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Brewster and pseudo-Brewster angles of uniaxial crystal surfaces and their use for determination of optical properties

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Brewster and pseudo-Brewster angles are defined for surfaces of transparent and absorbing uniaxial crystals parallel and perpendicular to the optic axis. Two Brewster angles of a transparent uniaxial crystal surface parallel to the optic axis, measured when the optic axis is oriented perpendicular and parallel to the plane of incidence, readily determine the ordinary and extraordinary indices N_o and N_e. N_o and N_e can also be obtained from two Brewster angles measured on a surface perpendicular to the optic axis in contact with two media of different refractive indices. Conditions for the existence of two Brewster angles are discussed. The complex N_o and N_e of an absorbing uniaxial crystal can be derived from pseudo-Brewster-angle and minimum-reflectance data obtained in two symmetrical orientations of a surface parallel to the optic axis. An approximate, but accurate, explicit inversion procedure is presented for this purpose.

1. INTRODUCTION

When p-polarized parallel light is incident upon the surface of an optically isotropic transparent medium, the reflected wave disappears at a specific angle of incidence, the well-known Brewster angle[1] ϕ_B. If the medium becomes absorbing, the intensity of the reflected wave reaches a nonzero minimum at the so-called pseudo-Brewster angle ϕ_{pB}. In this paper we extend the definition of Brewster and pseudo-Brewster angles to optically anisotropic surfaces, specifically surfaces of transparent and absorbing uniaxial crystals. We also discuss the use of such angles to determine the optical properties of the crystal.

2. BREWSTER ANGLES OF A TRANSPARENT UNIAXIAL CRYSTAL IN THREE SYMMETRICAL ORIENTATIONS

We consider two crystal surfaces, one parallel and the other perpendicular to the optic axis. For the surface parallel to the optic axis, we consider in this section the two orientations when the optic axis is parallel and perpendicular to the plane of incidence and defer to Section 3 the general orientation with the optic axis at an arbitrary angle ω from the plane of incidence. The three symmetrical orientations, which are shown in Fig. 1, are described as follows:

(1) For a surface parallel to the optic axis, oriented such that the optic axis is perpendicular to the plane of incidence, Fig. 1(a), the reflection coefficient of incident p-polarized light is given by[2]

$$R_{pp} = \frac{N_o{}^2 \cos \phi - nJ}{N_o{}^2 \cos \phi + nJ}, \tag{1}$$

$$J = (N_o{}^2 - n^2 \sin^2 \phi)^{1/2}, \tag{2}$$

where ϕ is the angle of incidence, n is the refractive index of the isotropic medium of incidence, and N_o and N_e are the ordinary and extraordinary refractive indices of the crystal. The Brewster angle ϕ_B is obtained by setting

$$R_{pp} = 0, \tag{3}$$

which gives

$$\tan \phi_B = N_o/n. \tag{4}$$

(2) For a surface parallel to the optic axis, oriented such that the optic axis is parallel to the plane of incidence, R_{pp} is given by[2]

$$R_{pp} = \frac{N_o N_e \cos \phi - nJ}{N_o N_e \cos \phi + nJ}. \tag{5}$$

$R_{pp} = 0$ gives a second Brewster angle ϕ_B' such that

$$\sin^2 \phi'_B = \frac{n^2 N_o{}^2 - N_o{}^2 N_e{}^2}{n^4 - N_o{}^2 N_e{}^2}. \tag{6}$$

(3) For a surface perpendicular to the optic axis,[2] the p-reflection coefficient is given by

$$R_{pp} = \frac{N_o{}^2 \cos \phi - n(N_e{}^2 - n^2 \sin^2 \phi)^{1/2}}{N_o{}^2 \cos \phi + n(N_e{}^2 - n^2 \sin^2 \phi)^{1/2}}. \tag{7}$$

R_{pp} is zero at a third Brewster angle ϕ''_B given by

$$\sin^2 \phi''_B = \frac{n^2 N_e{}^2 - N_o{}^4}{n^4 - N_o{}^4}. \tag{8}$$

In all three symmetrical orientations there is no $p \rightarrow s$ polarization conversion (the cross-reflection coefficient $R_{ps} = 0$), and the reflected beam is entirely extinguished at the Brewster angle, with no residual s-polarized component. Conditions for the existence of the Brewster angles ϕ'_B and ϕ''_B are discussed in Section 5.
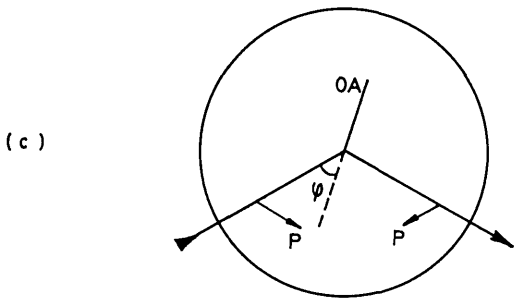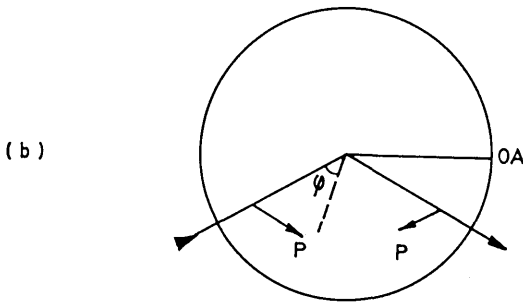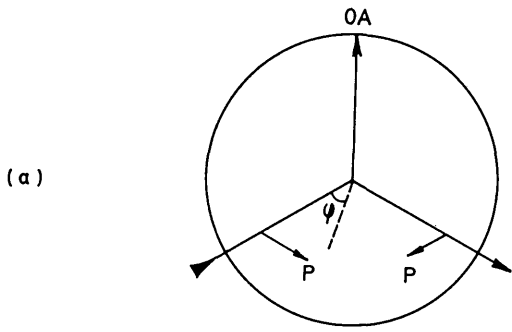
Fig. 1.   Three symmetrical orientations of a uniaxial crystal with the optic axis OA (a) parallel to the surface and perpendicular to the plane of incidence, (b) parallel to the surface and in the plane of incidence, and (c) perpendicular to the surface.   When the incident light is p polarized, the reflected light is likewise p polarized.
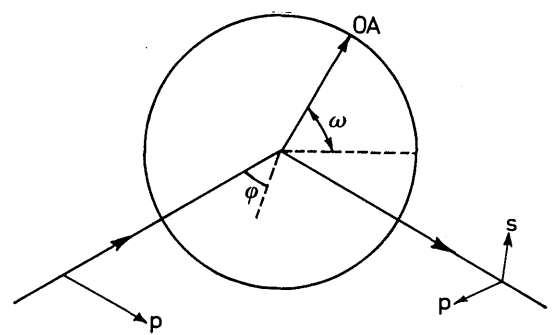

Fig. 2.   Uniaxial crystal surface parallel to the optic axis OA, which makes an arbitrary angle ω with the plane of incidence.   When the incident light is p polarized, the reflected light has both p- and s-polarized components.
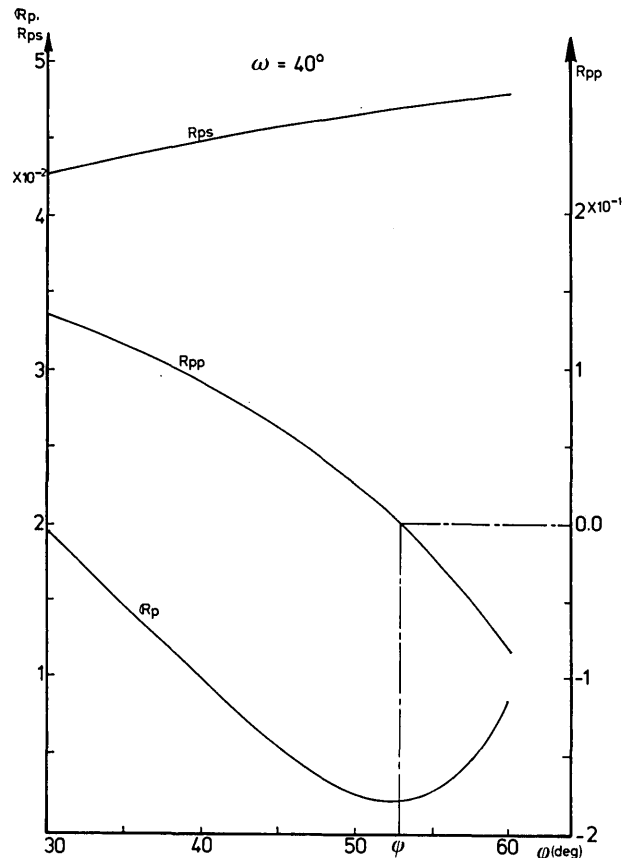

Fig. 3.   The reflection coefficients R_{pp} and R_{ps} and the net p reflectance $\mathcal{R}_{pp}$ versus the angle of incidence ϕ for a crystal surface of sodium nitrate parallel to the optic axis, which makes an angle $\omega = 40°$ with the plane of incidence at wavelength $\lambda = 5893$ Å.

3. PSEUDO-BREWSTER ANGLE OF A TRANSPARENT UNIAXIAL CRYSTAL WITH THE OPTIC AXIS PARALLEL TO THE SURFACE IN AN ARBITRARY ORIENTATION

When p-polarized light is incident upon a uniaxial crystal surface parallel to the optic axis, with the optic axis inclined at an angle ω with respect to the plane of incidence, Fig. 2, the reflected light has p- and s-polarization components that are determined by the following reflection coefficients[3]:

$$R_{pp} = \frac{A_1 B_4 + A_2 B_3}{A_1 + A_2}, \tag{9}$$

$$R_{ps} = \frac{A_1 A_2 (B_2 - B_1)}{(A_1 + A_2)}, \tag{10}$$

where

$$A_1 = J/[(n \sin^2 \phi + J \cos \phi)\tan \omega], \tag{11a}$$

$$A_2 = N_o \tan \omega (I + nN_o \cos \phi)/(IN_o \cos \phi + nJ^2), \tag{11b}$$

$$B_1 = (n \cos \phi - J)/(n \cos \phi + J), \tag{11c}$$

$$B_2 = (nN_o \cos \phi - I)/(nN_o \cos \phi + I), \tag{11d}$$

$$B_3 = (N_o^2 \cos \phi - nJ)/(N_o^2 \cos \phi + nJ), \tag{11e}$$

$$B_4 = (IN_o \cos \phi - nJ^2)/(IN_o \cos \phi + nJ^2), \tag{11f}$$

$$I = [N_o^2 N_e^2 - n^2 \sin^2 \phi (N_o^2 \sin^2 \omega + N_e^2 \cos^2 \omega)]^{1/2}, \tag{12}$$

and J is given by Eq. (2). The net parallel reflectance R_p is given by

$$\mathcal{R}_p = |R_{pp}|^2 + |R_{ps}|^2$$
$$= \mathcal{R}_{pp} + \mathcal{R}_{ps}, \qquad (13)$$

where $\mathcal{R}_{pp}$ and $\mathcal{R}_{ps}$ are the diagonal $(p \rightarrow p)$ and cross $(p \rightarrow s)$ reflectances, respectively.

We define two pseudo-Brewster angles for a given orientation ω. At one pseudo-Brewster angle $\phi_{pB}(\omega)$ the net p reflectance, Eq. (13), is minimum, and the reflected light has small residual p- and s-polarized components. The other pseudo-Brewster angle $\phi'_{pB}(\omega)$ is the angle of incidence of zero diagonal $p \rightarrow p$ reflectance. From Eq. (9) we see that $R_{pp} = 0$ is satisfied when

$$A_1/A_2 = -B_3/B_4, \qquad (14)$$

which can be solved for ϕ'_{pB} as a function of ω, N_o, and N_e. At the pseudo-Brewster angle ϕ'_{pB} incident p-polarized light is totally converted to purely s-polarized light on reflection. The residual reflected intensity is determined by $\mathcal{R}_{ps}$.

Figure 3 shows the variation of the reflection coefficients R_{pp} and R_{ps} and the net p reflectance $\mathcal{R}_p$ with the angle of
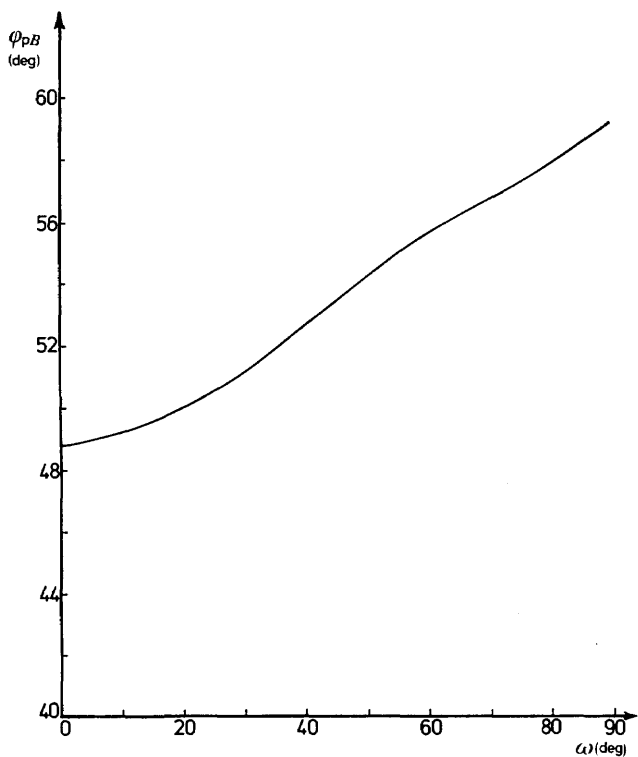

Fig. 4. The pseudo-Brewster angle ϕ_{pB} versus the orientation angle ω of the optic axis for a surface parallel to the optic axis of a sodium nitrate uniaxial crystal at $\lambda = 5893$ Å.

Table 1. Difference $\phi_{pB} - \phi'_{pB}$ of the Two Pseudo-Brewster Angles of a Uniaxial Crystal Surface of Sodium Nitrate Parallel to the Optic Axis at Discrete Orientation Angles ω of the Optic Axis[a]

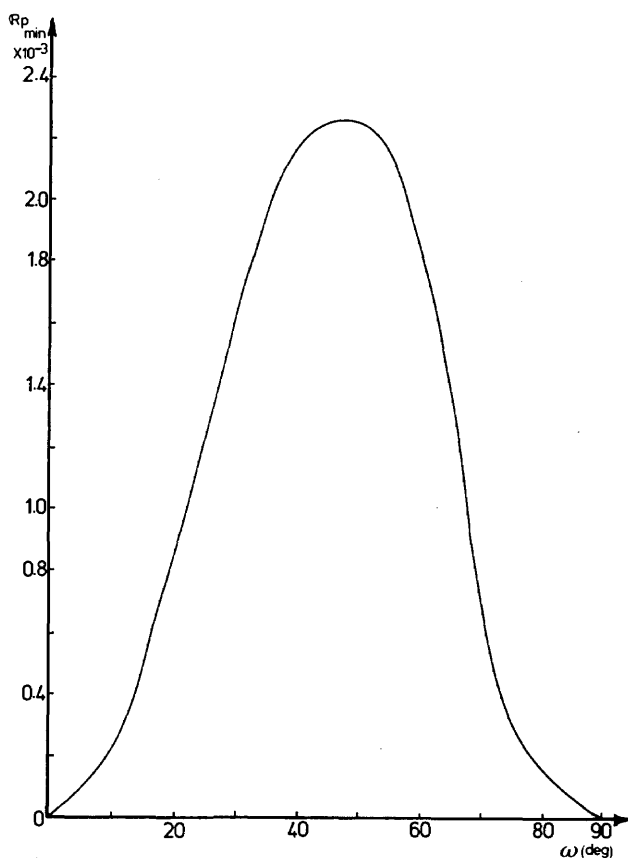| ω (deg)	0	10	20	30	50	70	80	90
$\phi_{pB} - \phi'_{pB}$ (deg)	0	$\simeq 0$	0.02	0.04	0.06	0.04	$\simeq 0$	0

[a] $\lambda = 5893$ Å.

Fig. 5. The residual minimum reflectance $\mathcal{R}_{p\,\text{min}}$ as a function of ω for a surface parallel to the optic axis of a sodium nitrate uniaxial crystal at $\lambda = 5893$ Å.

incidence ϕ for a uniaxial crystal surface of sodium nitrate ($N_o = 1.584$, $N_e = 1.336$, at wavelength $\lambda = 5893$ Å) (Ref.4) parallel to the optic axis that makes 40° with the plane of incidence. The angle of incidence of minimum net p reflectance ϕ_{pB} is nearly the same as that at which R_{pp} goes through zero, ϕ'_{pB}. Figure 4 shows the variation of the pseudo-Brewster angle ϕ_{pB} with the orientation angle ω of the optic axis for the sodium nitrate crystal. Figure 4 essentially gives also ϕ'_{pB} as a function of ω because of the closeness of ϕ_{pB} and ϕ'_{pB}. The difference $\phi_{pB} - \phi'_{pB}$ is given at discrete values of ω in Table 1. The maximum difference between these two pseudo-Brewster angles is about 0.06° and occurs at $\omega = 45°$. The two angles coincide for the symmetrical orientations $\omega = 0$ and 90°. Figure 5 shows the residual minimum reflectance $\mathcal{R}_{p\,\text{min}}$ at ϕ_{pB}, which nearly equals $\mathcal{R}_{ps}$ at ϕ'_{pB}, as a function of ω. The minimum residual reflectance is maximum when $\omega = 45°$.

4. PSEUDO-BREWSTER ANGLE OF AN ABSORBING UNIAXIAL CRYSTAL WITH THE OPTIC AXIS PARALLEL TO THE SURFACE

For an absorbing uniaxial crystal, only pseudo-Brewster angles can be defined since the reflectance of p-polarized light never reaches zero even in symmetrical orientations. We restrict our attention to the case of a surface parallel to the optic axis and take gallium selenide ($N_o = 3.7656 - j0.9062$, $N_e = 2.7986 - j0.4687$, at wavelength $\lambda = 3000$ Å) (Ref.5) as an example.

Figure 6 indicates a family of curves of the net p reflectance $\mathcal{R}_p$ [computed by using Eqs. (9)–(12)] as a function of the

angle of incidence ϕ at different orientation angles ω of the optic axis of a GaSe crystal. It is evident from Fig. 6 that the angle of incidence of minimum reflectance $\mathcal{R}_p$, the pseudo-Brewster angle ϕ_{pB}, shifts as the orientation angle of the optic axis ω changes. Figure 7 gives ϕ_{pB} as a function of ω.

Figure 8 shows the variation [computed by using Eqs. (9)–(12)] of the real and imaginary parts of the complex $p \to p$ and $p \to s$ reflection coefficients R_{pp} and R_{ps} with the angle of incidence ϕ for a fixed orientation of the optic axis $\omega = 40°$. Figure 8 also shows the curve of net p reflectance $\mathcal{R}_p$. Notice that the angle (ϕ) position of minimum $\mathcal{R}_p$ is nearly the same
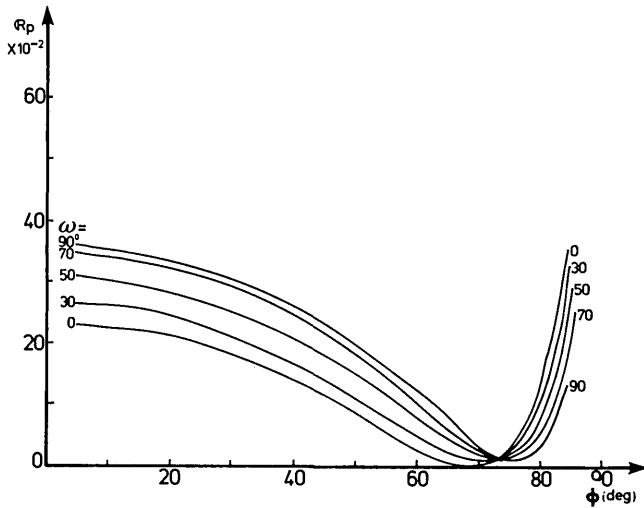

Fig. 6. The p reflectance $\mathcal{R}_p$ as a function of angle of incidence ϕ at various orientations ω of the optic axis for a surface parallel to the optic axis of a GaSe crystal at $\lambda = 3000$ Å.
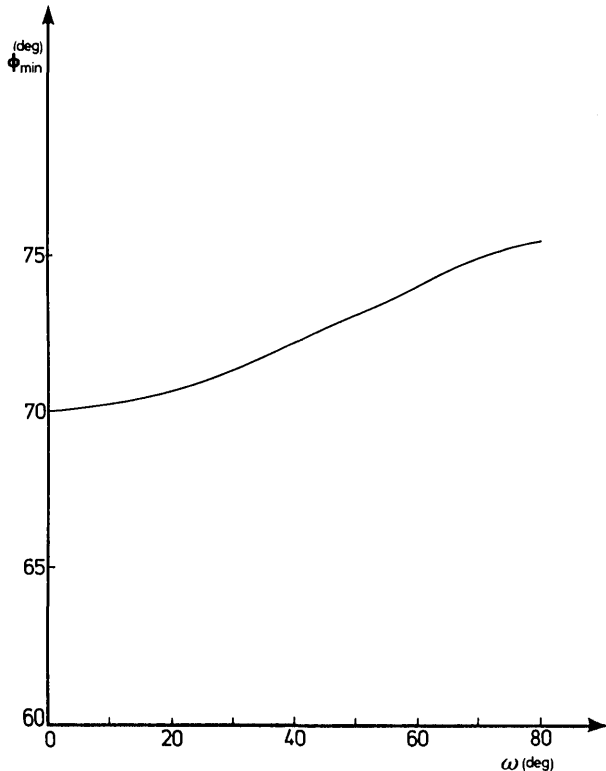

Fig. 7. The angle of minimum p reflectance in Fig. 6, the pseudo-Brewster angle ϕ_{pB}, as a function of ω.
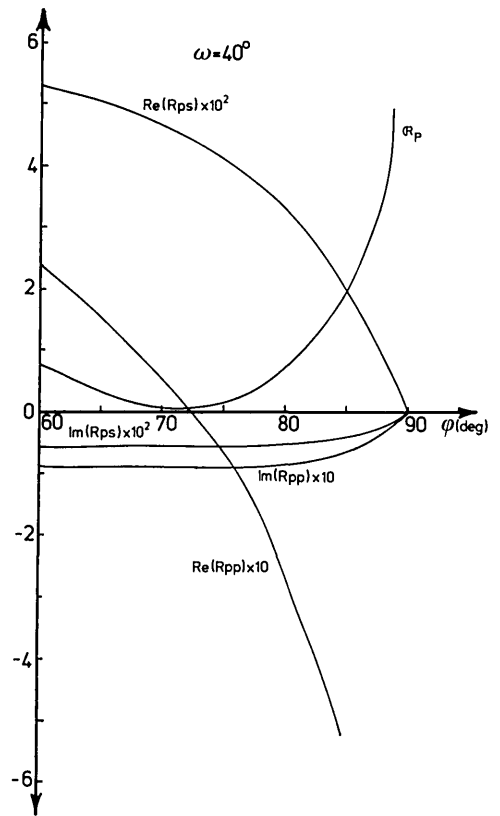

Fig. 8. The real and imaginary parts of the complex reflection coefficients R_{pp} and R_{ps}, and the net p reflectance $\mathcal{R}_p$ versus angle of incidence ϕ for a surface of GaSe crystal parallel to the optic axis, which is inclined by $\omega = 40°$ with respect to the plane of incidence; $\lambda = 3000$ Å.

as that at which the real part of R_{pp} goes through zero. This conclusion, which is confirmed by numerical results and which remains valid as ω changes from 0 to 90°, will be used in Section 5 as the basis of an approximate method to determine the optical properties N_o and N_e.

5. DETERMINATION OF THE ORDINARY AND EXTRAORDINARY REFRACTIVE INDICES OF A UNIAXIAL CRYSTAL FROM BREWSTER AND PSEUDO-BREWSTER ANGLES

Transparent Crystals

We describe two methods that involve measurement of two Brewster angles of one crystal surface. For a surface parallel to the optic axis, the Brewster angles ϕ_B and ϕ'_B are measured for the two symmetrical orientations of the optic axis perpendicular and parallel to the plane of incidence, respectively. From ϕ_B, the ordinary refractive index N_o is immediately derived by using Eq. (2). Subsequently N_e is obtained from ϕ'_B and N_o by using Eq. (6). We assume the same immersion medium of known refractive index n (e.g., air with $n = 1$).

The second method employs measurement of the Brewster angles ϕ''_{B1} and ϕ''_{B2} of a crystal face perpendicular to the optic axis and immersed in two media of different refractive indices n and n_2. Use of Eq. (8) twice yields two linear algebraic equations,

$$N_o{}^4 \cos^2 \phi''_{B1} - n_1{}^2 N_e{}^2 + n_1{}^4 \sin^2 \phi''_{B1} = 0,$$
$$N_o{}^4 \cos^2 \phi''_{B2} - n_2{}^2 N_e{}^2 + n_2{}^4 \sin^2 \phi''_{B2} = 0, \qquad (15)$$

which can be solved for $N_o{}^4$ and $N_e{}^2$; hence N_o and N_e are determined.

In the foregoing methods we took for granted the existence of all three Brewster angles ϕ_B, ϕ'_B, and ϕ''_B. While ϕ_B can always be found from Eq. (2), ϕ'_B and ϕ''_B may not exist for certain combinations of refractive indices n, N_o, and N_e. To discuss this point further, we define the normalized refractive indices:

$$\eta_o = N_o/n, \qquad \eta_e = N_e/n. \qquad (16)$$

In terms of η_o and η_e, Eqs. (6) and (8) become

$$\sin^2 \phi'_B = \frac{\eta_o{}^2 \eta_e{}^2 - \eta_o{}^2}{\eta_o{}^2 \eta_e{}^2 - 1}, \qquad (6')$$

$$\sin^2 \phi''_B = \frac{\eta_e{}^2 - \eta_o{}^4}{1 - \eta_o{}^4}. \qquad (8')$$

Because the squared sine of a real angle is confined between 0 and 1, Eqs. (6') and (8') indicate that η_o and η_e must be restricted to the hatched domains of the $\eta_o - \eta_e$ plane shown in Figs. 9(a) and 9(b) so that the Brewster angles ϕ'_B and ϕ''_B may exist (i.e., are real).
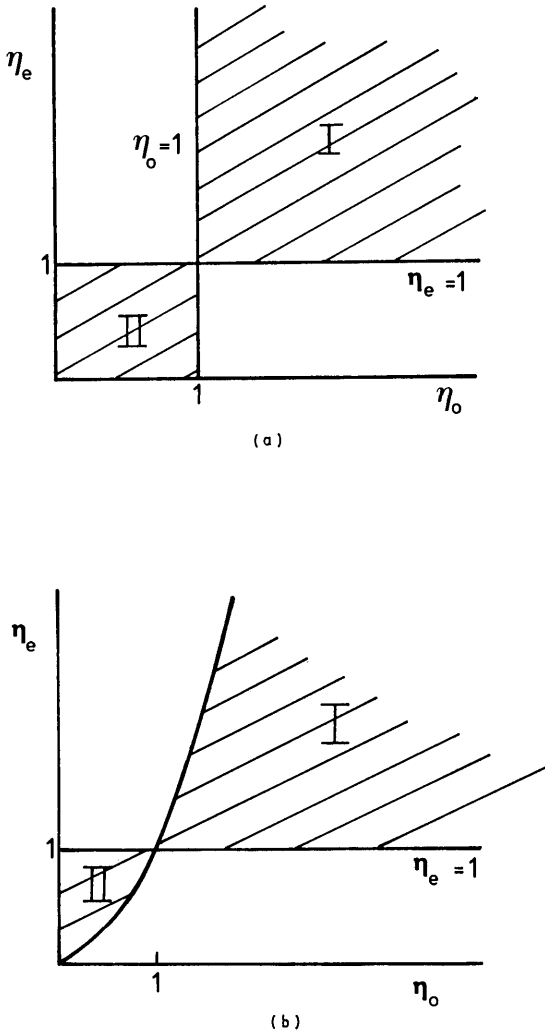

Fig. 9.   Domains of existence (shown hatched) of the Brewster angles (a) ϕ'_B and (b) ϕ''_B in the plane of the normalized ordinary and extraordinary refractive indices η_o and η_e.   Subdomains I and II correspond to external and internal reflection, respectively.

Absorbing Crystals

For a surface parallel to the optic axis the pseudo-Brewster angle and associated minimum reflectance are measured for the two symmetrical orientations with the optic axis perpendicular ($\omega = 90°$) and parallel ($\omega = 0$) to the plane of incidence. These are denoted by (ϕ_{pB}, $\mathscr{R}_{p\min}$) and (ϕ'_{pB}, $\mathscr{R}'_{p\min}$), respectively.   The complex ordinary refractive index N_o is derived from (ϕ_{pB}, $\mathscr{R}_{p\min}$) by making use of the condition that at ϕ_{pB}, the real part of the complex reflection coefficient R_{pp} is nearly zero, so that R_{pp} is essentially pure imaginary, and we could write[6]

$$R_{pp} = -j\sqrt{\mathscr{R}_{p\min}}. \qquad (17)$$

N_o is then obtained by inverting Eq. (1); this gives

$$N_o{}^2 = \rho n^2 [\rho \pm (\rho^2 - \sin^2 2\phi_{pB})^{1/2}]/2 \cos^2 \phi_{pB}, \qquad (18)$$

where

$$\rho = (1 + R_{pp})/(1 - R_{pp}). \qquad (19)$$

N_e is subsequently determined from N_o and (ϕ'_{pB}, $\mathscr{R}'_{p\min}$) by repeated use of the approximation $R'_{pp} = -j\sqrt{\mathscr{R}'_{p\min}}$ and inverting Eq. (3); this gives

$$N_e = n\rho'/N_o \cos \phi'_{pB},$$

where

$$\rho' = (1 + R'_{pp})/(1 - R'_{pp}). \qquad (20)$$

Simulated numerical calculations have verified the usefulness and accuracy of the previously described method of determining N_o and N_e of an absorbing uniaxial crystal from pseudo-Brewster-angle and minimum-reflectance data.   For GaSe, with $N_o = n_o - jk_o = 3.7656 - j0.9062$, $N_e = n_e - jk_e = 2.7986 - j0.4687$, the percentage errors of n_o, k_o, n_e, and k_e by using this procedure are about 0.1% of each.

More accurate inversion of the (ϕ_{pB}, $\mathscr{R}_{p\min}$), (ϕ'_{pB}, $\mathscr{R}'_{p\min}$) data to obtain N_o and N_e can always be attempted by using iteration and exact Eqs. (1) and (3), with N_o and N_e from Eqs. (18) and (20) serving as good initial values.

ACKNOWLEDGMENTS

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