Article

C-field Cosmological Model for Dust Distribution with Varying Λ in FRW Space-time

Raj Bali* & Seema Saraf

Department of Mathematics, University of Rajasthan, Jaipur-302004, India

Abstract

A cosmological model in creation field cosmology with varying Λ in the framework of FRW space-time is investigated. Following Hoyle and Narlikar (1964), we have assumed that universe is filled with dust distribution. To get the deterministic model satisfying conservation equation $T^j_{i;j} = 0$, we have assumed that $\Lambda=1/R^2$ where R is the scale factor and $T^i_j = \underset{(m)}{T}{}^j_i + \underset{(c)}{T}{}^j_i$, $\underset{(m)}{T}{}^j_i$ being energy-momentum tensor for matter and $\underset{(c)}{T}{}^j_i$ is the energy momentum tensor for C-field. We find that creation field increases with time and matter density is constant which is maintained due to continuous creation of matter. $\Lambda \sim 1/t^2$ and particle horizon does not exist. The model represents accelerating universe which matches with the result obtained by Riess et al. (1998) and Perlmutter et al. (1999).

Keywords: creation field, dust distribution, FRW space-time, accelerating universe.

1. Introduction

Astronomical observations in the late eighties have revealed that the predictions of Friedmann-Robertson-Walker type models do not always meet our requirements as was believed earlier (Smoot et al. (1992)). So alternative theories were proposed time to time – the most well known theory was steady state theory by Bondi and Gold (1948). In this approach, the universe does not have any singular beginning nor an end on the cosmic time scale. To maintain constancy of matter density, they envisaged a very slow but continuous creation of matter in contrast to explosive creation of standard model. But it suffered a serious disqualification by not giving any physical justification for continuous creation of matter and principle of conservation of energy was sacrificed in this formalism. To over come this difficulty, Hoyle and Narliker (1966) adopted a field theoretic approach introducing a massless and chargeless scalar field in the Einstein-Hilbert action to account for the matter creation. In C-field theory, there is no big-bang type singularity as in steady state theory of Bondi and Gold (1948).

If a model explains successfully the creation of positive energy matter without violating the conservation of energy then it is necessary to have some degree of freedom which acts as a negative energy mode. Thus a negative energy field provides a natural way for creation of matter. Narlikar (1973) has pointed out that, matter creation is accomplished at the expense of

*Correspondence: Raj Bali, Department of Mathematics, University of Rajasthan, Jaipur-302004, India.
E-mail: balir5@yahoo.co.in

negative energy C-field. Thus the introduction of a negative energy field may solve horizon and flatness problem faced by big-bang model. Narlikar and Padmanabhan (1985) have obtained the solution of modified Einstein field equation in presence of C-field. They have shown that cosmological model based on this solution is free from singularity, particle horizon and also provides a natural explanation to the flatness problem. Bali and Tikekar (2007) have investigated C-field cosmological model for dust distribution in FRW space-time with variable gravitational constant. Recently Bali and Kumawat (2009) have investigated C-field cosmological models using FRW space-time (with positive and negative curvature) with variable gravitational constant.

The non-trivial role of vacuum generates a Λ (cosmological constant) in Einstein's field equation which leads to inflationary scenario (Sakharov (1968)) which predicted that during an early exponential phase, the vacuum energy is treated as large cosmological constant as predicted by Glashow-Salam-Weinberg model (Langacker (1981)) and Grand Unified Theory (Sakharov (1968)). Therefore, the present day observations of smallness of cosmological constant ($\Lambda \leq 10^{-56}\ \text{cm}^{-2}$) support to assume the cosmological constant (Λ) time dependent. Two independent groups led by Riess et al. (1998) and Permutter et al. (1999) used type Ia supernovae to show that the universe is accelerating. Several cosmological models with varying cosmological constant (Λ) have been investigated by number of authors viz. Abdussattar and Vishwakarma (1996), Bronnikov et al. (2003), Singh and Chaubey (2006), Bali et al. (2007,2010), Singh and Baghel (2009), Ram and Verma (2010) and Singh et al. (2012).

In this paper, we have investigated C-field cosmological model with varying cosmological constant in the frame work of FRW space-time. To get the deterministic model of the universe, we have assumed that the universe is filled with dust distribution and $\Lambda=1/R^2$ where R is scale factor. We find that matter density is constant throughout and creation field increases with time. The model is free from particle horizon. The model represents accelerating universe.

2. The Metric and Field Equation

We consider the homogeneous and isotropic metric described by Robertson-Walker line-element in the form

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2\right] \tag{1}$$

where k = 0, ± 1

Einstein's field equation by introduction of C-field is modified by Hoyle and Narlikar [1964] as

$$R_i^j - \frac{1}{2} R\, g_i^j = -8\pi G\,[\underset{(m)}{T}{}_i^j + \underset{(c)}{T}{}_i^j] + \Lambda g_i^j \tag{2}$$

The energy momentum tensor $\underset{(m)}{T}{}_i^j$ for perfect fluid and $\underset{(c)}{T}{}_i^j$ for creation field are given by

$$\underset{(m)}{T}{}_i^j = (\rho + p)\, v_i\, v^j - p\, g_i^j \tag{3}$$

$$\underset{(c)}{T}{}_i^j = -f\left(C_i C^j - \frac{1}{2} g_i^j C^\alpha C_\alpha\right) \tag{4}$$

where f > 0 is the coupling constant between matter and creation field and $C_i = \frac{dC}{dx^i}$. The field equations (2) for the metric (1) lead to

$$\frac{3\dot{R}^2}{R^2} + \frac{3k}{R^2} = 8\pi G\left[\rho - \frac{1}{2} f\, \dot{C}^2\right] + \Lambda \tag{5}$$

$$\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{k}{R^2} = 8\pi G\left[\frac{1}{2} f\dot{C}^2 - p\right] + \Lambda \tag{6}$$

3. Solution of Field Equations

The conservation equation

$$[8\pi G\, T_i^j + \Lambda\, g_i^j]_{;j} = 0 \tag{7}$$

leads to

$$8\pi\dot{G}\left[\rho - \frac{1}{2} f\dot{C}^2\right] + 8\pi G\left[\dot{\rho} - f\dot{C}\ddot{C} + 3\rho\frac{\dot{R}}{R} - 3f\,\dot{C}^2\frac{\dot{R}}{R} + 3p\frac{\dot{R}}{R}\right] + \dot{\Lambda} = 0 \tag{8}$$

p being isotropic pressure.

Following Hoyle and Narlikar [1964], we have taken p = 0. The source equation of C-field. $C^i_{;i} = 0$ leads to C = t for large r. Thus $\dot{C} = 1$.

Using p = 0, equation (6) leads to

$$\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{k}{R^2} = 4\pi G\, f\, \dot{C}^2 + \Lambda \tag{9}$$

Using $\dot{C} = 1$, equation (5) and (6) lead to

$$\frac{3\dot{R}^2}{R^2} + \frac{3k}{R^2} = 8\pi G\left[\rho - \frac{1}{2} f\right] + \Lambda \tag{10}$$

$$\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{k}{R^2} = 4\pi G\, f + \Lambda \tag{11}$$

To get deterministic solution in terms of cosmic time t, we assume that $\Lambda = \frac{1}{R^2}$.

Thus equations (10) and (11) lead to

$$\frac{\ddot{R}}{R} + \frac{2\dot{R}^2}{R^2} + \frac{2k-1}{R^2} = 4\pi G\,\rho \qquad (12)$$

Using $\Lambda = \frac{1}{R^2}$, equation (11) leads to

$$\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{(k-1)}{R^2} = 4\pi G\, f \qquad (13)$$

which again leads to

$$2\ddot{R} + \frac{\dot{R}^2}{R} + \frac{(k-1)}{R} = 4\pi G\, f\, R \qquad (14)$$

To find the solution of equation (14), we assume

$$\dot{R} = F(R)$$

This leads to $\ddot{R} = F\,F'$ where $F' = \frac{dF}{dR}$.

Thus equation (14) leads to

$$\frac{dF^2}{dR} + \frac{1}{R}F^2 = 4\pi G\, R - \frac{(k-1)}{R} \qquad (15)$$

which leads to

$$F^2 = \frac{4\pi G f}{3} R^2 - (k-1) \qquad (16)$$

The integration constant has been taken zero for simplicity. The equation (16) leads to

$$\frac{dR}{\sqrt{R^2 + \frac{3(1-k)}{4\pi G f}}} = \sqrt{\frac{4\pi G f}{3}}\, dt \qquad (17)$$

which leads to

$$\frac{dR}{\sqrt{R^2 + \alpha^2}} = \beta\, dt \qquad (18)$$

where

$$\alpha^2 = \frac{3(1-k)}{4\pi Gf}, \beta = \sqrt{\frac{4\pi Gf}{3}} \tag{19}$$

Equation (19) leads to

$$R = \alpha \sinh \beta t \tag{20}$$

Thus

$$\frac{\dot{R}}{R} = \beta \coth \beta t \tag{21}$$

To get the deterministic value of R, we assume β = 1. Thus, we have

$$\Lambda = \frac{1}{R^2} = \frac{\operatorname{cosech}^2 t}{(1-k)} \tag{22}$$

and

$$R^2 = (1-k)\sinh^2 t \tag{23}$$

Equation (10) leads to

$$\rho = \frac{1}{8\pi G}\left[\frac{3\dot{R}^2}{R^2} + \frac{3k-1}{R^2}\right] + 4\pi Gf \tag{24}$$

Thus the metric (1), after using equation (23) leads to

$$ds^2 = dt^2 - (1-k)\sinh^2 t\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2\right] \tag{25}$$

Using p = 0, equation (8) leads to

$$8\pi[G\dot{\rho} + \dot{G}\rho] - 4\pi\dot{G} f\, \dot{C}^2 - 8\pi G f\, \dot{C}\ddot{C} - 24\pi G\frac{\dot{R}}{R} f\, \dot{C}^2 + 24\pi G\rho\frac{\dot{R}}{R} + \dot{\Lambda} = 0 \tag{26}$$

Since G = constant, equation (26) leads to

$$8\pi G\dot{\rho} - 8\pi Gf\, \dot{C}\ddot{C} - 24\pi G\frac{\dot{R}}{R} f\, \dot{C}^2 + 24\pi G\rho\frac{\dot{R}}{R} + \dot{\Lambda} = 0 \tag{27}$$

Now three different cases arise:

Case (i): K = 1

Equation (17) leads to

$$\frac{dR}{\sqrt{R^2}} = \beta\, dt \tag{28}$$

which leads to

$$R = e^{\beta t} \tag{29}$$

Substituting equations (29) and (24) into (27), we have

$$\frac{d\dot{C}^2}{dt} + 6\,\dot{C}^2 = 6 \tag{30}$$

Equation (30) leads to

$$\dot{C}^2 e^{6t} = 6\int e^{6t}\, dt = e^{6t} \tag{31}$$

From equation (31), we have

$$\dot{C} = 1 \tag{32}$$

which leads to

$$C = t \tag{33}$$

which agrees with the value used in the source equation. Thus creation field C increases with t.

Case (ii): K = – 1

Equation (17) leads to

$$\frac{dR}{\sqrt{R^2 + 2}} = dt \tag{34}$$

where $4\pi Gf/3 = 1$. Equation (34) leads to

$$R = \sqrt{2}\,\sinh t \tag{35}$$

Substituting equations (35) and (24) into equation (27), we have

$$\frac{d\dot{C}^2}{dt} + (6\coth t)\,\dot{C}^2 = 6\cot ht \tag{36}$$

Equation (36) leads to

$$\dot{C}^2 \sinh^6 t = 6\int \cot ht.\sinh^6 t\, dt \tag{37}$$

From equation (37), we have

$$\dot{C} = 1 \tag{38}$$

which leads to

$$C = t \tag{39}$$

which agrees with the value used in the source equation. Thus creation field C is proportional to time t.

Case (iii): K = 0

Equation (17) leads to

$$\frac{dR}{\sqrt{R^2+1}} = dt \tag{40}$$

$$R = \sin h\, t \tag{41}$$

Substituting equations (40) and (24) into equation (27) leads to

$$\frac{d}{dt}\dot{C}^2 + (6\coth t)\,\dot{C}^2 = 6\cot ht \tag{42}$$

Equation (42) leads to

$$\dot{C}^2 \sinh^6 t = 6\int \coth t \sinh^6 t\, dt \tag{43}$$

From equation (43), we have

$$\dot{C} = 1 \tag{44}$$

which leads to

$$C = t \tag{45}$$

which agrees with the value used in the source equation. Thus, creation field C increases with time.

4. Physical and Geometrical Aspects

The metric (1) for constrained mentioned above, leads to

$$ds^2 = dt^2 - (1-k)\sinh^2 t\left[\frac{dr}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2\right] \tag{46}$$

where $k \neq 1$.

The homogeneous mass density (ρ), the cosmological constant (Λ), the scalar factor (R) and the deceleration parameter (q) for the model (46) are given by

$$8\pi G\rho = \frac{2\coth^2 t + 4 - 6k}{1-k} \tag{47}$$

$$\Lambda = \frac{1}{(1-k)\sinh^2 t} \tag{48}$$

$$R = \sqrt{1-k}\sinh t \tag{49}$$

$$q = -\tan h^2 t \tag{50}$$

where $4\pi Gf = 3$ as $\beta = 1$ is assumed.

5. Conclusion

The homogeneous mass density $\rho > 0$ for $k = 0, -1$. The creation field (C) increases with time. The spatial volume (R^3) increases with time. Since the deceleration parameter $q < 0$, hence the model (46) represents an accelerating universe. The model (46) represents a singularity free model. The cosmological constant $\Lambda \sim 1/t^2$ which matches with the latest Astronomical observations. Also the coordinate distance (γ_H) to the horizon is the maximum distance a null ray could have travelled at time t starting from the infinite part i.e.

$$\gamma_H(t) = \int_{-\infty}^{t} \frac{dt}{R(t)}$$

We could extend the proper time t to ($-\infty$) in the past because of the non-singular nature of the space-time. Now

$$\gamma_H(t) = \int_0^t \frac{dt}{R(t)} = \int_0^t \frac{dt}{\sqrt{1-k}\sin ht}$$

The integral diverges at lower limit showing that the model (46) is free from horizon. Thus creation field cosmology (C-field cosmology) solves one of the outstanding problem. Horizon problem faced by Big-Bang cosmology. The big-bang model starts with a singular state while Creation field cosmological model is singularity free model.

References

1. Abdussattar and Vishwakarma, R.G. (1996): Int. J. Phys.B 70(4), 321.
2. Bali, R and Jain, S. (2007): Int. J. Mod. Phys. D 16, 11.
3. Bali, R. and Singh, J.P. (2008): Int. J. Theor. Phys. 47, 3288.
4. Bali, R. and Tikekar, R. (2007): Chin. Phys. Lett. 24, 3290.
5. Bali, R., Tinker, S. and Singh, P. (2010): Int. J. Theor. Phys. DOI 10.1007/s 10773-010-0322-5.
6. Bali,R. and Kumawat, M. (2009): Int. J. Theor. Phys. 48,3410.
7. Bondi, H. and Gold, T. (1948):Mon. Not. R. Astron. Soc. 108, 252.
8. Bronnikov, K.A., Dobosz, A. and Dymnikova, I.G. (2003): Class. Quant. Grav. 20, 3797.
9. Hoyle, F. and Narlikar, J.V. (1964): Proc. Roy. Soc. A 282, 178.
10. Hoyle,F. and Narlikar, J.V. (1966): Proc. Roy. Soc. A 290, 162.
11. Langacker, P. (1981): Phys. Rep. 72, 185.

12. Narlikar, J.V. (1973): Nature 242, 135.
13. Narlikar, J.V. and Padmanabhan, T. (1985): Phys. Rev. D 32,1928.
14. Perlmutter, S. et al. (1999): Astrophys. J. 517, 565.
15. Ram, S. and Verma, M.K. (2010): Astrophys. and Space-Science 330, 151.
16. Riess, A.G. et al. (1998): Astronomical J. 116, 1009.
17. Sakharov, A.D. (1968): Sov. Phys. Dokl. 12, 1040.
18. Singh, P., Singh, J.P. and Bali, R. (2012): Proc. Nat. Academy of Sciences, India, DOI 10.1007/s40010-012-0054-4.
19. Singh, T. and Chaubey, R.G. (2006): Pramana – J. Phys. 67,415.
20. Smoot, G.F. et al. (1992): Astrophys. J. 396, L.1.