Buckling of thin shells: Recent advances and trends

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This paper provides a review of recent research advances and trends in the area of thin shell buckling. Only the more important and interesting aspects of recent research, judged from a personal view point, are discussed. In particular, the following topics are given emphasis: (a) imperfections in real structures and their influence; (b) buckling of shells under local/non-uniform loads and localized compressive stresses; and (c) the use of computer buckling analysis in the stability design of complex thin shell structures.

1 INTRODUCTION

Thin-shell structures find wide applications in many branches of engineering. Examples include aircraft, spacecraft, cooling towers, nuclear reactors, steel silos and tanks for bulk solid and liquid storage, pressure vessels, pipelines and offshore platforms. Because of the thinness of these structures, buckling is often the controlling failure mode. It is therefore essential that their buckling behavior be properly understood so that suitable design methods can be established.

This paper provides a review of recent research advances and trends in the area of thin shell buckling. The paper is not intended to be an exhaustive review of the field, nor is it possible to do so in a single paper of limited length. Instead, only the more important and interesting aspects of recent research, judged from a personal viewpoint, will be discussed. In particular, the following topics are given emphasis: (a) imperfections in real structures and their influence; (b) buckling of shells under local/non-uniform loads and localized compressive stresses; and (c) the use of computer buckling analysis in the stability design of complex thin shell structures. The author wishes to apologize in advance for any inadvertent omission of relevant publications.

2 BRIEF HISTORICAL NOTES

2.1 General

Shell structures are widely used in many fields and have been studied actively for more than one hundred years (Calladine, 1988). The first shell buckling problem solved was cylindrical shells under axial compression (Lorenz, 1908; Timoshenko, 1910; Southwell, 1914). Early tests (Robertson, 1929; Flugge, 1932; Lundquist, 1933; Wilson and Newmark, 1933) indicated that real cylinders buckle at loads much lower than the classical buckling load, which is the linear bifurcation load based on the assumptions of simple supports and a membrane state of prebuckling stress distribution. Experimental buckling loads as low as 30% of the classical load are not uncommon. The search for reasons responsible for this discrepancy led to an enormous amount of research in the subsequent decades. Researchers have chiefly attributed this discrepancy to the effects of (a) boundary conditions, (b) prebuckling deformations, (c) geometric imperfections, and (d) load eccentricities. The effects of these factors have thus been investigated for many shell buckling problems. In the following, their effects are discussed briefly for axially compressed isotropic and stringer-stiffened cylinders respectively. This discussion not only illustrates the different roles these factors play in the two problems, but also constitutes a brief historical glimpse of thin shell buckling research, as it is fair to state that the foundations of shell stability theory were almost all laid in studying axially compressed cylinders.

2.2 Axially compressed isotropic cylinders

The effect of various boundary conditions, especially the inplane ones, on the buckling strength of cylindrical shells has been explored in detail using the membrane prebuckling stress assumption (Ohira, 1961, 1963; Stein, 1962; Hoff, 1965; Hoff and Rehfield, 1965; Hoff and Soong, 1965; Hoff, 1966; Almroth, 1966;). Although isotropic cylindrical shells with ends permitted to move in the circumferential direction have been found to buckle at a stress one half that of the classical prediction, it cannot explain the difference between the classical prediction and experiment because such boundary conditions rarely exist in actual shells.

The classical linear buckling theory assumes a membrane state of stress before buckling. In the case of a cylindrical shell under axial compression, this implies that the compressed cylinder is free to expand laterally. A free expansion is usually not possible in experiments or in real structures, so bending stresses and deformations are expected near the ends which can reduce the buckling load of the axially compressed cylinder. The effect of bending stresses and prebuckling deformations was first investigated by Fischer

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(1962, 1963). Stein (1962, 1964) also considered the prebuckling bending stresses and deformations and reported buckling strengths of the order of half the classical value. However, subsequent studies by Hoff and his co-workers (Hoff, 1966) revealed that these were due to the relaxed boundary conditions of freedom to displace circumferentially, and that similar reductions could be found using the membrane stress assumption. Later studies (Fischer, 1965; Almroth, 1966; Gorman and Evan-Iwanowski, 1970; Yamaki and Kodama, 1972) showed that the effect of prebuckling deformations is small and is not a primary reason for the difference between the classical prediction and experimental results and the great scatter of experimental results.

For axially compressed isotropic cylinders, small load eccentricities do not have a major influence on the buckling strength (Simitses, 1985 *et al*). The single dominant factor contributing to the discrepancy between theory and experiment for axially compressed isotropic cylinders is initial geometric imperfections. An enormous amount of research has therefore been carried out on the imperfection sensitivity of shell buckling. The most notable contributors in this research include von Kármán and Tsien (1941), Donnell and Wan (1950), Koiter (1945), Budiansky and Hutchinson (1966). For a thorough description of the buckling behavior of cylinders under various uniform loads, the book by Yamaki (1984) should be consulted. A recent review on imperfect cylinder buckling is that by Simitses (1986).

2.3 Axially compressed stringer stiffened cylinders

While the effect of initial geometric imperfections is the dominant factor responsible for the discrepancy between the classical theory and experiment, this effect is less pronounced in cylindrical shells with a significant amount of stringer stiffening. The imperfection sensitivity of stringer-stiffened cylinders depends on the geometry of stiffeners, particularly the ratio $A_s/(bt)$, where A_s, is the cross-section area of the stringer, b is the circumferential distance between stringers and t is the thickness of the shell skin (Weller and Singer, 1977). The effect of boundary conditions now becomes predominant (Weller, 1978). For example, axial restraint may raise the buckling load of a shell by 50% or more. Singer and his coworkers (Weller *et al*, 1974; Singer and Rosen, 1976; Weller and Singer, 1977; Weller, 1978; Singer and Abramovich, 1979; Singer, 1982a; 1982b; 1983) have contributed greatly to our knowledge of buckling of stiffened cylinders under axial compression, with special emphasis on the effects of boundary conditions and load eccentricities. The effect of load eccentricities for stringer-stiffened cylinders was first addressed by Stuhlman *et al* (1966), followed by extensive studies by Singer and his coworkers (Weller *et al*, 1974; Singer, 1983). These studies have shown that differences in buckling loads due to the effect of load eccentricities can be up to 50% for some practical configurations. Prebuckling deformations are only important when the shell is short enough for the non-uniformity caused by them to occur over most of the shell length (Weller *et al*, 1974).

The many studies by Singer and his co-workers have demonstrated conclusively that the influence of geometric imperfections should be introduced after effects of boundary conditions and load eccentricities have been determined to produce accurate buckling predictions. They have developed a non-destructive vibration correlation technique which can be used to define the boundary conditions and load eccentricities as well as to determine buckling loads directly (Singer and Abramovich, 1979; Singer, 1982a; Singer, 1982b; 1983).

2.4 Books and reviews on buckling of thin shells

Many other shell buckling problems have been studied in the past decades, and quite a number of books and review articles have been written on the subject. A recent article (Noor, 1990) provides an extensive list of books, conference proceedings and survey articles on shell structures. A number of books were published around ten years ago (Calladine, 1983a; Kollar and Dulacska, 1984; Yamaki, 1984; Bushnell, 1985) which provide a wealth of information on the buckling behavior and strength of thin shells. A handbook for shell stability design has recently been produced by Samuelson and Eggwertz (1992) which can be particularly useful to designers. Conference proceedings (Fung and Sechler, 1974; Koiter and Mikhailov, 1980; Ramm, 1982; Thompson and Hunt, 1983; Jullien, 1991) and paper collections (Zamrik and Dietrich, 1982; Harding *et al*, 1982; Narayanan, 1985; Galletly, 1995) are also sources of very useful information.

Many review articles on thin shell buckling have also been written. Nash (1960) summarized early achievements in shell buckling research. Hoff (1966) discussed the buckling behavior of cylinders with various boundary conditions. Work on elastic postbuckling and imperfection sensitivity has been reviewed in Hutchinson and Koiter (1970), Budiansky and Hutchinson (1966, 1979), Tvergaard (1976) and Citerley (1982). Babcock (1983) addressed the aspects of imperfection sensitivity, dynamic buckling, plastic buckling, experiments and computer buckling analysis in his review of shell buckling research in general. Babcock (1974) and Singer (1980, 1982b) surveyed experimental research on shell buckling. Bushnell (1982) examined many plastic buckling problems and the computer analysis of plastic buckling. The most recent review article is that of Simitses (1986) published almost a decade ago, in which the buckling and postbuckling behavior of imperfect cylinders was discussed in detail.

The past decade has seen further major advances in thin shell buckling research. This review is therefore prepared to provide a brief survey of the developments in the past decade and a more detailed discussion of several areas of intensive recent interest outlined earlier. Both tasks have not been undertaken in any previous review articles.

3 CURRENT STATUS

Despite extensive research over many decades, our knowledge of many shell buckling problems is still very limited. Consequently, shell stability design criteria contained in design codes for various structures such as tanks (*eg*, API 620, 1982), silos (*eg*, Trahair *et al*, 1983), pressure vessels (*eg*, ASME, 1986; BS 5500, 1988; AS 1210, 1989) and offshore platforms (*eg*, API RP2A, 1989) generally cover only the basic geometries of cylinders, cones, and spheres, and simple loading conditions such as uniform axial compression, uniform normal pressure, uniform torsion and bending, or a combination of some of them. This limitation also applies to the general code for steel shell buckling developed by the European Convention for Constructional Steelwork (ECCS, 1988).

This lack of knowledge has been due to the two main difficulties encountered in shell buckling research in the previous decades. First, the buckling phenomenon in shells is a highly complex one, described by nonlinear partial differential equations too difficult to solve except for a few simple cases before the computer era. Second, unlike beams and plates, buckling of shells is generally sensitive to small geometric imperfections induced in the fabrication process. Theoretical buckling loads obtained assuming a perfect geometry often greatly overestimate the actual strength of a shell, so design methods have relied heavily on extensive experimental data which are available only for a limited number of cases. Indeed, in the second edition of *Stability of Metal Structures: World View* (Beedle, 1991), it was pointed out that there was the greatest need for more experimental data in the area of shell buckling.

The availability of powerful computers and development of sophisticated finite element and other numerical techniques in recent years have changed the situation drastically (Bushnell, 1985; Yang *et al*, 1990). No longer is it impossible to numerically solve a specific complicated nonlinear buckling problem: indeed there are scores of proprietary and commercial codes based on the finite element method as well as other numerical methods (*eg*, Bushnell, 1976; Almroth and Brogan, 1978; Esslinger *et al*, 1984; Wunderlich *et al*, 1985; Combescure *et al*, 1987; Teng and Rotter, 1989a; 1989b; Ravichandran *et al*, 1994; Hibbit, Karlsson and Sorensen Inc., 1993) which can perform the task. Thus, the first difficulty seems to have disappeared. This does not imply that there is now no need of research on shell buckling, instead great strides can now be made in understanding the buckling behavior of shells for a much wider range of problems. The second difficulty remains and requires more urgent attention if numerical buckling analyses are to be applied directly in design or to replace much of the experimental work previously required in the development of design methods. The establishment of a reliable procedure to convert a numerically obtained buckling load to the design strength of a shell is one of the most important challenges facing the shell buckling research community (Samuelson, 1991a).

4 UNIFORM LOADING AND REAL IMPERFECTIONS

Simple uniform loads including axial compression, external pressure, torsion and their combinations have received the most attention in the past. Indeed, much of our understanding of shell buckling has come from research on unstiffened and stiffened cylindrical shells under various uniform loads. For perfect shells of revolution under uniform loading, their buckling loads can now be readily obtained from finite element analyses employing an axisymmetric shell element (*eg*, Teng and Rotter, 1989b) or other similar numerical analyses (*eg*, Bushnell, 1976). However, because of their sensitivity to initial imperfections, the central theme of current research is how real imperfections affect their buckling strengths.

Buckling of imperfect cylindrical shells thus remains a subject of active research, with special emphasis on the effect of real imperfections as well as of boundary conditions and load eccentricity. Most early research on imperfection sensitivity was concerned with idealized imperfection forms and imperfections in small scale laboratory models, but it has now been well recognized that these are generally not representative of real imperfections in full scale structures. Babcock, Arbocz, Singer and their colleagues (Arbocz, 1982; 1991; Arbocz and Babcock, 1981; Arbocz and Hol, 1991; Elishakoff *et al*, 1987; Singer, 1982b; Singer and Abramovich, 1995; Weller *et al*, 1986) have pioneered in the precise measurement of imperfections in laboratory and full scale aeronautical shells, and the development of statistically-based design methods using measured imperfections in the last fifteen years. An International Imperfection Data Bank was established with branches in Delft and Haifa for the evaluation of imperfection measurements and correlation studies (Arbocz, 1982; Singer, 1982b). Their work also demonstrated that the form and amplitude of imperfections are dependent on the fabrication process and quality. This work has recently been extended to offshore shells (Chryssanthopoulos *et al*, 1991a; 1991b; Chryssanthopoulos and Poggi, 1995) and silos and tanks (Ding *et al*, 1991; Coleman *et al*, 1992; Rotter *et al*, 1992). The development of shell imperfection measurement techniques has been reviewed in a recent article by Singer and Abramovich (1995).

Calladine (1995) recently added a new dimension to imperfection sensitivity research by suggesting that in addition to the assumed-to-be-stress-free geometric imperfections, locked-in stresses likely to occur in shells with fixed boundaries may also be important in reducing the load carrying capacity of shell structures. He also presented a thought-provoking review of the ideas deployed in understanding imperfection sensitivity of axially compressed cylindrical shells in the same article.

Many other interesting works have been or are being conducted on shells under uniform loads. Examples include buckling of unstiffened cylinders (*eg*, Galletly *et al*, 1987; Shen and Chen, 1991) and stiffened cylinders (*eg*, Agelidis *et al*, 1982; Miller and Vojta, 1984; Croll, 1985; Abramovich *et al*, 1991; Dowling, 1991) under combined loading; effect of lap joints (Rotter and Teng, 1989a; Teng, 1994a), an axisymmetric weld depression (Rotter and Teng, 1989b), residual stresses (Ravn-Jensen and Tvergaard, 1990) and thickness variations (Koiter *et al*, 1994) on the buckling of axially compressed cylinders; buckling of liquid filled conical shells (*eg*, Vandepitte *et al*, 1982), paraboloidal shells (*eg*, Chen and Xu, 1992; Ishakov, 1993), corrugated shells (*eg*, Yeh *et al* 1992; Ross and Humphries, 1993), externally-pressurized steel domes (*eg*, Blachut *et al*, 1991; Galletly and Blachut, 1991; Blachut and Galletly, 1995), externally-pressurized toriconical heads (*eg*, Wunderlich *et al*, 1987) and torispherical heads (*eg*, Lu *et al*, 1995), toroidal shells (*eg*, Panagiotopoulos, 1985; Wang and Zhang, 1991; Bielski, 1992; Galletly and Blachut, 1995) and cooling towers (*eg*, Combescure *et al* 1987; Aflak and Jullien, 1991; Kaluza and Gigiel, 1995; Radwanska and Waszczszyn, 1995); the effect of internal pressure and an elastic core on cylinder strength (*eg*, Knoedel and Schulz, 1988; Rotter and Zhang, 1990; Zhang and Ansourian, 1991; Limam *et al*, 1991; Teng and Rotter, 1992a; Knoedel *et al*, 1995; Knebel and Schweizerhof, 1995); the optimization of shell form to resist various loads (*eg*, Blachut, 1987; Jullien and Araar, 1991; Reitinger and Ramm, 1995); buckle propagation in submarine pipelines (*eg*, Kyriakides and Babcock, 1981; Winter *et al*, 1985; Kamalarasa and Calladine, 1988; Hahn *et al*, 1992; Lin, *et al* 1993); lower bound buckling loads (*eg*, Croll, 1985; Yamada and Croll, 1993; Croll, 1995); buckling of buried pipelines (*eg*, Yun and Kyriakides, 1990; Moore and Selig, 1990); thermal buckling (*eg*, Combescure and Brochard, 1991); creep buckling (*eg*, Arnold *et al*, 1989; Miyazaki, 1988, 1992; Sammari and Jullien, 1995); dynamic buckling (*eg*, Saigal *et al*, 1987; Birch and Jones, 1990; Florence *et al*, 1991; Lindberg, 1991; Wang *et al*, 1993a; 1993b; Pedron and Combescure, 1995); non-destructive testing (*eg*, Singer, 1982b; Nicholls and Karbhari, 1989; Souza and Assaid, 1991) and development of design codes and guidelines (*eg*, Odland, 1991; Samuelson, 1991a; Schmidt, 1991; Akiyama *et al*, 1991; Dulacska and Kollar, 1995).

5 CYLINDRICAL SHELLS SUBJECT TO WIND AND EARTHQUAKE LOADS

Steel silos and tanks, when empty or partially filled, are susceptible to buckling failure under wind pressure. Several such buckling failures have occurred in the past (Ansourian, 1992), leading to a substantial research effort in this area (*eg*, Resigner and Greiner, 1982; Jerath and Sadid, 1985; Uchiyama, 1987; Blackler and Ansourian, 1988; Ansourian, 1992; Greiner and Derler, 1995). Simple design methods have also been developed and included in some codes (*eg*, BS 2654, 1989). Some interesting aspects considered include the effect of a rectangular cutout on the buckling strength of wind loaded cylinders (Jerath, 1987), the buckling of tanks with unrestrained upper edge which occurs during the construction stage (Saal and Schrufer, 1991), buckling of cylinders under combined wind and snow loads (Kapania, 1990) and buckling of long cylinders for which axial and shear stresses become important (Greiner and Derler, 1995).

Buckling of liquid-filled storage tanks due to seismic excitations has received a great deal of research in recent years (*eg*, Niwa and Clough, 1982; Fischer, *et al*, 1985; Shih, 1987; Uras and Liu, 1990, 1991; Chiba *et al*, 1987a, 1987b; Peek, 1989; Fujita *et al*, 1990; Haroun and Mourad, 1990; Peek and El-Bkaily, 1991; Zhou *et al*, 1992; Manos, 1994). Rammerstorfer and Scharf (1990) presented a comprehensive survey of research on storage tanks subject to earthquake loading, while Liu *et al* (1991) reviewed research advances in dynamic buckling analysis of liquid-filled shells.

6 CYLINDRICAL SHELLS SUBJECT TO HORIZONTAL SHEAR OR NON-UNIFORM TORSION

Short cylindrical shells are susceptible to buckling in shear when subject to lateral loads. Shear buckling was first examined by Lundquist (1935). A solution for the elastic buckling of a perfect cantilever cylinder subject to transverse shear was presented by Schroeder (1972). Yamaki *et al* (1979) conducted experiments on polyester cylinders subject to transverse shear and internal pressure. Galletly and Blachut (1985a) performed plastic shear buckling experiments on steel cylinders and proposed a simple design equation. Further plastic shear buckling tests were conducted by Dostal *et al* (1987). The plastic shear buckling problem has been investigated further recently by Kulak, Elwi and their associates for large diameter fabricated tubes in a number of experimental (Bailey and Kulak, 1984; Obaia *et al*, 1992a) and finite element (Mok and Elwi, 1986; Roman and Elwi, 1989) studies, and an improved design equation has been established (Obaia *et al*, 1992b). The effect of imperfections was examined both numerically (Mok and Elwi, 1986; Roman and Elwi, 1989; Kokubo *et al*, 1993; Murakami and Yoguchi, 1991) and experimentally (Murakami and Yoguchi, 1991) and was found to be moderate. Roman and Elwi (1989) demonstrated that residual stresses due to a cold-bending fabrication process can lead to a large reduction in the ultimate load, but the effect of the residual stresses due to a longitudinal seam-weld is insignificant. The postbuckling load carrying mechanism for a cylindrical shell with end ring stiffeners may be regarded as a tension field anchored in the ring stiffeners (Bailey and Kulak, 1984) and modeled as an equivalent truss mechanism (Roman and Elwi, 1988). Plastic buckling of cylindrical shells under transverse shear in combination with other loads has also been considered (*eg*, Akiyama *et al*, 1987; Kokubo *et al*, 1993).

Ground-supported silos subject to unbalanced horizontal loads are usually subject to membrane shear stresses which vary approximately linearly in the axial direction (*eg*, Rotter and Hull, 1989). The elastic buckling of cylinders subject to shear stresses of linear longitudinal variation (non-uniform torsion) was examined by Jumikis and Rotter (1986) who also developed a design equation.

7 LOCALIZED CIRCUMFERENTIAL COMPRESSION

There is a class of shell structures in which localized circumferential compressive stresses arise under loads which are apparently tensile. Consequently, buckling failure is possible in these shells. Examples include pressure vessel heads, circular plates under a transverse load, and hemispheres under axial tension. These problems are normally not very sensitive to initial imperfections because the circumferential compression is localized and apart from this localized compression, both membrane stresses are tensile throughout the shell.

Torispherical shells have been studied extensively (*eg*, Adachi and Benicek, 1964; Galletly, 1985; Galletly and Blachut, 1985b; Roche and Autrusson, 1986; Galletly *et al*, 1990; Soric, 1990; 1995) and a design method has been developed and included into several codes including the ECCS (1988) code. Hagihara *et al* (1991) analyzed the bifurcation buckling of torispherical heads dynamically loaded by internal pressure. Studies into the collapse and buckling of cone-cylinder intersections under internal pressure have recently been undertaken and simple strength equations have been established (Teng, 1994b, 1995a, 1995b). Other problems already studied include truncated hemispheres under axial tension (Yao, 1963), spherical cargo tanks for liquid natural gas (Pedersen and Jensen, 1976; 1995), spherical caps with a movable edge under internal pressure (Shilkrut, 1983), circular plates under a central point load (Adams, 1993) and plate-end pressure vessels (Teng and Rotter, 1989c).

8 CYLINDRICAL SHELLS SUBJECT TO LOCAL AXIAL COMPRESSION

A cylinder in a column-supported silo or tank is a common example where buckling under local axial compression is important. Despite the extensive research efforts into shell buckling of the last few decades, only a few studies (Abir and Nardo, 1958; Bijlaard and Gallagher, 1959; Hoff *et al*, 1964; Johns, 1966; Libai and Durban, 1973, 1977; Peter, 1974) examined the linear bifurcation buckling of perfect cylinders under circumferentially varying axial loads before a recent resurgence of interest. A simple conclusion from this work might be that buckling of a perfect shell occurs under a circumferentially non-uniform distribution of axial stress when the maximum stress is slightly higher than the classical elastic critical value for uniform axial compression. Libai and Durban (1977) gave simple expressions which describe the increase in buckling stress above this simple rule, but the strength gains are generally small for thin shells. Li (1990) studied the linear bifurcation buckling of simply supported cylinders under many equally-spaced discrete axial forces as an imperfect realization of uniform axial compression.

The above studies are entirely restricted to linear bifurcation in perfect cylinders, and thus do not address the practical buckling strength question. The loading and boundary conditions considered also do not represent discrete column forces on shells. In order to develop stability design criteria for cylinders under local axial compression, particularly for metal column-supported tanks and silos, many intensive studies have been carried out recently. Teng and Rotter (1991a) appears to be the first to investigate the nonlinear buckling behavior of column-supported cylinders numerically, followed by several other studies (Ramm and Butcher, 1991; Rotter *et al*, 1991; Teng and Rotter, 1992b; Guggenberger, 1991; Dhanens *et al*, 1993; She and Rotter, 1993). A summary of this work has been given in Rotter (1993) and a design proposal was submitted for inclusion in the ECCS (1988) code (Rotter *et al*, 1993). The nonlinear buckling of column-supported imperfect cylinders has also been analyzed by Gould *et al* (1994, 1995) recently using their local-global method (Gould and Ravichandran, 1993; Harintho and Gould, 1994; Ravichandran *et al*, 1992, 1994) which can also deal with other local effects efficiently. So far, the problem treated is that of a cylinder directly supported on columns or under local axial edge loads. Many other questions remain to be answered and these include the strength of shells with more complicated arrangements of local supports (Gorenc, 1985).

9 CYLINDERS SUBJECT TO OTHER LOCAL/NON-UNIFORM LOADS OR WITH LOCAL GEOMETRIC DISTURBANCES

The effect of local disturbances and loads on axially compressed cylinders has been the topic of several recent studies. Research has been carried out on axially compressed cylindrical shells with cutouts (Tennyson, 1968; Almroth and Holmes, 1972; Almroth *et al* 1973; Starnes, 1974; Montague and Horne, 1981; Miller, 1982; Toda, 1983; Knoedel and Schulz, 1985; Allen *et al*, 1990), and design procedures have been formulated (Eggwertz and Samuelson, 1991a; Samuelson and Eggwertz, 1992). The effect of a local vertical stiffener (Eggwertz and Samuelson, 1991b), additional local loads (Samuelson, 1985, 1991b) and a single or multiple localized deep dents as may be caused by collision damage (Krishnakumar and Foster, 1991a, 1991b) has also been addressed.

The effect of a single deep longitudinal dent on the buckling of externally pressurized cylinders has been studied by Schmidt (1986) and Guggenberger (1995) which showed that the effect of a single longitudinal dent on buckling strength is small and tends to vanish with increasing dent amplitude.

Early studies on the buckling of cylinders subject to non-uniform external pressure were carried out by Almroth (1962), Weingarten (1962) and Uemura and Morita (1971). Recent investigations into the buckling of cylinders subject to non-uniform or partial external pressure have been undertaken by Wei and Shun (1988), Chiba *et al* (1989), Ramm and Butcher (1991), and Sengupta and Ansourian (1994). Sengupta and Ansourian (1994) considered the case of external pressures which are non-uniform around the circumference but uniform longitudinally and derived a simple design formula from finite element results.

A significant number of studies have examined stability problems in horizontal storage vessels and some simple design methods have been developed (Saal, 1982; Tooth and Susatijo, 1983; Krupka, 1987, 1991a, 1991b, 1991c, 1992; Ansourian and Sengupta, 1993).

Plastic buckling of cylinders in bending has received much attention in recent years (Bushnell, 1981; Calladine, 1983b; Kyriakides and Shaw, 1987; Ju and Kyriakides, 1991a, 1991b, 1992; Kyriakides and Ju, 1992; Murray and Bilston, 1992; Li and Molyneaux, 1994). Many other local or non-uniform load problems need further research, including silos and tanks subject to uneven foundation settlement (Palmer, 1992) and silos under eccentric discharge (Rotter, 1986; Bucklin *et al*, 1990; Rotter, 1993).

10 PLASTICITY MODELS IN PLASTIC BIFURCATION ANALYSIS

The problems discussed in previous sections include both elastic and plastic buckling, so a separate discussion of the latter is not given here. The review paper by Bushnell (1982) on plastic buckling may be consulted for more information. However, it is interesting to mention several recent studies that attempt to resolve the difference between flow theory and deformation theory of plasticity in predicting plastic bifurcation buckling loads.

For plastic bifurcation buckling of plates and shells, the paradox remains that the analytically less satisfactory deformation theory of plasticity gives results in closer agreement with experimental results than the more rigorous and well accepted flow theory. The many available explanations are still inconclusive, and recent results once again confirm the better agreement between deformation theory and experiment (Ore and Durban, 1989, 1992; Galletly *et al*, 1990; Giezen *et al*, 1991). As bifurcation analyses using the deformation theory usually also predict lower buckling loads than those from a flow theory analysis, the deformation theory option has been included in some computer codes (*eg*, Bushnell, 1976; Teng and Rotter, 1989b). A third alternative is to use the shear modulus for deformation theory in the flow theory (*eg*, Bushnell, 1976; Teng and Rotter, 1989b), as it has been shown that the difference in shear modulus in the inelastic range for the two theories is solely responsible for the difference in the obtained torsional buckling load of a cruciform column. Roche and Autrusson (1986), Bodner and Naveh (1988) and Combescure (1991) have also discussed some other possible stress-strain relations.

11 RING STIFFENERS

Ring stiffeners are often an integral part of shell structures. One common mode of failure is the out-of-plane buckling of rings (Bushnell, 1977), especially when they are attached to a shell intersection. Several recent studies have been conducted on rings at shell intersections (Jumikis and Rotter, 1983; Rotter and Jumikis, 1985; Rotter, 1987; Sharma *et al*, 1987; Teng and Rotter, 1988, 1989d, 1991b, 1992c; Greiner, 1991; Teng and Lucas, 1994). Esslinger and Geier (1993) and Louca and Harding (1994) investigated the torsional buckling strength of ring stiffeners on externally pressurized cylinders.

In the finite element modeling of ring-stiffened shells, rings can be modeled either as shell branches using shell theory or as discrete rings (Bushnell, 1985). Among the existing nonlinear shell theories, that due to Sanders' (1963) is the most widely used. The accuracy of Sanders' (1963) theory has been generally accepted as being satisfactory for thin shells for all practical purposes. However, Rotter and Jumikis (1988) recently pointed out that while Sanders' theory works well when buckling is dominated by normal displacements of the shell surface which is the case for most problems, it can lead to erroneous results when applied in a conventional manner to problems where buckling is dominated by in-plane displacements. This is because nonlinear strains arising from in-plane displacements are not properly accounted in Sanders' theory. One example is the buckling of ring-stiffened shells of revolution in which the ring buckles in its own plane. Rotter and Jumikis (1988) derived a new nonlinear theory for thin shells of revolution which includes nonlinear strain terms arising from in-plane displacements. They demonstrated that their theory is the only one which gives the correct classical ring buckling load for the in-plane buckling of an annular plate if the annular plate is modeled using axisymmetric shell elements (Rotter and Jumikis, 1988).

12 SHELL STABILITY DESIGN BY NUMERICAL BUCKLING ANALYSIS

The obvious difficulty an engineer faces in directly using numerical shell buckling analysis in design is how to convert his numerical buckling load based on any of the several types of buckling analysis to the design strength of his particular structure. Until this problem is satisfactorily solved, direct use of computer buckling analysis in design is difficult. Several approaches have been considered by various researchers and code writing committees (Schmidt and Krysik, 1991; Speicher and Saal, 1991; Samuelson and Eggwertz, 1992; Teng and Rotter, 1995).

In the first approach, a linear elastic bifurcation buckling analysis is carried out to determine the bifurcation load of the perfect structure. Reduction factors can then be applied to account for geometric imperfections and plasticity. This approach has perhaps the greatest appeal to the designer as it is similar to current code rules for simple load cases, and a linear bifurcation analysis is relatively easy to perform. The difficulty lies in deriving appropriate reductions factors for different loading and support conditions (Schmidt and Krysik, 1991; Samuelson and Eggwertz, 1992).

On the other hand, a fully nonlinear analysis can be carried out with large deflections, geometric imperfections and plasticity properly modeled. Here, the difficulty is in establishing the form and amplitude of imperfections to be used in the nonlinear analysis. Speicher and Saal (1991) have proposed that an equivalent imperfection of the same form as the first bifurcation mode be used and have also deduced the required magnitude of this equivalent imperfection to produce a safe design, based on existing test results for cylindrical shells. This method of determining the amplitude and form of imperfection is perhaps the best possible if no information is available on the amplitude or form of real imperfections in the structure under consideration. The most rational approach, however, will be the establishment of a statistically based imperfection model for a particular class of shell structures fabricated by the same process, based on extensive measurements of real geometric imperfections on full scale structures (Arbocz, 1991; Rotter *et al*, 1992). Such an imperfection model distinguishes between high quality and low quality shells, and defines strength in terms of reliability. While this approach is rational, its establishment relies on extensive measurement data which are currently not available (Rotter *et al*, 1992).

Intermediate approaches where one of the two factors (ie imperfections and plasticity) is included in the numerical analysis have also been discussed, but the two approaches described above seem to be the most promising. However, for axisymmetric problems, a nonlinear bifurcation analysis, considering prebuckling large deflections and plastic material behavior, may be used as a compromise between a linear bifurcation and a fully nonlinear analysis if axisymmetric imperfections are dominant or effective axisymmetric imperfections can lead to good results (Almroth *et al*, 1970; Teng and Rotter, 1995). For example, the simple axisymmetric weld depression proposed by Rotter and Teng (1989b) has been shown to represent the dominant components of imperfections in full scale steel silos and to produce buckling strengths comparable to those from current design criteria when its amplitude is of the order of shell wall thickness. It is expected that this weld depression (or its extension) may be used to obtain a good approximation in other shells of revolution under predominantly meridional compression of uniform or non-uniform distribution. This approach, including both plasticity and imperfection, can lead to a close estimate of the ultimate strength directly. The amplitude of weld depression in any analysis still needs to be carefully chosen based on past measurements or engineering judgment, or deduced from existing experimental results or design criteria.

Accurate modeling of boundary conditions does not seem to have been addressed in the context of stability design by numerical buckling analysis, although this is also important (Singer and Rosen, 1976). As pointed out by Bushnell (1985), the difficulty in defining accurately the boundary conditions can be largely avoided if all parts of the structure, or all parts between stations at which there is no doubt about the actual boundary conditions, are included in the model, so assumptions are not made about boundary conditions and load eccentricities. When some assumptions have to be made about uncertain boundary conditions, a conservative model should be used to ensure safety.

13 CONCLUSION

A review of recent research on thin shell buckling has been presented. Research on buckling of thin shell is still very active, and will remain so for a long time to come because of the many unsolved practical problems. In particular, buckling of shells under local or non-uniform loads, the effect of real imperfections, the direct application of numerical buckling analysis in design, and the development of simple design methods for commonly encountered engineering problems will be some of the most important themes of future investigations.

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