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Research Paper

Calculating the Centrality Values According to the Strengths of Entities Relative to their Neighbours and Designing a New Algorithm for the Solution of the Minimal Dominating Set Problem

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Abstract— The dominating set problem in graph theory is an NP-complete problem for an arbitrary graph. Many approximation-based studies are in the literature to solve the dominating set problems for a given graph. Some of them are exact algorithms with exponential time complexities and some of them are based on approximation without robustness concerning obtained solutions. In this study, the Malatya centrality value was used and a new Malatya centrality value was defined to solve the dominating set problem for a given graph. The improved algorithms have polynomial time and space complexities.

Keywords: *Dominating Set, Malatya Centrality Value, Graph Theory, NP-Complete Problem*

1. Introduction

Graphs are topological models to model many problems such as computer networks, social networks, biological molecular interactions, etc. Many graph problems include minimum vertex cover, minimum dominating set, maximum independent set, maximum sized clique, etc. This study focuses on investigating and improving algorithm(s) for minimum dominating sets.

A dominating set of a graph $G = (V, E)$ is a set V_D of nodes of G such that every node of G is an element of V_D or it has at least one neighbour in V_D. The dominating set can be defined in many types such as minimum dominating set, minimum independent dominating set, minimum connected dominating set, etc. The focus of this study is to investigate the minimum dominating set and minimum independent dominating set.

Definition 1: *Assume that $G = (V, E)$ is a simple graph, and $V_D \subseteq V$. $\forall v_i \in V$ and $v_i \in V_D$, $v_j \in N(v_i)$ and $v_j \in V_N$, where $N(v_i)$ is the set of neighbour nodes of v_i except itself. The set V_D with minimum cardinality is called minimum dominating set and all elements of set V_N have at least one adjacent node in V_D.*

Definition 2: *Assume that $G = (V, E)$ is a simple graph, and $V_D \subseteq V$. $\forall v_i \in V$ and $v_i \in V_D$, $v_j \in N(v_i)$ and $v_j \in V_N$, where $N(v_i)$ is the set of neighbour nodes of v_i except itself. The set V_D with minimum cardinality is called minimum independent dominating set where all elements of V_D are not adjacent.*

Goddarda and Henning gave the details of studies on independent dominating set problems. The independent dominating set and ranges for independent domination number were given in detail based on the studies focused on this problem (Goddard and Henning, 2013). This study focused on Albert's study including an algorithm for Dominating Set of size k which is an NP-Complete problem (Hagerup, 2012). The study (Bourgeois et al, 2013) focused on the minimum independent dominating set problem solving, and they advised an algorithm of time complexity as exponential. The contribution of this study is to improve the running time of the algorithm based on the approximation (Bourgeois et al, 2013). The study (Rooij and Bodlaender, 2011) focused on the dominating set and independent set problem solving based on the measure and conquer. The proposed algorithm in this study consists of a series of branch and reduce algorithms (Rooij and Bodlaender, 2011). The study (Khamis et al, 2009) includes algorithm to demonstrate a randomized algorithm for determining a dominating set in a given graph having a maximum degree of five. The study (Grandoni, 2006) focused on the complexity of the minimum dominating set algorithm advised based on set cover paradigm. The study (Guha and Khuller, 1998) presented two polynomial time algorithms that achieve approximation factors $2\mathrm{H}(\Delta(\mathrm{G}))+2$ and $\mathrm{H}(\Delta(\mathrm{G}))+2$ where $\Delta(\mathrm{G})$ is the maximum node degree in graph G, and $\mathrm{H}(..)$ is the harmonic series. The study (Wawrzyniak, 2015) improved an approximation based algorithm for a minimum dominating set. The study (Khuller and Yang, 2019) focused on the Guha and Khuller study (Guha and Khuller, 1998), and they tried to include the local information about graph for greedy criteria (Khuller and Yang, 2019).

In this study, Malatya Centrality Value (Karci et al, 2022) is used as a basic building block to develop a novel algorithm to find the dominating set for a given graph (any type of simple graph), since there are some methods for solving dominating set problem with exponential time complexity, or polynomial time complexity and executable for small size graph. We will develop a novel algorithm with polynomial time complexity and polynomial space complexity, and it is executable for large graphs.

2. The First Malatya Centrality Value

The minimum dominating set problem is an NP-complete problem, and there is no deterministic algorithm to solve this problem for all types of graphs. There are some exact methods for specific graph types. Karci et al (Karci et al, 2022) proposed an algorithm for finding minimum vertex-cover problem solving which is approximation-based and effective algorithm of polynomial time complexity. Karci and his friends proposed this method for finding solution to minimum vertex-cover problem. We will use this method as a building block for a new algorithm to find solution for the minimum dominating set.

Principle 1: *The stronger an entity is relative to its neighbours, the more powerful it is within that community locally.*

Definition 3: *(Karci et al, 2022) Assume that $G=(V,E)$ is a simple graph, and $V_N,V_D \subseteq V$, $V_N \cap V_D=\emptyset$ and $V_N \cup V_D=V$. $\forall v_i \in V$, $v_i \in V_D$ and $v_j \in N(v_i)$, where $N(v_i)$ is the set of neighbour nodes of v_i except itself. The summation of a specific node degree over its neighbour node degree separately is called The First Malatya Centrality Value, and it is denoted as Malatya[1] or $\Psi_1(\ldots)$, it can be computed by using Eq.1.*

Karci and his friends (Karci et al, 2022) defined a new centrality metric which was called Malatya Centrality Value, and then they used this metric for solving minimum vertex-cover problem. Throughout this paper, we will call this centrality value as the First Malatya Centrality Value (Malatya[1]), its definition is given in Eq.1.

Algorithm 1: `FirstMalatyaCentralityValue(G,`Ψ_1`)`

```
Input: Adjacency matrix of G is A and G = (V, E)
// G is a Graph

Output: Ψ1

   1. Initialize the first Malatya centrality
      values to zero.
   2. Compute the First Malatya Centrality values
      for all nodes by using adjacency matrix
      (Eq.1)

   3. Output the First Malatya centrality values
      as an array
```

$$Malatya^1 = \Psi_1(v_i) = \sum_{\forall v_j \in N(v_i)} \frac{d(v_i)}{d(v_j)} \quad (1)$$

The calculation of Malatya[1] for a specific node is handled by using its degree and degrees of whose neighbours. Dividing corresponding node degree to degree of its neighbours separately, and then the summation of these ratios will give us Malatya[1] (Ψ_1). The algorithm for computing Malatya[1] values for all nodes is given in Algorithm 1.

3. The Proposed Method: The Second Malatya Centrality Value and Solution for Dominating Set Problem

The Second Malatya Centrality Value (Malatya[2] - $\Psi_2(\ldots)$) was defined in this paper for the first time. The Malatya[1] values for all nodes in given graph are computed by using Algorithm 1. The second step is to calculate Malatya[2] values for all nodes in given graph. The Malatya[2] value for a specific node is calculated by dividing corresponding node Malatya[1] value by all its neighbours' Malatya[1] values separately and then suming the obtained ratios. The formal definition is given in Definition 4.

Principle 2: *When the strength of an entity relative to its neighbours is compared with the strength of its neighbours, it reveals the strength of that entity in the community globally.*

Definition 4: *Assume that $G = (V, E)$ is a simple graph and $\forall v_i \in V$, $\Psi_1(v_i)$ are computed with respect to Algorithm 1 (Eq.1). $\forall v_i \in V$, $\Psi_2(v_i)$ are computed by using Eq.2, and obtained results are called as the Second Malatya Centrality Value ($Malatya^2 = \Psi_2(\ldots)$).*

$$Malatya^2 = \Psi_2(v_i) = \sum_{v_j \in N(v_i)} \frac{\Psi_1(v_i)}{\Psi_1(v_j)} \frac{1}{d(v_i)} \frac{d_{active}(v_i)}{d(v_i)} \quad (2)$$

$d_{aktive}(v_i)$ is the degree of node v_i which are related to node not covered by selected nodes. The strength of any node such as v_i is calculated by using the summation of $\Psi_1(v_i)$ over $\Psi_1(v_j)$, $v_j \in N(v_i)$. By using this principle, after calculating the Second Malatya Centrality Value (Malatya[2] - Ψ_2) for all nodes in the given graph (Algorithm 2), the node with maximum Ψ_2 value is selected to Dominating Set (V_D) and its neighbours are added to V_N (neighbours set). The selected node and its neighbours are removed from given graph. Again, the Ψ_2 values for remaining nodes in the given graph are calculated, and the node with maximum Ψ_2 value is selected for set V_D and its neighbours are added to set V_N. This process continues until all nodes in the graph are assigned to set V_D or set V_N (Algorithm 3).

Algorithm 2: `SecondMalatyaCentralityValue(G,`Ψ_2`)`

```
Input: Adjacency matrix of G is A and G = (V, E)
Output: Ψ2
   1. FirstMalatyaCentralityValue(G,Ψ1)
   2. Compute the second Malatya centrality values
      for all nodes of given graph
   3. Output the second Malatya centrality values
      as an array
```

The complexities of Algorithm 1, Algorithm 2 and Algorithm 3 are as follows:

Assume that $G = (V, E)$, $|V| = n$ and $|E| = m$, $\Delta(G)$=Maximum node degree in graph G, $\delta(G)$=Minimum node degree in graph G.

Algorithm 1: $T_1(n) = n\Delta(G) = O(n^2)$

Algorithm 2: $T_2(n) = T_1(n) + n\Delta(G) = O(n^2)$

Algorithm 3: Assume that the size of dominating set is k:

$$T_3(n) = k(T_2(n) + n\Delta(G)) = O(kn^2)$$

Algorithm 3: OptimalDominatingSet(G,V_D)

```
Input: Adjacency matrix of G is A and G = (V, E)
Output: V_D ⊆ V, V_D is optimal dominating set
   1. V_N←∅, V_D←∅
   2. While the union of V_D and V_N is not equal to V do
   3.    Compute the SecondMalatyaCentralityValue(G,Ψ2)
      for all nodes of given graph
   4. Add arg max{Ψ2} to V_D
   5. Remove the neighbours of arg max{Ψ2} and arg
      max{Ψ2} itself from given graph with incident
      edges
   6. Add neighbours of arg max{Ψ2} to V_N
   7. Revise the Adjacency matrix (A) with respect to
      arg max{Ψ2} and its neighbours
   10.Output=V_D
```

4. Experimental Results

The experimental results will be illustrated on Path and Ring, separately.

Path graph: Assume that $G = (V, E)$ is a path graph of size n.

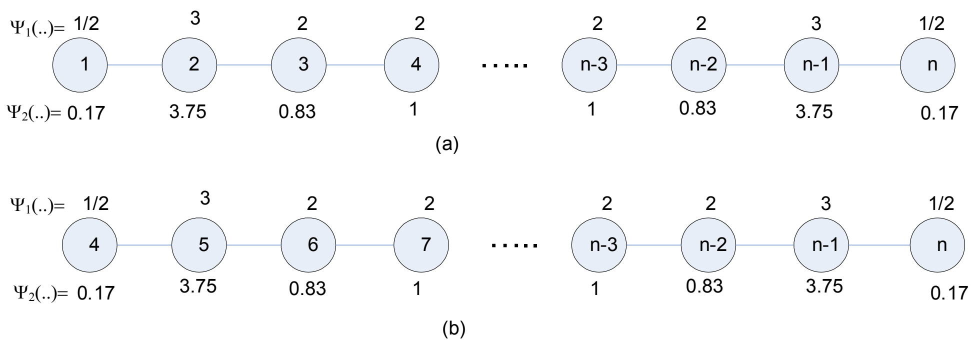

Figure 1. Path graph and its Malatya centrality values for each node. (a) Malatya centrality values for original graph, (b) Malatya centrality values for updated graph.

To obtain the optimal dominating set for graph given in Fig.1, Algorithm 1 should be executed on this graph. The results of this algorithm are given in Fig.1. After computation of the first Malatya centrality values, the second Malatya centrality values are computed by using Algorithm 2 for the first step of Algorithm 3. Fig.1 (a) illustrates the Malatya centrality values for original graph, and Fig.1 (b) illustrates the Malatya centrality values for updated graph.

First Way:

Step 1: $V_D = \{2\}$, and $V_N = \{1,3\}$, $G_1 = G_0 - \{2\} - \mathrm{N}(2)$

Step 2: $V_D = \{2,5\}$, and $V_N = \{1,3,4,6\}$, $G_2 = G_1 - \{5\} - \mathrm{N}(5)$

Step 3: $V_D = \{2,5,8\}$, and $V_N = \{1,3,4,6,7,9\}$, $G_3 = G_2 - \{5\} - \mathrm{N}(8)$

……… Until $V_D \cup V_N = V$.

Second Way: If there are more than one node whose Ψ_2 is maximum and at least one child of that node is leaf, if the distance between these nodes is greater than 2, then all these nodes are selected for optimal dominating set.

Step 1: $V_D = \{2, \mathrm{n}-1\}$, and $V_N = \{1,3,\mathrm{n},\mathrm{n}-2\}$, $G_1 = G_0 - \{2,\mathrm{n}-1\} - \mathrm{N}(2) - \mathrm{N}(\mathrm{n}-1)$

Step 2: $V_D = \{2,\mathrm{n}-1,5,\mathrm{n}-4\}$, and $V_N = \{1,3,4,6,\mathrm{n},\mathrm{n}-2,\mathrm{n}-3,\mathrm{n}-5\}$, $G_2 = G_1 - \{5,\mathrm{n}-4\} - \mathrm{N}(5) - \mathrm{N}(\mathrm{n}-4)$

Step 3: $V_D = \{2,\mathrm{n}-1,5,\mathrm{n}-4,8,\mathrm{n}-7\}$, and $V_N = \{1,3,4,6,\mathrm{n},\mathrm{n}-2,\mathrm{n}-3,\mathrm{n}-5,7,9,\mathrm{n}-8,\mathrm{n}-6\}$, $G_3 = G_2 - \{8,\mathrm{n}-7\} - \mathrm{N}(8) - \mathrm{N}(\mathrm{n}-7)$

…..

Until $V_D \cup V_N = V$.

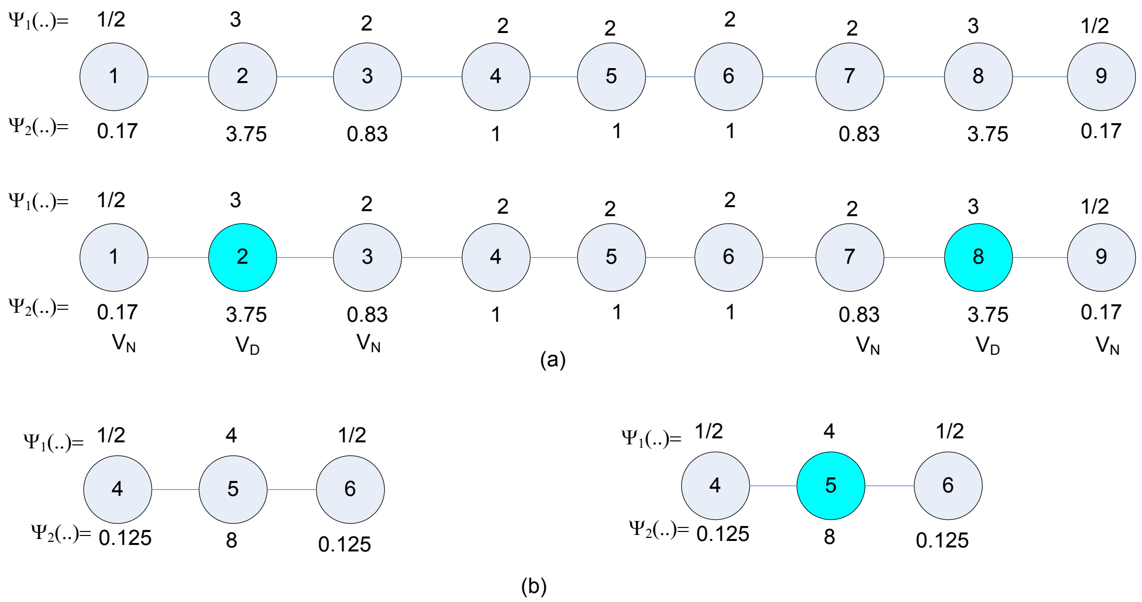

Figure 2. Minimum dominating set for path graph of size 9.

Fig.2 illustrates the execution of Algorithm 3 for a path graph of size 9. The two steps execution of selecting nodes for V_D concluded in minimum dominating set.

Cycle graph: Assume that $\mathrm{G} = (\mathrm{V}, \mathrm{E})$ is a cycle graph of size n.

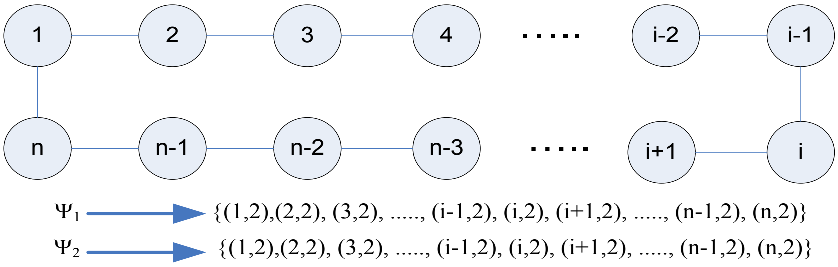

Figure 3. Cycle graph and its Malatya centrality values for each node. The centrality sets $\Psi_1 \rightarrow \{(v_i,\Psi_1(v_i)) \mid v_i \in V$, and $\Psi_1(v_i)$ is Malatya[1], $\Psi_2 \rightarrow \{(v_i,\Psi_2(v_i)) \mid v_i \in V$, and $\Psi_2(v_i)$ is Malatya[2]$\}$.

To obtain the optimal dominating set for graph given in Fig.3 (cycle graph), Algorithm 1 should be executed on this graph. The results of this algorithm are given in Fig.2 as {(1,2),(2,2), (3,2), ..., (i-1,2), (i,2), (i+1,2), ..., (n-1,2), (n,2)}. After computation of the first Malatya centrality values, the second Malatya centrality values are computed by using Algorithm 2 for the first step of Algorithm 3 as {(1,2), (2,2), (3,2), ..., (i-1,2), (i,2), (i+1,2), ..., (n-1,2), (n,2)}. Fig.3 illustrates the Malatya centrality values for original graph. The Ψ_2 values are all equal, since cycle graphs are also regular graph. One of the nodes can be selected as the first node in dominating set, and then this node and its neighbours are removed from graph. The resultant graph is a path graph and we know that this algorithm obtains minimum dominating set for a path graph. Fig.4 depicts the second step for Algorithm 3.

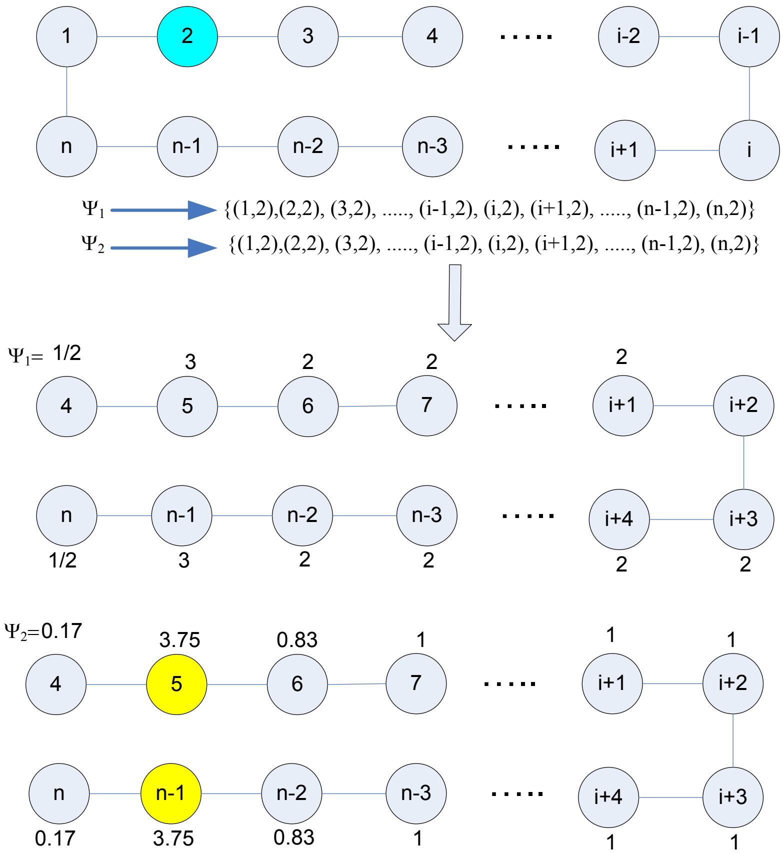

Figure 4. Cycle graph and its Malatya centrality values for each node.

5. Conclusions

The proposed method in this study is based on the power of an entity concerning its neighbours' power. The first Malatya centrality value is based on a node and its neighbours. This case can be considered as the first level strength of entities. The second Malatya centrality value can be considered as the second-level strength of entities.

The algorithms proposed in this study based on strengths of entities has the capabilities of finding minimum dominating sets for Path and Cycle graphs.

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