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# Business and Default Cycles for Credit Risk

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# Business and Default Cycles for Credit Risk\*

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# Business and Default Cycles for Credit Risk

## Abstract

Various economic theories are available to explain the existence of credit and default cycles. There remains empirical ambiguity, however, as to whether these cycles coincide. Recent papers suggest by their empirical research set-up that they do, or at least that defaults and credit spreads tend to co-move with macro-economic variables. If true, this is important for credit risk management as well as for regulation and systemic risk management. In this paper, we use 1933–1997 U.S. data on real GDP, credit spreads, and business failure rates to shed new light on the empirical evidence. We use a multivariate unobserved components framework to disentangle credit and business cycles. We distinguish two types of cycles in the data, corresponding to periods of around 6 and 11-16 years, respectively. Cyclical co-movements between GDP and business failures mainly arise at the longer frequency. At the higher frequency of 6 years, co-cyclicality is less clear-cut. We also show that spreads reveal a positive and negative co-cyclicality with failure rates and GDP, respectively. This pattern disappears, however, if we concentrate on the post World War II period. We comment on the implications of our findings for credit risk management.

**Key words:** credit cycles; business cycles; defaults; credit risk; procyclicality; multivariate unobserved component models.

**JEL Codes:** C19; G21.

## 1 Introduction

Credit risk research has considerably gained momentum over the last decade, see for example Caouette, Altman, and Narayanan (1998) and Allen and Saunders (2003) for an overview.<sup>1</sup> Spurred by regulatory developments, different classes of models have been put forward to measure, manage, and price credit risk. In this paper we study the dynamic behavior of two important determinants of credit risk, namely the default rate and the credit spread, in their relation to business cycle developments. We use a multivariate unobserved components approach to disentangle long-term patterns

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<sup>1</sup>See also the collection of papers at <http://www.defaultrisk.com>.

from shorter term cyclical patterns. We are particularly interested in testing whether cycles in credit risk factors coincide with business cycles. To answer this question, our model explicitly allows for different cyclical movements in credit risk factors and economic activity, as measured by real GDP.

Early credit risk models focus on the prediction of the likelihood to default (credit scoring) using, e.g., Altman's Z-score, logit and probit models, and neural networks, see Altman (1983) and Caouette et al. (1998). These models usually emphasize the cross-sectional rather than the time-series dimension of the sample to distinguish 'good' from 'bad' companies. The time-series or dynamic behavior of credit risk, however, has become increasingly important over the last few years among academics, practitioners, and regulators. Three reasons for this appear important.

First, the market for credit risk has become much more liquid, see for example Patel (2003). Asset backed securities like Collateralized Bond and Loan Obligations (CBOs and CLOs), as well as credit derivatives, allow financial institutions to mitigate their credit risk exposure without breaking up client-relationships. Appropriate pricing and hedging of these new generation credit instruments, however, requires an adequate description of the dynamic behavior of interest rates, default and recovery rates, and credit spreads. Typical examples include Jarrow and Turnbull (1995), Jarrow, Lando, and Turnbull (1997), and Duffie and Singleton (1999), but see also the earlier work of Merton (1974). To identify the dynamic behavior of the relevant economic variables, one can either use directly observed historical data on the variables themselves, or use implied models based on prices of liquid credit sensitive instruments like credit default swaps, see for example Duffie and Singleton (1999). The increased flexibility in managing a portfolio of credits through derivatives or securitization complements the well-known credit scoring methodology. Moreover, it entails a shift in attention from cross-sectional, point-in-time predictions of default to a dynamic credit management perspective.

A second reason for the attention for credit risk dynamics lies in the adoption of a portfolio perspective to credit risk, see Gupton, Finger, and Bhatia (1997), Credit Suisse (1997), and Wilson (1997a,b). Whereas the models of, e.g., Jarrow and Turnbull (1995) and Duffie and Singleton (1999) can in principle be used both for single-name and multi-name credit risky instruments, there is a crucial difference as to the type of risk that is important. Making the standard distinction between idiosyncratic and systematic risk, it is the systematic risk that is most important at a portfolio level, see for example

Jarrow, Lando, and Yu (2003), Frey and McNeil (2001), Lucas, Klaassen, Spreij, and Straetmans (2001), and Giesecke and Weber (2003). The idiosyncratic risk can be largely diversified. Portfolio models like *CreditMetrics* of Gupton, Finger, and Bhatia (1997) and *CreditRisk<sup>+</sup>* of Credit Suisse (1997) pay little attention to the dynamic behavior of the systematic risk factor, though extensions of these models are possible, see Finger (1999) and Li (1999). An exception is the *CreditPortfolioView* model, see Wilson (1997a,b). Systematic credit risk factors are usually thought to correlate with macro-economic conditions. This appears both from theoretical models on real business cycles, like for example Williamson (1987), Kiyotaki and Moore (1997), Bernanke, Gertler, and Gilchrist (1999), and Kwark (2002), and from empirical evidence, see for example Wilson (1997a,b), Nickell, Perraudin, and Varotto (2000), Bangia, Diebold, Kronimus, Schagen, and Schuermann (2002), and Kavvathas (2001). There is of course much experience in modeling the dynamic behavior of macro-economic variables. If, therefore, a link can be established between the macro-economic environment and systematic credit risk factors, knowledge on the state and direction of macro variables may help in assessing portfolio credit risk over time.<sup>2</sup>

The third reason for the interest in the dynamics of credit risk lies in regulatory developments, see Basel Committee on Bank Supervision (2003). In the new proposals of the Basle Capital Accord, banks have to link their capital requirements directly to the creditworthiness of counterparties. The creditworthiness is assessed through default probabilities and collateral values. Default probabilities can be taken implicitly from ratings issued by the official rating agencies, or explicitly from banks' own internal rating models. A major concern with the new regulatory framework is that it may lead to pro-cyclical capital requirements, see Basel Committee on Banking Supervision (2002), and in this way to exacerbated business cycle fluctuations. The argument is that during an upswing of the economy, banks may lower their capital levels. Such a decrease in capital may be spurred by risk sensitive capital requirements based on recent estimates of default probabilities, see Altman and Saunders (2001) and Borio, Furfine, and Lowe (2001). As a result, capital levels may be too low at the peak of the cycle to cope with the subsequent downswing. The capital accumulation during the downswing may also be too slow. Moreover, the increases in capital may result

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<sup>2</sup>The reverse may also be true: knowledge on the state of credit risk markets may help to predict macro-economic developments, see for example Kwark (2002) and Guha and Hiris (2002).

in a credit crunch and thus worsen already adverse economic conditions, see Laeven and Majnoni (2002). The issue of pro-cyclicality highlights the need to assess whether ratings, default rates and spreads, and other credit risk drivers are pro-cyclical or not. The empirical evidence appears inconclusive. Whereas Altman and Saunders (2001) find ratings lagging the business cycle, D'Amato and Furfine (2003) claim that business cycle conditions influence new ratings much more than existing ratings. Moreover, using a theoretical model, Gorton and He (2003) show that credit cycles may have their own dynamics distinct from business cycles.

Given the importance of dynamic credit risk modeling and the controversy on the exact relation of credit risk drivers with the state of the business cycle as mentioned above, we set out in this paper to build a multivariate time-series model for business failure rates, credit spreads, and real GDP growth. Empirical models that link default rates to macro variables can be found in Wilson (1997a,b), Nickell et al. (2000), Bangia et al. (2002), Kavvathas (2001), and Pesaran, Schuermann, Treutler, and Weiner (2003). The general conclusion of these models is that defaults probabilities tend to be higher in recession states, see also Allen and Saunders (2003). Empirical evidence linking credit spreads to the business cycle can be found in for example Fama and French (1989), Chen (1991), and Stock and Watson (1989). There, the general conclusion is that risk premia on bonds contain a countercyclical component and that credit spreads are good predictors for future business cycle conditions.

Our paper contains the following contributions. First, we build a tri-variate model including both default rates and spreads in their relation to economic growth. Though bivariate analyses using either spreads or default rates in a combination with economic growth rates have been more prevalent in the literature, the empirical evidence mentioned earlier suggests that an analysis based on all three series simultaneously is more appropriate. In this way, we can investigate the claimed lead-relationship of credit spreads over growth, the (in)congruence between credit and business cycles, and the dynamics of default rates in one unified framework. The joint behavior of these series can moreover be used as an input to credit risk models in much the same vein as in Pesaran et al. (2003).<sup>3</sup> Our second contribution lies in

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<sup>3</sup>Note that the credit spread can be regarded as a proxy for the risk neutral expected loss, see Merton (1974) and Jarrow, Lando, and Turnbull (1997). It is a noisy proxy, however, as the spread also contains various other components (including for example a compensation for a lack of liquidity of the underlying bonds). See Elton, Gruber, Agrawal,

the fact that we use an unobserved components model, see Harvey (1989) and Durbin and Koopman (2001). In this way, we are able to disentangle long-term (co)-movements from short-term cyclical movements in a clear and interpretable way. By focusing on the time-series dimension of our series, we also complement the existing literature by considering a long time span of data: 1933–1997. By contrast, papers like Nickell et al. (2000) and Bangia et al. (2002) focus much more on the cross-sectional dimension to estimate their models, typically using time series of 20 to 25 years for a large number of companies. Our longer time span allows for repeated observations on business cycles and therefore helps to test for the presence and co-variation of cyclical patterns in credit risk factors. The importance of the time-series dimension in credit risk analysis is also stressed in Gordy and Heitfield (2002).

Our empirical findings reveal a rich and diverse view on the dynamic relations between the three series considered. After filtering out the long-term trends from the data, we find two types of cycles for the 1933–1997 period. The first type of cycle has a frequency of around 6 years. There is clear (positive) co-cyclicity between spreads and business failures and (negative) between spreads and GDP. The relation between GDP and business failures is insignificant at this frequency. The second type of cycle has a longer period of around 11 years. For this frequency, there is a clear positive relation between spreads and failures, and a negative relation between GDP on the one hand and spreads and failures on the other hand. Contemporaneous correlations between the innovations for each of the components in our model show the same intuitive pattern.

We check the robustness of our results in three ways. First, we lag GDP and credit spreads, as one may argue that business failures only react with a lag to these variables. The effects remain robust, with the exception that co-cyclicity between GDP and business failures at the 6-year frequency becomes counter-intuitively positive. We also test the robustness of our findings by concentrating on the post World War II period. Interestingly, for this shorter period, co-cyclicity only appears statistically significant between GDP and business failures. The longer cycle now has a period of around 16 years. Again, the co-cyclicity appears economically most important at this lower frequency than at the higher frequency of 6 years. Third, we use quarterly data over the post-war period and corroborate the results for the annual data.

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and Mann (2001) and Huang and Huang (2003) for empirical analyses on this issue. Our model explicitly allows for measurement errors in the variables.



Our results support the findings in, for example, Nickell et al. (2000) and Bangia et al. (2002). At the same time, however, the double cyclicity found in our analysis for both the long and short data sample indicate that the relation between defaults and GDP may be more complicated than assumed in earlier papers on credit risk. By using a univariate classification scheme to distinguish recession from expansion default probabilities, one may miss some of the more intricate dynamics of the system. This holds in particular if such classifications are based on standard business cycles of around 6 years, whereas our results suggest that the economically most important correlations are at a somewhat longer frequency. It is also interesting to see that credit spreads appear to contain little predictive information with respect to business failures in the post World War II sample. This appears especially puzzling given the usual timeliness of financial markets' information. The co-cyclicity between default rates, credit spreads, and GDP also has a possible impact on the pro-cyclicality debate mentioned earlier. Though a thorough investigation of this issue is beyond the scope of the current paper, the empirical patterns emerging from our analysis illustrate that more research is needed to uncover the intricate dynamic relations between credit and default cycles and their potential impact.

The paper is set-up as follows. In Section 2, we discuss the data and our modeling approach. The empirical results are contained in Section 3. Section 4 concludes.

## 2 Data and modeling approach

We use three data series in our analysis: real GDP, credit spreads, and business failure rates. The first series, real GDP, is taken from the data base of the Federal Reserve Bank of St.Louis (FRED). The series contains GDP in chained 1996 dollars. From the same site, we also obtain Moody's yields on Baa corporate bonds and the yield on government bonds with a maturity exceeding 10 years. These are used to construct annual credit spreads, defined as the difference between the two yields. Our third series is from Dun and Bradstreet (1998) and contains U.S. business failure rates per 10,000 companies over the period 1927–1997. After 1997, the series was discontinued. Following the description of Dun and Bradstreet (1998), the numbers indicate businesses that ceased operations after assignment or bankruptcy; ceased operations with losses to creditors after such actions as foreclosure or attachment; voluntarily withdrew leaving unpaid debts; were

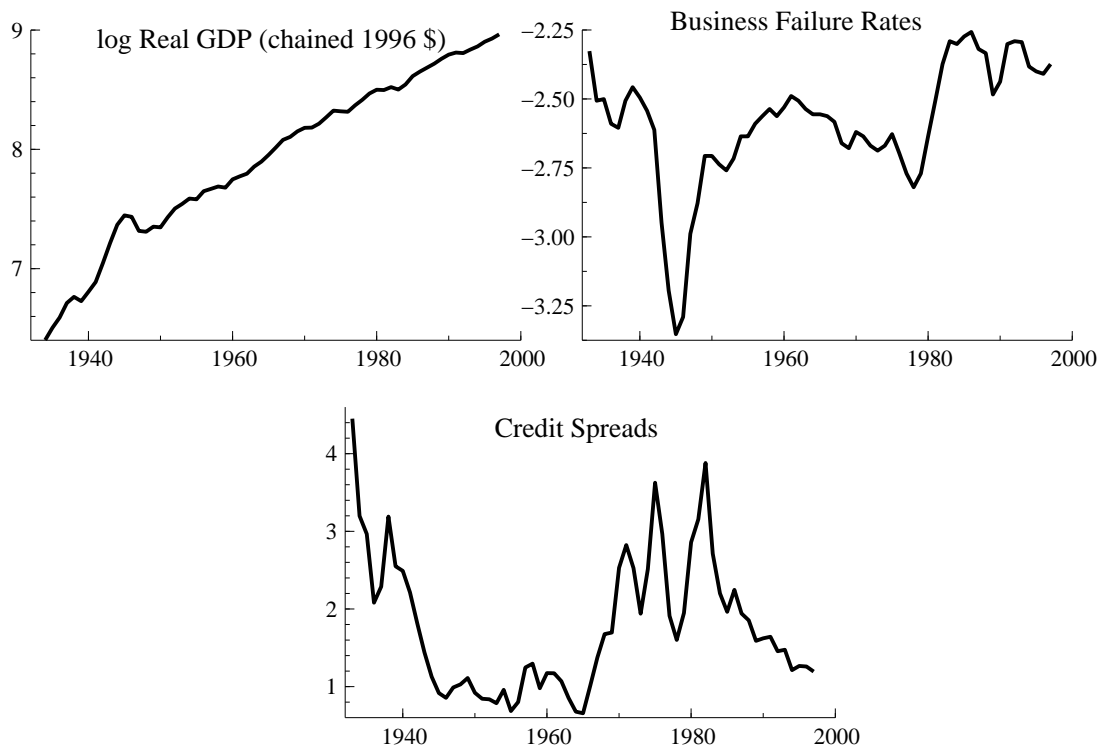


Figure 1: Logarithm of Real GDP, Dun&Bradstreet business failure rates, and credit spreads (Moody’s Baa minus long term government bonds).

involved in court actions such as receivership, reorganization or arrangement; or voluntarily compromised with creditors. As such, the business failure rate may be an underestimate of the default rate, because defaulting investment projects within a business may be compensated by well-performing projects within that same business, see also Kwark (2002). In this paper, however, we are not as much involved with the level of the default rate, but with its dynamic behavior over time and its co-variation with other variables included in the model. Given the difficulty in obtaining reliable default rate statistics from competing sources, we take the business failure rate as a proxy for describing default rate dynamics.<sup>4</sup> Combining all three series, our sample runs from 1933 up to 1997. The data is presented in Figure 1 for the two sample periods used in this paper.

To describe the dynamic behavior of the three series as well as their interdependencies, we introduce an unobserved components model, see Harvey

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<sup>4</sup>One additional potential complication is the change in data collection by Dun & Bradstreet after 1984. This increased both the number of businesses and business failures. The failure rate, however appears relatively unaffected. We tested for possible effects by including a dummy variable for a level break in defaults from 1984 onwards. This variable turned out to be insignificant.

(1989) and Durbin and Koopman (2001). Our basic specification is

$$y_t = \mu_t + A\gamma_t + B\psi_t + e_t, \quad e_t \stackrel{\text{i.i.d.}}{\sim} N(0, \Sigma_e), \quad t = 1, \dots, n, \quad (1)$$

where  $y_t$  represents the time series observation vector as given by

$$y_t = \begin{bmatrix} y_t^R \\ y_t^S \\ y_t^D \end{bmatrix} = \begin{bmatrix} \text{real GDP (GDP)} \\ \text{business failures (DEFLT)} \\ \text{credit spreads (SPRD)} \end{bmatrix}, \quad t = 1, \dots, n, \quad (2)$$

with  $t = 1$  for 1933 and  $t = n = 65$  for 1997. The irregular component  $e_t$  in (1) is included to allow for measurement noise in the observations. The trend component  $\mu_t$  is given by

$$\begin{aligned} \mu_t &= \mu_{t-1} + \beta_t, \\ \beta_t &= \beta_{t-1} + \eta_t, \end{aligned} \quad (3)$$

where  $\eta_t$  is a zero mean normal innovation with variance matrix  $\Sigma_\eta$ . The  $\eta_t$ 's are serially independent and, moreover, independent of the other innovations in the model. The role of  $\mu_t$  is to filter out the low frequency or long term dynamics from the data. Alternatively, we could use a the Hodrick-Prescott (HP) filter for this, which is a restricted version of (3). As shown by Harvey and Jaeger (1993), our current procedure is more flexible. This may be important in our current context, where we have multiple time series. The HP filter is optimized to filter out the low frequency patterns from GDP type data. It is not clear a priori, however, that this filter is also optimal for non-GDP data like defaults and credit spreads. Therefore, we prefer the inclusion of  $\mu_t$  over pre-filtering the data using the HP filter.

We can now interpret  $y_t - \mu_t$  as the cyclical components in our data. After some preliminary experimentation, a model with two cyclical components appeared empirically the most promising. The cycles correspond to short business cycle type frequencies of around 6 years, and a somewhat longer frequency between 11 and 16 years, respectively. We label the cycles  $\gamma_t$  and  $\psi_t$ , respectively. We normalize  $\gamma_t$  to be the cycle with the shorter period. Note that both  $\gamma_t$  and  $\psi_t$  are tri-variate. In this way we can test whether the cycle in real GDP is the same as that in credit spreads or business failures. This is done by an inspection of the ‘cycle loadings’ matrices  $A$  and  $B$ . Various specifications for the stationary cycle components can be considered. For example, the cycles can be modelled as an autoregressive process of order 2, in short AR(2), with the polynomial autoregressive coefficients selected in

the complex range. To enforce this restriction we can represent the model as a trigonometric process, that is

$$\begin{pmatrix} \psi_t \\ \psi_t^* \end{pmatrix} = \phi_\psi \left( \begin{bmatrix} \cos \lambda_\psi & \sin \lambda_\psi \\ -\sin \lambda_\psi & \cos \lambda_\psi \end{bmatrix} \otimes I_3 \right) \begin{pmatrix} \psi_{t-1} \\ \psi_{t-1}^* \end{pmatrix} + \begin{pmatrix} \omega_t \\ \omega_t^* \end{pmatrix}, \quad (4)$$

$$\begin{pmatrix} \omega_t \\ \omega_t^* \end{pmatrix} \stackrel{\text{i.i.d.}}{\sim} N(0, I_2 \otimes D_\omega),$$

with frequency  $\lambda_\psi$  and persistence parameter  $|\phi_\psi| < 1$ . The disturbances  $\omega_t$  and  $\omega_t^*$  are serially and mutually uncorrelated and normally distributed, where  $D_\omega$  is a diagonal  $3 \times 3$  matrix. A similar specification holds for the short cycle component  $\gamma_t$ . This stochastic cycle specification generates 3 stationary cyclical processes with a common period of  $p = 2\pi/\lambda_\psi$ . The factor loading matrices  $A$  and  $B$  scale the cycles  $\gamma_t$  and  $\psi_t$  for each of the individual series. For the identification of  $D_\omega$ ,  $B$  is restricted to a lower triangular matrix with unity as diagonal elements. A similar restriction applies to  $A$ .

It is well known that lead and lag relationships between macro-economic time series may exist. To allow for this, we perform several robustness checks of our benchmark results later on in terms of lagging some of the variables in the analysis.

The multivariate unobserved components model (1) to (4) can be put into the state space form

$$\begin{aligned} \alpha_t &= T\alpha_{t-1} + \nu_t, \\ y_t &= Z\alpha_t + \varepsilon_t, \end{aligned} \quad (5)$$

where the state vector contains the mean parameters including the unobservables, that is  $\alpha_t = (\mu_t, \gamma_t', \gamma_t^{*'}, \psi_t', \psi_t^{*'})'$ . The system matrices  $Z$  and  $T$  are constructed according to the specifications implied by the model. The state disturbance vector  $\nu_t$  contains the disturbances, e.g.,  $e_t$ ,  $\omega_t$ , etc. The unknown coefficients of the model can be estimated by numerically maximising the log-likelihood function of the model for a given set of observations  $y_1, \dots, y_n$ . The log-likelihood function can be computed via the Kalman filter; see, for example, Durbin and Koopman (2001) for details of the Kalman filter and associated methods and techniques. Once the parameters are estimated, the unobserved components  $\gamma_t$  and  $\psi_t$  can be extracted from the observations using the Kalman filter and the associated smoother. These estimates, together with confidence intervals, can be graphically presented. Diagnostic statistics and graphs can be obtained as by-products of the Kalman filter and can be used to test the underlying assumptions of the model such as

normality and independence of the disturbances. Finally, standard goodness-of-fit statistics can be computed for each equation of the multivariate model. We estimate our model using the software package STAMP, see Koopman, Harvey, Doornik, and Shephard (2000).

### 3 Empirical results

In an attempt to provide clear evidence on the existence and commonality of credit cycles and business cycles in our data, we consider the decomposition model explained in the previous section. Our benchmark results are presented in the first three columns of Table 1.

The diagnostics of the model appear adequate given the limited number of observations. The data show clear evidence of a cyclical patterns. We start our discussion with the short business cycles  $\gamma_t$ . Note that  $\gamma_t$  is tri-variate, such that there are three cycles. These cycles have a period of 6.3 years and are all significant as can be seen from the elements of  $D_\gamma$ . The cycles are persistent with a damping factor  $\phi_\gamma$  of 0.917. Given the period length, it is tempting to label this cycle the typical business cycle. To determine whether there is any co-cyclicality, we look at the load matrix  $A$ . There is no evidence of a common short cycle between real GDP and defaults. By contrast, real GDP and credit spreads have a common cycle given the significant loading of -14.5 on the GDP cycle in the credit spread equation. The loading has the intuitive negative sign: if economic activity declines, credit risks and thus credit spreads increase. Credit spreads also share some of the cyclicality in defaults given the significant loading on 6.12 of the default cycle. Again, this loading has the intuitive sign: default activity and credit spreads appear to be positively correlated.

The second set of cycles,  $\psi_t$ , has a period of 11.6 years and is also highly persistent with a damping factor of 0.924. Again, all three cycles are statistically significant given the elements of  $D_\psi$ . By comparing  $D_\psi$  and  $D_\gamma$  we see that  $\psi_t$  is economically more important than  $\gamma_t$  for GDP and defaults. For credit spreads, by contrast, the both cycles are about equally important. This is also clearly seen in Figure 2, which plots the data and smoothed components for the different series. The load matrix  $B$  of  $\psi_t$  shows significant co-cyclicality between all series at this longer frequency. The signs are intuitively plausible: positive between defaults and spreads, and negative between GDP and spreads and defaults. The covariance matrices  $\Sigma_\eta$  and  $\Sigma_e$  (not completely shown in the table) also have the same intuitive signs.

Table 1: Parameter Estimates

The table contains parameter estimates for the model

$$\begin{aligned}
 y_t &= \mu_t + A\gamma_t + B\psi_t + e_t, & e_t &\stackrel{\text{i.i.d.}}{\sim} N(0, \Sigma_e), \\
 \Delta\mu_{t+1} &= \beta_t = \beta_{t-1} + \eta_t, & \eta_t &\stackrel{\text{i.i.d.}}{\sim} N(0, \Sigma_\eta), \\
 \begin{pmatrix} \gamma_t \\ \gamma_t^* \end{pmatrix} &= \phi_\gamma \begin{pmatrix} \cos(\lambda_\gamma)I & \sin(\lambda_\gamma)I \\ -\sin(\lambda_\gamma)I & \cos(\lambda_\gamma)I \end{pmatrix} \begin{pmatrix} \gamma_{t-1} \\ \gamma_{t-1}^* \end{pmatrix} + \begin{pmatrix} \kappa_t \\ \kappa_t^* \end{pmatrix}, & (\kappa_t, \kappa_t^*)' &\stackrel{\text{i.i.d.}}{\sim} N(0, I_2 \otimes D_\gamma), \\
 \begin{pmatrix} \psi_t \\ \psi_t^* \end{pmatrix} &= \phi_\psi \begin{pmatrix} \cos(\lambda_\psi)I & \sin(\lambda_\psi)I \\ -\sin(\lambda_\psi)I & \cos(\lambda_\psi)I \end{pmatrix} \begin{pmatrix} \psi_{t-1} \\ \psi_{t-1}^* \end{pmatrix} + \begin{pmatrix} \omega_t \\ \omega_t^* \end{pmatrix}, & (\omega_t, \omega_t^*)' &\stackrel{\text{i.i.d.}}{\sim} N(0, I_2 \otimes D_\psi),
 \end{aligned}$$

with  $y_t$  containing real GDP (GDP), Dun and Bradstreet (1998) business failures (DEFLT) and credit spreads (SPRD), respectively. The failure rates are transformed using a probit transformation. The  $Q(k)$  statistics have a  $\chi^2$  distribution with  $k$  degrees of freedom, while the normality test has a  $\chi^2(2)$  distribution. Parameter significance is denoted by <sup>a</sup> (20%), <sup>b</sup> (10%), or <sup>c</sup> (5%). Panels labeled lagged are performed on lagged real GDP, business failures, and lagged credit spreads.

	No lags, 1933 - 1997			Lags, 1933 - 1997			No lags, 1948 - 1997			Lags, 1948 - 1997		
period ( $2\pi/\lambda_\gamma$ )	6.324 <sup>c</sup>			5.630 <sup>c</sup>			6.096 <sup>c</sup>			5.567 <sup>c</sup>		
$\phi_\gamma$	0.917 <sup>c</sup>			0.887 <sup>c</sup>			0.833 <sup>c</sup>			0.870 <sup>c</sup>		
variance $D_\gamma$	0.0072 <sup>b</sup>	0.0117 <sup>c</sup>	0.0598 <sup>c</sup>	0.0045 <sup>c</sup>	0.0104 <sup>c</sup>	0.0005 <sup>a</sup>	0.0071 <sup>c</sup>	0.0123 <sup>c</sup>	0.0011	0.0052 <sup>c</sup>	0.0115 <sup>c</sup>	0.0008 <sup>a</sup>
load matrix $A$	$\gamma_{gdp}$	$\gamma_{def}$	$\gamma_{spr}$	$\gamma_{gdp}$	$\gamma_{def}$	$\gamma_{spr}$	$\gamma_{gdp}$	$\gamma_{def}$	$\gamma_{spr}$	$\gamma_{gdp}$	$\gamma_{def}$	$\gamma_{spr}$
GDP	1	0	0	1	0	0	1	0	0	1	0	0
DEFLT	-0.148	1	0	0.290 <sup>b</sup>	1	0	-1.830 <sup>c</sup>	1	0	-0.101	1	0
SPRD	-14.5 <sup>c</sup>	6.12 <sup>c</sup>	1	-27.4 <sup>c</sup>	4.20 <sup>c</sup>	1	-0.215	-0.013	1	-0.245	-0.005	1
period ( $2\pi/\lambda_\psi$ )	11.58 <sup>c</sup>			10.40 <sup>c</sup>			16.36 <sup>c</sup>			15.46 <sup>c</sup>		
$\phi_\psi$	0.924 <sup>c</sup>			0.917 <sup>c</sup>			0.974 <sup>c</sup>			0.931 <sup>c</sup>		
variance $D_\psi$	0.0198 <sup>b</sup>	0.0395 <sup>c</sup>	0.0610 <sup>c</sup>	0.0178 <sup>c</sup>	0.0179 <sup>c</sup>	0.0	0.0083 <sup>b</sup>	0.0140 <sup>c</sup>	0.0	0.0088 <sup>b</sup>	0.0242 <sup>c</sup>	0.0
load matrix $B$	$\psi_{gdp}$	$\psi_{def}$	$\psi_{spr}$	$\psi_{gdp}$	$\psi_{def}$	$\psi_{spr}$	$\psi_{gdp}$	$\psi_{def}$	$\psi_{spr}$	$\psi_{gdp}$	$\psi_{def}$	$\psi_{spr}$
GDP	1	0	0	1	0	0	1	0	0	1	0	0
DEFLT	-1.16 <sup>c</sup>	1	0	-1.77 <sup>c</sup>	1	0	-3.298 <sup>c</sup>	1	0	-2.235 <sup>c</sup>	1	0
SPRD	-1.16 <sup>c</sup>	1.88 <sup>c</sup>	1	-1.21 <sup>c</sup>	6.84 <sup>c</sup>	1	-0.110	0.0248	1	-0.0986	0.0340	1
diagonal $\Sigma_e$	0.00007	0.00002	0.0277	0.00012	0.00026	0.0222	0.0	0.00014	0.0	0.00013	0.00012	0.000001
diagonal $\Sigma_\eta$	0.00006	0.00037	0.0028	0.00007	0.00066	0.0025	0.000001	0.00001	0.0	0.000002	0.00007	0.0
$R_d^2$	0.36	0.36	0.17	0.41	0.35	0.25	0.26	0.33	0.26	0.24	0.33	0.30
Normality:	9.68	8.33	4.61	12.5	9.16	2.27	2.03	2.04	6.40	3.53	0.15	8.40
1 <sup>st</sup> order autocorr.	0.12	-0.01	0.07	0.16	0.09	0.07	0.12	0.20	-0.01	0.13	-0.10	-0.02
Q(13):	20.6	8.91	5.30	22.5	7.93	5.26	7.20	14.7	9.49	7.25	8.29	8.37

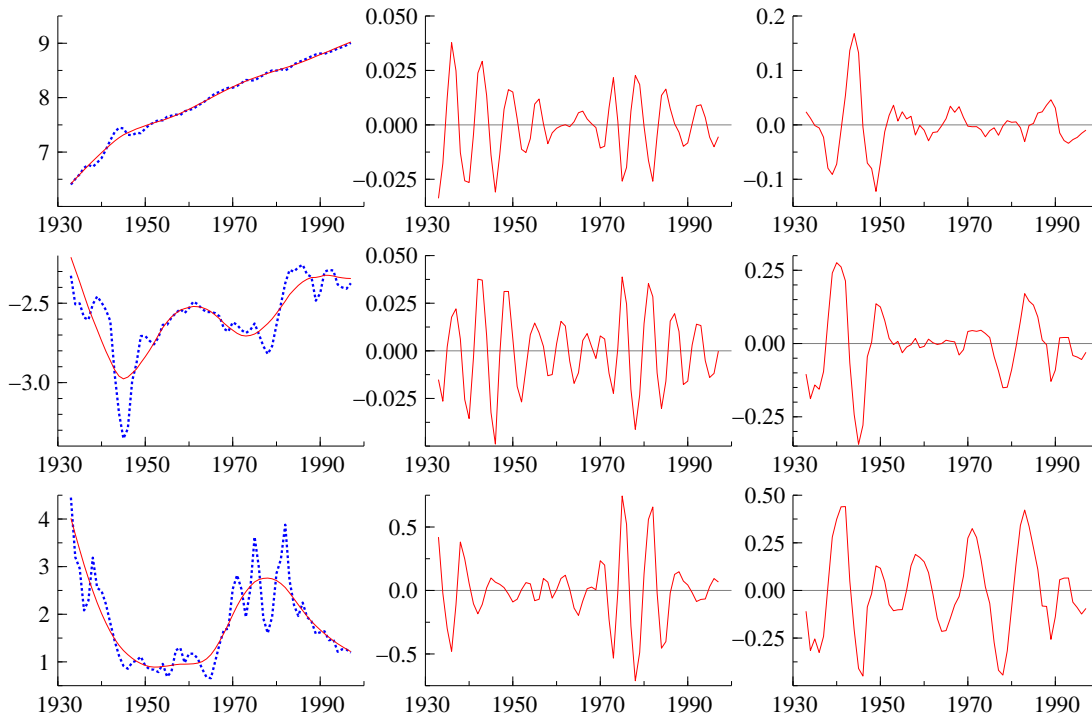


Figure 2: Time series with trend ( $\mu_t$ ), “business” cycle ( $\gamma_t$ ) and longer cycle ( $\psi_t$ ) components in real GDP, business failure rates and the credit spread. This figure presents the fit (smoothed estimates) of the model estimated in Table 1 for the sample 1933-1997, variables not lagged. The top row of graphs indicates the real GDP (top-left) indicated by the dotted line, and the trend component (solid line,  $\mu_t$ ). The graphs for business failure rates and credit spreads are in the middle and bottom graphs, respectively. The middle column of graphs contains the shorter cyclical component ( $\gamma_t$ ) of the three series considered. The last column of graphs contains the longer cyclical component ( $\psi_t$ ) of the three series considered.

Interestingly, though,  $\Sigma_e$  has rank one, such that effectively there is only one irregular component affecting all three series simultaneously. Given the magnitude of the diagonal of  $\Sigma_e$ , it appears that this is the irregular component for the credit spread. The other two variances are negligible compared to the variances of the cyclical components. The matrix for the slope coefficient is full rank, but also shows a large correlation of -0.94 between the GDP and default equation. Therefore, also the long term trend between these two series appears to be correlated. By contrast, the slope innovation’s correlation between GDP and spreads is only -0.41.

The main conclusion following from the discussion of our benchmark results is that there appears co-cyclicity between defaults, credit spreads, and GDP. The co-cyclicity is strongest at frequencies of around 12 years rather than at ‘typical’ business cycle frequencies of 6 years. To test the robustness

of these findings, we perform three additional analyses. First, one may argue that default activity only reacts with a lag to economic indicators such as GDP (growth) and credit spreads. To test this, we estimate the same model, but with real GDP and spreads lagged one year. The results are in the second set of three columns in Table 1.

The diagnostics of this model are similar as for our benchmark model. Also the results appear to carry over. There are three main differences. First, both the 5.6 and 10.4 cycles have reduced rank as one of the elements of  $D_\gamma$  and  $D_\psi$  is estimated to be (almost) zero. Moreover, the magnitude of the elements in  $D_\gamma$  and  $D_\psi$  is generally lower in the model with lagged variables than in the model without lags. The cyclical part of the model, therefore, explains less of the dynamics. This is supported by the fact that the variances of the irregular component (see the diagonal of  $\Sigma_e$ ) have increased. Finally, the loading on the short GDP cycle in the default equation has turned from negative and insignificant into positive and significant. This finding is counter-intuitive. We come back to it when discussing the results of the quarterly data.

As another important robustness test, we re-estimate the model using only data from the post World War II period 1948–1997. The data and trends are presented in the left-hand column of graphs in Figure 3. By dropping the first 15 years of the sample, the shape of the time series for defaults and credit spreads becomes substantially different. The estimation results are in the third set of columns in Table 1. With respect to the benchmark model, there are three main differences. First, the period of the longer cycle has increased from 12 to 16 years. The cycle ( $\psi_t$ ) has also become slightly more persistent given its decay factor  $\phi_\psi$  of 0.97. Second, as was the case with lagged variables for the 1933–1997 sample, the long cycle has reduced rank. To some extent, this also holds for the short cycle. With respect to the co-cyclicity issue, there is a strong and negative correlation between the GDP and the default cycle. The cyclicity between credit spreads and GDP and defaults, however, has become statistically insignificant. This constitutes an important difference with our previous analysis. This finding is in line with Collin-Dufresne, Goldstein, and Martin (2001), who find that changes in credit spreads are not well explained by systematic factors such as business climate and term structure variables. The results are similar if we lag GDP and spreads by one year for the short sample as well, see the last three columns of Table 1. In that case, only the correlation between the short GDP and default cycles decreases in magnitude and has a much higher  $p$ -



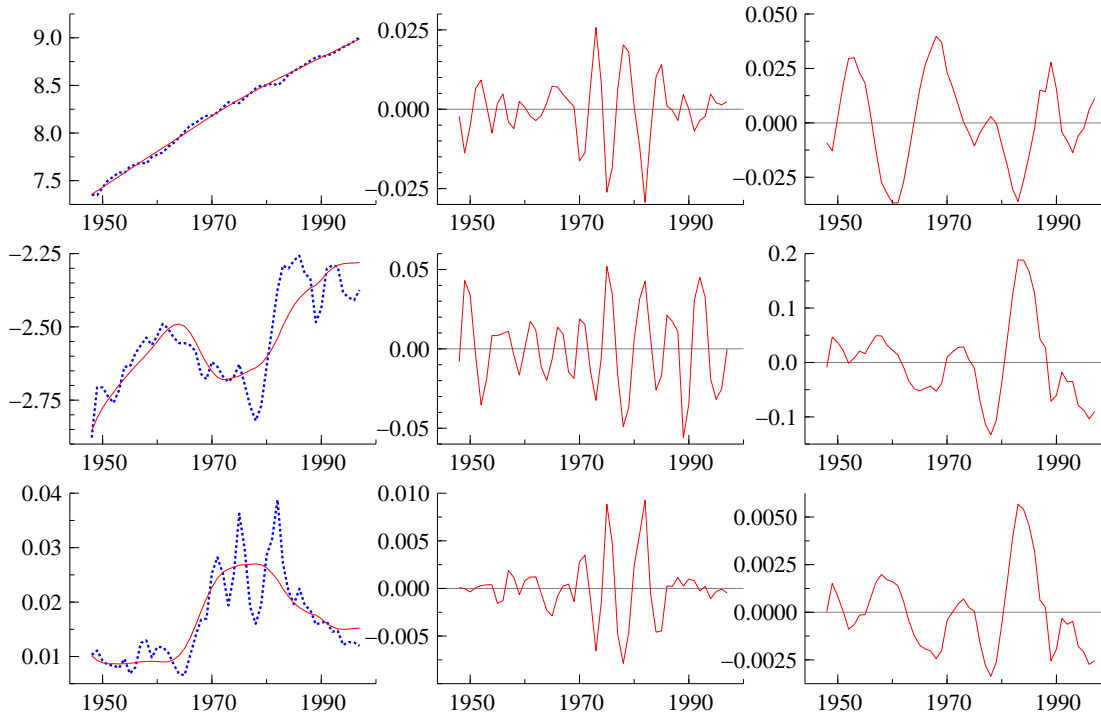


Figure 3: Time series with trend ( $\mu_t$ ), “business” cycle ( $\gamma_t$ ) and longer cycle ( $\psi_t$ ) components in real GDP, business failure rates and the credit spread. This figure presents the fit (smoothed estimates) of the model estimated in Table 1 for the sample 1948-1997, variables not lagged. The top row of graphs indicates the real GDP (top-left) indicated by the dotted line, and the trend component (solid line,  $mu_t$ ). The graphs for business failure rates and credit spreads are in the middle and bottom graphs, respectively. The middle column of graphs contains the shorter cyclical component ( $\gamma_t$ ) of the three series considered. The last column of graphs contains the longer cyclical component ( $\psi_t$ ) of the three series considered.

value.

Our results so far point out two results. First, co-cyclicalities between economically most important and empirically most robust between real GDP and defaults at frequencies that are higher than typical business cycle frequencies of 6 years. At this shorter frequency, negative co-cyclicalities between GDP and defaults is only found if the two variables enter the model contemporaneously. Second, co-cyclicalities between credit spreads and the other variables in the model does not appear to be a robust phenomenon. In particular, for post-war data, cycle loadings for the GDP and default cycles in the credit spread equation are not significant. They do have the correct signs, however, except for the link between the short (6 year) credit spreads and default cycle, which is very near zero.

To conclude the analysis, we investigate the shift in the correlation be-

tween the short cyclical component in GDP and defaults if we lag GDP and credit spreads in the model. To do this, we use quarterly data for 1948–1997. The data are obtained from the same sources as the original annual data. The only problem is caused by the default series, for which we only have annual data. As a rough approximation, we spread the default frequency in each years evenly over the four quarters. Using the quarterly data, we estimated the model without lags. Next, we lag GDP 1 up to 3 quarters, and re-estimate the model. The results are given in Table 2.<sup>5</sup>

Obviously, given the different type of data the estimation results differ somewhat from those in Table 1. The broad picture, however, in terms of signs of cycle loadings, lengths of cycles and damping factors is fairly consistent. The correlation between GDP and default cycles on the one hand, and credit spreads on the other hand is never significant. At the short frequency between 5 and 6 years, the correlation between the GDP cycle and the default cycle is negative and strongest for lags of zero or one quarters. At lag 2, the correlation drops significantly given a decrease in factor loading from 1.27 to 0.82. At a lag of 3 quarters, the correlation has dropped to zero and has become insignificant. By contrast, the corresponding loading ( $B$ ) for the long cycle becomes stronger, a phenomenon not found for the annual data. It appears that the lead/lag times between defaults and economic predictors should not be set too high. Lags of up to one quarter appear acceptable, but at longer lags common features between defaults and other variables may be corrupted. This holds in particular for co-cyclicity at higher frequencies.

## 4 Conclusions

In this paper we used a multivariate unobserved components approach to describe the dynamic behavior of credit risk factors in their relation to the real economy. We depart from other approaches in credit risk modeling in that we focus on the time-series behavior rather than the cross section dimension of default related data. Moreover, we model credit spreads and business failure rates jointly with macro-economic developments. By adopting the unobserved components approach, we were able to disentangle medium and

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<sup>5</sup>Possibly due to the rough approximation of the quarterly default series from the annual data, the estimation procedure ran into some numerical difficulties. We solved these by fixing the period of the long cyclical component to 16 years, which appears adequate given the estimation results in the last six columns of Table 1.

Table 2: Parameter Estimates

The table contains parameter estimates for the model

$$\begin{aligned}
 y_t &= \mu_t + A\gamma_t + B\psi_t + e_t, & e_t &\overset{\text{i.i.d.}}{\sim} \text{N}(0, \Sigma_e), \\
 \Delta\mu_{t+1} &= \beta_t = \beta_{t-1} + \eta_t, & \eta_t &\overset{\text{i.i.d.}}{\sim} \text{N}(0, \Sigma_\eta), \\
 \begin{pmatrix} \gamma_t \\ \gamma_t^* \end{pmatrix} &= \phi_\gamma \begin{pmatrix} \cos(\lambda_\gamma)I & \sin(\lambda_\gamma)I \\ -\sin(\lambda_\gamma)I & \cos(\lambda_\gamma)I \end{pmatrix} \begin{pmatrix} \gamma_{t-1} \\ \gamma_{t-1}^* \end{pmatrix} + \begin{pmatrix} \kappa_t \\ \kappa_t^* \end{pmatrix}, & (\kappa_t, \kappa_t^*)' &\overset{\text{i.i.d.}}{\sim} \text{N}(0, I_2 \otimes D_\gamma), \\
 \begin{pmatrix} \psi_t \\ \psi_t^* \end{pmatrix} &= \phi_\psi \begin{pmatrix} \cos(\lambda_\psi)I & \sin(\lambda_\psi)I \\ -\sin(\lambda_\psi)I & \cos(\lambda_\psi)I \end{pmatrix} \begin{pmatrix} \psi_{t-1} \\ \psi_{t-1}^* \end{pmatrix} + \begin{pmatrix} \omega_t \\ \omega_t^* \end{pmatrix}, & (\omega_t, \omega_t^*)' &\overset{\text{i.i.d.}}{\sim} \text{N}(0, I_2 \otimes D_\psi),
 \end{aligned}$$

with  $y_t$  containing real GDP (GDP), Dun and Bradstreet (1998) business failures (DEFLT) and credit spreads (SPRD), respectively. The failure rates are transformed using a probit transformation. The sample consists of quarterly observations from 1948 to 1997. Parameter significance is denoted by <sup>a</sup> (20%), <sup>b</sup> (10%), or <sup>c</sup> (5%).

16

	No lags, 1948 - 1997			GDP, lag 1			GDP, lag 2			GDP, lag 3		
period $2\pi/(4\lambda_\gamma)$	5.036 <sup>c</sup>			4.932 <sup>c</sup>			5.177 <sup>c</sup>			5.096 <sup>c</sup>		
$\phi_\gamma$	0.938 <sup>c</sup>			0.952 <sup>c</sup>			0.934 <sup>c</sup>			0.922 <sup>c</sup>		
variance $D_\gamma$	0.00533 <sup>c</sup>	0.0	0.00113 <sup>a</sup>	0.0045 <sup>c</sup>	0.0	0.0009 <sup>b</sup>	0.0053 <sup>c</sup>	0.0045 <sup>c</sup>	0.0	0.0062 <sup>b</sup>	0.0078 <sup>c</sup>	0.0
load matrix $A$	GDP	DEFLT	SPRD	GDP	DEFLT	SPRD	GDP	DEFLT	SPRD	GDP	DEFLT	SPRD
$\gamma_{gdp}$	1	-1.199 <sup>c</sup>	-0.199	1	-1.273 <sup>c</sup>	-0.195	1	-0.820 <sup>c</sup>	-0.080	1	0.024	0.017
$\gamma_{def}$	0	1	0.031	0	1	0.015	0	1	0.245	0	1	0.17
$\gamma_{spr}$	0	0	1	0	0	1	0	0	1	0	0	1
period $(2\pi/\lambda_\psi)$	16.0			16.0			16.0			16.0		
$\phi_\psi$	0.979 <sup>c</sup>			0.982 <sup>c</sup>			0.979 <sup>c</sup>			0.982 <sup>c</sup>		
variance $D_\psi$	0.0038 <sup>c</sup>	0.0139 <sup>c</sup>	0.0	0.0036 <sup>c</sup>	0.0086 <sup>c</sup>	0.0	0.0038 <sup>c</sup>	0.0097 <sup>c</sup>	0.0	0.0038 <sup>c</sup>	0.0114 <sup>c</sup>	0.0
load matrix $B$	GDP	DEFLT	SPRD	GDP	DEFLT	SPRD	GDP	DEFLT	SPRD	GDP	DEFLT	SPRD
$\psi_{gdp}$	1	-1.344 <sup>c</sup>	-0.114	1	-1.762 <sup>c</sup>	-0.122	1	-1.873 <sup>c</sup>	-0.117	1	-2.123 <sup>c</sup>	-0.106
$\psi_{def}$	0	1	0.036	0	1	0.034	0	1	0.041	0	1	0.040
$\psi_{spr}$	0	0	1	0	0	1	0	0	1	0	0	1

short-term cyclical movements from longer term developments in credit risk factors. In this way, we could retrace some of the earlier empirical evidence on the relation between credit risk and the macro-economy. For our longest data sample, there appeared to be strong co-cyclicalities between spreads and defaults and between spreads and GDP at typical business frequencies of 6 years. At longer frequencies of 11 years, there was also significant co-cyclicalities between GDP and defaults. The latter result is robust to a variety of changes in the model specification. The other two effects are not. In particular, looking at more recent, i.e., post-war data, co-cyclicalities between credit spreads and the other two series has become statistically insignificant. Moreover, the co-cyclicalities between GDP and defaults at typical business cycle frequencies of 6 years is easily corrupted if GDP is lagged by more than one quarter.

Our results corroborate some of the earlier results in the empirical credit risk literature. It also opens up some new possibilities for making default probabilities dependent on the state of the economy. This may prove useful in for example a dynamic credit risk management setting, where default scenarios are needed over a variety of economic conditions. An important extension of our current research would be to test whether our results hold up for more recent data. Of particular interest is the lack of correlation between the credit spread and the other variables in our model. The advantage of more recent data is that we may come up with better measurements of defaults and credit spreads using available data bases. The obvious drawback, however, will be the relatively short time span available for a proper time series analysis of the type presented in this paper.

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