Bulk Locality and Quantum Error Correction in AdS/CFT

Daniel Harlow

Princeton University - PCTS

June 25, 2015

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- Today however I will be interested in understanding the duality at a fixed time in the Schrodinger representation; this is essential if we wish to understand the relevance of entanglement for the emergence of the bulk theory.
- We will see that formulating the definition this way leads to some surprising consequences, which can be naturally understood in the language of "quantum error correction", a subject first developed as part of quantum computation theory. Almheiri/Dong/Harlow, Harlow/Pastawski/Preskill/Yoshida

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- Introduce a discrete model of AdS/CFT, which realizes many of its interesting features in an exactly soluble context.

Altogether I believe this adds up to a new, and more precise, understanding of "how" holography works in AdS/CFT.

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- The Hamiltonians are equivalent, as are the other generators of the AdS symmetries.
- For any bulk field $\phi(x)$, as we pull it to the boundary it becomes a CFT local operator:

$$\lim_{r\to\infty} \phi(t,r,\Omega)r^{\Delta} = \mathcal{O}(t,\Omega).$$

This is sometimes called the "extrapolate dictionary".

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This will ultimately be important, but I will ignore it for now and see how far we can go before getting into trouble.

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- Obeys the bulk equation of motion as an operator equation.
- Is consistent with the extrapolate dictionary.

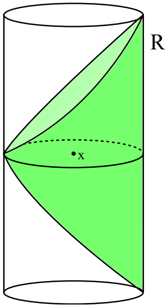
These two conditions give us a PDE that we can hope to solve uniquely, at least order by order in $1/N$.

Banks/Douglas/Horowitz/Martinec, Hamilton/Kabat/Lifschytz/Low, Heemkerk/Marolf/Polchinski/Sully

This procedure leads to formulas like:

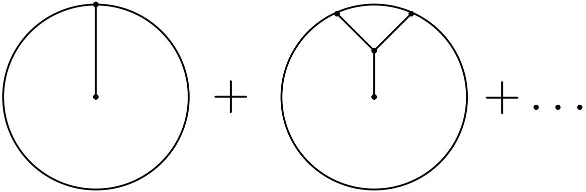
$$\phi(x) = \int_R dX\ K(x;X)\mathcal{O}(X) + O(1/N),$$

where $K(x;X)$ is a "smearing function".



This is often called *global reconstruction*.

The $1/N$ corrections can be computed diagrammatically:



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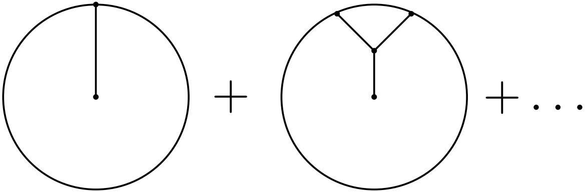

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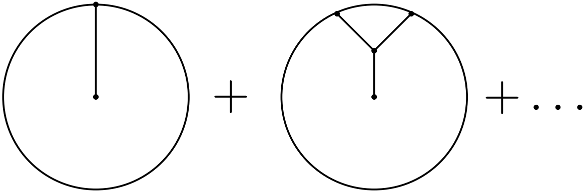

They are important in understanding how this construction implements backreaction; for example if we consider a state with a planet in it then, as in electrodynamics, there will be an infinite subclass of diagrams that we should resum to correct the smearing function to be a solution in the new background.
(This is *not* "state-dependence" of the type that is sometimes argued to be relevant for the black hole interior, and it is quite consistent with the linearity of quantum mechanics. Harlow, Marolf/Polchinski)

Bulk algebra in the CFT

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- One example of something that would break is the *algebra* of the operators; for example we want to have

$$\langle \Omega | \phi \dots [\phi(x), \phi(y)] \dots \phi | \Omega \rangle = 0 \qquad (x-y)^2 > 0,$$

but this usually won't be true in the CFT unless we use the right EOM. Kabat/Lifschytz/Lowe

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- In fact there is a simple argument that this type of commutator cannot vanish, or even be small, as a quantum operator.

  Almheiri/Dong/Harlow

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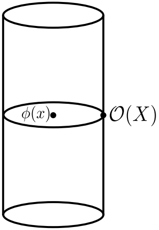

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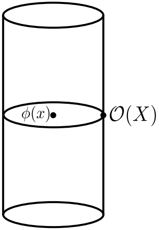

Here $\mathcal{O}(X)$ is some arbitrary local boundary operator. Do we have

$$[\phi(x), \mathcal{O}(X)] = 0?$$

No!

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This would be inconsistent with a standard property of quantum field theory, which is called the "time-slice axiom" (or "primitive causality"):

- For any $\epsilon > 0$, any bounded operator that commutes with all local operators in a time slice of thickness ϵ about some Cauchy surface Σ must be proportional to the identity operator. Streater/Wightman, Haag

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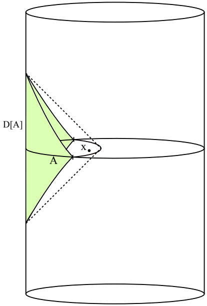
But then how do we express it? More generally, how do we think about the emergence of the bulk algebra?

Subregion duality

To proceed, I need one more tool; the *AdS-Rindler reconstruction*

Hamilton/Kabat/Lifschytz/Lowe, Morrison:



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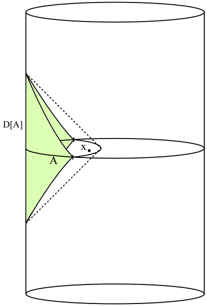

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In the bulk it is equivalent to the global reconstruction; they are related by a Bogoliubov transformation.

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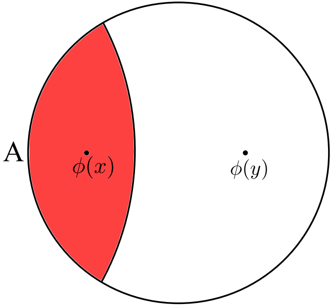

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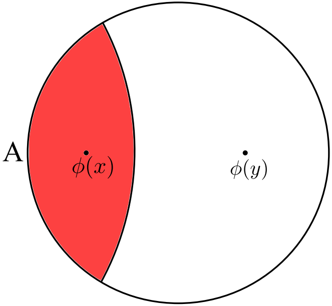

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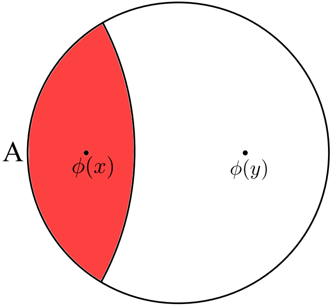

The operator $\phi(x)$ can be represented on A, but the operator $\phi(y)$ cannot. This is a fairly precise realization of "subregion-subregion duality"

Bousso/Freivogel/Leichenauer/Rosenhaus/Zukowski, Czech/Karczmarek/Nogueira/Van Raamsdonk, Hubeny/Rangamani.

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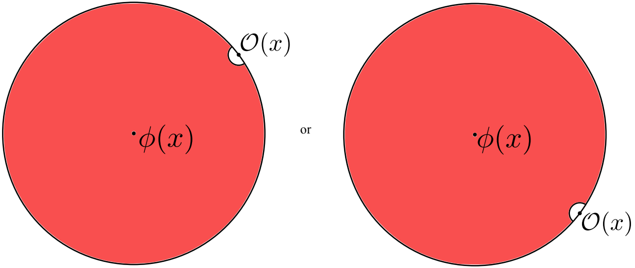

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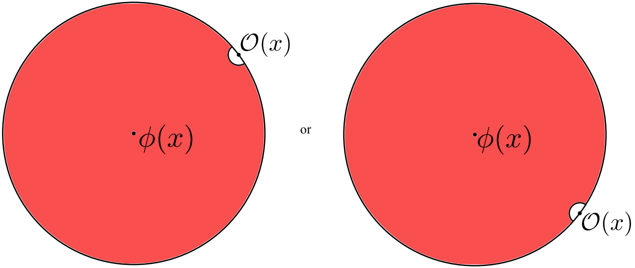

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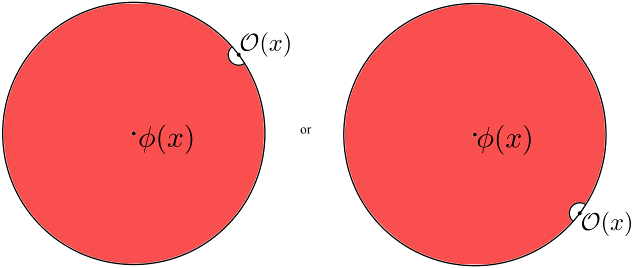
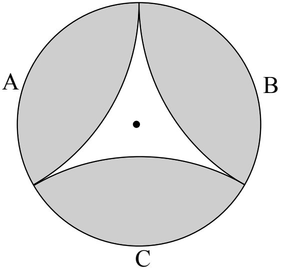


We can always find a wedge reconstruction of $\phi(x)$ such that $[\phi(x), \mathcal{O}(X)] = 0$.
This can only be consistent with the time-slice axiom if the different representations aren't actually equal as operators!

Another illustration:



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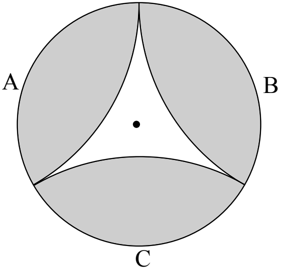

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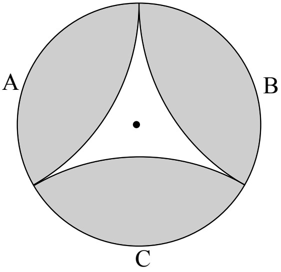

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Something interesting is going on here, but what is it?

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- Nonetheless, there is a way of a encoding the state which protects it against postal corruption - quantum error correction.
- QEC was first developed as a necessary part of building a quantum computer: decoherence of your memory is almost inevitable, so you need a way to fix it!

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The idea is to instead send you three qutrits in the state

$$|\widetilde{\psi}\rangle = \sum_{i=0}^{2} C_i |\widetilde{i}\rangle,$$

where $|\widetilde{i}\rangle$ is a basis for a special subspace of the full 27-dimensional Hilbert space, which is called the *code subspace*.

Explicitly, we take

$$|\widetilde{0}\rangle = \frac{1}{\sqrt{3}}\left(|000\rangle + |111\rangle + |222\rangle\right)$$
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- In other words, any single qutrit has *no information* about the encoded state $|\widetilde{\psi}\rangle$.

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- This entanglement leads to the interesting property that in any state in the subspace, the density matrix on any one of the qutrits is maximally mixed, ie is given by $\frac{1}{3}\left(|0\rangle\langle 0| + |1\rangle\langle 1| + |2\rangle\langle 2|\right)$.
- In other words, any single qutrit has *no information* about the encoded state $|\widetilde{\psi}\rangle$.
- This leads to the remarkable fact that we can *completely recover* the quantum state from any two of the qutrits!

To see this explicitly, we can define a two-qutrit unitary operation U_{12} that acts as

$$\begin{array}{lll} |00\rangle \rightarrow |00\rangle & |11\rangle \rightarrow |01\rangle & |22\rangle \rightarrow |02\rangle \\ |01\rangle \rightarrow |12\rangle & |12\rangle \rightarrow |10\rangle & |20\rangle \rightarrow |11\rangle \\ |02\rangle \rightarrow |21\rangle & |10\rangle \rightarrow |22\rangle & |21\rangle \rightarrow |20\rangle \end{array} .$$

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- By symmetry there must also exist U_{13} and U_{23}.

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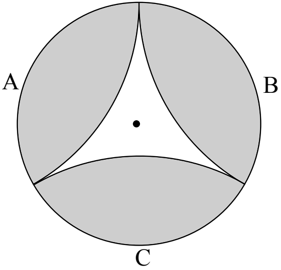
Generically this operator will have nontrival support on all three qutrits, but using our U_{12} we can define

$$O_{12} \equiv U_{12}^{\dagger} O_1 U_{12},$$

which acts nontrivially only on the first two but still implements O on the code subspace.

The point now is that we can interpret O_{12}, O_{13}, and O_{23} as being analogous to the representations of $\phi(0)$ on AB, AC, and BC in this example:



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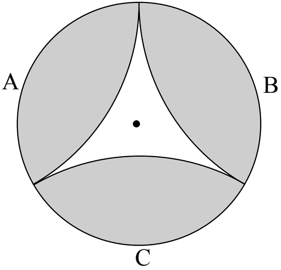

By using the entanglement of the code subspace, we can replicate the paradoxical properties of the AdS-Rindler reconstruction.

We can also make contact with the commutator puzzle: let's compute

$$\langle\widetilde{\psi}|[\widetilde{O}, X_3]|\widetilde{\phi}\rangle,$$

where X_3 is some operator on the third qutrit and $|\widetilde{\phi}\rangle$, $|\widetilde{\psi}\rangle$ are arbitrary states in the code subspace.

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This is the lesson to learn for AdS/CFT; the bulk algebra of operators *holds only on a subspace of states*!

Moreover this subspace must be *highly entangled* from the point of view of the local CFT degrees of freedom at fixed time.

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- Say that we have n physical qubits, and we want to protect a k-qubit message from an erasure of ℓ or fewer of the physical qubits. Then we need
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  and in fact for a typical 2^k-dimensional code subspace this is *sufficient*.

This is quite intuitive; sending a bigger message that is better protected requires more qubits!

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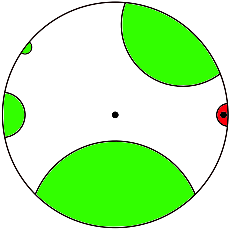

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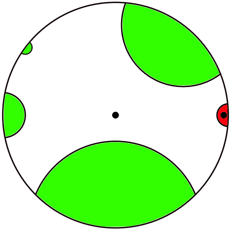

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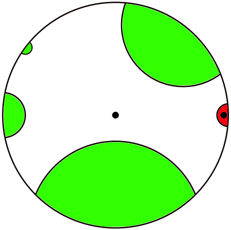
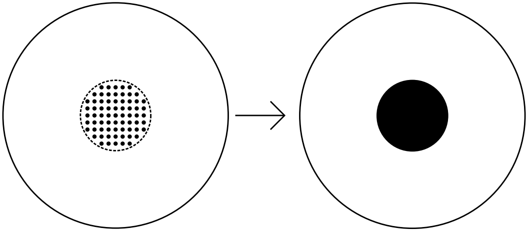


Indeed we need $\approx 1/2$ of the system to reconstruct the center.

Notice however that if we are NOT in the center we correct less well: this is a precise realization of the "radial direction $\leftrightarrow$ scale" correspondence.

We can now ramp up k:



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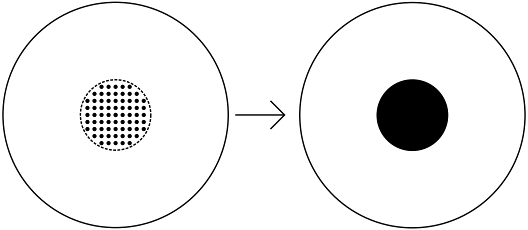

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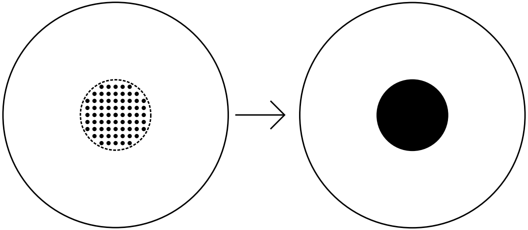

Clearly the answer will not change from $1/2$ until $k \sim N^2$, but on the bulk side this is just when we expect to create a huge black hole in the center! Thus we see that we are able to push our reconstruction of bulk operators just until the point where the old holographic arguments become relevant.

In fact recently I learned that this phenomenon can be experimentally realized:

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- They are based on methods developed in condensed matter theory and quantum information theory, called *tensor networks*.
- The basic idea is to replace the CFT by a spin system and then just write down a set of states whose entanglement structure closely resembles that of the low energy states of a CFT.

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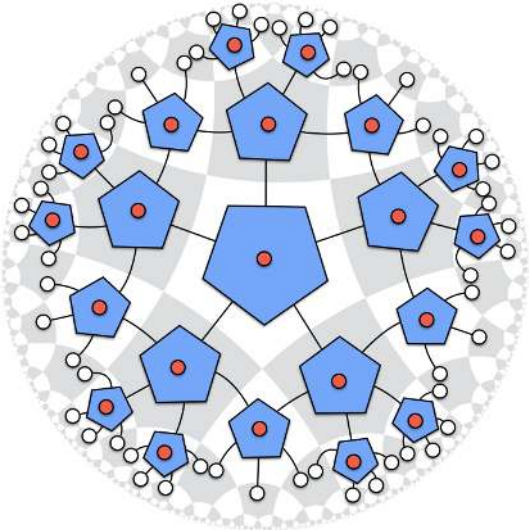
- The tensor network builds a big tensor $T_{i_1 \ldots i_n, j_1 \ldots j_k}$, which we use to define the subspace via

$$\langle i_1 \ldots i_n | j_1 \ldots j_k \rangle = T_{i_1 \ldots i_n, j_1 \ldots j_k}.$$

We build this big tensor from a tiling of the hyperbolic plane with pentagons, each of which has a special six-leg tensor in the center:



These six-leg tensors are special cases of what we call *perfect tensors*; any balanced bipartition gives a unitary transformation.

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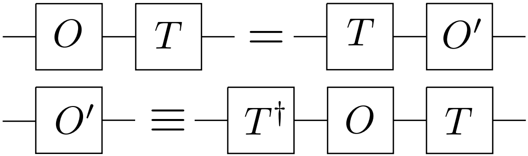
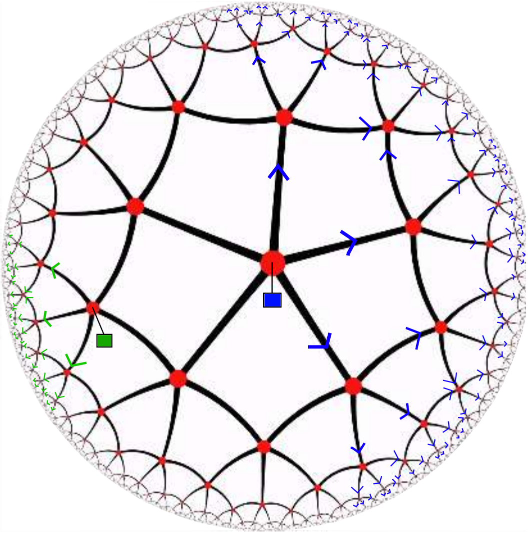
This allows us to "push operators through the tensors":

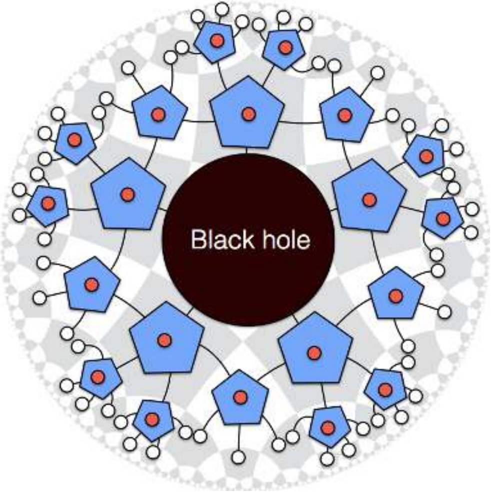


We can use this operation to do operator reconstruction:



This produces a full boundary realization of the bulk algebra!

You might ask about what happened to the rest of the Hilbert space. Our claim is that it is accounted for by black holes:



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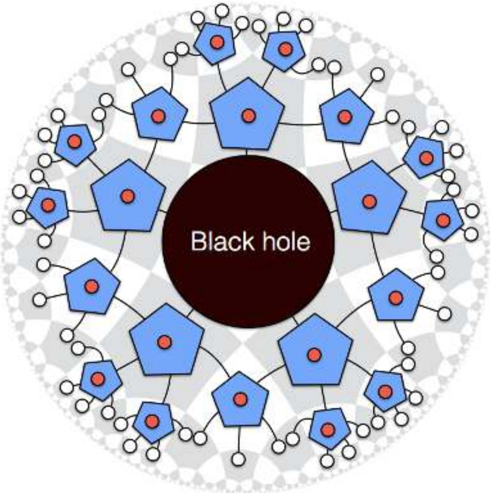

The entropy scales as the area, and we can still describe perturbative quanta outside of the horizon.

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Thanks for listening!

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- Quantum error correction gives a precise way of understanding "the emergence of spacetime from entanglement" that does not involve any violations of the linearity of quantum mechanics; this makes me more optimistic about this set of ideas.
- For big AdS black holes, whether the infalling observer sees a singularity at the horizon or the singularity is a sub-AdS scale question.