

# Calculation of Ramp Response of Lossy Transmission Lines Using Two-port Network Functions

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**Abstract**—In this paper, we present a new analytical approach for computing the ramp response of an RLC interconnect line with a pure capacitive load. The approach is based on the two-port representation of the transmission line and accounts for the output resistance of the driver and the line inductance. The results of our analysis are compared with the results of HSPICE simulations demonstrating the high accuracy of our solution under various values of driver, interconnect, and load impedances.

## 1. INTRODUCTION

With the exponential reduction in the feature size, the delays due to interconnections have become the dominating factor in determining the circuit performance. Due to aggressive scaling of interconnects, even an average length metal line may have significant resistance compared to the driver resistance. Thus the distributed nature of the interconnect must be modeled. Furthermore, the IC operating frequency nears multi-gigahertz requiring the interconnect inductance to be properly modeled.

Approximation techniques for estimating the time domain response of interconnect structures have been proposed. AWE [1] provides one approximation of general RLC interconnect model and has been successfully applied to analyze on-chip signal propagation. AWE begins with the differential state equations of a lumped linear time-invariant circuit and then obtains the Laplace transform solution of the homogeneous equation. This solution is expanded in a McLaurin series, and the time-domain moments are computed from this series and are matched to an approximating function consisting of a linear combination of exponential functions. REX [2] is another approach for rapidly estimating the transient response of lossy transmission line which expands the reciprocal of transfer function of the system. For critical underdamped interconnects, this method provides better results compared to AWE. Both of these approaches suffer from inaccuracy especially in high speed integrated circuits. Liao and Dai [3] proposed using an S-parameter based macromodel as a two-port network for modeling the interconnect structures. Another way of obtaining the time domain response of an interconnect line is to solve the Telegrapher's equations. Kahng and Muddu [4] used this approach for a distributed RC interconnection under the ramp excitation. They assumed that a finite number of reflections (namely four) is sufficient for generating a result very close to SPICE simulation. The authors however do not consider the inductive effect of interconnect line in their model and assume that the exciting voltage source has zero valued output resistance.

In this paper, we begin with a two-port model of the transmis-

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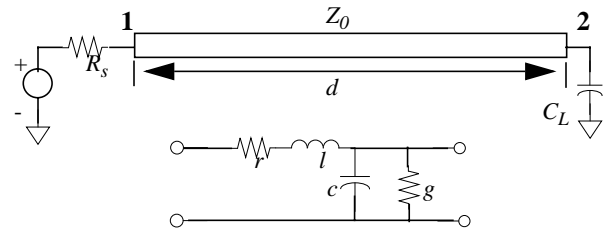
sion line and obtain the time-domain expression of the ramp response for a finite-length RLC lines. The effects of wire inductance and the resistance of CMOS driver of the interconnect are considered in our method. Section 2 summarizes the background knowledge about the Telegrapher's equations. Section 3 presents

our analytical method for computing the ramp response of a lossy transmission line. We present our experimental results and concluding remarks in sections 4 and 5, respectively.

## 2. BACKGROUND

We give some definitions and terminology first. A linear circuit belongs to the class of linear time invariant systems. Hence it can be completely characterized by its impulse response. The transient behavior of any linear system is contained in its system function which is the Laplace transform of the impulse response.

A uniform transmission line with capacitive load is depicted in Fig. (1). The transmission line has the property that a signal propagates over the interconnection medium as a wave. Fig. (1.b) depicts the electrical model of the transmission line.



**Fig. (1).** Uniform transmission line. (a) Distributed transmission line of length  $d$  with a load. (b) The electrical model.

Let  $r, l, c, g$  be the resistance, inductance, capacitance, and conductance values per unit length of a uniform transmission line. The Telegrapher's equations for such a transmission line is [5]:

$$\frac{\partial^2 v(x, t)}{\partial x^2} = lc \frac{\partial^2 v(x, t)}{\partial t^2} + (lg + rc) \frac{\partial v(x, t)}{\partial t} + rgv(x, t) \quad (1)$$

Eq. (1) is the fundamental relationship governing wave propagation along a uniform transmission line. The shunt conductance is often negligible, hence we set  $g=0$ . The boundary and initial conditions for Eq. (1) are:

$$\text{BoundaryCondition: } v(0, t) = e(t)$$

$$\text{InitialConditions: } v(x, 0^-) = 0, \quad \frac{\partial v(x, 0^-)}{\partial t} = 0$$

At each point  $x$  on the transmission line, the voltage is the sum of incident and reflected components of the wave.

In the subsequent analysis, we model the input voltage as a ramp in the  $[0, t_{rise}]$  interval and a delayed step function in the  $[t_{rise}, \infty)$  interval. We obtain the output response for each of the inputs separately. However when obtaining the output response for second part we use the initial condition imposed by the first part of the input waveform.

### 3. INTERCONNECT TRANSFER FUNCTION

Since we are interested in calculating the waveform at the output of the interconnect line, we do not go through complicated details of wave reflections through the endpoints of transmission line. Instead we resort to the two port representation of transmission lines whereby chain parameters are used for relating the port variables. More precisely we have [5], [6]:

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} e^{\lambda d} + e^{-\lambda d} & \frac{e^{-\lambda d} - e^{\lambda d}}{Y_0(s)} \\ (e^{-\lambda d} - e^{\lambda d})Y_0(s) & e^{\lambda d} + e^{-\lambda d} \end{bmatrix} \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} \quad (3)$$

where  $\lambda = \lambda(s) = \sqrt{sC(sL+R)}$ ,  $Y_0(s) = \frac{sC}{sL+R}$ , and  $d$  denotes the length of the transmission line.  $L$ ,  $R$ , and  $C$  denote the total line inductance, line resistance, and line capacitance, i.e.  $L=dl$ ,  $R=dr$ , and  $C=dc$ .

The voltage at the output port 2 (cf. Fig. (1)) is related to the current at that port by the load capacitor equation. Hence the transfer function of the interconnection loaded by a capacitor  $C_L$  is obtained as:

$$H(s) = \frac{V_2}{V_1} = \frac{2}{(e^{\lambda d} + e^{-\lambda d})(1 + Z_0(s)(C_L s) \tanh(\lambda d))} \quad (4)$$

where  $Z_0(s) = 1/Y_0(s)$ . The inverse of the first parenthesis in the denominator term of Eq. (4) is a limit summation of a power series. The transfer function can thus be written in the following form:

$$H(s) = \frac{\sum_{n=1}^{\infty} (-1)^{n+1} e^{-(2n-1)\lambda d}}{(1 + Z_0(s)(C_L s) \tanh(\lambda d))} \quad (5)$$

As can be seen from Eq. (5),  $\lambda(s)$  and  $Z_0(s)$  depend on the square root of the frequency variable  $s$ . Consequently, the inverse Laplace transform consists of the error function which does not result in a simple formula for the time domain representation of the output waveform. So we extend the McLaurin series of  $\lambda(s)$  and  $Z_0(s)$  about  $s=\infty$ , and then based on practical values of parameters, truncate the series into the first two terms of the series. A good approximation for  $\lambda(s)$  and  $Z_0(s)$  is then obtained as follows:

$$\lambda(s) = \sqrt{LC}s + \frac{1}{2}R\sqrt{\frac{C}{L}} \text{ and } Z_0(s) = \sqrt{\frac{L}{C}}\left(1 + \frac{R}{2Ls}\right) \quad (6)$$

Notice that neglecting the resistive term in  $Z_0(s)$  expression yields the well known characteristic impedance for a lossless transmission line and that the propagation delay of wave through the interconnect media is completely captured in the approximation to  $\lambda(s)$ . Combining Eqs (5) and (6), the transfer function of a lossy transmission line is obtained as:

$$H(s) = \frac{\sum_{n=1}^{\infty} (-1)^{n+1} e^{-(2n-1)\frac{R}{2}\sqrt{\frac{C}{L}}} e^{-(2n-1)\sqrt{LC}s}}{(1 + Z_0(s)(C_L s) \tanh(\lambda d))} \quad (7)$$

The above approximation for  $\lambda(s)$  causes a large change in the DC value of the transfer function. We alleviate this error by adding a gain compensation factor to the transfer function. To find an effective gain, let  $H_I(s)$  be defined as:

$$H_I(s) = \frac{1}{(1 + Z_0(s)(C_L s) \tanh(\lambda d))} \quad (8)$$

The output expression is then composed of the delayed versions of  $h_I(t)$ , the inverse Laplace transform of  $H_I(s)$  is calculated as:

$$v_{out}(t) = \sum_{n=1}^{\infty} (-1)^{n+1} e^{-\frac{(2n-1)R}{2}\sqrt{C/L}} h_1[t - (2n-1)T] \quad (9)$$

where  $T = \sqrt{LC}$  is the time of flight of the wave. Since  $T$  is very small compared to temporal changes of  $h_I(t)$ , we can assume that  $T$  is negligible, factorize  $h_I(t)$ , and put it outside the summation. We therefore come up with the following equation:

$$v_{out}(t) \cong h_1(t) \left\{ \sum_{n=1}^{\infty} (-1)^{n+1} e^{-\frac{(2n-1)R}{2}\sqrt{C/L}} \right\} \quad (10)$$

The limit of the power series in Eq. (10) gives us an estimate of the steady-state value of  $v_{out}(t)$  which is interpreted as  $V_{out}(0)$  in the  $s$ -domain (the final-value theorem [8]). Doing this we obtain:

$$v_{out}(t) \cong h_1(t) \frac{e^{-\frac{R}{2}\sqrt{C/L}}}{1 + e^{-R\sqrt{C/L}}} \quad (11)$$

The actual steady-state value of  $h_I(t)$  is one. The error is due to the second term in the right hand-side of Eq. (11). Considering the practical values of interconnect parasitics, we see that  $\exp(-R\sqrt{C/L}) \ll 1$ . Consequently the compensating gain is set to  $\exp((R/2)\sqrt{C/L})$ . The modified transfer function after taking this multiplicative factor into consideration is written as:

$$H(s) = H_I(s) \cdot \sum_{n=1}^{\infty} (-1)^{n+1} e^{-\frac{(n-1)R}{2}\sqrt{C/L}} e^{-(2n-1)\sqrt{LC}s} \quad (12.a)$$

$$v_{out}(t) = \sum_{n=1}^{\infty} (-1)^{n+1} e^{-(n-1)R\sqrt{C/L}} h_1[t - (2n-1)T] \quad (12.b)$$

$H_I(s)$  depends upon the Laplace transform of voltage at port (cf. Fig. (1)). Now we do further manipulation to make the analysis more efficient. We propose the following piece-wise linear function as an approximation to  $\tanh(\cdot)$ :

$$\tanh(\lambda l) = \begin{cases} 1 & \lambda d \geq 2 \\ 0.5(\lambda d) & 0 \leq \lambda d \leq 2 \end{cases} \quad (13)$$

We will obtain the transfer function and the ramp response of the lossy interconnect for each of these cases in the following subsections.

$H(s)$  in Eq. (12) denotes the relation between the voltages at the output port (i.e. port 2) and the input port (i.e. port 1) of the interconnect line. If we wish to have the relation between the output voltage of the interconnect and the source voltage  $e(t)$ , we have to consider the voltage division between the driver impedance and the input impedance  $Z_i(s)$  seen by looking into the interconnect line. We know from [6] that  $Z_i(s)$  is:

$$Z_i(s) = z_{11}(s) - \frac{z_{12}z_{21}}{z_{22} + Z_L} \quad (14)$$

where  $z_{11}$ ,  $z_{22}$ ,  $z_{21}$ ,  $z_{12}$  are the two-port open-circuit impedance parameters and  $Z_L$  is the load impedance. By knowing the chain parameters, any of the other sets of two-port parameters, such as the  $z$ -parameters, can be computed [6]. Hence the input impedance of the interconnect can be expressed in terms of the parameters of interconnect as follows:

$$Z_i(s) = \frac{Z_L(1 + Z_0 Y_L \tanh(\lambda d))}{1 + Z_L Y_0 \tanh(\lambda d)} \quad (15)$$

We could use a similar piece-wise linear approximation for  $\tanh(\cdot)$  which was used in Eq. (13). Using this approximation would however result in a fourth-order source to the output transfer func-

tion. To avoid this complexity, we use a different approximation.

Essentially we ignore the  $e^{-\lambda d}$  term in comparison with the  $e^{\lambda d}$  term. In the following we show that this approximation does not cause a large error.

We can rewrite Eq. (15) in the following form:

$$Z_i(s) = \frac{Z_L}{1 + Z_L Y_0 \tanh(\lambda d)} \cdot \frac{1}{H_1(s)} \quad (16)$$

where  $H_1(s)$  was given in Eq. (8). Since we are concerned about the magnitude of error we can write the magnitude of the frequency response of Eq. (16) as:

$$|Z_i(j\omega)| \leq |Z_L(j\omega)| \cdot \frac{1}{|H_1(j\omega)|} \quad (17)$$

$|Z_L(j\omega)|$  is very small in today's high-speed circuits. Furthermore, as frequency increases  $|Z_L(j\omega)|$  becomes even smaller. Any error in approximating  $H_1(s)$  is multiplied by this small value. In practice, usually  $\exp(\lambda d)$  is about 10 times greater than  $\exp(-\lambda d)$ . For instance, using the interconnect parameters for a  $0.18\mu\text{m}$  CMOS technology and assuming a  $1\text{mm}$  of Metal1 wire, a typical value for  $\lambda d$  at 500MHz clock frequency is around 1.3 [7]. For global interconnect lines, this value is even larger.

Based on the above approximation, we come up with the following expression for  $Z_i(s)$ :

$$Z_i(s) = \frac{1}{Y_0(s)} = Z_0(s) \quad (18)$$

Consequently, the output voltage of the interconnect line is related to source voltage,  $e(t)$ , by a simple voltage division made by  $Z_i(s)$  and  $R_s$ .

$$V_1(s) = E(s) \frac{Z_i(s)}{Z_i(s) + R_s} = \frac{Z_0(s)}{R_s + Z_0(s)} E(s) \quad (19)$$

In the following subsections each of the two cases for approximating  $\tanh(\cdot)$  are considered separately. The output waveform is obtained for each of two cases.

#### CASE I. $\tanh(\lambda d) = 1$ :

In this case  $H_1(s)$  is represented by a first-order rational function of  $s$  as follows:

$$H_1(s) = \frac{1}{\sqrt{\frac{L}{C}} C_L \left( s + \frac{R}{2L} + \frac{1}{C_L \sqrt{\frac{L}{C}}} \right)} \quad (20)$$

Comparing Eq. (20) with the actual value of  $H_1(s)$ , again we see that the actual DC value of  $H_1(s)$  differs from the DC value of the approximated expression of  $H_1(s)$ . This difference will affect the steady state value of the output voltage. To overcome this, we can add a constant multiplicative gain so that the DC value of Eq. (20) becomes unity value. Therefore we obtain:

$$H_1(s) = (1/\tau_L)/(s + 1/\tau_L) \quad (21)$$

$$\text{where: } 1/\tau_L = \frac{1}{C_L \sqrt{\frac{L}{C}}} + \frac{R}{2L}$$

As stated before, we break up the input waveform into two parts: (i) ramp input  $e_r(t)$  (ii) step input  $e_s(t)$ . Hereafter we use the convention that any voltage variable with index  $r$  is related to the ramp input, and any voltage variable with index  $s$  is related to the step section of the input. Output response is computed for each of these parts.

Let's consider the first part of the input. From Eq. (19) we are

able to obtain the input voltage to transmission line:

$$V_r(s) = \frac{V_{DD}}{t_{rise}} \frac{\sqrt{L/C}}{\sqrt{L/C} + R_s} \cdot \frac{s + \frac{R}{2L}}{(s + 1/\tau_s)s^2} \quad (22)$$

$$\text{where: } 1/\tau_s = \frac{\sqrt{L/C}(R/(2L))}{\sqrt{L/C} + R_s}$$

We apply partial fraction expansion to  $V_r(s)$  and then, after obtaining the response to each of the fractional terms, simply utilize the superposition property to calculate the final value of the output response as follows. Eq. (23) represents the partial fraction expansion of  $V_r(s)$ :

$$V_r(s) = \frac{V_{DD}/t_{rise}}{s^2} - \frac{R_s}{(R/(2L))\sqrt{L/C}} \left( \frac{1}{s} - \frac{1}{s + 1/\tau_s} \right) \quad (23)$$

As can be seen from the above equation, three terms are present in the partial fraction expansion of the voltage at port 1. We name each of the terms as  $V_{r1}(s)$ ,  $V_{r2}(s)$ , and  $V_{r3}(s)$ , respectively. The Laplace transform of the output voltage at port 2 is composed of the response to each of the three terms. We name each of the output terms as  $V_{r1}^o(s)$ ,  $V_{r2}^o(s)$ , and  $V_{r3}^o(s)$ , respectively. It is well known that the Laplace transform of the system response is a product of the system transfer function and the Laplace transform of the input[8]. Consequently, we have:

$$V_{r1}^o(s) = H_1(s) V_{r1}(s) = \frac{V_{DD}}{t_{rise}} \cdot \frac{1/\tau_L}{s^2(s + 1/\tau_L)} \quad (24)$$

Applying partial fraction expansion, then taking inverse Laplace transformation, we come up with the temporal waveform of the output:

$$v_{r1}^o(t) = \frac{V_{DD}}{t_{rise}} \cdot tu(t) - \left( \frac{V_{DD}}{t_{rise}} \cdot \tau_L \right) (1 - e^{-t/\tau_L}) \quad (25)$$

Similarly, we repeat the above steps to obtain the timing waveforms of  $v_{r2}^o(t)$ ,  $v_{r3}^o(t)$  as follows:

$$v_{r2}^o(t) = - \left( \frac{V_{DD}}{t_{rise}} \cdot \frac{R_s}{(R/(2L))\sqrt{L/C}} \right) (1 - e^{-t/\tau_L}) \quad (26)$$

$$v_{r3}^o(t) = \frac{V_{DD}}{t_{rise}} \cdot \frac{2L}{R} \cdot \frac{\tau_s/\tau_L}{\left( 1 + \frac{\tau_L}{R_s C_L} \right)} (e^{-t/\tau_s} - e^{-t/\tau_L}) \quad (27)$$

The output voltage is composed of the algebraic summation of the three components, i.e.:

$$v_r^o(t) = v_{r1}^o(t) + v_{r2}^o(t) + v_{r3}^o(t) \quad (28)$$

For the time interval  $[t_{rise}, \infty)$  the input has a step form,  $e_s(t)$ . The output voltage is made up of the step input and the initial condition imposed by the ramp input. As a result of the continuity condition, the interconnect response to the second part of the input at time  $t_{rise}$  must be equal to the response of the interconnect to the first part at that same time. The same relationship exists for voltages at all other points especially the voltage at the input port of the interconnect, that is:

$$v_r(t = t_{rise}) = v_s(t = t_{rise}) \quad (29)$$

As defined before,  $v_r(t)$  is the input voltage of port 1 when the excitation is in the form of a ramp whereas  $v_s(t)$  is the input voltage of port 1 when the excitation is a step function.  $v_r(t)$  is simply determined by taking the inverse Laplace transform of Eq. (21):

$$v_r(t) = \frac{V_{DD}}{t_{rise}} r(t) - \left( \frac{V_{DD}}{t_{rise}} \cdot \frac{R_s}{(R/(2L))\sqrt{L/C}} \right) (1 - e^{-t/\tau_s}) \quad (30)$$

$V_s(s)$  is determined by substituting  $E(s)$  in Eq. (19) with the Laplace transform of the step function. Furthermore we should include an extra term which specifies the response under the initial condition. After taking the inverse Laplace transform, we obtain the following expression:

$$v_s(t) = V_0 e^{-t/\tau_s} + V_{DD} \left( 1 - \frac{R_s}{R_s + \sqrt{L/C}} e^{-t/\tau_s} \right) \quad (31)$$

In Eq.(31) the unknown variable  $V_0$  is determined by satisfying Eq. (29). We can subsequently write:

$$v_s(t) = \frac{V_{DD}}{t_{rise}} \cdot \frac{R_s}{(R/(2L))\sqrt{L/C}} (1 - e^{t_{rise}/\tau_s}) e^{-t/\tau_s} + V_{DD} \quad (32)$$

We take the same steps as was taken for obtaining the response for interval  $[0, t_{rise}]$  to derive the response for  $[t_{rise}, \infty)$ .  $v_s(t)$  is composed of two terms. One is exponentially rising in time and asymptotically goes toward a constant value, while the other is constant in time. We name  $v_{s1}^o(t)$ ,  $v_{s2}^o(t)$  as the system responses to each of the above portions of the  $v_s(t)$ . We have:

$$v_{s1}^o(t) = V_{DD} (1 - e^{-t/\tau_L}) \quad (33)$$

$$v_{s2}^o(t) = \frac{V_{DD}}{t_{rise}} \cdot \frac{2L}{R} \cdot \frac{\tau_s/\tau_L}{\left(1 + \frac{\tau_L}{R_s C_L}\right)} (1 - e^{t_{rise}/\tau_s}) \cdot (e^{-t/\tau_s} - e^{-t/\tau_L}) \quad (34)$$

The final expression of the output voltage is composed of the two functions of Eq. (33), and Eq. (34):

$$v_s^o(t) = v_{s1}^o(t) + v_{s2}^o(t) \quad (35)$$

The system response is the algebraic summation of  $v_s^o(t)$  and  $v_r^o(t)$ .

CASE II.  $\tanh(\lambda d) = 0.5(\lambda d)$ :

In this case we expect that the equations become more complicated since the order of the transfer function is increased. As we will show this complexity also appears in the output voltage waveform so that the waveform asymptotically goes to its final value with ringing. Similar to the previous case, we change the DC value of the transfer function such that it becomes identical to the DC value of the actual transfer function:

$$H_1(s) = \frac{\omega_n^2}{s^2 + 2\alpha s + \omega_n^2} \quad (36)$$

where  $\omega_n^2 = \frac{1}{2} \left( \frac{R}{2L} \right)^2 + \frac{1}{LC_L}$  and  $\alpha = \frac{R}{4L}$ .

The denominator has always two complex conjugate poles, hence the response has underdamped behavior and we should observe ringing effect on the output waveform. The method for obtaining the output waveform is the same as that used for case I. So we do not present the details of computations. For interval  $[0, t_{rise}]$ , the output is composed of three terms:

$$v_{r1}^o(t) = \frac{V_{DD}}{t_{rise}} r(t) - \left( \frac{2V_{DD}}{t_{rise}} \cdot \frac{\alpha}{\omega_n^2} \right) u(t) - \frac{V_{DD}}{t_{rise}} \cdot \frac{e^{-\alpha t}}{\omega_d} \cos(\omega_d t - \theta) \quad (37)$$

where  $\omega_d = \sqrt{\omega_n^2 - \alpha^2}$ ,  $\theta = \arctan\left(\frac{\alpha^2 - \omega_d^2}{2\alpha\omega_d}\right)$ ,

$$v_{r2}^o(t) = - \left( \frac{V_{DD}}{t_{rise}} \cdot \frac{R_s}{(R/(2L))\sqrt{L/C}} \left( 1 - \frac{\omega_n}{\omega_d} e^{-\alpha t} \cos(\omega_d t - \phi) \right) \right) \quad (38)$$

where  $\phi = \arctan\left(\frac{\alpha}{\omega_d}\right)$ , and

$$v_{r3}^o(t) = \frac{V_{DD}}{t_{rise}} \cdot \frac{R_s}{2L\sqrt{L/C}} \cdot H_1\left(\frac{-1}{\tau_s}\right) \cdot \left( e^{-t/\tau_s} - e^{-\alpha t} \cos(\omega_d t - \xi) \right) \quad (39)$$

where  $\xi = \arctan\left(\frac{\alpha - (1/\tau_s)}{\omega_d}\right)$ .

The output voltage is again stated as an algebraic summation of  $v_{r1}^o(t)$ ,  $v_{r2}^o(t)$ ,  $v_{r3}^o(t)$ . Similar to case I, the output for  $[t_{rise}, \infty)$  is composed of two terms,  $v_{s1}^o(t)$ ,  $v_{s2}^o(t)$ , due to the two separable parts of the input:

$$v_{s1}^o(t) = V_{DD} \left( 1 - \frac{\omega_n}{\omega_d} e^{-\alpha t} \cos(\omega_d t - \phi) \right) \quad (40)$$

$$v_{s2}^o(t) = V_{s0} \cdot H_1\left(\frac{-1}{\tau_s}\right) \cdot \left( e^{-t/\tau_s} - e^{-\alpha t} \cos(\omega_d t - \xi) \right) \quad (41)$$

where  $V_{s0} = \frac{V_{DD}}{t_{rise}} \cdot \frac{R_s}{R\sqrt{L/C}} (1 - e^{t_{rise}/\tau_s})$ . Again  $v_s^o(t) = v_{s1}^o(t) + v_{s2}^o(t)$ .

Finally the system response is  $v_{s+r}^o(t) = v_s^o(t) + v_r^o(t)$ .

From Eq. (12.b), the final value of the output is a summation of delayed versions of the computed output voltage. Since the attenuation factor is large, we only consider a few terms of Eq. (12.b) to calculate the final value of the voltage. Experiments have shown that four terms are sufficient to produce high accuracy. With this approximation, the final value of the output voltage will be:

$$v_{out}(t) = \sum_{n=1}^4 (-1)^{n+1} \exp\left(- (n-1)R\sqrt{\frac{C}{L}}\right) v_{s+r}^o[t - (2n-1)\sqrt{LC}] \quad (42)$$

## 4. EXPERIMENTAL RESULTS

We examined several examples with different values of interconnect length, technology parameters, and load and source impedances. Among these experiments four representative examples are reported.

Table 1. compares the rise-time of the waveforms derived by our analysis with those derived by HSPICE simulation for a number of different transmission lines. Rows 1 through 4 correspond to the transmission lines depicted in Fig. (2) through Fig. (5). The remaining rows represent new data points. In table 1,  $WD$  represents the width of each conductor,  $HT$  is the height of conductor, and  $TH$  is thickness of the conductor.

Figs (2) and (4) show the ramp response of two lossy interconnects with  $\lambda$  less than two. The analysis of case II is used for these two figures. Fig. (2.a) depicts the output waveform obtained by our analysis whereas Fig. (2.b) shows the output waveform of the same configuration by HSPICE simulation. Similarly Figs (4.a) and (4.b) show the output response of another interconnect obtained by our analysis and HSPICE simulation respectively. Figs (3) and (5) show the ramp responses of lossy interconnects with  $\lambda$  greater than two. The analysis of case I is used for these two cases.

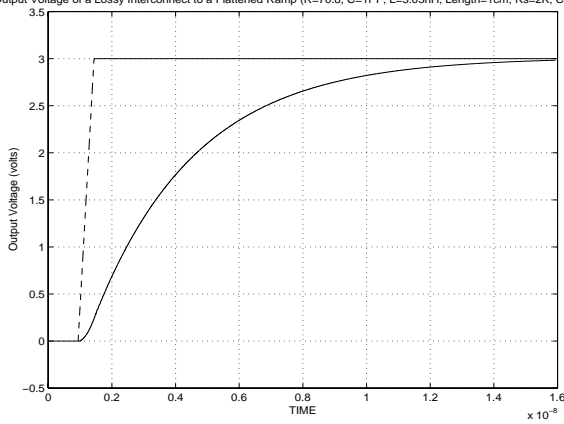
## 5. CONCLUSION

In this paper we proposed a new method for obtaining the analytical expression for the ramp response of a lossy interconnect. The inductive effects of the wire line, and most importantly the output resistance of the wire driver were considered in our analysis. We started with the two-port representation of the transmission line. Among various two-port parameters the chain matrix was selected. Notice that this property is very useful when analyzing the ramp response of a cascade of interconnect segments with different widths. This kind of two-port matrix allows us to obtain the two port matrix of any number of cascade connections of different wires with different wire sizes very easily by simply multiplying their two-port chain matrices. We then obtained the ramp response of the system by doing some further simplification. The results show that this method is able to obtain the ramp response of the lossy interconnect with small error.

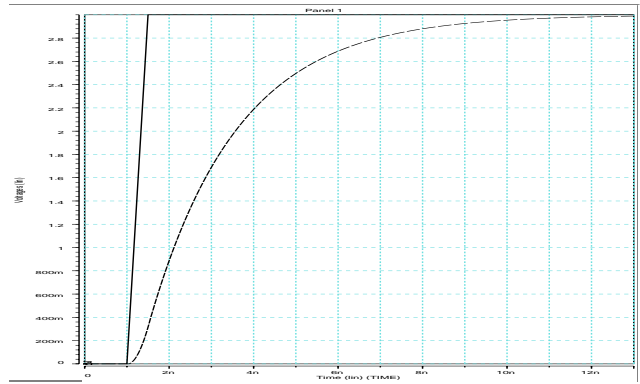
**Table 1: Comparison between rise-times of our analysis and HSPICE**

$WD$ ( $\mu\text{m}$ )	$HT$ ( $\mu\text{m}$ )	$TH$ ( $\mu\text{m}$ )	$R_s$ $\text{k}\Omega$	$C_L$ $\text{pF}$	$R$ $\Omega$	$C$ $\text{pF}$	$L$ $\text{nH}$	$d$ $\text{mm}$	$t_{rise}(\text{ours})$ $\text{nsec}$	$t_{rise}(\text{HSPICE})$ $\text{nsec}$	<i>error</i>
3	2	0.8	2	0.03	70.8	1	3.05	10	0.49	0.65	24.6%
0.2	2.5	0.25	2	0.01	680	0.0736	1.45	2	0.3	0.39	23%
0.3	2.5	0.25	2	0.01	227	0.043	0.635	1	0.23	0.22	4.35%
0.25	2.5	0.25	2	0.01	540	0.077	1.4	2	0.38	0.4	5%
0.2	3	1	2	0.01	170	0.068	1.29	2	0.45	0.4	11%
0.6	2	0.5	2	0.01	113	0.1053	1.035	2	0.74	0.63	14.9%
0.6	2	0.1	2	0.01	567	0.0965	1.21	2	0.46	0.5	8%
0.25	1.0	0.25	2	0.01	544	0.1	1.07	2	0.5	0.55	9.1%
0.25	1.75	0.25	2	0.05	544	0.086	1.26	2	0.57	0.69	17.4%

Output Voltage of a Lossy Interconnect to a Flattened Ramp ( $R=70.8$ ,  $C=1\text{pF}$ ,  $L=3.05\text{nH}$ , Length= $1\text{cm}$ ,  $R_s=2\text{K}$ ,  $C_L=0.03\text{pF}$ )



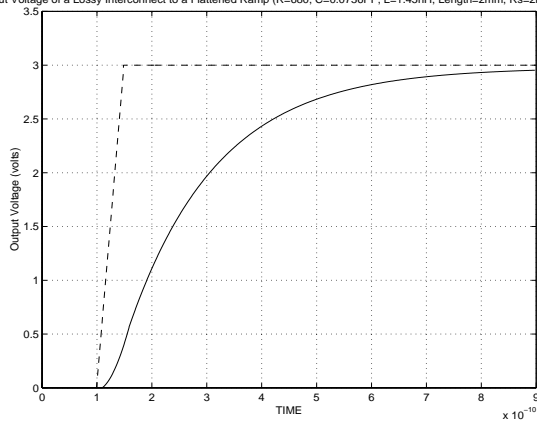
(a)



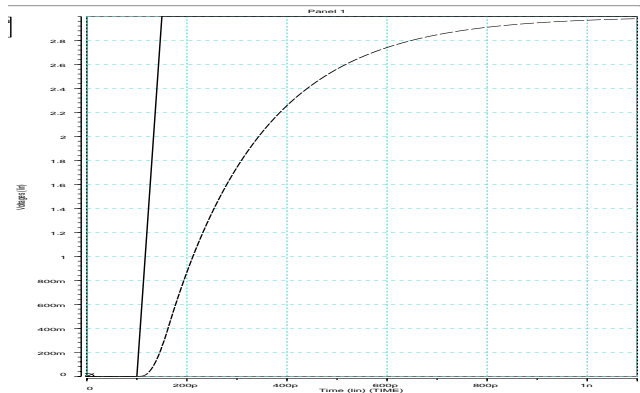
(b)

**Fig. (2).** Ramp response of a lossy interconnect (Length= $1\text{cm}$ ,  $R=70.8$ ,  $C=1\text{pF}$ ,  $L=3.05\text{nH}$ ) excited by a ramp input with  $R_s=2\text{K}$  as the source resistance and  $C_L=0.03\text{pF}$  as the load capacitance. (a) The result obtained by our method. (b) The result obtained from HSPICE simulation.

Output Voltage of a Lossy Interconnect to a Flattened Ramp ( $R=680$ ,  $C=0.0736\text{pF}$ ,  $L=1.45\text{nH}$ , Length= $2\text{mm}$ ,  $R_s=2\text{K}$ ,  $C_L=0.01\text{pF}$ )



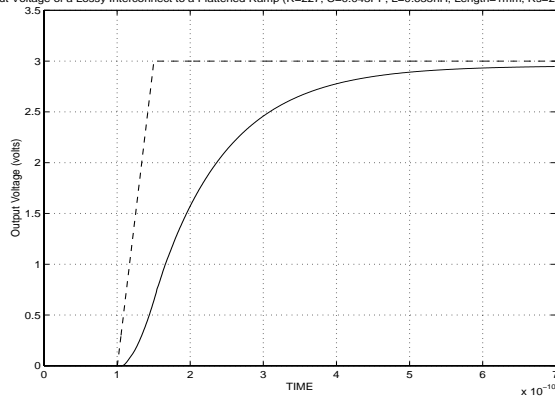
(a)



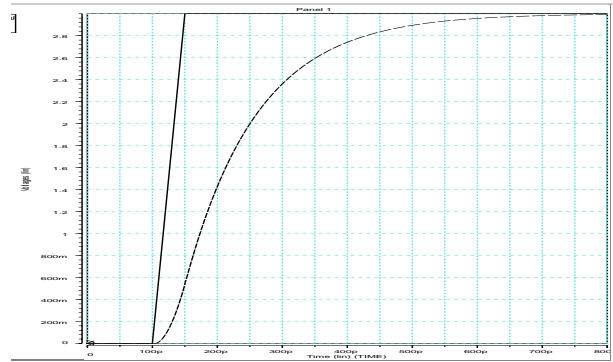
(b)

**Fig. (3).** Ramp response of a lossy interconnect (Length= $2\text{mm}$ ,  $R=680$ ,  $C=0.0736\text{pF}$ ,  $L=1.45\text{nH}$ ) excited by a ramp input with  $R_s=2\text{K}$  as the source resistance and  $C_L=0.01\text{pF}$  as the load capacitance. (a) The result obtained by our method. (b) the result obtained from HSPICE simulation.

Output Voltage of a Lossy Interconnect to a Flattened Ramp ( $R=227$ ,  $C=0.043\text{PF}$ ,  $L=0.635\text{nH}$ , Length=1mm,  $R_s=2\text{K}$ ,  $C_L=0.0$ )



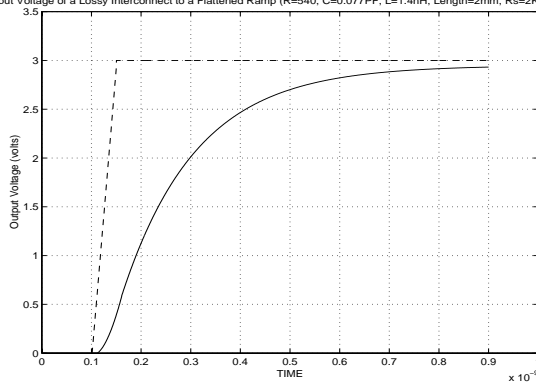
(a)



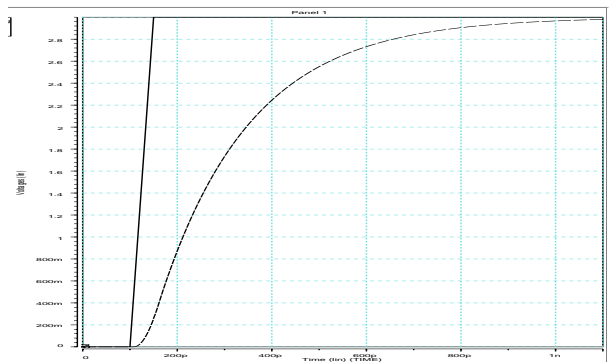
(b)

**Fig. (4).** Ramp response of a lossy interconnect (Length=1mm,  $R=227$ ,  $C=0.043\text{PF}$ ,  $L=0.635\text{nH}$ ) excited by a ramp input with  $R_s=2\text{K}$  as the source resistance and  $C_L=0.01\text{pF}$  as the load capacitance (a) The result obtained by our method. (b) the result obtained from HSPICE simulation.

Output Voltage of a Lossy Interconnect to a Flattened Ramp ( $R=540$ ,  $C=0.077\text{PF}$ ,  $L=1.4\text{nH}$ , Length=2mm,  $R_s=2\text{K}$ ,  $C_L=0.01\text{PF}$ )



(a)



(b)

**Fig. (5).** Ramp response of a lossy interconnect (Length=2mm,  $R=540$ ,  $C=0.077\text{PF}$ ,  $L=1.4\text{nH}$ ) excited by a ramp input with  $R_s=2\text{K}$  as the source resistance and  $C_L=0.01\text{pF}$  as the load capacitance. (a) The results obtained by our method. (b) the result obtained from HSPICE simulation.

## 6. REFERENCES

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