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Abstract

It is shown that the ΔI=2 staggering effect recently discovered in superdeformed rotational bands could be explained by a phenomenological theory of C_{4v} bifurcation. In this scenario, the energy staggering is associated with the alignment of the total nuclear angular momentum along the axis perpendicular to the long deformation axis of the prolate nucleus.

21.10.Re, 21.60.Fw, 23.20.Lv, 27.60.+j

The development of large γ-ray detector arrays has allowed experimentalists to find new nuclear phenomena at high angular momentum. For example, it has recently been discovered [1,2] that rotational bands built on superdeformed intrinsic configurations may exhibit weak $\Delta I=2$ energy oscillations where states differing by $4\hbar$ of angular momentum are perturbed in the same direction. The magnitude of the staggering effect is extremely small and only the analysis of high-fold γ-ray coincidences has allowed the determination of the γ-ray energies with the precision required to reveal a systematic effect. It has been proposed [1] that this staggering effect could be due to the presence of a perturbation which has C_4 symmetry. The experimental data suggest an increase of the staggering amplitude with spin. This feature is also a characteristic of the C_{2v} bifurcation in normal-deformed rotational bands of odd-A nuclei [3]. In this case, a small-amplitude $\Delta I=1$ staggering at low spin results from the tunneling of the angular-momentum vector between two equivalent energy minima; after a critical spin, a large-amplitude staggering involves the precession of the angular-momentum vector around the C_2-symmetry axis. In this Rapid Communication, we will use the phenomenological theory of the C_{4v} bifurcation in rotational spectra [3] in order to explain the experimental data.

Let us consider only the perturbation terms of the rotational Hamiltonian, which according to the C_{4v} symmetry can be written as an expansion

$$H_{C_{4v}} = \sum_{k=0}^{\infty} \sum_{m=0,4,\ldots}^{2k} \left[t_{km} T_{2k,m} + (-1)^m t^*_{km} T_{2k,-m}\right] \tag{1}$$

in terms of irreducible spherical tensor operators T. These operators depend on the body-fixed components (I_α with $\alpha = 1,2,3$) of the total angular momentum operator. In order to describe collective rotation and the alignment of $\mathbf{I}$ along the 1-axis which is perpendicular to the long 3-axis of the prolate nuclear shape, the tensor operators are most conveniently taken as

$$T_{l,-m} = I_-^m u_{lm}\left(\mathbf{I}^2, I_1\right) = (-1)^m T^+_{l,m},\ m \geq 0, \tag{2}$$

where $I_\pm = I_2 \pm i I_3$ are the ladder operators. The explicit expressions for the real functions

u are given, for example, in Ref. [4]. Considering the 1-axis, which coincides with the C_4-symmetry axis, as the axis of quantization, we can regroup the terms of the sum (1) and rewrite it in the form

$$H_{C_{4v}} = \sum_{k=0}^{\infty} \left[I_+^{4k} f_k^* \left(\mathbf{I}^2, I_1 \right) + f_k \left(\mathbf{I}^2, I_1 \right) I_-^{4k} \right], \tag{3}$$

where the f_k functions depend on the coefficients t. The operator (3) represents the general Hamiltonian which is invariant with respect to time reversal, inversion of the body-fixed frame, and rotation by 90° around the 1-axis.

The classical rotation energy surface defined on the space of dynamical variables (phase space) is a fundamental tool in the investigation of rotational dynamics [5]. The phase space of the rotational motion is a sphere of radius I centered at the origin. The spherical angles θ and φ of a point on this sphere determine the direction of the vector $\mathbf{I}$ in the body-fixed frame

$$I_1 = I\cos\theta, I_2 = I\sin\theta\cos\varphi, I_3 = I\sin\theta\sin\varphi. \tag{4}$$

With the help of the Hamiltonian (3) we can express the rotational energy surface

$$\begin{aligned} \varepsilon(\theta,\varphi) &= 2f_0(I,\cos\theta) + \textstyle\sum_{k=1}^{\infty}[f_k^*(I,\cos\theta)e^{4ik\varphi} \\ &\quad + f_k(I,\cos\theta)e^{-4ik\varphi}]\sin^{4k}\theta, \end{aligned} \tag{5}$$

as a function of the spherical angles (θ,φ) and the angular momentum I. The lines at constant energy on this surface represent the classical trajectories of the tip of the vector $\mathbf{I}$. They coincide with the trajectories obtained from the equation of motion

$$\frac{dI_\alpha}{dt} = \{H_{C_{4v}},\ I_\alpha\}, \tag{6}$$

where $\{\ldots\}$ is the Poisson bracket. We will investigate the motion in a small region of the phase space near the 1-axis. In other words, we will consider the rotation of a nucleus around an axis whose direction is very close to that of the C_4-symmetry axis. The rotation around such an axis is stationary because $\{H_{C_{4v}}, \mathbf{I}\} = 0$. The *aligned* stationary state $\mathbf{I}(I,0,0)$

corresponds to the fixed point $\theta = 0$ of the classical energy surface (5). Considering this surface near the fixed point, we can expand the energy $\varepsilon(\theta, \varphi)$ in a power series of the small angle θ

$$\varepsilon(\theta, \varphi) = \varepsilon_0(I) - a_1\theta^2 + (a_2 + 2c\cos 4\varphi)\,\theta^4, \tag{7}$$

where $\varepsilon_0(I)$ is the energy of rotation around the 1-axis. The coefficients a_1, a_2 and c depend on the angular momentum I. The polar angle θ plays the role of a symmetry parameter according to the Landau theory of second-order phase transitions [6]. The aligned stationary state has a *higher* symmetry because it is invariant under the C_{4v} transformation. Assuming that the angular-momentum vector $\mathbf{I}$ becomes parallel to the 1-axis at the critical value $I = I_c$, then, for the stationary states with the minimal energy, $a_1 < 0$ and $a_1 > 0$ for $I > I_c$ and $I < I_c$, respectively. Before the critical angular momentum, the stationary states with the minimal energy correspond to an angle $\theta > 0$. Since it has different sign on either side of I_c, the coefficient a_1 must vanish at the critical point $I = I_c$. Near the critical angular momentum we can approximate a_1 by $\alpha(I - I_c)$ with $\alpha < 0$. The spin I_c corresponds to the critical point of the energy surface (7) because the second derivative

$$\left(\frac{\partial^2 \varepsilon}{\partial \theta^2}\right)_{\theta=0} = -2a_1(I) = -2\alpha(I - I_c), \tag{8}$$

vanishes in this point. Eq. (7) is the canonical form of a catastrophe function for the Hamiltonian system with the C_{4v} symmetry [7]. It properly describes the behavior of the energy surface near the fixed point $\theta = 0$ as I varies.

The catastrophe function (7) is a useful guide for the determination of the quantum Hamiltonian that describes the staggering phenomenon. Accordingly, we will consider the four-parameter Hamiltonian

$$H_{C_{4v}} = \alpha(I - I_c)\frac{I_1^2 - I^2}{I^2} + a_2\left(\frac{I_1^2 - I^2}{I^2}\right)^2 + c\frac{I_+^4 + I_-^4}{I^4}, \tag{9}$$

where α, a_2, and c are assumed to be independent of I. This Hamiltonian is invariant under the C_{4v} point-symmetry group and also under the transformation $c \rightarrow -c$ with the simultaneous rotation of 45° around the 1-axis. The latter invariance allows us to consider only the positive values of c. The catastrophe Hamiltonian (9) can be used to describe the states in yrast region, which corresponds to the small area of the phase space previously discussed. The energy of these states is calculated relative to $\varepsilon_0(I)$.

The classical trajectories obtained with the Hamiltonian (9) have a different topology depending on the coefficients a_2 and c [3]. If $\mid a_2 \mid > 2c$, the trajectories correspond to the precession of $\mathbf{I}$ in the small region confined near the north pole ($\theta = 0$) of the phase sphere, and the bifurcation has a local character which involves a modification of the precessional motion. As the spin decreases, the precession around the 1-axis transforms itself into a precession around four equivalent axes located symmetrically near the north pole and rotated by an angle of 90° relative to each other. The classical energy surfaces corresponding to the Hamiltonian (9) depicted in Figs. 1a and 1b demonstrate this critical phenomenon. In the quantum case, the bifurcation manifests itself in the rearrangement of the lowest levels of the rotational multiplets. For even values of I, the lowest levels of the Hamiltonian (9) form a quartet of states with symmetry A ($A_1 + A_2$), B ($B_1 + B_2$), and E of the C_{4v} point-symmetry group. The bifurcation transforms a system of approximately equidistant quasi-degenerate doublets $A_1 + A_2$, $B_1 + B_2$ and degenerate doublets E into the six-fold quasi-degenerate clusters of states $A_1 + A_2 + B_1 + B_2 + E$ or regroups them in the initial structure of doublets (e.g. see Fig. 3.5 of Ref. [3]). The six-fold quasi-degenerate clusters are the result of the delocalized quantum precession, which involves the tunneling of $\mathbf{I}$ across the potential barriers that separate the four stable precessional axes. For $\mid a_2 \mid < 2c$, the bifurcation is non-local and cannot explain the increase of the staggering amplitude with spin. Therefore, this range of coefficients will not be discussed in this Rapid Communication.

The local bifurcation is accompanied by the alignment of $\mathbf{I}$ along the C_{4v}-symmetry axis as I increases. In this case $\alpha < 0$. As a first step, we will suppose that the C_{4v} quantum numbers are good. To form the ΔI=2 staggering pattern for a superdeformed band we can consider only the fully symmetric A_1 states. Fig. 2 shows the result of a χ^2 minimization for the staggering data of the yrast superdeformed band in ^{149}Gd. The four parameters of the Hamiltonian (9) have been varied and the calculated points have been obtained by diagonalizing this Hamiltonian for integer spins. The lowest A_1 eigenvalues have been used to form the theoretical staggering pattern. All the important features of the experimental data are reproduced: 1) there is a spin region where the staggering effect is small followed by 2) large ΔI=2 oscillations and 3) an inversion of the oscillating pattern at high spin. As pointed out in Ref. [1], in the experimental data this inversion could simply be due to an accidental degeneracy with another band having the same parity and signature; such a degeneracy has already been observed in a superdeformed band (band c) of ^{149}Gd and three points were affected in ΔE_γ [8]. However, such a scenario is improbable in the case of the yrast superdeformed band of ^{149}Gd since the configuration of this band is expected to remain yrast up to the highest spins populated in the experiment. The theoretical result shown in Fig. 2 should be considered as an example of calculation and not as a rigorous fit to the experimental data, since the calculation was performed for integer spins and the Hamiltonian (9) is expected to be valid only when the total angular momentum I is close to its critical value I_c. Furthermore, due to the magnitude of the experimental uncertainties there could be acceptable χ^2 values with different sets of parameters.

The staggering mechanism is different before and after the critical angular momentum I_c. In the region $I < I_c$, the staggering is due to the tunneling of $\mathbf{I}$ between the four minima of the energy surface. A qualitative analysis of this mechanism has been performed in Ref. [9] by evaluating the tunneling matrix element between adjacent minima. The imaginary part of this matrix element gives the staggering amplitude. In our case, it is proportional to $\exp\{-sI(I_c - I)\}$ where s is independent of I. The fact that the amplitude depends on $I_c - I$ explains the increase of the oscillations when I approaches I_c. The real part of the matrix element produces the modulation of the amplitude and can generate inversions of the staggering pattern. It should be noted that the inversion point close to critical spin I_c is always the last inversion point in a staggering pattern.

The region $I > I_c$ involves a non-tunneling staggering mechanism. As can be seen in Fig. 1b, there is only one energy minimum in this region and there is no delocalized precession.

The total nuclear angular-momentum vector precesses around the C_4 axis which coincides with the minimum of the rotational energy surface. The precessional motion has a simple form where the spin projection K_1 on the 1-axis is approximately a good quantum number. The energy of the lowest levels inside a multiplet is approximately given by

$$E_{IK} = \omega(I)(I - K_1), \tag{10}$$

where the precessional frequency is equal to $\omega = 2\alpha(I_c - I)/I$ in the high-spin limit. Eq. (10) means that the energy splitting between the two lowest levels is equal to ω. Depending on I, these levels have different quantum numbers of the C_{4v}-symmetry group. For even values of I, the lowest level is A for the sequence of spins $I = 4n$ and B for $I = 4n + 2$ [10], where n is integer. Thus, the alternating order of the A and B states produces the ΔI=2 staggering and the amplitude of this type of staggering is approximately equal to $4\alpha(I_c - I)/I$. Fig. 3 shows the spin dependence of the energy spacing between the two lowest levels involved in the staggering phenomenon. The two staggering regions and the inversion point can be seen in this figure.

The Hamiltonian (9) allows us to consider another rotational regime recently studied by Hamamoto and Mottelson [9]. In this regime, the axis of rotation is perpendicular to the C_4-symmetry axis i.e. the C_4-symmetry axis is parallel to the long 3-axis of the superdeformed prolate nuclear shape. The parameters of the Hamiltonian have to be constrained in order to ensure that the angular momentum vector remains perpendicular to the 3-axis at all spins [11]:

$$a_1 > 2a_2\frac{I-1}{I} - 2c\frac{2I^2-5}{I^2}, \tag{11}$$

where the parameter a_1 has been used instead of the expression $\alpha(I - I_c)$. There is no bifurcation in this rotational regime and the angular-momentum vector is localized in the neighborhood of four energy minima in the plane perpendicular to the C_4 axis. The tunneling mechanism results in a staggering amplitude proportional to $\exp(-sI)$. It should be pointed out that the same mechanism is responsible for the formation of clusters in molecular rotational spectra [5]. An example of staggering pattern calculated with the Hamiltonian

(9) in this rotational regime can be seen in Fig. 4. It presents a fast damping of the staggering amplitude with increasing spin which is not observed in the experiment. However, this feature could be related to our parameterization of the $H_{C_{4v}}$ Hamiltonian. For the moment, we do not know the microscopic origin of the C_4 perturbation and it is therefore impossible to deduce the spin dependence of the parameters.

The two different rotational regimes of the Hamiltonian (9) suggest different microscopic origins for the ΔI=2 staggering. On one hand, the Hamamoto and Mottelson regime could be realized by the presence of a static hexadecapole deformation ε_{44} in the nuclear shape. The $I_+^4+I_-^4$ term in the effective Hamiltonian would be a direct consequence of this deformation. However, calculations for superdeformed bands in ^{149}Gd performed by Ragnarsson with the modified oscillator potential suggest that the hexadecapole deformation ε_{44} is very small at all spins [12].

On the other hand, the rotational regime investigated in the present Rapid Communication suggests a dynamical origin for the C_{4v} perturbation. As suggested by Åberg and Nazarewicz [1,13], it could be related to nuclear shape fluctuations. In this case, the $I_+^4+I_-^4$ term in the Hamiltonian (9) would originate from the presence of a hexadecapole phonon with an angular momentum $\mathbf{J}$ which becomes aligned along the rotation axis in the high-spin limit. The corresponding rotation-vibration effective Hamiltonian would have the form

$$H = A_1(\mathbf{I}-\mathbf{J})^2 + (A_3-A_1)(I_3-J_3)^2 + H_{C_{4v}}(\mathbf{J}), \tag{12}$$

where A_1 and A_3 are rotational constants and $H_{C_{4v}}$ is the C_{4v}-invariant vibrational Hamiltonian (not the effective Hamiltonian (9)). The Hamiltonian (12) is not four-fold invariant. The term proportional to $(I_3-J_3)^2$ violates the C_{4v} symmetry and mixes the states A and B. This perturbation would be reduced when considering only the motion with small values of I_3-J_3, i.e. with the rotational angular momentum $\mathbf{R}=\mathbf{I}-\mathbf{J}$ close to the direction of the 1-axis. The corresponding classical trajectories in the phase space would be quasi-symmetrical. In the quantum case, these orbits produce the scars of the C_{4v} symmetry in the rotational bands of some superdeformed nuclei. The symmetry property is no longer associated with a particular state, but is reflected in the modulation of the whole spectrum. This phenomenon is typical of non-linear dynamics. As an example, the simplest system exhibiting the scars of the SO(2,2) symmetry is the hydrogen atom in a magnetic field [14]. The magnetic field plays an analogous role to the C_{4v}-symmetry breaking term $(I_3 - J_3)^2$ in Eq. (12). If this scenario is responsible for the staggering effect in superdeformed bands, then one could expect to observe the scars of the C_{3v} symmetry in nuclei with well-developed octupole vibrations.

In summary, the ΔI=2 staggering observed in superdeformed bands has been interpreted in terms of a phenomenological theory of C_{4v} bifurcation. The comparison of the two possible rotational regimes of superdeformed nuclei that occur in the presence of C_{4v} symmetry tends to suggest that the staggering effect does not originate from a static hexadecapole deformation. Rather, it arises from a dynamical effect that involves the alignment of an angular-momentum vector. The concept of the scars of the C_{4v} symmetry provides a promising scenario for pursuing the investigation of this staggering effect.

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[10] More precisely, the lowest states in the multiplets with even spin are the $A_1 - A_2$ and $B_1 - B_2$ inversion doublets. The energy splitting of the inversion doublets is small relative to the splitting between the A and B states.

[11] The Hamiltonian used in Ref. [9] coincides with the Hamiltonian (9). After the permutation of the axis labels (1,2,3)→(3,1,2) in Eq. (9), one can obtain the following relationships between the parameters of both Hamiltonians: $B_1 = 4c/I^4, B_2 = (a_2 - 2c)/I^4, A = a_1/I^2 + 2a_2/I^3 - 10c/I^4, \varepsilon_0(I) = AI^2 + B_1 I(2I-1) + B_2 I^2$. With these formulas, the inequality $A > 2B_2 I^2$ converts into Eq. (11).

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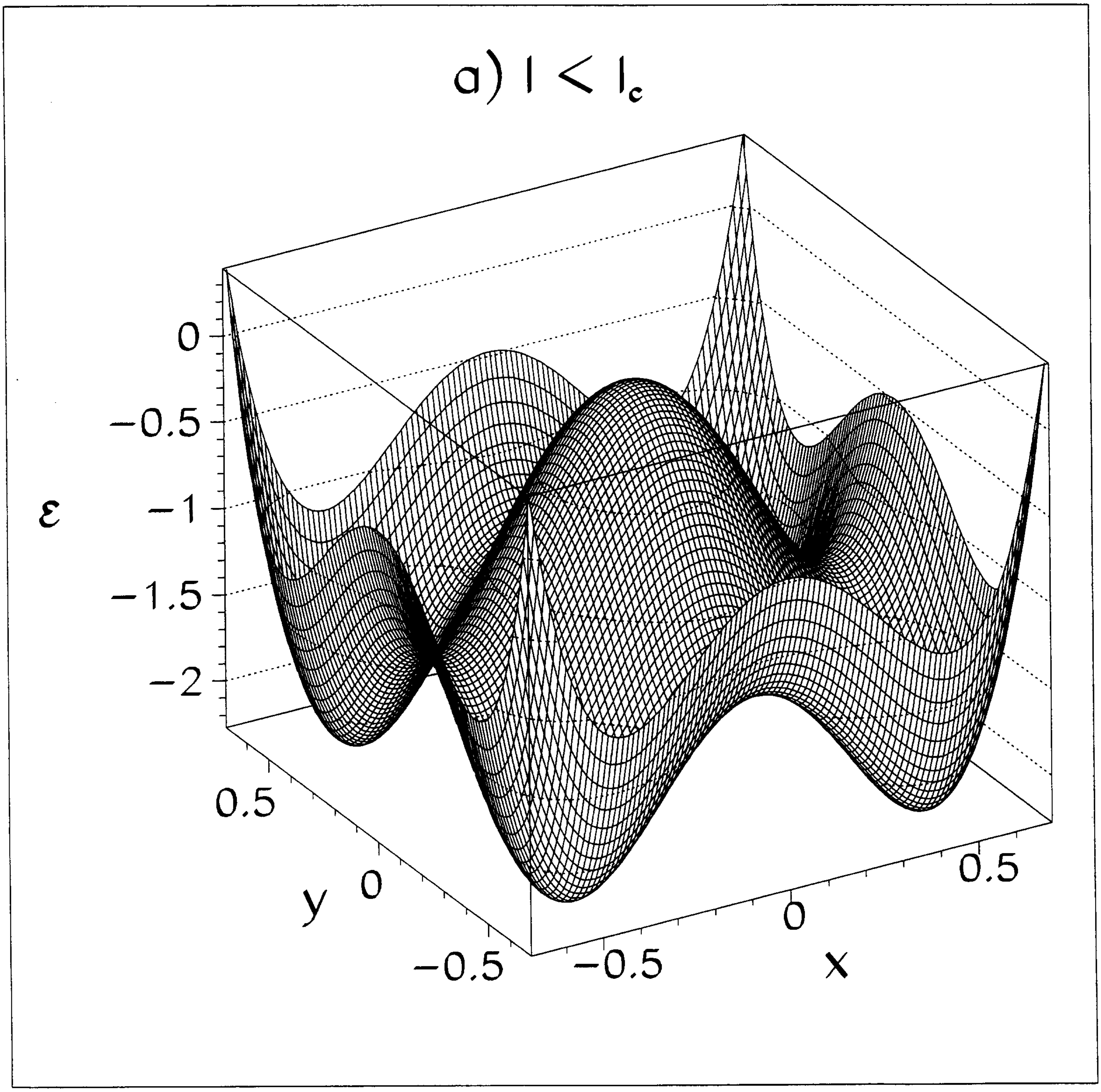
FIGURES

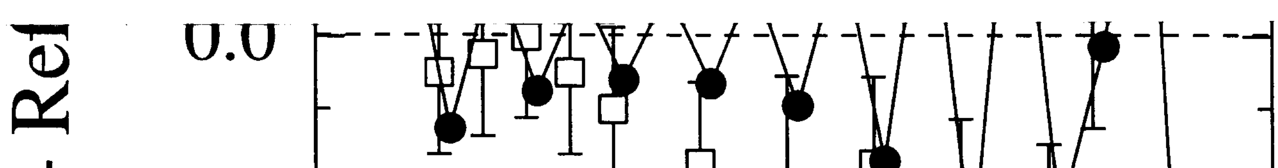
FIG. 1. Typical rotational energy surfaces of the effective Hamiltonian (9) a) before and b) after the critical point I_c. The parameter $a_1 = \alpha(I - I_c)$ was fixed to $a_1 = 10$ and $a_1 = -10$ for $I < I_c$ and $I > I_c$, respectively. In both cases, the remaining parameters were fixed to $a_2 = 15$ and $c = 2$.

FIG. 2. Energy differences ΔE_γ between two consecutive γ-ray transitions of the superdeformed band in ^{149}Gd as a function of angular momentum after subtraction of a smooth reference given by $\Delta E_\gamma^{\rm ref}(I) = [\Delta E_\gamma(I+2) + 2\Delta E_\gamma(I) + \Delta E_\gamma(I-2)]/4$ [1]. Empty squares refer to the experimental data assuming the theoretical spin assignments of Ragnarsson [15]. Filled circles correspond to a calculation performed with the effective Hamiltonian (9) with the parameters $\alpha = -0.6$, $I_c = 45$, $a_2 = 725$, and $c = 354$.

FIG. 3. Energy difference between the two lowest levels of the rotational multiplet. Filled (empty) circles correspond to states with angular momentum $I = 4n$ ($I = 4n + 2$), where n is integer. In the calculation, the parameters of the Hamiltonian (9) were the same as those used in Fig. 2.

FIG. 4. Example of staggering pattern (see Fig. 2) in the Hamamoto and Mottelson regime of rotation. The calculation has been performed with the Hamiltonian (9) with the parameters $a_1 = \alpha(I - I_c) = 2$, $a_2 = 2000$, and $c = 1000$.


a) I < I_c
ε
0
−0.5
−1
−1.5
−2
y
0.5
0
−0.5
x
−0.5
0
0.5


Rel
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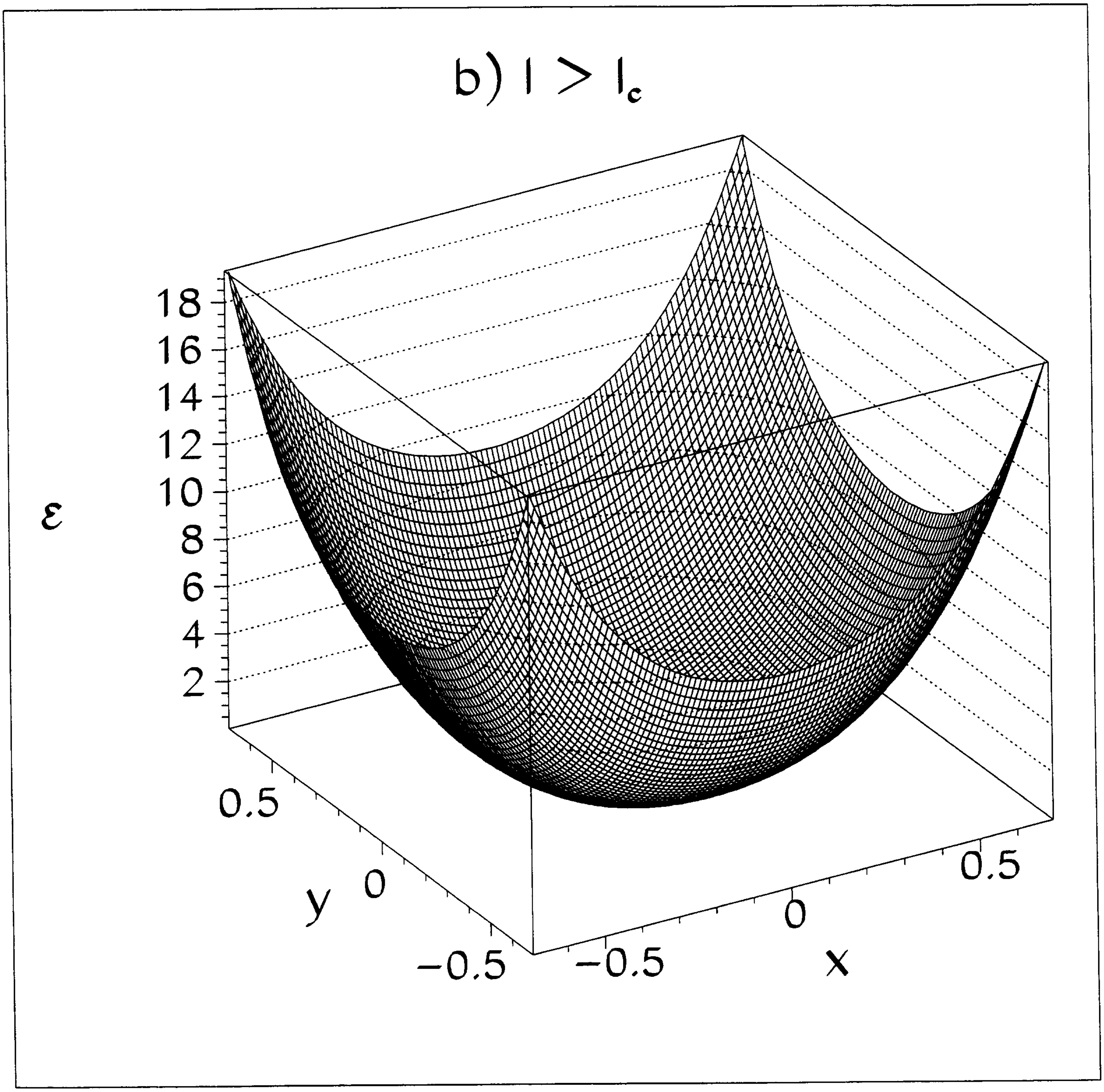
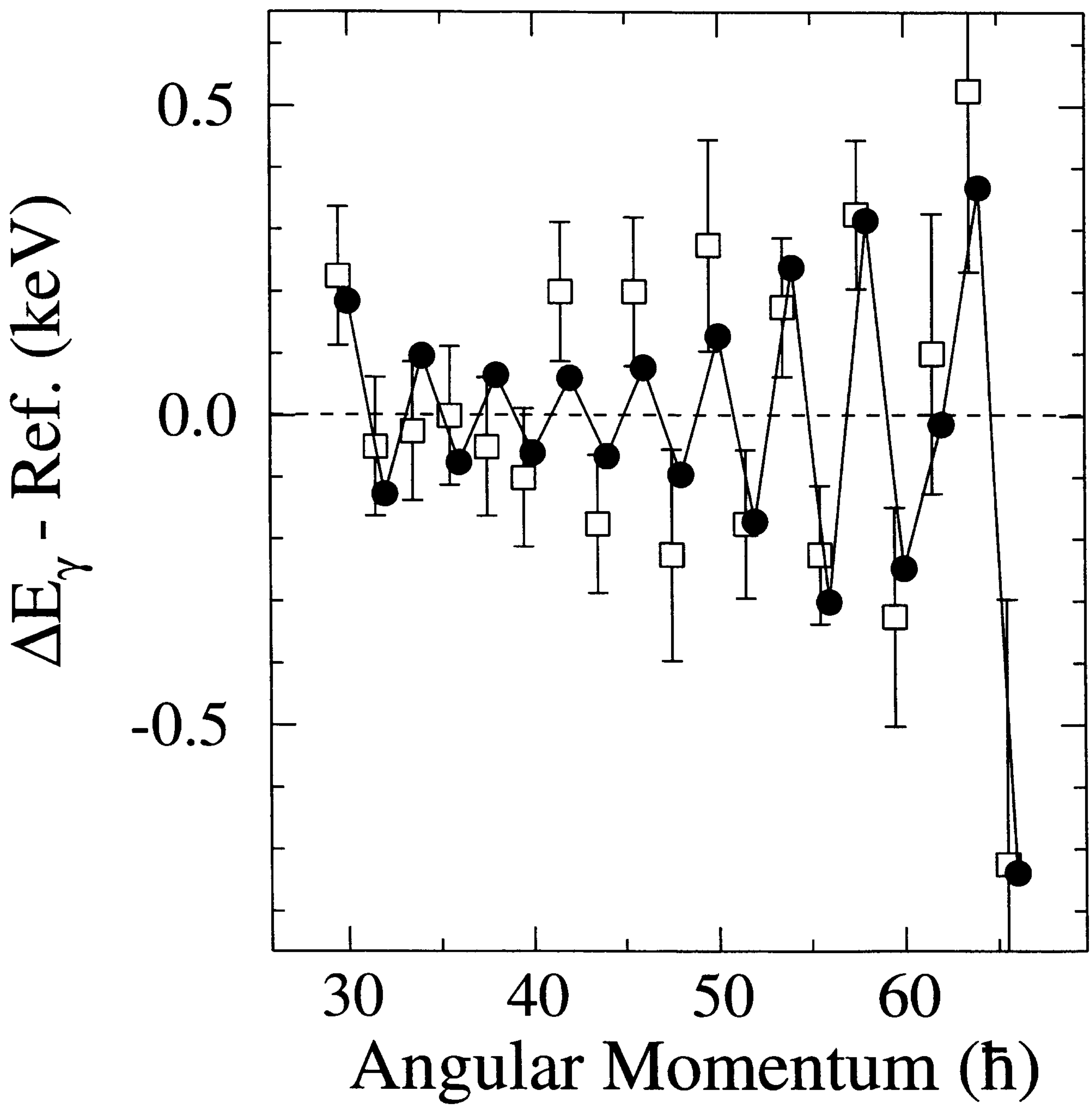

b) I > I$_c$
18
16
14
12
10
ε
8
6
4
2
0.5
y 0
−0.5
−0.5
0
0.5
x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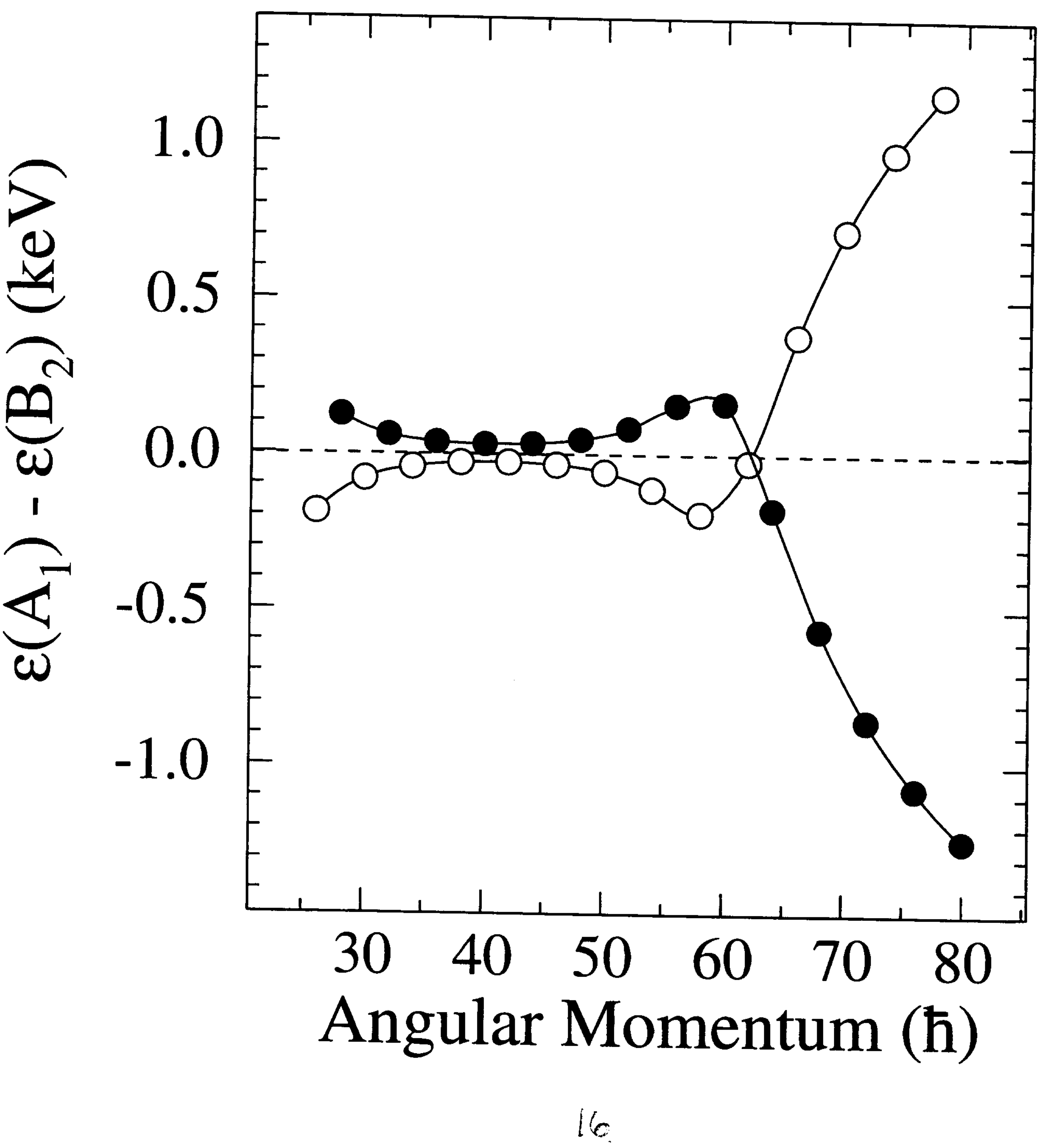
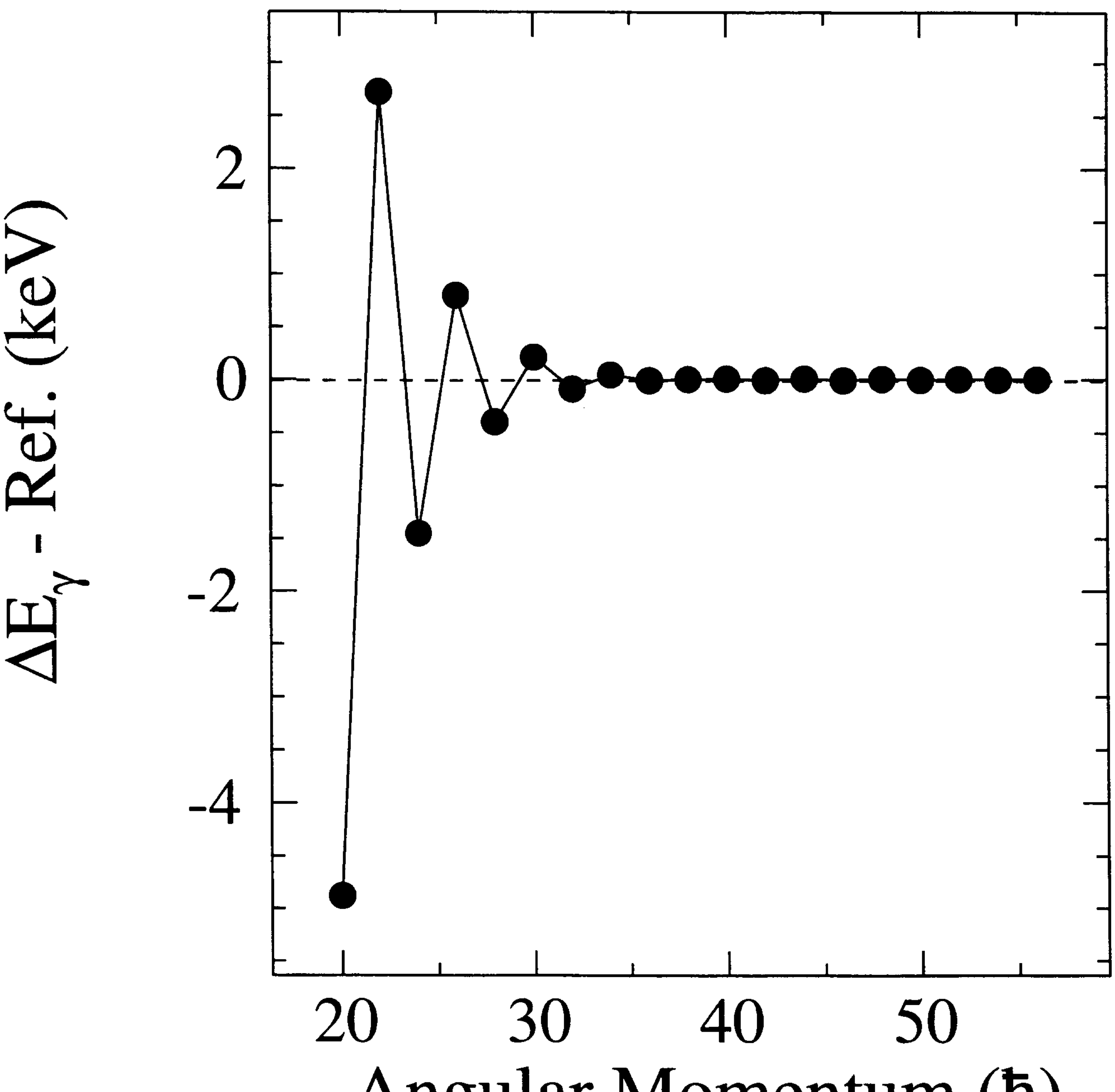
0.0
-0.5
ΔE_γ - Ref. (keV)
30
40
50
60
Angular Momentum (ħ)



1.0
0.5
0.0
-0.5
-1.0
ε(A1) - ε(B2) (keV)
30
40
50
60
70
80
Angular Momentum (ħ)


2
0
-2
-4
ΔE_γ - Ref. (keV)
20
30
40
50
Angular Momentum (ħ)