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Broadband Wireless Using Single Carrier and Frequency Domain Equalization

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Abstract

The application of frequency domain equalization techniques makes single carrier modulation a potentially valuable alternative to OFDM, especially in regard to its robustness to RF implementation impairments. In this overview paper we survey recent results on single carrier systems with frequency domain equalization (SC-FDE), including comparisons and compatibilities with OFDM, and extensions via decision feedback equalization and overlap-save processing.

Keywords

Equalization, frequency domain equalization, single carrier, OFDM, broadband wireless

INTRODUCTION

Wireless communications systems offering broadband access at bit rates of 20 Mb/s and more exist now for wireless LANs, and are under intense research and standardization for outdoor fixed and mobile application environments. In these environments, non line of sight (NLOS) coverage is commonplace, causing significant multipath delay spread. The resulting intersymbol interference patterns at high bit rates may span 40 or more data symbols. Orthogonal Frequency Division Multiplexing (OFDM) is currently a popular multi-carrier solution to this problem, since it uses computationally-efficient Fast Fourier Transform (FFT) operations to transmit and receive multiple narrowband non-interfering data streams over parallel subcarriers without intersymbol interference [1]. Traditional *single carrier* modulation schemes using adaptive time domain equalization would have much higher signal processing complexity than comparable OFDM approaches if expected channel impulse response lengths exceed about 10 to 20 data symbols. However the complexity of the single carrier approach becomes similar to that of OFDM if *frequency domain equalization* is employed in the single carrier receiver. OFDM and single carrier systems with frequency domain equalization (SC-FDE) both have their respective advantages. In terms of their signal processing operations in the frequency domain, they are closely related, and to a certain extent compatible.

Moreover, the single carrier approach can reduce some of the RF implementation problems to which OFDM is prone.

SINGLE CARRIER MODULATION WITH FREQUENCY DOMAIN ADAPTIVE EQUALIZER PROCESSING

A single carrier (SC) system is a traditional digital transmission scheme in which data symbols are transported as a fixed-symbol-rate serial stream of amplitude and/or phase-modulated pulses, which in turn modulate a sinusoidal carrier. A linear frequency domain equalizer (FDE) performs receiver filtering in the frequency domain to minimize time-domain intersymbol interference. Its function is the same as that of a time-domain equalizer. However for channels with severe delay spread it is computationally simpler because equalization is performed on a block of data, and the operations on this block involve an efficient FFT operation and a simple channel inversion operation, just as is done in OFDM. Frequency-domain equalization of single-carrier-modulated signals has been known since the early 1970's (see [2], [3] and references therein). Sari et al [4], [5], [6] pointed out that when combined with FFT processing and the use of a cyclic prefix (which makes convolutions appear circular), single carrier systems with frequency domain equalization (SC-FDE) have essentially the same low complexity as OFDM systems.

For comparison purposes, Figure 1 shows block diagrams of an OFDM system and of a single carrier system with frequency domain equalization (SC-FDE) and cyclic prefix insertion (CPI). Note that similar FFT and other signal processing operations are present in these two systems, albeit with different orders of appearance. In each of these frequency domain systems, data is organized in blocks, whose length M , is typically at least 8 to 10 times the maximum expected channel impulse response length. In the case of OFDM, each transmitted block is processed by an inverse FFT (IFFT) to implement multicarrier modulation. In the SC case, the IFFT operation is at the output of the receiver's equalizer. A cyclic prefix, which is a copy of the last part of the transmitted block, is prepended to each block. The length of the cyclic prefix is the maximum

expected length of the channel impulse response. In both single carrier and OFDM receivers, the received cyclic prefix is discarded, and FFT processing is done on each M -symbol block. The cyclic prefix transmitted at the beginning of each block has two main functions: (1) it prevents contamination of a block by intersymbol interference from the previous block; and (2) it makes the received block appear to be *periodic with period M* . This produces the appearance of circular convolution, which is essential to the proper functioning of the FFT operation. For SC-FDE systems, the cyclic prefix and its consequent overhead requirement, can be eliminated by using overlap-save processing at the receiver, at the expense of slightly

degradations resulting from nonlinear distortion [11], OFDM systems require highly linear transmitter power amplifiers and/or several dB more power backoff than do comparable single-carrier systems with the same average power output [12]. For given cell coverage requirements, this translates into significantly higher RF front-end costs for OFDM systems, especially for mobile and portable terminals [13]. For given power amplifier specification, it translates into lower cell coverage. This power backoff penalty is especially important for subscribers near the edge of a cell, with large path loss, where lower-level modulation such as BPSK or QPSK modulation must be used.

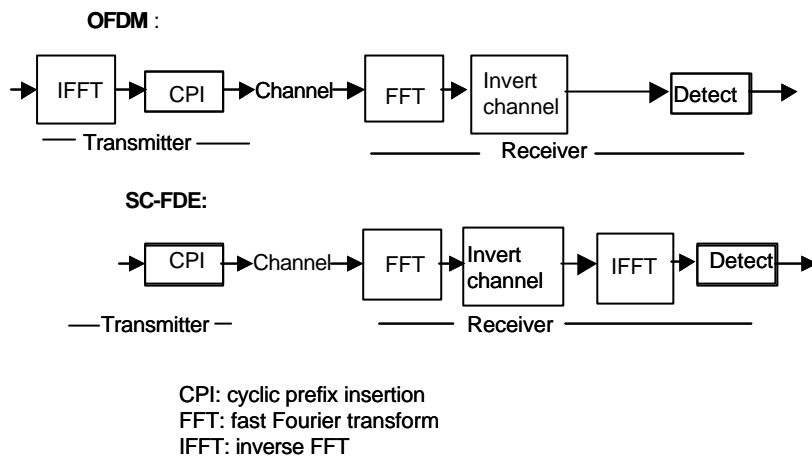


Figure 1. OFDM and SC-FDE - signal processing similarities and differences

increased complexity, as will be seen later. Coding and interleaving is typically employed in both OFDM and SC systems. Coding is in fact essential for OFDM systems operating on frequency selective channels, since otherwise, data transported on those subchannels with low SNR would be error-prone.

Coded linear SC-FDE systems have been shown to offer similar, (and in high code rate/high SNR case) better, bit error rate performance to coded OFDM [5], [6], [7], [8]. Both use efficient FFT and IFFT operations and simple one-tap equalization operations on each frequency component. Their equalization signal processing complexities are similar: on the order of $\log_2 M$ operations per data symbol, for block length M . For channels with multipath delay spreads spanning more than about 10 data symbols, this complexity is far less than that typically required by a conventional time domain equalization approach [9], [8].

WHY USE SC-FDE INSTEAD OF OFDM?

SC-FDE systems have several RF implementation advantages over OFDM. In particular, a disadvantage of OFDM is that its RF signal suffers from high envelope fluctuations due to the transmission of many sub-carriers. To avoid significant spectral re-growth [10] or BER

A further sensitivity of OFDM, not shared to the same degree by single carrier, is phase noise and frequency offsets, due to the close spacing in frequency of its subcarriers. This sensitivity leads to tighter local oscillator requirements for OFDM systems [14], [13].

SC-FDE WITH DECISION FEEDBACK EQUALIZATION

Decision feedback equalization (DFE) gives better performance for frequency-selective radio channels than does linear equalization [15]. In conventional time domain DFE equalizers, symbol-by-symbol data symbol decisions are made, filtered, and immediately fed back to remove their interference effect from subsequently detected symbols. Because of the delay inherent in the block FFT signal processing, this immediate filtered decision feedback cannot be done easily in a frequency domain DFE, which uses frequency domain filtering of the fed-back signal. A hybrid time-frequency domain DFE approach, which avoids the abovementioned feedback delay problem would be to use frequency domain filtering only for the forward filter part of the DFE, and to use conventional transversal filtering for the feedback part. The transversal feedback filter is relatively simple in any case, since it does not require complex multiplies, and it could be made as short or long as is required for adequate performance. Figure 2 illustrates such a hybrid time-frequency domain DFE topology. Complexity is minimized by making the feedback taps few in number and sparse, corresponding to the largest channel impulse response echoes. This also tends to minimize possible DFE error propagation problems.

Known Channels

Data is transmitted in blocks of M data symbols $\{a_k\}$ at a symbol rate of $1/T$ per second. Each block is preceded by a cyclic prefix. We consider a single-carrier frequency domain DFE that processes blocks of MI received samples $\{r_m\}$ at a time, using a MI -point FFT, where I is the number of receiver input samples per data symbol, and M is

the number of data symbols per FFT block. The choice of $I > 1$ gives a fractionally-spaced equalizer whose performance is relatively insensitive to sampling phase; good performance can also be obtained for $I = 1$ sample per symbol with an optimal sampling phase derived from a symbol timing subsystem.

The data symbols are assumed to be normalized

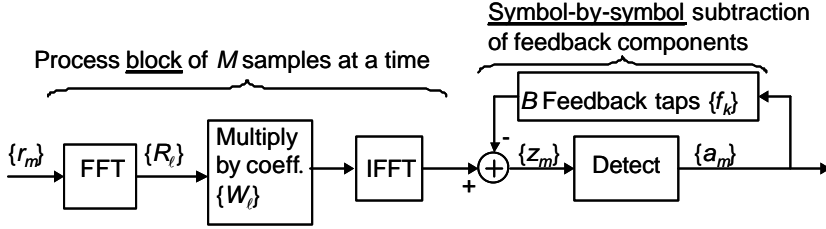


Figure 2. SC-FDE decision feedback equalizer

uncorrelated complex random variables derived from a discrete alphabet such as QPSK or 16QAM, with zero mean, and unit variance. The forward filter has MI complex frequency-domain coefficients $\{W_\ell\}$. After the inverse FFT operation, its time domain output is sampled once per symbol interval. There are B complex feedback coefficients $\{f_k^*\}$, $k \in F_B$, where F_B is a set of non-zero indices that correspond to the delays (in symbol periods) of the B feedback coefficients. For example, the indices F_B could correspond to the relative estimated delays of the largest channel impulse response echoes; for example, $B=1$ and F_B has just one non-zero index – the relative delay of the largest echo. For linear equalization (FD-LE) $B=0$, and F_B is a null set.

With this notation, the m th time domain output sample, obtained by decimating the sampled forward filter output by $1/I$, is

$$z_m = \frac{1}{MI} \sum_{\ell=0}^{MI-1} W_\ell R_\ell \exp(j \frac{2\pi}{M} \ell m) - \sum_{k \in F_B} f_k^* a_{m-k}, \quad (1)$$

where $m = 0, 1, 2, \dots, (M-1)$,

$$\text{and where } R_\ell = \sum_{m=0}^{MI-1} r_m \exp(-j \frac{2\pi}{MI} \ell m), \quad (2)$$

where $\ell = 0, 1, 2, \dots, (MI-1)$,

is the FFT of the received MI -sample sequence $\{r_m\}$. Complex conjugates are denoted by asterisks. The error at the m th sample is

$$e_m = z_m - a_m, \quad (3)$$

and the mean squared error $E(|e_m|^2)$ is to be minimized

with respect to the $\{W_\ell\}$ and $\{f_k\}$.

The received complex samples $\{r_m\}$, sampled at rate I/T , are expressed as

$$r_m = \sum_{k=0}^{M-1} a_k h(mT/I - kT) + n(mT/I), \quad (4)$$

for $m = 0, 1, 2, \dots, (MI-1)$,

where $h(t)$ is the channel's impulse response (including transmit filtering), and $\{n(mT/I)\}$ are samples of additive noise, assumed to be uncorrelated, have zero mean, and variance σ^2 . Because of the presence of the cyclic prefix,

the data symbols $\{a_k\}$ can be assumed to be periodic ($a_k = a_{k \pm LM}$, for any integer L), as can the impulse response samples ($\{h(mT/I) = h((m/I \pm LM)T)\}$).

In the discrete frequency domain, (4) becomes

$$R_\ell = H_\ell A_\ell + V_\ell \quad (5)$$

where, for $\ell = 0, 1, 2, \dots, (MI-1)$,

$$H_\ell = \sum_{m=0}^{MI-1} h(mT/I) \exp(-j2\pi \frac{m\ell}{MI}) \quad (6a)$$

$$A_\ell = \sum_{m=0}^{M-1} a_m \exp(-j2\pi \frac{m\ell}{M}) \quad (6b)$$

$$\text{and } V_\ell = \sum_{m=0}^{MI-1} n(mT/I) \exp(-j2\pi \frac{m\ell}{MI}) \quad (6c)$$

The mean squared error-minimizing solution for the optimum frequency domain forward filter coefficients can be expressed as [16], for $\ell = 0, 1, 2, \dots, (MI-1)$,

$$W_\ell = \frac{H_\ell^* [1 + \sum_{k \in F_B} f_k^* \exp(-j2\pi \frac{k\ell}{M})]}{\mathbf{s}^2 + |\hat{H}_\ell|^2}. \quad (7)$$

where we have defined, for $\ell = 0, 1, 2, \dots, (MI-1)$,

$$|\hat{H}_\ell|^2 = \frac{1}{I} \sum_{k=0}^{I-1} |H_{(\ell+kM) \bmod MI}|^2.$$

Optimization of the feedback coefficients $\{f_k, k \in F_B\}$ in F_e , results in a set of B linear equations in the optimum $\{f_k, k \in F_B\}$ that minimize the mean squared error, which can be expressed in the following matrix form [16]:

$$\mathbf{V}\mathbf{f} = -\mathbf{v} \quad (8a)$$

$$\text{where } \mathbf{f} = (f_{k_1}, f_{k_2}, \dots, f_{k_B})', \quad (8b)$$

$$\mathbf{v} = (v_{k_1}, v_{k_2}, \dots, v_{k_B})', \quad (8c)$$

$$\mathbf{V} = \begin{bmatrix} v_0 & v_{k_1-k_2} & \cdot & v_{k_1-k_B} \\ v_{k_2-k_1} & v_0 & v_{k_2-k_3} & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ v_{k_B-k_1} & \cdot & \cdot & v_0 \end{bmatrix}, \quad (8d)$$

$$\text{and } v_k = \frac{\mathbf{s}^2}{M} \sum_{\ell=0}^{M-1} \frac{\exp(-j2\pi \frac{\ell k}{M})}{\mathbf{s}^2 + |\hat{H}_\ell|^2}. \quad (8e)$$

To minimize complexity, B , the number of feedback coefficients, should be as low as possible. This can be

accomplished if the indices k_i $\hat{\mathbf{I}}F_B$ correspond to the relative estimated delays of the largest channel impulse response echoes. For example, $B=1$ and F_B has just one non-zero index – the relative delay of the largest echo. $B=0$ corresponds to linear equalization.

Training for Channel estimation

The estimation of the equalizer parameters $\{W_\ell\}$ and $\{f_k\}$, can be done using a least squares (LS) approach, from the reception of N consecutive training blocks, each consisting of a sequence of P known transmitted training symbols $\{a_k, k=0, 1, \dots, P-1\}$. The length of a training block, P , may be equal to or less than the length of a data block M , and it is preceded by a cyclic prefix. If it is less than M , P is picked to be at least equal to the maximum expected channel impulse response length in data symbol intervals. With no loss of generality, we assume that the same training sequence is transmitted in every training block.

The m th sample of the n th P -symbol training block can be expressed as

$$e_m^{(n)} = \frac{1}{P} \sum_{\ell=0}^{P-1} \left[\frac{1}{I} \sum_{k=0}^{I-1} \tilde{W}_{\ell+kP} R_{\ell+kP}^{(n)} \right] \exp(j \frac{2\mathbf{P}}{P} \ell m) - \sum_{k \in F_B} \tilde{f}_k^* a_{m-k} - a_m, \quad (9)$$

where $m = 0, 1, 2, \dots, (P-1), n = 1, 2, \dots, N$.

We define, for $\ell=0, 1, 2, \dots, (P-1)$,

$$\tilde{\mathbf{W}}_\ell = (\tilde{W}_\ell \quad \tilde{W}_{\ell+P} \quad \dots \quad \tilde{W}_{\ell+(I-1)P})' \quad (10a)$$

$$\text{and } \tilde{\mathbf{H}}_\ell^{(n)} = \frac{1}{I} \begin{pmatrix} R_\ell^{(n)} & R_{\ell+P}^{(n)} & \dots & R_{\ell+(I-1)P}^{(n)} \\ A_\ell & A_\ell & \dots & A_\ell \end{pmatrix} \quad (10b)$$

$$\text{and the operation } \langle X \rangle = \frac{1}{N} \sum_{n=1}^N X^{(n)}. \quad (10c)$$

It is clear that $\tilde{\mathbf{H}}_\ell^{(n)}$ is an estimate of a version of \mathbf{H}_ℓ for FFT blocks of length P .

Interpolation from P forward equalizer coefficients to M coefficients is done in the frequency domain: the inverse FFT, of length P , of each component of the vector $\tilde{\mathbf{H}}_\ell^{(n)}$ is computed, the resulting sequences are padded with zeroes to length M , and the FFT is taken; the resulting version of $\tilde{\mathbf{H}}_\ell^{(n)}$ is of length M , and is used to compute the forward and feedback equalizer coefficients as follows:

After the interpolation to length- M blocks, we can write the sum of squared errors, which is to be minimized with respect to the equalizer coefficients as

$$\begin{aligned} & \sum_{m=0}^{M-1} \langle |e_m^{(n)}|^2 \rangle \\ &= \frac{1}{M} \sum_{\ell=0}^{M-1} |A_\ell|^2 \tilde{\mathbf{W}}_\ell^H \tilde{\mathbf{U}}_\ell \tilde{\mathbf{W}}_\ell \\ & - \frac{2}{M} \text{Re} \left[\sum_{\ell=0}^{M-1} |A_\ell|^2 \tilde{\mathbf{W}}_\ell^H \tilde{\mathbf{H}}_\ell^* \tilde{F}_\ell \right] + \frac{1}{M} \sum_{\ell=0}^{M-1} |A_\ell|^2 |\tilde{F}_\ell|^2 \end{aligned} \quad (11)$$

where superscript H stands for complex conjugate transpose, and

$$\tilde{\mathbf{U}}_\ell = \frac{1}{I} \langle \tilde{\mathbf{H}}_\ell^{(n)} \tilde{\mathbf{H}}_\ell^{(n)H} \rangle, \quad \tilde{\mathbf{H}}_\ell = \langle \tilde{\mathbf{H}}_\ell^{(n)} \rangle,$$

$$\text{and } \tilde{F}_\ell = 1 + \sum_{k \in F_B} \tilde{f}_k^* \exp(-j2\mathbf{P} \frac{\ell k}{M}). \quad (12)$$

Minimizing (11) with respect to $\tilde{\mathbf{W}}_\ell$ and $\{\tilde{f}_k\}$ we get

$$\tilde{\mathbf{W}}_\ell = (\tilde{\mathbf{U}}_\ell)^{-1} \tilde{\mathbf{H}}_\ell^* \tilde{F}_\ell \quad \text{for } \ell = 0, 1, 2, \dots, (M-1), \quad (13)$$

and

$$\tilde{\mathbf{V}} \tilde{\mathbf{f}} = -\tilde{\mathbf{v}} \quad (14a)$$

$$\text{where } \tilde{\mathbf{f}} = (\tilde{f}_{k_1}, \tilde{f}_{k_2}, \dots, \tilde{f}_{k_B})', \quad (14b)$$

$$\tilde{\mathbf{v}} = (\tilde{v}_{k_1}, \tilde{v}_{k_2}, \dots, \tilde{v}_{k_B})', \quad (14c)$$

$$\tilde{\mathbf{V}} = \begin{bmatrix} \tilde{v}_0 & \tilde{v}_{k_1-k_2} & \cdot & \tilde{v}_{k_1-k_B} \\ \tilde{v}_{k_2-k_1} & \tilde{v}_0 & \tilde{v}_{k_2-k_3} & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \tilde{v}_{k_B-k_1} & \cdot & \cdot & \tilde{v}_0 \end{bmatrix}, \quad (14d)$$

$$\text{and } \tilde{v}_k = \sum_{\ell=0}^{M-1} |A_\ell|^2 \left[1 - \tilde{\mathbf{H}}_\ell^H \tilde{\mathbf{U}}_\ell^{-1} \tilde{\mathbf{H}}_\ell \right] \exp(-j2\mathbf{P} \frac{\ell k}{M}), \quad (14e)$$

Equations (12)-(14) summarize the least squares equalizer training algorithm. For the special case of $I=1$ sample per data symbol, vectors and matrices become scalars, and

$$\tilde{U}_\ell = \frac{1}{N} \sum_{n=1}^N |\tilde{H}_\ell^{(n)}|^2, \quad \tilde{H}_\ell = \frac{1}{N} \sum_{n=1}^N \tilde{H}_\ell^{(n)}, \quad (15a)$$

$$\text{and } \tilde{W}_\ell = \frac{\tilde{H}_\ell^*}{\tilde{U}_\ell} \left[1 + \sum_{k \in F_B} \tilde{f}_k^* \exp(-j2\mathbf{P} \frac{\ell k}{M}) \right] \quad (15b)$$

$$\text{Also, } \tilde{v}_k = \sum_{\ell=0}^{M-1} |A_\ell|^2 \left[1 - \frac{|\tilde{H}_\ell|^2}{\tilde{U}_\ell} \right] \exp(-j2\mathbf{P} \frac{\ell k}{M}) \quad (15c)$$

The sequence of P transmitted training symbols $\{a_k, k=0, 1, \dots, P-1\}$ is known as a *unique word*. Ideally, its discrete Fourier transform $\{A_\ell \text{ for } \ell=0, 1, \dots, P-1\}$ should have equal, or nearly equal magnitude for all indices ℓ ; the corresponding cyclic autocorrelation function of $\{a_k, k=0, 1, \dots, P-1\}$ should ideally be zero for non-zero lags. Such an ideal training sequence ensures that each frequency component of the channel is probed uniformly to provide the estimates of $\{H_\ell\}$ and of $\{|H_\ell|^2 + \sigma^2\}$. For unique word lengths P which are powers of two, such as 64 or 256,

polyphase Frank-Zadoff sequences [17] or Chu sequences [18] are suitable. If binary-valued sequences are more desirable from a hardware implementation standpoint, length 2^l-1 , pn sequences can be modified by adding a small dc value in quadrature, as suggested by Milewski [19]. Each of these types of sequences have the desired property that A_d is a constant value. Typical training consists of four or eight repetitions of a 64-symbol Frank sequence. Time-averaging takes place over the four or eight training blocks, and the results are interpolated to say, $M=1024$ or 2048 coefficients by transforming the 64-length blocks to the time domain, padding with zeroes to 1024 or 2048, and then transforming to the frequency domain.

The use of unique words for equalizer training, as well as interpolation in the frequency domain is the counterpart of the use of pilot tones and frequency domain interpolation in OFDM systems. Estimation of the frequency domain equalizer coefficients can also be done from the unique words in the time domain – crosscorrelating the received unique word segments with unique words in order to estimate the channel impulse response, followed by transformation to the frequency domain and use of equations (7) – (8). In this case, a separate estimate of the noise variance σ^2 is also necessary.

BER PERFORMANCE

To compare the performance of SC-FDE and OFDM systems in frequency-selective fading channels, we consider 5 Msymbol/s transmission over a "SUI-5" channel model, one of six multipath channel models adopted by the IEEE 802.16a task group for evaluating broadband wireless systems in 2-11 GHz bands [20]. Its profile consists of three independent Rayleigh-fading echoes at delays of 0, 5 and 10 μ s, and relative powers of 0, -5 and -10 dB, respectively. Thus the multipath spans 50 data symbols. QPSK, 16QAM and 64QAM single carrier and OFDM systems were simulated against this model for a range of received signal to noise ratios, each with 20,000 random channel realizations. For each channel realization, obtained by Monte Carlo simulation, the BER was computed, and then the BER was, in turn, averaged over all channel realizations. BER results were compiled for interleaved convolutionally coded systems with various code rates, obtained by Monte Carlo simulation. The coding scheme was bit-interleaved coded modulation (BICM) [21], in which coded bits were interleaved and mapped into transmitted M-ary QAM data symbols using Gray mapping. OFDM performance was based on optimum linear equalization and optimally weighted soft decision (log APP) MLSE decoding. Code rates greater than 1/2 were realized by optimally puncturing

[22] a standard rate 1/2, constraint length 7 code with generator polynomials (133,171). Each FFT block consisted of 512 QAM symbols. Row-column block interleaving was used within each FFT block, where the data bits are written by row, and mapped to QAM symbols by column, each row consisting of 32 bits. The raised cosine rolloff factor used for the single carrier systems was 10%.

Figure 3 shows the average bit error probability evaluated over a range of average signal to noise ratios using a rate 1/2 code and for the three (4-16-64) QAM constellations, for the following system configurations: (1) single carrier modulation using frequency domain linear equalization (FD-LE); (2) OFDM, based on optimum linear equalization and optimally weighted soft decision MLSE decoding; (3) Single carrier modulation using ideal frequency domain decision feedback equalization (FD-DFE), assuming an infinite-length feedback filter and correct feedback (no decision errors). For an upper bound comparison, the matched filter bound (MFB) (performance with a matched filter receiver and no intersymbol interference). In these simulations, perfect channel and output SNR estimation was assumed for all systems. Results for practical channel estimation schemes and small, finite numbers of feedback

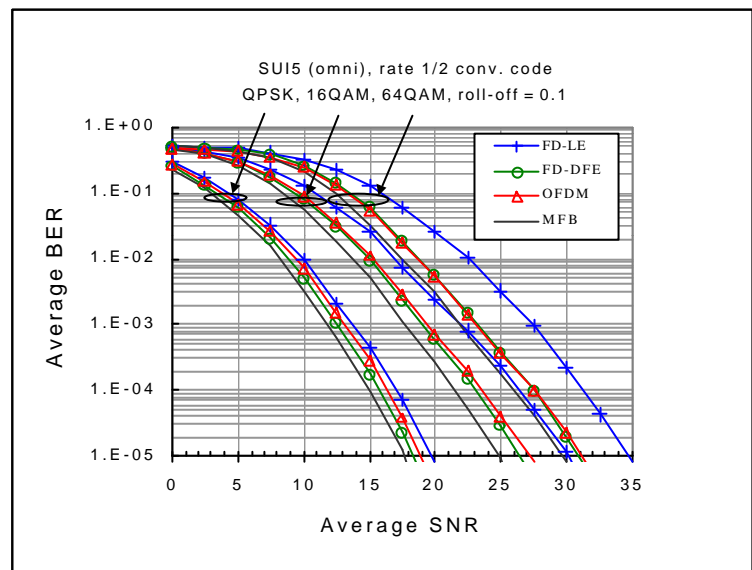


Figure 5. Performance comparison for systems with perfect channel knowledge on SUI-5 channel for code rate 1/2

tap coefficients are found in [16] and [8].

The results of Figure 3 suggest that for a channel operating at lower average SNR's - where QPSK modulation is appropriate, OFDM, FD-LE and ideal FD-DFE SC systems all perform within about 1.5-2 dB of one another. The ideal FD-DFE performs to within about 1 dB of the ideal matched filter bound for QPSK. OFDM performed slightly

better than FD-LE, and slightly worse than the ideal FD-DFE. For 16QAM and 64QAM, there is a somewhat larger spread among the results, but with the same relative rankings. In particular, the spread between the BER performance of FD-LE and that of the other systems becomes larger for higher level modulations.

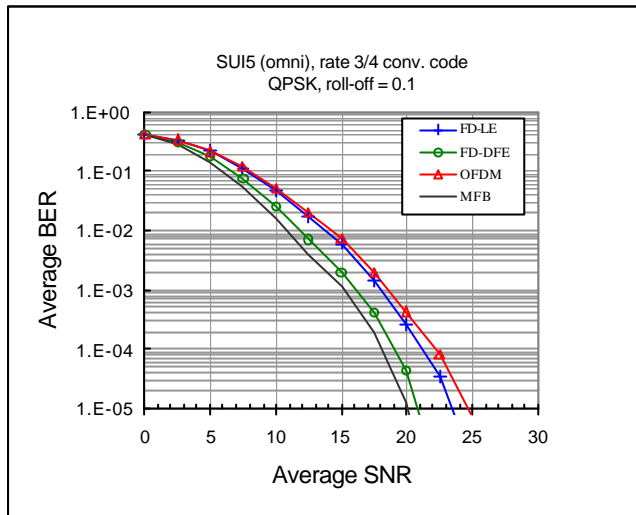


Figure 4. Performance comparison for systems with perfect channel knowledge on SUI-5 channel model for QPSK with code rate 3/4

Both OFDM and FD-LE suffer from noise enhancement in severe frequency selective Rayleigh fading channels, such as SUI-5, but their corresponding decoders operate and perform somewhat differently. For the FD-LE, the noise enhancement loss increases with the average input SNR, i.e. when the channel has deep nulls and the SNR is high (typically required for high level modulation), the linear equalizer will try harder to invert the nulls and, as a result, the noise in those null locations is also amplified. In contrast, OFDM can exploit the *independent* (Rayleigh-distributed) *known* (from channel estimation measurements) gain and phase of each subchannel and combine the useful energy across all subchannels through coding and interleaving. However, the performance of OFDM in frequency selective fading is sensitive to the code rate (and strength of the code) used.

Figure 4 shows the performance over the SUI-5 channel using QPSK and higher code rate (3/4). For this and higher rates, OFDM actually performs slightly worse than the FD-LE [16],[8]. For an uncoded system, the BER performance of OFDM is far inferior on frequency-selective channels to that of the linear and DFE single carrier systems, since without coding, the Rayleigh fading on each OFDM subchannel presents the appearance of flat Rayleigh fading to the OFDM symbol detector.

Figure 5 shows the simulated performance of a SC frequency domain equalizer, with training, on the SUI-5 channel. 64-QAM symbols with rate 3/4 BICM are used in the data payload. $N=2, 4, \text{ and } 8$. Frank-Zadoff sequence unique word training blocks, each of length 64, are used to estimate the forward coefficients of a linear equalizer. The channel estimation procedure uses frequency domain interpolation to extrapolate from 64-training symbol FFT blocks to the 1024-symbol FFT blocks that are used for frequency domain equalization in this example. The performance measure used in Figure 5 is the probability (over an ensemble of 20,000 SUI-5 channel realizations) that the bit error rate of the rate 3/4, 64QAM coded system is worse than 10^{-6} . Call this probability (that a minimal BER is not maintained) an ‘outage probability’.. Figure 5 also shows that the performance loss relative to perfect channel estimation is on the order of 1 to 2 dB or less for four training blocks. In all cases, the channel is assumed unknown before the use of the training symbols to estimate the channel. What’s more, only the training symbols are used to estimate the channel: no decision direction of payload symbols is used to progressively improve the accuracy of the channel estimates. Note that 8 training blocks, each of length 64 symbols, represents a total of one half the length of a single FFT block (1024 symbols) in this example. In a burst environment, one or more of these FFT blocks may compose a single burst.

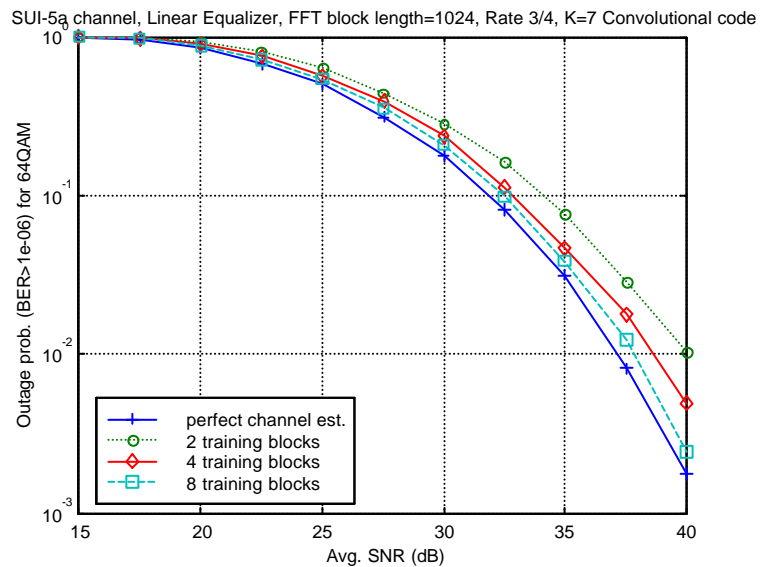


Figure 5. Outage probability for SC-frequency domain equalization for SUI-5 Rayleigh fading channel. training over 2, 4, and 8 blocks, each consisting of 64 symbols.

OVERLAP-SAVE PROCESSING IN SC-FDE:

There is a possible incentive to avoid the cyclic prefix when using frequency-domain equalization; this would

reduce overhead, and would also allow computationally-efficient frequency domain receiver processing to be applied to existing single carrier air interfaces, which do not incorporate a cyclic prefix.

The overhead created by the cyclic prefix in SC-FDE systems could be eliminated by using well-known frequency domain overlap-save or overlap-add processing methods [23]. Such methods were proposed for example in early papers on SC-FDE [2], [3]. Overlap-save frequency domain processing is mathematically equivalent to N -tap time-domain equalization in the following sense: the set of $2N$ frequency domain equalizer coefficients are the discrete Fourier transform of N time domain coefficients which have been padded out to $2N$ with N zeroes. Length - $2N$ FFT blocks are then overlapped by N . Thus the performance (MMSE or BER) of the overlap-save approach, with $2N$ coefficients equals that of the corresponding time domain equalizer with N optimal coefficients.

On the other hand, block frequency domain processing, where each block is preceded by a cyclic prefix (CP) avoids any inter-block interference. Furthermore, block frequency domain processing with CP achieves equalization performance expected from using an *infinite* number of time domain equalizer taps; e.g. a linear FDE with cyclic prefix could perfectly invert the cyclic frequency response of a channel, whereas a time domain equalizer in general could achieve this feat only with an infinite number of taps.

Thus an overlap-save equalizer cannot exceed the BER performance of a corresponding cyclic prefix FDE. This has been verified in simulation studies comparing BER performance of SC-FDE systems with cyclic prefix and with M frequency domain taps against time domain equalizers with M time domain taps (the equivalent of overlap-save equalizers with $2M$ frequency domain taps). On channels with multipath delay spreads much less than M , their performances were similar; for delay spreads on the same order of magnitude as M , even the linear cyclic prefix SC-FDE equalizers gave significantly lower BER at high SNR than the corresponding time domain (and therefore overlap-save) DFE equalizers. The conclusion we draw is that SC-FDE systems with cyclic prefix and overlap-save equalizers would both need to use FFT block lengths M which are much longer (e.g. at least 8 to 10 times) than the maximum expected channel delay spread: the cyclic prefix SC-FDE systems to ensure that the cyclic prefix adds minimal overhead, and the overlap-save systems to ensure that BER performance is not degraded.

Calculation of an overlap-save equalizer's parameters from the corresponding time domain equalizer's minimum MSE parameters is unattractive because of the latter's high computational and adaptation complexity for long channel impulse responses. A simpler method is to *approximate* the

overlap-save equalizer's frequency domain coefficients by using those found for the *cyclic prefix* FDE from the FDE/CP training method described earlier. The absence of a preceding cyclic prefix will contaminate the initial portion (spanning the duration of the channel's impulse response) of a block of data received from a channel. This contamination can be considered as a noise sequence added to the first L data symbol intervals of the block, where L is the channel impulse response length. In the time domain, the equalizer will smear this noise sequence over a longer time span whose duration is the sum of L and of the effective time duration of the equalizer's time domain response. For this reason, it is essential to make the FFT block length long enough that this contaminating smear is mostly confined to the first half of the block, and its effect rendered negligible by retaining the equalizer's output only for the last half of the block.

The following overlap-save approach was found to be effective: for channels such as SUI-5 with long (e.g. 50 symbol intervals) delay spread, use 1024 to 2048-symbol FFT blocks for linear equalization; 1024-symbol blocks suffice for decision feedback equalization; shift the phase of the FDE equalizer forward coefficients so that there is $\frac{1}{4}$ block delay in the output decisions. Discard the first half of each equalizer output block, and overlap successive blocks must overlap by one half.

Figure 6 shows the mean squared error for cyclic prefix and overlap-save equalizers, averaged over 1000 realizations of the SUI-5 channel, as a function of the average received SNR. The FFT block length is $M=1024$ and there are 2 feedback taps. Both the cyclic prefix equalizer and the overlap-save equalizer use the same frequency domain forward equalizer parameters, and also the same feedback taps, all derived from four 64-symbol training blocks preceded by one 64-symbol cyclic prefix. There are four curves shown: (1) cyclic prefix equalizer performance with known channel (perfect channel estimation, no training); (2) calculated output SNR for the trained cyclic prefix equalizer; (3) SNR estimated from simulation of the trained cyclic prefix equalizer; (4) SNR estimated from simulation of the trained overlap-save equalizer. The latter three curves nearly coincide, indicating that in this case the overlap-save equalizer which uses the same parameters as the trained cyclic prefix equalizer, suffers only a small performance loss (at high SNR) relative to its cyclic prefix counterpart, for 1024-symbol FFT block length. Furthermore, the cyclic prefix equalizer, given perfect channel estimation improves the output SNR only a little over 1 dB relative to its four-block-trained counterpart.

One of the possible applications of overlap-save frequency domain equalization is to wideband CDMA systems. In this case, computationally efficient frequency domain processing could be done at the chip level in the receiver to perform the MMSE equalization function to restore

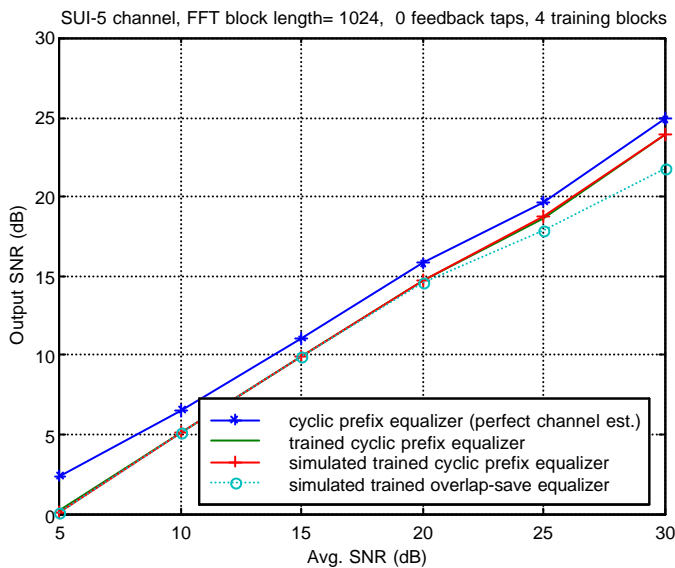


Figure 6. Comparative output SNR performance of cyclic prefix and overlap-save frequency domain equalizers with ideal channel estimates and with training

orthogonality of spreading codes, or for time-invariant spreading codes to simultaneously mitigate multipath and multi-user interference [24], [25].

SC-FDE AND OFDM: COEXISTENCE AND MIXED MODES

As is evident from Figure 1, SC-FDE and OFDM incorporate essentially the same signal processing operations, but distribute them differently between transmitter and receiver. Thus in principle, a software-defined modem could easily accommodate both single carrier and OFDM modes of operation. Moreover, there may be a distinct advantage to using *both* modes in future cellular wireless systems:

for example OFDM in the downlink (base to mobile terminal) and SC in the uplink, as suggested in Figure 7. This arrangement has the following potential advantages:

- (1) The mobile terminal's transmitter is single carrier, and thus is less costly and inherently more efficient in terms of power consumption.
- (2) Concentrating most of the signal processing complexity at the hub, or base station. The hub has two inverse FFT operations and one FFT, while the subscriber has just one FFT for receiving the downlink OFDM signal, as shown in Figure 6.
- (3) The uplink TDMA single carrier mode is simple and efficient; short

MAC messages can be transmitted in very short-duration bursts, whereas OFDM burst lengths must be multiples of the FFT block length.

SUMMARY

SC-FDE shares with OFDM the signal processing efficiency advantages of FFT frequency domain processing. Both of these frequency domain modulation and equalization schemes are ideal for severely frequency selective fading channels whose impulse responses may span tens or even hundreds of information bits. Over such channels, coded SC-FDE (with or without decision feedback) and OFDM have similar bit error rate performance, with OFDM being slightly better for low code rates, and SC-FDE being slightly better for higher code rates (and much better for uncoded systems). Furthermore the performance of SC-FDE is enhanced when it is combined with simple sparse time-domain decision feedback equalization. Both of these systems can employ adaptive modulation and coding schemes. For example, both OFDM and single carrier air interfaces with adaptive modulation and coding schemes are defined in the draft IEEE 802.16a standard [26].

However, OFDM is much more sensitive to power amplifier nonlinearities, phase noise and frequency offsets than single-carrier modulation. As a result, single carrier systems generally require several dB less transmitter power amplifier backoff, and therefore can use less expensive power amplifiers than OFDM. Alternatively, with the same power amplifier, running with lower backoff than OFDM, a SC system will have higher received SNR and therefore a better link budget (which may offset the small SNR disadvantage of linearly equalized SC-FDE at low code rates). The reduced power amplifier backoff requirement is

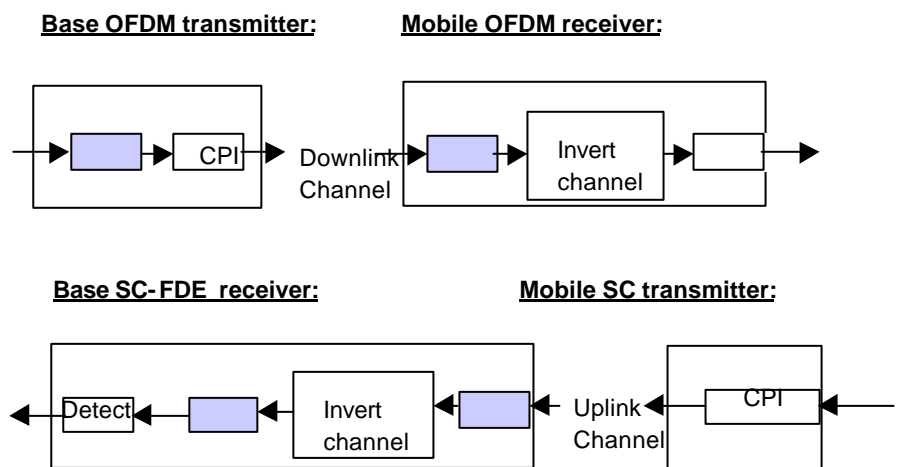


Figure 7. Downlink OFDM, uplink SC-FDE

single carrier's major advantage over OFDM.

Also, as we have seen, single carrier and OFDM systems can potentially coexist for mutual benefit and cost reduction, because of the obvious similarities in their basis frequency domain signal processing functions. In particular, an architecture using OFDM in the downlink and SC-FDE in the uplink has clear advantages for cost-sensitive fixed or mobile subscriber terminals.

At the expense of slightly higher signal processing complexity, overlap-save receiver processing can be used in SC-FDE systems to eliminate the need for a cyclic prefix. This can reduce overhead and make receiver frequency domain processing usable for existing non-cyclic prefix single carrier schemes. The FFT block length M for SC-FDE schemes, with or without cyclic prefix, (and also for OFDM schemes) is typically on the order of 10 times the cyclic prefix length or the maximum expected multipath span. Efficient training of equalizer parameters can be carried out using simple least squares methods in the frequency domain, extrapolating from relatively short unique word lengths to the full FFT block length.

Furthermore, single carrier techniques can be easily combined with receiver antenna arrays [27], [28] and MIMO (multiple-input, multiple-output) techniques [29], in which both transmitting and receiving ends use arrays of antenna elements.

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