CALCULATION OF SHEAR CAPACITY OF RECYCLED CONCRETE BEAMS WITH WEB REINFORCEMENT BASED ON MODIFIED COMPRESSION FIELD THEORY

by

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Based on the modified compression field theory, this paper considers the interlocking force between recycled concrete aggregates and their effective grain sizes, and establishes a shear model to describe accurately the actual force of recycled concrete beams with web reinforcement. Mathematical formulas for calculation of the interlocking force are deduced, and the shear capacity of the recycled concrete beams can be estimated. The data of five test beams and 20 test beams in open literature are in good agreement with the theoretical prediction, which proves that this method can be used to calculate the ultimate shear capacity for practical applications.

Key words: *recycled aggregate, modified compression field theory, recycled concrete beams with web reinforcement, shear capacity*

Introduction

The concrete structure is the most widely used structures in the world. A damaged concrete structure needs to be demolished and reconstructed, resulting in a large number of abandoned concrete. Many researchers proposed to crush the waste concrete in a factory, and then removed a large number of its impurities to produce recycled aggregates [1, 2]. Concrete made of the recycled aggregate is called the recycled aggregate concrete (RAC) [3].

The research on RAC is mainly divided into two aspects: material properties and structural performance [4]. O'Mahony [5] found that the shear strength of RAC can meet the shear requirements of concrete. Gonzalez and Martinez [6] found that the shear capacity of beams is proportional to the stirrup ratio. Huang [7] found that the web reinforcement is beneficial to the shear capacity of RAC beams, and proposed the calculation model of shear capacity of RAC beams. At present, the existing calculation methods of shear capacity of RAC beams do not consider the characteristics of recycled aggregate itself, which cannot describe accurately the actual situation. Based on the modified compression field theory (MCFT), a

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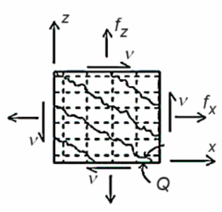
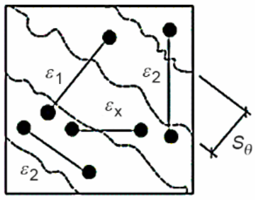
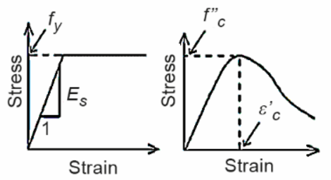
new shear model is proposed in this paper. By considering the particle size of the recycled aggregate and the crack width of RAC beams, the stress at the crack is analyzed, and the method of ultimate shear capacity of RAC beam with web reinforcement is proposed.

Modified compression field theory

The MCFT puts forward 15 basic formulas, as shown in tab. 1. Through these formulas, the shear performance of various structures can be analyzed.

Table 1. Formulas in the MCFT

		
Equation of equilibrium mean stresses $f_x = \rho_x f_{sx} + f_1 - v\cot\theta$ (1) $f_z = \rho_v f_{sz} + f_1 - v\tan\theta$ (2) $v = (f_1 + f_2)/(\tan\theta + \cot\theta)$ (3)	Geometrical compatibility mean strain $\tan^2\theta = \dfrac{\varepsilon_x + \varepsilon_2}{\varepsilon_z + \varepsilon_2}$ (6) $\varepsilon_1 = \varepsilon_x + \varepsilon_z + \varepsilon_2$ (7) $\gamma_{xz} = 2\lvert\varepsilon_x + \varepsilon_2\rvert\cot\theta$ (8)	Stress-strain relationship: steel bar: $f_{sx} = E_s\varepsilon_x \le f_{yx}$ (11) $f_{sz} = E_s\varepsilon_z \le f_{yv}$ (12)
Stress at cracks $f_{sxcr} = \dfrac{f_x + v\cot\theta + v_{ci}\cot\theta}{\rho_x}$ (4) $f_{szcr} = \dfrac{f_z + v\tan\theta - v_{ci}\tan\theta}{\rho_z}$ (5)	Crack width: $w = \varepsilon_1 s_\theta$ (9) $s_\theta = \dfrac{1}{\left\lvert\dfrac{\sin\theta}{s_x} + \dfrac{\cot\theta}{s_z}\right\rvert}$ (10)	Concrete: $f_2 = \dfrac{f'_c}{0.8 + 170\varepsilon_1}\left\lvert\dfrac{2\varepsilon_2}{\varepsilon_0} - \left\lvert\dfrac{\varepsilon_2}{\varepsilon_0}\right\rvert^2\right\rvert$ (13) $f_1 = \dfrac{0.33\sqrt{f'_c}}{1 + \sqrt{500\varepsilon_1}}$ (14)
		Shear stress at cracks: $v_{ci} \le \dfrac{0.18\sqrt{f'_c}}{0.31 + \dfrac{24w}{d_a + 16}}$ (15)

Note: f_x and f_z are longitudinal stress and vertical extrusion force, respectively, ρ_x and ρ_v – the longitudinal reinforcement ratio and stirrup reinforcement ratio, respectively, f_{sx} and f_{sz} – the stress of longitudinal and transverse reinforcement, respectively, f_{sz} – the mean tensile stress perpendicular to the crack direction, v – the shear strength, θ – the crack angle, f_2 – the average compressive stress parallel to fracture orientation, f_{sxcr} and f_{szcr} – the stress at longitudinal and transverse cracks, respectively, v_{ci} – the shear stress transmitted along the fracture surface, ε_x – the average longitudinal strain, ε_2 – the principal compressive strain along fracture orientation, ε_z – the average vertical strain, ε_1 – the principal tensile strain perpendicular to fracture surface, γ_{sz} – the shear strain, w – the average crack width, S_θ – the average diagonal crack spacing, S_x and S_z – the spacing of cracks perpendicular to x-direction and y-direction, respectively, E_s – the elastic modulus of steel, f_{yx} and f_{yy} – the longitudinal reinforcement yield strength and stirrup yield strength, respectively, $f'_c = 0.8 f_{cu,m}$, $f_{cu,\,m}$ – the average compressive strength of concrete cube, ε_0 – the strain corresponding to peak stress of concrete, and d_a – the maximum aggregate size.

Calculation method of the shear capacity of recycled concrete beams with wed reinforcement

Shear model

In this paper, the shear model shown in fig. 1 is used to calculate the shear capacity of RAC beams with web reinforcement.

In order to make MCFT easy to use in practical engineering design, Mitchell *et al.* [8] proposed a simplified calculation method. The expression of the shear capacity for beams with web reinforcement can be written:

$$V_u = \beta\sqrt{f_c^{'}}b_w d_v + \frac{A_v f_{yv}}{s} d_v \cot\theta \qquad (16)$$

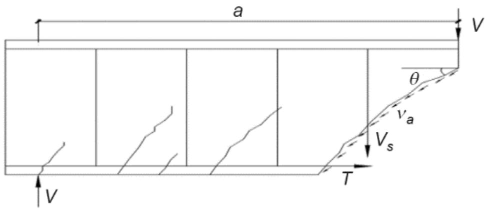


Figure 1. Shear capacity model

where β is:

$$\beta = \frac{f_1 \cot\theta}{\sqrt{f_c^{'}}} = \frac{0.33\cot\theta}{1+\sqrt{500\varepsilon_1}} \le v_{ci} = \frac{0.18}{0.3 + \dfrac{24w}{d_a + 16}}$$

where A_v is the whole section area of each limb of stirrups arranged in the same section, s – the distance of stirrups along component length, and d_v – the effective shear height of component section.

For the convenience of calculation and analysis, the shear capacity calculation formula of the recycled concrete beams with web reinforcement in this paper will be simplified:

$$V_\mathrm{u} = V_\mathrm{a} + V_\mathrm{s} = v_\mathrm{a} b_\mathrm{w} d_v + \rho_v f_{yv} b_\mathrm{w} d_v \cot\theta \qquad (17)$$

where V_u is the ultimate shear capacity, V_a – the aggregate interlocking force, V_s – the shear capacity provided by stirrups, ρ_v – the stirrup reinforcement ratio, and b_w – the component width.

Calculation of the equivalent diameter of the recycled aggregate

The equivalent coefficient of the recycled aggregate particle size is introduced to achieve the purpose of equivalent mechanical properties of natural aggregate with the same particle size. Therefore, the calculation expression of the equivalent particle size of the recycled aggregate is proposed:

$$d_\mathrm{a}' = \xi d_\mathrm{a} \qquad (18)$$

d_a is the maximum particle size of recycled aggregate.

Since the analysis of a single aggregate is too complex, this paper is simplified to analyze the volume of the recycled aggregate, the volume of the natural aggregate and the total volume of aggregate in RAC, and the equivalent coefficient of the recycled aggregate particle size is proposed:

$$\xi = \frac{V_{\mathrm{RAC}}^{\mathrm{NA}}}{V_{\mathrm{RAC}}} \tag{19}$$

where $V_{\mathrm{RAC}}^{\mathrm{NA}}$ is the volume of the natural aggregate and V_{RAC} is the total aggregate volume in the RAC. According to the literature [9], the following formulation can be drawn:

$$V_{\mathrm{RAC}}^{\mathrm{NA}} = V_{\mathrm{RAC}} - V_{\mathrm{RAC}}^{\mathrm{RA}} = V_{\mathrm{RAC}} - rC_{\mathrm{RM}}V_{\mathrm{RAC}} = (1 - rC_{\mathrm{RM}})V_{\mathrm{RAC}} \tag{20}$$

where r is the aggregate replacement rate, C_{RM} – the residual mortar content, $V_{\mathrm{RAC}}^{\mathrm{RA}}$ – the volume of the recycled aggregate, and C_{RM} can be calculated according to the literature [10]:

$$C_{\mathrm{RM}} = 1.37 d_{\mathrm{a}}^{-0.426} \tag{21}$$

Substituting eqs. (20)-(21) into eq. (19), we can obtain:

$$\xi = \frac{V_{\mathrm{RAC}}^{\mathrm{NA}}}{V_{\mathrm{RAC}}} = \frac{(1 - rC_{\mathrm{RM}})V_{\mathrm{RAC}}}{V_{\mathrm{RAC}}} = (1 - rC_{\mathrm{RM}}) \tag{22}$$

Therefore, by substituting eq. (22) into eq. (18), the calculation formula of the equivalent particle size of the recycled aggregate can be obtained:

$$d_{\mathrm{a}}' = (1 - rC_{\mathrm{RM}})d_{\mathrm{a}} \tag{23}$$

Calculating the crack width

In MCFT, the crack width is related to the assumed crack spacing, and the crack spacing is related to the crack angle. The crack width can be taken as the product of the main tensile strain and the oblique crack spacing, namely:

$$w = \varepsilon_1 s_{m\theta} \tag{24}$$

According to fig. 2, the mathematical equations of spacing of diagonal crack and the spacing of vertical and horizontal cracks are established:

$$s_{m\theta} = \frac{1}{\dfrac{\sin\theta}{s_{mx}} + \dfrac{\cos\theta}{s_{mv}}} \tag{25}$$

where s_{mx} is the crack spacing in the vertical x-direction and s_{mv} is the crack spacing in the vertical y-direction.

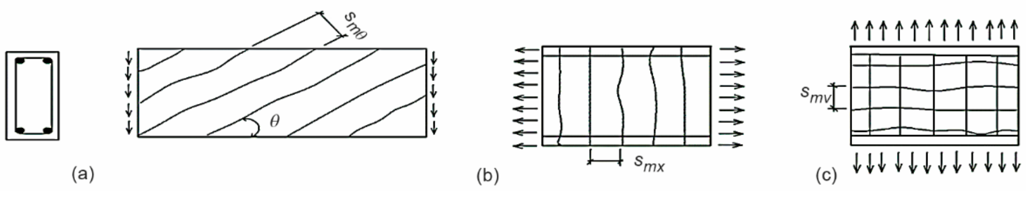


Figure 2. Crack spacing of regenerated concrete beams; (a) diagonal crack, (b) vertical crack, and (c) horizontal fracture

According to CEB-FIP mode specification, the calculation formula of the vertical and horizontal crack spacing can be obtained:

$$s_{mx} = 2\left(c_x + \frac{s_x}{10}\right) + 0.25k_1 \frac{d_{bx}}{\rho_x}, \quad s_{mv} = 2\left(c_v + \frac{s_v}{10}\right) + 0.25k_1 \frac{d_{bv}}{\rho_v} \tag{26}$$

where $\rho_x = A_s / A_c$, $\rho_v = A_v / b_w s$.

In eq. (26), d_{bx} and d_{by} are diameters of steel bars in *x*- and *y*-directions, respectively, fig. 2, c_x and c_y – the concrete cover in *x*- and *y*-directions, respectively, s_x and s_y – the spacing of bars in *x*- and *y*-directions respectively, k_1 – the coefficient, 0.4 for the deformed bar and 0.8 for the round bar, A_s – the longitudinal reinforcement area, and A_c – the section area of the concrete.

According to eq. (1), the inclination angle of cracks in RAC beams can be calculated:

$$\tan\theta = \frac{\rho_v f_{sv} + f_1}{v} \tag{27}$$

where $v = V/b_w d$ and V is the outside load.

The strain of the longitudinal tensile reinforcement can be obtained according to fig. 3.

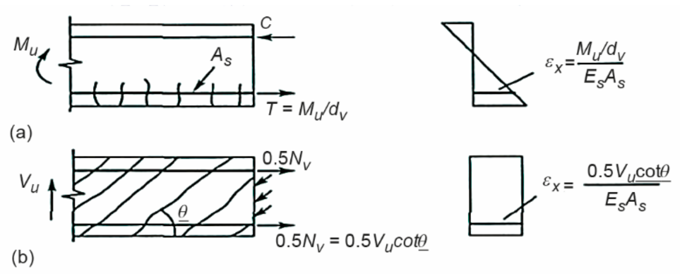


Figure 3. Determination of strain

$$\varepsilon_{sx} = \frac{\dfrac{M_u}{d_v} + 0.5V_u \cot\theta}{E_s A_s} \tag{28}$$

where $0.5V_u \cot\theta$ is the longitudinal reinforcement tension caused by the shear force. For safety consideration, $0.5\cot\theta$ is approximately taken as 1. The simplified formula is:

$$\varepsilon_{sx} = \frac{\dfrac{M_u}{d_v} + V_u}{E_s A_s} \tag{29}$$

where M_u is the design moment, V_u – the design shear, and E_s – the elastic modulus of the longitudinal reinforcement. The ε_x can be approximately taken as half of the longitudinal tensile steel strain:

$$\varepsilon_x = \frac{\varepsilon_{sx}}{2} \tag{30}$$

For the recycled aggregate concrete beams with web reinforcement, the vertical extrusion force is very small, which can be ignored, namely $f_z = 0$ [11]. It is assumed that the stirrups at and between cracks reach the yield strength when the shear failure occurs, namely $f_{szcr} = f_{sv} = f_{yv}$, so eqs. (2) and (3) are combined to obtain:

$$f_2 = f_1 \cot^2 \theta + \rho_v f_{sv} (1 + \cot^2 \theta) \tag{31}$$

Referring to the study on the stress-strain curve of the RAC under a uniaxial compression in [12], the average principal compressive strain can be obtained by taking $\varepsilon_0 = 0.002$ into eq. (13) formula:

$$\varepsilon_2 = 0.002 \left(1 - \sqrt{1 - f_2 \frac{0.8 + 170\varepsilon_1}{0.8 f_c^{'}}} \right) \tag{32}$$

The principal tensile strain ε_1 in the equation can be derived from eqs. (6) and (7) in tab. 1:

$$\varepsilon_1 = \varepsilon_x (1 + \cot^2 \theta) - \varepsilon_2 \cot^2 \theta \tag{33}$$

Calculation of the aggregate interlock force

For the shear analysis of recycled concrete beams, the aggregate interlocking force can be calculated by the shear stress transferred along the crack surface in the modifying compression field theory, namely:

$$V_a = v_a b_w d_v \tag{34}$$

According to CSA Committee A23.3 (Design of concrete structures (CSA A23.3-04), Mississauga: Canadian Standards Association, 2004) and AASHTO LRFD (Bridge design specifications and commentary, Washington, D.C.: American Association of State Highway Transportation Officials, 2004), d_v 0.9d, and d is the effective height of the component, v_a is the shear stress transmitted along the fracture surface. According to eqs. (15) and (18), we can obtain:

$$v_a = \frac{0.18\sqrt{f_c^{'}}}{0.3 + \dfrac{24w}{16 + d_a^{'}}} \tag{35}$$

Applicability analysis

Experimental verification

Five RAC beams with shear span ratio of 2, length of 2300 mm and section dimension of 200 mm × 300 mm were produced in the test. The stirrup ratio of all recycled concrete beams was 0.22%. The HRB335 hot rolled steel bars were used for the longitudinal tensile reinforcement, and HPB235 hot rolled steel bars were used for the web reinforcement. The detailed parameters of the test beams were shown in tab. 2, where RBS represents the inclined section shear test beam and A in RBS-A represents the recycled aggregate replacement rate (0%, 30%, 50%, 80%, and 100%).

Table 2. The main characteristic parameters of the test beam

Number	Recycled aggregate replacement ratio [%]	Stirrup	Stirrup reinforcement ratio [%]	Shear span ratio, λ	Cubic compressive strength, $f_{cu,k}$ [MPa]	Tensile strength, f_{tk} [MPa]	Slump [mm]
RBS-0	0	Ø6 at 150	0.22	2	37.29	2.28	67
RBS-30	30	Ø6 at 150	0.22	2	33.25	2.12	60
RBS-50	50	Ø6 at 150	0.22	2	39.84	2.39	46
RBS-80	80	Ø6 at 150	0.22	2	34.77	2.20	43
RBS-100	100	Ø6 at 150	0.22	2	34.42	2.18	41

To verify the rationality of the calculation method of the shear capacity of the RAC beams with web reinforcement based on MCFT proposed in this paper, the test data and calculation data are compared and analyzed. The specific data results are shown in tab. 3.

Table 3. Comparison between theoretical value and experimental value

Test specimen number	Replacement rate	Crack width [mm]	V_{test}	V_u	V_u/V_{test}
RBS-2-0	0	0.8	139.751	133.890	0.96
RBS-2-30	30%	1.0	131.710	124.348	0.94
RBS-2-50	50%	1	134.323	126.890	0.94
RBS-2-80	80%	1.7	121.071	116.897	0.97
RBS-2-100	100%	1.5	117.184	111.234	0.95

Comparison with literature data

To verify the applicability of the shear capacity calculation formula proposed in this paper, 20 groups of shear capacity test data of simply supported beams with web reinforcement under concentrated load in open literature were collected. The section width b of RAC beam was 235-400 mm, the effective section width b_w was 120-300 mm, the cube compressive strength f_{cu} was 29.2-107.8 MPa, the stirrup ratio ρ_v was 0.12%-1.31%, the shear span ratio λ was 1.2-4.2. The calculation method of shear capacity of RAC beams with web reinforcement is used to calculate the data, and the specific calculation data are shown in tab. 4.

Conclusions

The theoretical analysis method is used to establish the shear model of recycled concrete beams with web reinforcement by the modified compression field, which fully considers the factor of the aggregate interlock force. Through the verification of test data, the model is more suitable for the analysis of the shear bearing capacity of RAC than the current standard calculation model.

Considering the influence of the geometric size of the old mortar and stone damage on the properties of the recycled aggregate, the calculation method of the equivalent coefficient for particle sizes of recycled aggregate is established, which truly reflects the role of the recycled aggregate interlocking force in the shear process.

Table 4. Comparison of experimental and calculated values

Literature	Replacement rate	Cube crushing strength	Experimental value, V_{test}	Calculated value, V	V/V_{test}
[13]	50	29.6	110.00	104.5	0.95
		29.6	112.00	108.64	0.97
		29.6	116.50	114.17	0.98
	100	29.2	113.50	106.69	0.94
		29.2	108.00	100.44	0.93
		29.2	117.50	104.58	0.89
[14]	100	34.6	64.00	76.16	1.19
		56.4	78.00	94.38	1.21
		40.1	81.50	90.47	1.11
		60.2	68.00	86.36	1.27
		105.3	83.00	110.39	1.33
		59.6	118.50	112.58	0.95
		89.1	121.00	122.21	1.01
		35.8	120.50	109.655	0.91
		59.6	119.00	116.62	0.98
		107.8	130.50	134.42	1.03
[15]	100	30.5	166.22	169.54	1.02
		31.3	172.11	173.83	1.01
		30.5	191.78	187.94	0.98
		31.3	188.11	186.23	0.99

By using the test verification method, the calculation method of the bearing capacity proposed in this paper is compared with the values calculated by the ordinary concrete specification and the recycled concrete technical specification with the test values. The results show that the proposed method in this paper has higher consistency with the test values and smaller data dispersion, which meets the calculation accuracy requirements of the shear bearing capacity of RAC beams.

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