Brownian motion in AdS/CFT

Masaki Shigemori
Institute for Theoretical Physics, University of Amsterdam
Valckenierstraat 65, 1018 XE Amsterdam, The Netherlands

1 Introduction and motivation

Since its advent, the holographic AdS/CFT correspondence [1, 2, 3, 4] has been exploited to study the physics of non-Abelian quark-gluon plasmas at finite temperature using gravitational physics of AdS black holes, and *vice versa*. The dual gravitational description of strongly coupled gauge theories provides an efficient way to study the thermodynamic properties such as the phase structure of the gauge theory. More recently, this correspondence was furthered to the hydrodynamical level, as was originally proposed in [5] and has been significantly developed afterward (see [6] and references therein for earlier work on hydrodynamics in the AdS/CFT context). Namely, the long-wavelength physics of the field theory plasma described by a hydrodynamical Navier–Stokes equation is holographically dual to the long-wavelength fluctuation of the horizons of AdS black holes. This correspondence allows for a detailed quantitative study of the plasma from the bulk, and *vice versa*.

Generally, thermodynamics and hydrodynamics are macroscopic effective description obtained by coarse-graining the underlying microscopic physics which is more fundamental. It is thus very natural to make steps toward more microscopic non-equilibrium aspects of the AdS/CFT correspondence, underpinning the above-mentioned thermodynamical/hydrodynamical correspondence. Historically, Brownian motion [7] played a crucial role in connecting the macrophysics and the underlying microphysics. This peculiar motion is due to collisions with the fluid particles in random thermal motion, and gave crucial evidence for the atomic theory of nature; for example, it allowed to determine the Avogadro number. Brownian motion is a ubiquitous phenomenon; any particle immersed in a fluid at finite temperature exhibits such Brownian motion, from a small pendulum suspended in a dilute gas to a heavy particle in quark-gluon plasma. Therefore, study of Brownian motion in the AdS/CFT setup is expected to help us understand the microscopic workings of strongly coupled plasmas. Conversely, we should be able to learn about quantum physics of AdS black holes from the boundary side.

The AdS/CFT correspondence has already been successfully and fruitfully used for understanding features of quark-gluon plasmas. For example, the KSS bound [8, 6] on the ratio of shear-viscosity to entropy density for relativistic hydrodynamic systems, $\eta/s \geq 1/4\pi$ played a major role in understanding the properties of quark-gluon plasmas. Furthermore, studies of the motion of quarks, mesons, and baryons in

the quark-gluon plasma have been carried out in the holographic framework starting with the seminal papers [9, 10, 11, 12, 13, 14, 15, 16], by considering the dynamics of probe strings and D-branes in asymptotically AdS black hole spacetime—for a sample of recent reviews on the subject, see [17]. The general philosophy in these discussions was to use the probe dynamics to extract the rates of energy loss and transverse momentum broadening in the medium, which bear direct relevance to the physical problem of motion of quarks and mesons in the quark-gluon plasma. In such computations, the motion of an external quark in the quark-gluon plasma is assumed to be described by a Langevin equation [18].

Therefore, one can say that the most basic data of the Langevin equation describing Brownian motion in the CFT plasma are already available. However, rather than taking such approaches which are phenomenological in some sense, one could study more fundamental aspects of Brownian motion in the AdS/CFT context. For example, in the first place, why does an external quark exhibit Brownian motion, and why is the motion described by a Langevin equation? What is the bulk meaning of the fluctuation-dissipation theorem relating γ and κ? The main purpose of the current article is to elucidate the AdS/CFT physics of Brownian motion, by addressing such questions. For example, the computation of the random force in [13, 14, 16] using the GKPW prescription [2, 3] does not immediately explain what the bulk counterpart of the random force is. We will see that it corresponds to a version of Hawking radiation in the bulk.[1]

More recent papers discussing aspects of Brownian motion in the AdS/CFT context include [20, 21, 22, 23].

2 Brownian motion

Let us start the discussion from the field theoretic, or boundary, side of the story in the AdS/CFT context, by briefly reviewing the Langevin dynamics that describes the Brownian motion. A general formulation of non-relativistic Brownian motion is based on the generalized Langevin equation [24, 25], which takes the following form in one dimension:

$$\dot{p}(t) = -\int_{-\infty}^{t} dt'\, \gamma(t-t')\, p(t') + R(t) + K(t), \tag{1}$$

where $p = m\dot{x}$ is the (non-relativistic) momentum of the Brownian particle at position x, and $\dot{} \equiv d/dt$. The first term on the right hand side of (1) represents (delayed) friction, which depends on the past trajectory of the particle via the memory kernel $\gamma(t)$. The second term corresponds to the random force which we assume to have the

[1]For an earlier discussion of the relation between Hawking radiation and diffusion the context of AdS/QCD, see [19].

following average:

$$\langle R(t)\rangle = 0, \qquad \langle R(t)R(t')\rangle = \kappa(t-t'), \tag{2}$$

where $\kappa(t)$ is some function. $K(t)$ is an external force that can be added to the system. The separation of the force into frictional and random parts on the right hand side of (1) is merely a phenomenological simplification—microscopically, the two forces have the same origin (collision with the fluid constituents). One can think of the particle as losing energy to the medium due to the friction term and concurrently getting a random kick from the thermal bath modeled by the random force. The two functions $\gamma(t)$ and $\kappa(t)$ completely characterize the Langevin equation (1). Actually, $\gamma(t)$ and $\kappa(t)$ are related to each other by the fluctuation-dissipation theorem [26].

The time evolution of the displacement squared of a Brownian particle obeying (1) goes as follows [27]:

$$\langle s(t)^2\rangle \equiv \langle [x(t)-x(0)]^2\rangle \approx \begin{cases} \dfrac{T}{m}t^2 & (t \ll t_{\rm relax}) \quad : \text{ ballistic regime} \\ 2Dt & (t \gg t_{\rm relax}) \quad : \text{ diffusive regime} \end{cases} \tag{3}$$

The crossover time $t_{\rm relax}$ between two regimes is given by

$$t_{\rm relax} = \frac{1}{\gamma_0}, \qquad \gamma_0 \equiv \int_0^\infty dt\, \gamma(t), \tag{4}$$

while the diffusion constant D is given by

$$D = \frac{T}{\gamma_0 m}. \tag{5}$$

In the ballistic regime, $t \ll t_{\rm relax}$, the particle moves inertially ($s \sim t$) with the velocity determined by equipartition, $\dot{x} \sim \sqrt{T/m}$, while in the diffusive regime, $t \gg t_{\rm relax}$, the particle undergoes a random walk ($s \sim \sqrt{t}$). This is because the Brownian particle must be hit by a certain number of fluid particles to get substantially diverted from the direction of its initial velocity. The time $t_{\rm relax}$ between the two regimes is called the relaxation time which characterizes the time scale for the Brownian particle to forget its initial velocity and thermalize.

Another physically relevant time scale, the microscopic (or collision duration) time $t_{\rm coll}$, is defined to be the width of the random force correlator function $\kappa(t)$. Specifically, let us define

$$t_{\rm coll} = \int_0^\infty dt\, \frac{\kappa(t)}{\kappa(0)}. \tag{6}$$

If $\kappa(t) = \kappa(0)e^{-t/t_{\rm coll}}$, the right hand side of this precisely gives $t_{\rm coll}$. This $t_{\rm coll}$ characterizes the time scale over which the random force is correlated, and thus can be

thought of as the time elapsed in a single process of scattering. In many cases,

$$t_{\rm relax} \gg t_{\rm coll}. \tag{7}$$

There is also a third natural time scale $t_{\rm mfp}$ given by the typical time elapsed between two collisions. In the kinetic theory, this mean free path time is typically $t_{\rm coll} \ll t_{\rm mfp} \ll t_{\rm relax}$; however in the case of present interest, this separation no longer holds, as we will see. For a schematic explanation of the timescales $t_{\rm coll}$ and $t_{\rm mfp}$, see Figure 1.

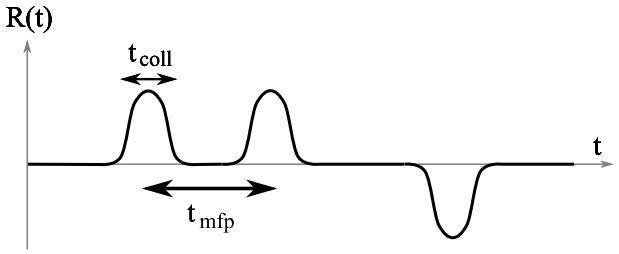

Figure 1: The A sample of the stochastic variable $R(t)$, which consists of many pulses randomly distributed.

3 Brownian motion in AdS/CFT

The AdS/CFT correspondence states that string theory in AdS_d is dual to a CFT in $(d-1)$ dimensions. In particular, the (planar) Schwarzschild-AdS black hole with metric

$$ds_d^2 = \frac{r^2}{\ell^2}\left[-h(r)\,dt^2 + d\vec{X}_{d-2}^2\right] + \frac{\ell^2}{r^2 h(r)}\,dr^2, \qquad h(r) = 1 - \left(\frac{r_H}{r}\right)^{d-1} \tag{8}$$

is dual to a CFT at a temperature equal to the Hawking temperature of the black hole,

$$T = \frac{1}{\beta} = \frac{(d-1)\,r_H}{4\pi\,\ell^2}. \tag{9}$$

In the above, ℓ is the AdS radius, and t, $\vec{X}_{d-2} = (X^1, \ldots, X^{d-2}) \in \mathbf{R}^{d-2}$ are the boundary coordinates. The black hole horizon at $r = r_H$. In this black hole geometry (8), let us consider a fundamental string suspended from the boundary at $r = \infty$, straight down along the r direction, into the horizon at $r = r_H$; see Figure 2. In the boundary CFT, this corresponds to having a very heavy external charged particle. The $\vec{X}_{d-2}$ coordinates of the string at $r = \infty$ in the bulk give the boundary position

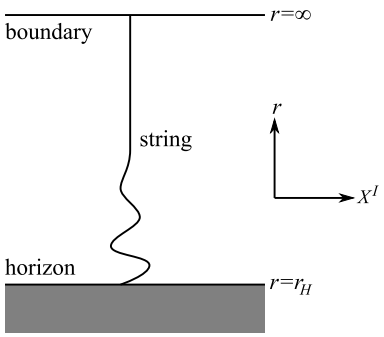

Figure 2: The bulk dual of a Brownian particle: a fundamental string hanging from the boundary of the AdS space and dipping into the horizon. The AdS black hole environment excites the modes on the string and, as a result, the string endpoint at infinity moves randomly, corresponding to the Brownian motion on the boundary.

of the external particle. As we discussed above, such an external particle at finite temperature T is expected to undergo Brownian motion.

The physical mechanism for Brownian motion in the gravitational picture is as follows. Any fields in a black hole environment get excited by the Hawking radiation. The transverse position of a string is also a field, although restricted to live on the string worldsheet, and is thus subject to Hawking radiation. Intuitively, the string is randomly kicked by the quantum effects on the horizon. This fluctuation propagates along the string worldsheet and, consequently, the endpoint of the string at $r = \infty$ exhibits a Brownian motion which can be modeled by a Langevin equation. This whole process is dual to the quark doing Brownian motion due to collisions with the constituents of the quark-gluon plasma. We will refer to the string dual to the Brownian particle on the boundary as the Brownian string.

We study this motion of a string in the probe approximation where we ignore its backreaction on the background geometry. Also, if we take the string coupling g_s to be very small, the interaction of the Brownian string with the thermal gas of closed strings in the bulk of the AdS space can be ignored. The reason is as follows. How much the closed strings in the bulk of the AdS space is excited and how much the fluctuation modes on the Brownian string are excited are both controlled by g_s. Furthermore, the interaction between those closed string modes and the fluctuation modes on the Brownian string is controlled by g_s. So, the magnitude of interaction effects is down by a power of g_s as compared to the physics of the fluctuation itself and can be ignored. As a result, the only part of the Brownian string that interacts is the near-horizon part.

As the action of the Brownian string, we use the Nambu-Goto action. If we parametrize the string worldvolume by $x^\mu = t, r$, the transverse position is a field

$\vec{X}_{d-2}(x)$. The Nambu-Goto action expanded in X is

$$S_{NG} = \text{const} + S_2 + S_4 + \cdots, \tag{10}$$

$$S_2 = -\frac{1}{4\pi\alpha'}\int dt\, dr\, \left[\frac{\dot{X}^2}{h(r)} - \frac{r^4 h(r)}{\ell^4}X'^2.\right] \tag{11}$$

We will consider small fluctuation and use the quadratic part S_2 as the action. This corresponds to the nonrelativistic limit of the Brownian motion. This quadratic action is analogous to Klein-Gordon action and, as such, can be studied using mode expansion just like Klein-Gordon scalar. Explicitly, the mode expansion is

$$X(t,r) = \int_0^\infty d\omega\, \left[f_\omega(r)e^{-i\omega t}a_\omega + \text{h.c.}\right] \tag{12}$$

where $f_\omega(r)$ is the complete basis of positive energy solutions to the equation of motion derived from the quadratic action (11):

$$\left[\omega^2 + \frac{h(r)}{\ell^4}\partial_r\left(r^4 h(r)\partial_r\right)\right] f_\omega(r) = 0 \tag{13}$$

and a_ω are annihilation operators satisfying the usual commutation relation $[a_\omega^\dagger, a_{\omega'}] = 2\pi\delta(\omega-\omega')$. For $d=3$, this equation (13) can be solved exactly while for $d>3$ it can be solved in the low frequency limit.

The equation (13) must be solved with an appropriate boundary condition. Near the boundary $r=\infty$, we put a UV[2] cutoff at $r_H = r_c$ and impose Neumann boundary condition $\partial_r X|_{r_c} = 0$. This can be thought of as putting a "flavor brane" at $r = r_H$ and it makes the mass of the Brownian particle finite. This is necessary because if the Brownian string were infinitely long then the mass of the Brownian particle would be infinite and there would be no Brownian motion.

As we have said, the Brownian string gets excited near the horizon, and the resulting fluctuation propagates to the UV endpoint and leads to the boundary Brownian motion. Therefore, there is a direct relation between the fluctuation of the Brownian string near the horizon and the position of the boundary Brownian particle. To understand the precise dictionary, let us look at the wave-functions of the world-sheet fields in the two interesting regions: (i) near the black hole horizon and (ii) close to the boundary. Near the horizon ($r \sim r_H$), the expansion (12) becomes

$$X(t, r\sim r_H) \approx \int_0^\infty \frac{d\omega}{\sqrt{2\omega}}\left[\left(e^{-i\omega(t-r_*)} + e^{i\theta_\omega}e^{-i(t+r_*)}\right)a_\omega + \text{h.c.}\right] \tag{14}$$

where r_* is the tortoise coordinate $dr_* = (\ell^2/r^2 h(r))dr$. The first term in the parentheses of (14) corresponds to the mode emerging out from the black hole while the

[2]We use the terms "UV" and "IR" with respect to the boundary energy. In this terminology, in the bulk, UV means near the boundary and IR means near the horizon.

second term correspond to the mode falling into the black hole. θ_ω is a phase shift. Eq. (14) means that the oscillators a_ω is directly related to the amplitude of the modes $e^{-i\omega(t-r_*)}$ coming out of the horizon. On the other hand, at the UV cut-off $r = r_c$, which we have chosen to be the location of the regularized boundary, (12) becomes

$$x(t) \equiv X(t, r_c) = \int_0^\infty d\omega \left[f_\omega(r_c) e^{-i\omega t} a_\omega + \text{h.c.} \right]. \tag{15}$$

We will interpret this as the position of the Brownian particle in the boundary theory. Here, the operators a_ω are related to the Fourier coefficients of $x(t)$.

Using the above relation between (14) and (15), one can predict the correlators for the outgoing modes near the horizon, $\langle a_{\omega_1} a^\dagger_{\omega_2} \ldots \rangle$ *etc.*, from the boundary correlators $\langle x(t_1)\, x(t_2) \ldots \rangle$ in field theory. In particular, if we would be able to make a very precise measurement of Brownian motion in field theory, we could in principle predict the precise state of the radiation that comes out of the black hole. In the semiclassical approximation, the radiation is purely thermal as Hawking showed, but in the quantum theory the state of the radiation must reflect the quantum gravity microstate which the black hole is in. In this way, we can learn about the physics of quantum black holes in the bulk from the boundary data. This of course requires us to compute the correlation function for the test particle's position in a strongly coupled medium, which is a difficult task that we will not undertake here.

Semiclassical analysis

However, at the semiclassical level, we can utilize this dictionary to rather go from the bulk to the boundary and learn about the boundary Brownian motion from the bulk data. This is possible because, semiclassically, the state of the outgoing modes near the horizon is given by the usual Hawking radiation [28, 29]. Namely,

$$\langle a^\dagger_\omega a_{\omega'} \rangle = \frac{2\pi \delta(\omega - \omega')}{e^{\beta\omega} - 1}. \tag{16}$$

In the $d = 3$ (AdS_3) case, where we can solve the equation of motion (13) exactly, we can use the expectation value (16) and the dictionary (14), (15) to accurately compute the correlators for x, the position of the Brownian particle. Namely, we can use the AdS/CFT correspondence to precisely predict the form of the Brownian motion that the external quark undergoes.

Explicitly, the displacement squared, regularized by normal ordering, for $d = 3$ becomes

$$s^2(t) \equiv \langle :[x(t) - x(0)]^2: \rangle = \frac{4\alpha' \beta^2}{\pi^2 \ell^2} \int_0^\infty \frac{d\omega}{\omega} \frac{1 + (\frac{\beta\omega}{2\pi})^2}{1 + \omega^2 \ell^4} \frac{\sin^2 \frac{\omega t}{2}}{e^{\beta\omega} - 1}. \tag{17}$$

By evaluating the integral for $r_c \gg r_H$, we find the following behavior:

$$s^2(t) \approx \begin{cases} \dfrac{T}{m}t^2 & (t \ll t_{\rm relax}), \\ \dfrac{\alpha'}{\pi\ell^2 T}t & (t \gg t_{\rm relax}). \end{cases} \tag{18}$$

where $m = r_c/2\pi\alpha'$ is the mass of the external quark. So, we observe two regimes, the ballistic and diffusive regimes, exactly as for the standard Brownian motion (3). The crossover time $t_{\rm relax}$ is given by

$$t_{\rm relax} \sim \frac{\alpha' m}{\ell^2 T^2}. \tag{19}$$

The diffusion constant that can be read off from the diffusive regime (see (3)) is consistent with the known results for test quarks moving in the thermal $\mathcal{N} = 4$ super Yang-Mills plasma [9, 10, 11, 13].

Using these data, we can determine $\gamma(t), \kappa(t)$ that characterize the Langevin equation and thus Brownian motion in the $d = 3$ case [30]. The result is

$$\frac{1}{\gamma[\omega] - i\omega} = \frac{\alpha'\beta^2\, m}{2\pi\,\ell^2}\,\frac{1 - i\nu/\rho_c}{1 - i\rho_c\nu}, \qquad \kappa(\omega) = \frac{4\pi\ell^2}{\alpha'\beta^3}\,\frac{1+\nu^2}{1+\rho_c^2\nu^2}\,\frac{\beta|\omega|}{e^{\beta|\omega|} - 1}, \tag{20}$$

where $\nu = \beta\omega/2\pi$, $\rho_c = r_c/r_H$. Divergences have been regularized by normal ordering. Also,

$$\kappa(\omega) = \int_{-\infty}^{\infty} dt\, \kappa(t)\, e^{i\omega t}, \qquad \gamma[\omega] = \int_0^{\infty} dt\, \gamma(t)\, e^{i\omega t}. \tag{21}$$

It is possible to show that these satisfy the fluctuation-dissipation theorem; more details and a proof that the fluctuation-dissipation theorem holds in general for any d are presented in [30].

4 Time scales

In the above, we discussed the time scales associated with Brownian motion: the relaxation time $t_{\rm relax}$, the collision duration time $t_{\rm coll}$, and the mean-free-path time $t_{\rm mfp}$, which are important quantities characterizing the properties of the plasma. $t_{\rm relax}$ and $t_{\rm coll}$ can be straightforwardly read off from the parameters $\gamma(t), \kappa(t)$ characterizing the Langevin equation. The result is [30]

$$t_{\rm relax} \sim \frac{m}{\sqrt{\lambda}\,T^2}, \qquad t_{\rm coll} \sim \frac{1}{T}, \tag{22}$$

where we borrowed the standard notation for the 't Hooft coupling for four dimensional $\mathcal{N} = 4$ super Yang-Mills and set $\lambda = \ell^4/\alpha^2$. The time scale $t_{\rm mfp}$ can be thought of as the relaxation time for a plasma constituent, because any particle in thermal environment, including the plasma constituent itself, will undergo Brownian motion. In a CFT, a quasi-particle in plasma should have energy of order T, because it is the only dimensional scale available. So, $t_{\rm mfp}$ can be estimated by replacing m in $t_{\rm relax}$ by T:

$$t_{\rm mfp} \sim \frac{1}{\sqrt{\lambda}\, T}. \tag{23}$$

In the weak coupling case, $\lambda \ll 1$, we have $t_{\rm relax} \gg t_{\rm mfp} \gg t_{\rm coll}$ and the conventional kinetic theory picture where the Brownian particle is hit by fluid particles once in a while is good. On the other hand, in the strong coupling case, we have $t_{\rm coll} \gg t_{\rm mfp}$ which implies that a plasma particle is interacting with many other plasma particles simultaneously. This is presumably closely related to the observed small thermalization time of the QCD quark-gluon plasmas. This fact is also reminiscent of the conjecture [31, 32] that black holes can scramble information very fast, whose dual picture is that a degree of freedom in the boundary theory interacts with a huge number of other degrees of freedom simultaneously.

One can argue that the mean-free-path time $t_{\rm mfp}$ can be directly computed from the connected 2- and 4-point functions of the random force $R(t)$ [30]. From the bulk point of view, connected 4-point functions receive contribution from the quartic terms that appear in the expansion of the Nambu-Goto action as in (10). Such 4-point functions should be computable [30, 33] making use of the formalism for Lorentzian AdS/CFT correspondence [34, 35].

Acknowledgment

I would like to thank Ardian Atmaja, Jan de Boer, Veronika Hubeny, Mukund Rangamani, and Koenraad Schalm for collaborations, including [30], which the current article is based on.

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