comment on flavor models with other non-Abelian discrete symmetries which gives the non-vanishing d effectively. One is the flavor model based on $\Delta(27)$ group which is given by Grimus and Lavoura.[35] The trimaximal mixing is enforced by the soft broken discrete symmetry. In this model, we find the relation $d = e^{i\pi/3}c$, where a, b, c, d are complex. As seen in Ref. 35) the large $\sin\theta_{13}$ is expected. Another example is the flavor twisting model in the five-dimensional framework.[33),34)] In this model, the flavor symmetry breaking is triggered by the boundary conditions of the bulk right-handed neutrino in the fifth spatial dimension. The parameters a, b, c, d involve the bulk neutrino masses and the volume of the extra dimension. In the case of the S_4 flavor symmetry,[34)] there appears one relation among these four parameters, so that the general allowed region is further restricted as in the modified A_4 model. By putting the experimental data of $\Delta m^2_{\rm atm}$ and $\Delta m^2_{\rm sol}$, $\sin\theta_{13}$ is predicted to be around 0.18 (~ 0) in the case of the normal (inverted) hierarchy.

§4. Summary

The T2K and Double Chooz will soon present new data of $\sin\theta_{13}$. If we expect $\sin\theta_{13}$ to be $0.1 - 0.2$, which is close to the present experimental upper bound, we should not persist in the paradigm of the tri-bimaximal mixing.

As a promising model of the left-handed Majorana mass matrix which produces large $\sin\theta_{13}$, we have discussed Eq. (2·3) and examine its general predictions. The expected $\sin\theta_{13}$ is close to the experimental upper bound 0.2 for the normal or inverted hierarchical neutrino mass spectrum. On the other hand, $\sin^2\theta_{23}$ and $\sin^2\theta_{12}$ are expected to be not far from 1/2 and 1/3, respectively. Furthermore, our A_4 model, which is the modified version of the Altarelli and Feruglio model, is discussed in detail. In this model, $\sin\theta_{13}$ is expected to be around 0.15 in the case of the normal hierarchical neutrino masses $m_3 \gg m_2, m_1$, whereas $\sin\theta_{13} \approx 0.2$ in the case of the inverted hierarchical neutrino masses $m_3 \ll m_2, m_1$.

The mass matrix form of Eq. (2·3) is suggestive of other kinds of flavor symmetries as well. For example, $\Delta(27)$, S_3 and S_4 flavor symmetries can also realize such a structure.

It is emphasized that this specific pattern for breaking of the tri-bimaximal mixing, $\sin\theta_{13} \simeq 0.2$ with $\sin^2\theta_{23} \simeq 1/2$ and $\sin^2\theta_{12} \simeq 1/3$, is only successfully given by the non-Abelian discrete symmetry for flavors. If experiments observe this breaking of the tri-bimaximal mixing, one expects the non-Abelian discrete symmetry such as A_4, $\Delta(27)$, S_3 and S_4 for the underlying theory of flavors. Otherwise this mixing pattern is considered to be an accidental one. Such a breaking of the tri-bimaximal mixing will be soon tested at T2K and Double Chooz in the near future.

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