Buyer Search Costs and Endogenous Product Design

Dmitri Kuksov

Technical Appendix: Continuous Distribution of Buyers

Here I consider aggregate uncertainty in markets with continuously distributed tastes. The only difference between the model considered here and the model in the paper is that instead of two consumer segments (consumer valuation V or $V+t$), we have continuous distribution of consumers (valuation from V to $V+t$) a la Hotelling. The purpose of this appendix is to show that the main implications of the two-segment model hold in the case of continuous distribution of buyers.

The following summarizes the model. Consumer preferences are distributed uniformly on the unit segment with "transportation" cost of having product away from the consumer location linear in the distance (with coefficient of proportionality t) a la Hotelling. Consumer valuation unadjusted for the above stated preference (transportation cost) distribution is $V+t$. Firms have uncertainty about V. The prior distribution on V is that $V=\underline{V}$ or $\overline{V}$ each with probability 1/2. Further, firms receive individual signals x of V. A particular x a firm receives remains private knowledge to that firm. Signal x could be one of the two forms: a) "no information", denoted by $x=\emptyset$, and b) exact value: $x=V$. In the former case, posterior belief on V is same as prior belief, and in the latter case, posterior belief is equal to x. Firms then simultaneously set price ($p=p(x)$). Consumers know the locations of firms (valuation of the products offered by the firms) but have to incur a search cost to find the price beyond the first price that they receive at zero cost. When firms are able to change the location, they have to do that before receiving the signal x of the valuation parameter V (due to the long time required to redesign the product). Consumers know their valuation, and therefore know the parameter V.

I will assume in the following models that t is large compared to $2s$ for simplification of graphical display of the regions, i.e. that the search cost for price is small relative to the amplitude of valuations of consumers.

Denote: $\delta=\overline{V}-\underline{V}$. Case $2s>\delta$ leads to monopoly pricing regardless of differentiation, and hence is not interesting. Cases $4s\geq\delta$ and $4s<\delta$ lead to prices being lower than monopoly for a smaller and larger set of $\underline{V}$ respectively. I will first consider $4s<\delta$, which is a more difficult case, and make notes on what would change in the other case along the way. $\underline{V}$ remains a parameter on which the results of the model (the way search costs enter the equilibrium prices and profits) will depend.

Model with no Product Differentiation

Consider an undifferentiated market with products of both firms located at 0 of the Hotelling line segment. In order to consider the competitive behavior of firms, we first need to consider what would a monopoly do in this situation (alternatively, one can think of the monopoly case as the case when search costs are so high that the firms are not concerned about the competition, since the decrease in demand is then just a scaling parameter).

If $x = V$, then the monopoly price is

$$p_0^m(x) = \begin{cases} x/2 & \text{if } x \leq 2t, \\ x - t & \text{if } x > 2t, \end{cases}$$

(the subscript 0 stands for the no-differentiation case, and upper-script m stands for monopoly).

If $x = \emptyset$, then monopoly price is still piecewise linear, but has more points where the slope changes: price is $\mathrm{E}V/2 = (\underline{V} + \overline{V})/4$ until the consumer at 1 buys in the case $V = \overline{V}$ but gets no surplus. Then price increases with the slope 1 as a function of $\underline{V}$ as to keep the consumer at 1 marginal in the case $V = \overline{V}$ until the firm finds it optimal to go for the consumers that are marginal in case of $V = \underline{V}$. Then the price increases with the slope 1/2 again until the consumer at 1 becomes marginal in the case $V = \underline{V}$. Then the price increases with slope 1, so that all consumers buy, with consumer at 1 buying just barely if $V = \underline{V}$. Solving the conditions above, one finds that the formula for the monopoly price when $x = \emptyset$ as a function of $\underline{V}$ is (0 denotes no differentiation case):

$$p_0^m(\emptyset) = \begin{cases} \underline{V}/2 + \delta/4 & \text{if } \underline{V} < 2t - 3\delta/2, \\ \underline{V} - t - \delta & \text{if } \underline{V} \in [2t - 3\delta/2, 3t - 2\delta], \\ \underline{V}/2 + t/2 & \text{if } \underline{V} \in [3t - 2\delta, 3t], \\ \underline{V} - t & \text{if } \underline{V} > 3t. \end{cases}$$

Consider now the competitive equilibrium. Since given the valuation, there are two equally probable signals x, in a pure strategy equilibrium, consumers search if they see a price more than $2s$ higher than the price given the lower signal. Note that if one consumer finds it beneficial to search, all do, and vice versa. If consumers search, they don't come back since they find a price that is at least as good. Therefore, if consumers search, the firm loses all the demand. Therefore, firms set the price so that consumers do not search. It follows that if $x = \underline{V}$, the price $p_0^e(\underline{V})$ is the monopoly price $p_0^m(\underline{V})$; if $x = \emptyset$, the equilibrium price $p_0^e(\emptyset)$ is the minimum of the monopoly price $p_0^m(\emptyset)$ and the no-search constraint $p_0^e(\underline{V}) + 2s$, and if $x = \overline{V}$, the equilibrium price $p_0^e(\overline{V})$ is again the minimum of the monopoly price $p_0^m(\overline{V})$ and the no-search constraint $p_0^e(\emptyset) + 2s$.

Model with Product Differentiation

Now consider the case when one firm is located at 0, and the other at 1 (restricting ourselves to the case of two firms).

Since product fit is known, consumers closer to one of the location obtain the price of that location first. Again, I first consider the collusive case, when each firm sells only to the consumers that are closer to it. I will still use the upper-script m for the collusive case as for the monopoly case before, but will use subscript d for differentiation.

The collusive price given signal $x = V$ or $x = \emptyset$ is derived as before. The only difference is that the consumer population is on the segment of length 1/2 instead of 1. One obtains

$$p_d^m(V) = \begin{cases} V/2 & \text{if } V < t, \\ V - t/2 & \text{otherwise,} \end{cases}$$

and

$$p_d^m(\emptyset) = \begin{cases} \underline{V}/2 + \delta/4 & \text{if } \underline{V} < t - 3\delta/2, \\ \underline{V} - t/2 - \delta & \text{if } \underline{V} \in [t - 3\delta/2, 3t/2 - 2\delta], \\ \underline{V}/2 + t/4 & \text{if } \underline{V} \in [3t/2 - 2\delta, 3t/2], \\ \underline{V} - t/2 & \text{if } \underline{V} > 3t/2. \end{cases}$$

Just as before, the collusive price is the equilibrium price if the search cost is too high (so that the difference in the monopoly prices is never more than $2s$; this happens when $2s \geq \delta$).

When $2s < \delta$, if the firms set the collusive prices above, some consumers will search. However, only consumers close to 1/2 will search, since the difference in the fit makes it not optimal for the rest to switch even if they find price which is lower by δ. Therefore, a firm may see it optimal to set the price such that some consumers search, and the equilibrium price derivation is somewhat more complicated than in the no-differentiation case.

Again, when the signal is $x = \underline{V}$, the price is at the monopoly level: $p^e_d(\underline{V}) = p^m_d(\underline{V})$ (the firm is not afraid that consumers will search as they can not hope to find a lower price, and no consumers from the other side can buy, since the price is above the valuation for them).

If the signal is $x = \emptyset$, consumers do not search out (search when they see price $p(\emptyset)$): they only can search if they hope that the other price is lower, i.e. set by a firm that has $x = \underline{V}$. That is only possible if actual V is $\underline{V}$, but then the price offered by the other firm is above their valuation due to lower fit. However, the demand is increased over a certain range of $\underline{V}$ since some consumers from the competitor side search if $V = \underline{V}$ and other $x = \underline{V}$. This leads to the following equilibrium price (to find it one solves the simultaneous equations with equilibrium price given $x = \underline{V}$):

$$p^e_d(\emptyset) = \begin{cases} \underline{V}/2 + \delta/4 & \text{if } \underline{V} < t - 3\delta/2, \\ \underline{V} - t/2 - \delta & \text{if } \underline{V} \in [t - 3\delta/2, 3t/2 - 2\delta], \\ \underline{V}/2 + t/4 & \text{if } \underline{V} \in [3t/2 - 2\delta, 3t/2 - 2\delta + 4s], \\ 3\underline{V}/5 + t/10 + \delta/5 - 2s/5 & \text{if } \underline{V} \in [3t/2 - 2\delta + 4s, V_0], \\ 3\underline{V}/7 + 5t/14 - 2s/7 & \text{if } \underline{V} \in [V_0, 3t/2 - 4s], \\ \underline{V}/2 + t/4 & \text{if } \underline{V} \in [3t/2 - 4s, 3t/2], \\ \underline{V} - t/2 & \text{if } \underline{V} > 3t/2, \end{cases}$$

where $V_0 = 3t/2 - 7\delta/6 + 2s/3$. The fourth and the fifth segment are the segments at which the price is above the collusive price due to expected increase of demand due to consumers that are closer to the other firm.

Now, consider a firm having the signal $x = \overline{V}$. Assume $4s < \delta$. Then the following is an equilibrium price:

$$p^e_d(\overline{V}) = \begin{cases} \overline{V}/2 & \text{if } \underline{V} < t - \delta, \\ \overline{V} - t/2 & \text{if } \underline{V} \in [t - \delta, V_0], \\ \underline{V}/7 + 11t/14 + 4s/7 & \text{if } \underline{V} \in [V_0, 3t/2 - 4s], \\ \underline{V}/2 + t/4 + 2s & \text{if } \underline{V} \in [3t/2 - 4s, 3t/2], \\ \underline{V} - t/2 + 2s & \text{if } \underline{V} > 3t/2. \end{cases}$$

Note that some consumers search when $V = \overline{V}$, $\underline{V}$ is in the third segment above, and they see the higher price ($p = p^e_d(\overline{V})$). Namely, the consumers that are at most

$$\frac{p^e_d(\overline{V}) - p^e_d(\emptyset) - 2s}{2t} = \frac{3t - 2v - 8s}{8}.$$

At the beginning of the range of $\underline{V}$, that's $2/5(\delta - 4s)$ consumers, at the end the amount decreases to 0. Also, some consumers search at the end of the second range on $\underline{V}$. The equilibrium and collusive prices in the differentiated case are plotted in Figure

Proof. To prove that the above is an equilibrium, note that in the first range of $\underline{V}$ in the formula above, consumers don't search. Therefore, the monopoly price is optimal.

In the second range, some consumers may search (towards the end of the range), but the increase in demand due to lower search is at most $\Delta p/2t/2$, where $2t$ in the denominator is because of the fit adjustment, and additional 2 for probability 1/2 that the other price is lower (that the other firm received signal $x = \emptyset$). Hence (arithmetics using the previously derived $p^e_d(\emptyset)$ shows), it is suboptimal for the firm to reduce price. It is also suboptimal to increase price, as then some consumers will not buy due to valuation (the price is collusive).

In the third range, the marginal benefit of reducing price due to additional demand (which is linear in Δp) is exactly equal to the cost of losing Δp on each consumer in the expected demand (which is increasing as p decreases). At higher end of the third range, the price is lowered so that it is only $2s$ above the other possible lower price of $p^e_d(\emptyset)$.

In the range 4 and 5, the price is bounded by the no-search condition, i.e. $p^e_d(\underline{V}, \emptyset) + 2s$. $\square$

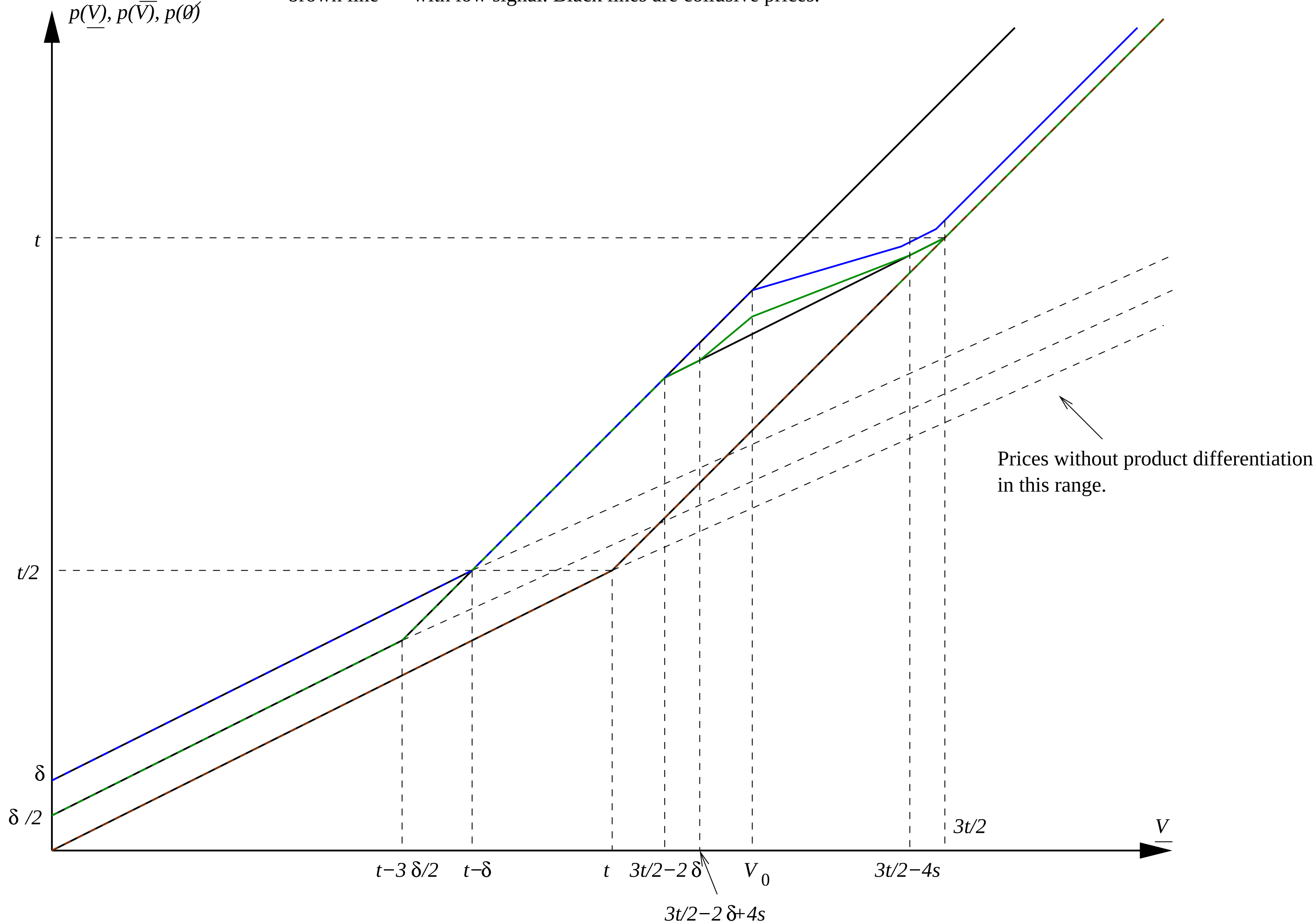
If $4s > \delta$ then the third region does not exist, and consumers never search.

The symmetric equilibrium is unique.

Price Dispersion

Price dispersion in the above models is given by the intersection of the vertical line $\underline{V} = V_0$ with the graphs of equilibrium prices given the signal. In the no-differentiation case, the price distribution between firm with signals $\underline{V}$ and $\emptyset$; and $\emptyset$ and $\overline{V}$ are either 0 or 2s. In the differentiated case, price dispersion on some range is larger and does not depend on s, and is $2s$ or 0 for high $\underline{V}$. Hence, the expected price dispersion in a given market (given V) can

The equilibrium price in the model with uniformly distributed consumer preferences. Blue line is the competitive price with high signal, green line −− with no signal, brown line −− with low signal. Black lines are collusive prices.



be higher in differentiated case than in not differentiated case. Similarly to the two-segment model (Corollaries 1 and 2), given the level of product differentiation, expected prices and price dispersion decreases as s decreases.

Benefit of Differentiation and Effects on Prices, Profits, and Price Dispersion

Since for smaller V, differentiated equilibrium is as monopoly, and prices in non-differentiated market reduce from the monopoly level as s decreases, benefit of differentiation increases as s decreases. For larger values of V (namely, for $\underline{V} > 3t$), benefit of differentiation is independent of s: benefit from differentiation is $t/4$ per firm that comes from having products with better fit, and benefit of higher search costs is $s/4$ per firm (both benefits are for the firms). Hence, for all parameter values, benefit of differentiation does not decrease and may increase as s decreases. This is similar to Proposition 3. It follows from this that as s decreases, prices, price dispersion, and profits *may* increase due to the increase in product differentiation. Note, that just as in the case of two-segment buyer distribution, the increase in prices and price dispersion does not come from the better product, but rather from products being more *different*. That is, prices and price dispersion may increase as buyer search costs decrease even after accounting for product quality.

Consumer Surplus

Consumers are worse off from differentiation for any s, since firms are able to extract all the additional surplus, and the average consumer becomes closer to the marginal in the differentiated case. However, when $V < 3t$, the higher the search cost, the less consumers stand to lose due to differentiation. This is also similar to the result of the two-segment model.