Calibration of a conceptual rainfall–runoff model using a genetic algorithm integrated with runoff estimation sensitivity to parameters

Shiang-Jen Wu, Ho-Cheng Lien and Che-Hao Chang

ABSTRACT

This study proposes a parameter-calibration method (SA_GA) for the conceptual rainfall–runoff model using a real-value coding genetic algorithm (GA) which takes into account runoff estimation sensitivity to model parameters; this process is carried out using the standardized regression equation. The proposed SA_GA method treats the standardized values of model parameters as the real-value code and adopts a multinomial trial process with a probability of selecting genes for the crossover and mutation resulting from the runoff estimation sensitivity to the model parameters. A 19-parameter conceptual rainfall–runoff model, Sacramento Soil Moisture Accounting (SAC-SMA) model, and seven rainstorm events recorded in the Baj-Hang River watershed of South Taiwan are applied in the model development and validation. The results indicate that SA_GA is superior to a simple genetic algorithm (SGA) as regards the calculation of fitness values associated with the optimal parameters under various GA operators. In addition, by comparing the performance indices of estimated runoff with the calibrated optimal parameters by SA_GA and SGA with the different number of calibration rainstorm events, SA_GA can provide efficient and robust optimal parameters. These parameters not only estimate reliable and accurate runoff, but also capture the varying trends of discharge in time.

Key words | genetic algorithm, Monte Carlo simulation, Sacramento soil moisture accounting (SAC-SMA) model

Shiang-Jen Wu (corresponding author)
Ho-Cheng Lien
National Center for High-performance Computing,
Hsinchu 30076,
Taiwan,
R.O.C.
E-mail: sjwu@nchc.narl.org.tw

Che-Hao Chang
Department of Civil Engineering,
National Taipei University of Technology,
Taipei 10608,
Taiwan,
R.O.C.

INTRODUCTION

Conceptual rainfall–runoff models have extensive parameters that are not directly measurable and must be estimated through model calibration by fitting the estimated outputs of the model to the observed data (Abdulla & AL-Badranih 2000). The genetic algorithm (GA) has been successful in the contexts of searching, optimization and teaching (Goldberg 1989) and can also be applied to the parameter calibration of conceptual rainfall–runoff models (e.g. Wang 1991, 1997; Franchini 1996, Franchini & Galeati 1997, Franchini et al. 1998; Seibert 2000; Madsen et al. 2002; Agrawal & Singh 2003; Jain & Srinivasulu 2004; Siriwardene & Perera 2006). The simple GA (SGA) consists of four steps: fitness, reproduction, crossover and mutation steps. These steps allow for flexibility, but make it difficult

to obtain the optimal model parameters. Additionally, they require more computation time in a complicated system associated with a number of parameters. Numerous investigations are proposed to modify GA in order to fast yield a more accurate optimum solution. For example, Krishnakumar (1989) proposed a micro-genetic algorithm[(μMA)], which begins with a small population (generally 5–50), and concluded that as compared with the simple GA, the micro-genetic algorithm is suitable for the parameter calibration of a complex rainfall–runoff model. Wang (1991, 1997) developed a GA which, combined with a standard local search method, can provide an efficient and robust method for calibrating conceptual rainfall–runoff models using hypothetical examples and real data. Franchini (1996)

unitized the sequential quadratic programming to calibrate parameters of conceptual rainfall–runoff models with the starting points of parameters resulting from the optimal parameters obtained by Wang's GA. Franchini and co-workers (1997, 1998) compared the pattern search (PS) method with GA by using an 11-parameter conceptual rainfall–runoff model, a distributed model (ADM), and concluded that PS elicits a slightly superior performance to GA. Furthermore, they proposed a modified GA integrated with the Sequential Quadratic Programming (SQP), named GA-SQP, and compared it with the PS-SQP, which is the PS combined with SQP, as well as the shuffled complex evolution (SCE-UA) using theoretical and real-world cases. Their results indicated that the SCE-UA is better than GA-SQP and PS-SQP in the theoretical cases; however, in practical applications, the associated optimal model parameters are limited by the boundary of their own range. Notably, Ndiritu & Daniell (2001) calibrated conceptual rainfall–runoff by using a modified traditional binary-code with three strategies pertaining to automatic search space, the fine-turning, hill climbing and independent subpopulation search shuffling. As compared with the shuffled complex evolution (SCE-UA), the improved GA elicited more efficient results. Siriwardene & Perera (2006) performed a sensitivity analysis for GA operators, i.e. the population size, number of generations and number of model parameters, and summarized that the adaptive selection of GA operations is strongly helpful to calibrate the parameters of urban drainage models associated with extensive parameters. Work has also been undertaken in the context of calibrating the model parameters using the minimum population size (e.g. Goldberg 1989; Siriwardene & Perera 2006). Basically, the proper population size depends on the number of model parameters. For example, Goldberg (1989) suggested a population size of 25–50, but Siriwardene & Perera (2006) presented a population of 100 for models with more than five parameters. Although increasing the population size can result in more reliable optimal parameters, this would increase computation time.

Recently, sensitivity and uncertainty analyses have been widely employed to review the variations in model behaviour attributed to climate change or hydrological uncertainties, such as spatial variability of rainfall (e.g. Chaubey *et al.* 1999; Yu *et al.* 2001; Hossain *et al.* 2004; Cullmann *et al.* 2006). The results of the sensitivity and uncertainty analyses could be helpful in deciding how the model structure may be simplified and which parameters may be fixed during calibration. Therefore, to calibrate effect and robust optimal parameters, this study proposes a parameter-calibration method that integrates the GA and sensitivity analysis for the calibration of the conceptual rainfall–runoff model with a large number of parameters, i.e. the SA_GA method.

METHODOLOGY

Genetic algorithm integrated with sensitivity analysis

Basic concept

Genetic algorithm (GA) is a widely used stochastic search method originally developed by Holland (1975), and it is more flexible than most other search methods, which require only information concerning the quality of the solution produced by each parameter set. The simple GA (SGA) is composed of four operations: (1) *parameter representation*: encodes the model parameters or finds specific magnitudes to represent the model parameter values; (2) *selection and reproduction*: selects the fittest individual chromosomes, which are a series of genes, to be recombined in order to produce better off-chromosomes. Here, a probabilistic mechanism (e.g. a roulette wheel) is used to allocate greater survival to the best individuals; (3) *crossover*: recombines (exchanges) genes from randomly selected pairs of individuals with a certain ratio and (4) *mutation*: randomly changes genes in the chromosomes with a certain (small) ratio. These steps aim at keeping the population diverse and take preventative measures against premature convergence onto a local optimum.

As SGA arbitrarily selects the genes in the crossover and mutation operators, it may take a large number of repetitions to achieve the optimal values of model parameters. Increasing the population size is useful to obtain model parameters, but this could require lengthy computation time. In general, model outputs must be influenced by specific model parameters (Cullmann *et al.* 2006; Castaings *et al.* 2007; Milivojevic *et al.* 2009). That is, the model outputs are sensitive to model parameters. Therefore,

to effectively calibrate model parameters, this study modifies SGA by taking into account the estimated runoff sensitivity to model parameters for the conceptual rainfall–runoff model with a large number of parameters, defined as SA_GA. In the proposed SA_GA method, a sensitivity analysis for the model parameters would be carried out using the standardized regression equation. Note that a 19-parameter conceptual rainfall–runoff model, Sacramento Soil Moisture Accounting (*SAC-SMA*) model, is adopted in the model development and validation.

Sensitivity analysis for model parameters

As most parameters of conceptual rainfall–runoff models are sensitive to high flow conditions regardless of time scale and watershed (Tang *et al.* 2007), this study first performs the sensitivity analysis to assess the effect of the parameters of *SAC-SMA* model on the estimation of runoff characteristics, i.e. the runoff volume, the peak discharge, time to peak runoff and the volume at the rising and recession limbs.

Most of the sensitivity analysis methods can be grouped into two types (Subret 2008): the first is a regression-based method based on a linear regression of the model output on the input vector, such as the standardized regression coefficient (SRC), which is exactly the same as the correlation coefficient in a linear regression. The second is the variance-based methods in which the observed variance of the model responses is partitioned into components induced by the respective variables, such as the analysis of variance (ANOVA) or *Sobol* indices methods. This study employs the SRC method to explore the sensitivity of the *SAC-SMA* model parameters. The standardized regression equation can be expressed as follows (Speed & Yu 1993):

$$\frac{Y - \bar{Y}}{s_Y} = \sum_{i=1}^{n} \beta_i \frac{X_i - \bar{X}_i}{s_{X_i}} \tag{1}$$

where Y and X are the model output and inputs; $\bar{Y}$ and $\bar{X}$ are the mean of the model output and inputs; s_Y and s_X are the corresponding standard deviation; and β_i denotes the SRCs. The model output sensitivity to model inputs can be quantified based on the SRC β_i. In detail, the greater absolute value of the coefficient has a more significant effect on the model output. If the model input has a positive coefficient, this implies that the model input is directly proportional to the model output. Eventually, the sensitive parameters can be identified according to the associated coefficients of the standardized regression equation.

Development of SA_GA model

Similar to the simple GA (SGA), in which parameter representation, fitness, selection and reproduction and crossover as well as mutation are implemented, the proposed SA_GA method is introduced below:

Parameter representation

In the SGA, the first step is the parameter representation or encoding, which can represent the model parameters. Generally speaking, there are two types of the parameter representation used in the GA, binary and real-value coding. The variables are commonly encoded as the binary representation, in which the variable would be encoded as a series of binary digits (i.e. zero and one), called the chromosome. Notably, binary coding only provides the nearest defined combination of model parameters (Mayer *et al.* 1999). However, real-value coding could efficiently obtain more consistent and accurate results (Janikow & Michalewicz 1991), and has proven to be superior to binary coding, especially for the high-dimensional search problem (Salomon 1996; Back *et al.* 1997; Wardlaw & Sharif 1999; Bessaou & Siarry 2001; Avila *et al.* 2004; Fang & Ball 2007). Although real-value coding can significantly reduce the risk of producing strong chromosomes (such as a large range of parameters) by generating a large initial number of population size, this situation may result in undesirable computation time. Therefore, to effectively calibrate model parameters, this study proposes an alternative method to convert the model parameters into standardized values as the real-value code through the following equation:

$$z_{\theta_i} = \frac{\theta_i - \bar{\theta}_i}{s_{\theta_i}} \tag{2}$$

in which z_{θ_i} is the standardized value of the model parameter θ_i, and $\bar{\theta}_i$ and s_{θ_i} denote the mean and standard

deviation of model parameters, respectively. Furthermore, this study regards the standardized values of model parameters as the genes, which are combined to become the chromosomes, and which are applied in three GA operators. As the range of standardized variables is $[-1,1]$, the resulting optimal model parameters can be obtained under the consideration of the reasonable mean and standard deviation of model parameters.

Reproduction, fitness and selection

After encoding the model parameters, the next GA operator reproduces genes. As the proposed SA_GA method is expected to be used in the calibration of the multi-parameter rainfall–runoff model, this study employs the multivariate Monte Carlo simulation method (Wu *et al.* 2006) to generate standardized values of model parameters and convert them into real values. The runoff can be estimated by the rainfall–runoff model with the generated model parameters and the corresponding fitness values can be computed. Then, the chromosome and associated genes can be selected in the crossover and mutation according to the corresponding fitness values. Theoretically, the chromosome associated with the larger fitness value has a higher probability of being used in the crossover and mutation.

In general, the optimal parameters of a rainfall–runoff model can be determined based on the objective function. The objective functions for the parameter calibration of hydrologic models can be based on four criteria: (1) a good agreement between the average simulated and observed catchment runoff volumes; (2) a good overall agreement of the shape of the hydrographs; (3) a good agreement of the peak flow with respect to time and volumes; and (4) a good agreement for low flows (Madsen 2000). Based on the above four criteria, the objective function F_{obj}, the weighted mean square error, is widely used in the parameter calibration of hydrological models as:

$$F_{obj}(\theta) = \text{Min}\left\{ \left[\frac{1}{n_f}\sum_{t=1}^{n_f} w_t(\theta) \times (q_{obs,t} - q_{est,t}(\theta))^2\right]^{0.5} \right\}$$

$$w_t(\theta) = \frac{\bar{q}_{obs} + q_{obs,t}}{2\bar{q}_{obs}}$$

(3)

where n_f is the runoff duration; $q_{obs,t}$ and $q_{est,t}(\theta)$ are the observed and estimated discharge at time t using model parameter θ; and $\bar{q}_{obs}$ is the mean of observed runoff volume. The less objective function value indicates that the estimated runoff fits better to observed data. That is, the fitness value of estimated runoff increases. As a result, the fitness value $f(\theta_i)$ is negatively related to the objective function value $F_{obj}(\theta_i)$:

$$f(\theta_i) = 1/F_{obj}(\theta_i)$$

(4)

In general, GA selects chromosomes in which the associated genes are used in the crossover and mutation using various selection schemes, e.g. the roulette wheel selection, proportionate selection (Goldberg 1989) or linear ranking and tournament selection (Siriwardene & Perera 2006). As Goldberg & Deb (1991) indicated that no one selection scheme is superior to the other schemes, this study proposes an alternative scheme, i.e. the multinomial trial process with the probability of selecting the chromosome $P_{chrom}(\theta_i)$ based on the fitness value as follows:

$$P_{chrom}(\theta_i) = \frac{f(\theta_i)}{\sum_{i=1}^{n_{chrom}} f(\theta_i)}$$

(5)

where n_{chrom} denotes the number of chromosomes, i.e. population size. Referring to Equation (5), as the chromosome has a larger P_{chrom}, the associated genes have a higher chance of being used in the crossover and mutation. Notably, a multinomial trial process is a sequence of independent, identically distributed random variables $X = (X_1, X_2,…, X_k)$. The sequence X is formed by sampling from the common probability density function of the trial variables.

Crossover and mutation

In SGA, the genes used in the crossover and mutation are randomly determined in the selected chromosomes, which probably increases the computation time and obtains inadequate optimal parameters. The above-mentioned standardized regression equation indicates that the variables with the larger absolute regression coefficients might contribute to more effect on model outputs. Hence, this study calculates the probability of extracting genes used in the crossover and mutation according to the coefficients of a

standardized regression equation:

$$P_{\text{gene}} = \frac{|\beta_i|}{\sum_{i=1}^{n_{\text{par}}} |\beta_i|} \qquad (6)$$

in which β_i is the regression coefficient of the ith model parameter and n_{par} is the number of model parameters. As the gene in a chromosome selected for the crossover and mutation has a large P_{gene}, it has a high likelihood of being exchanged in the crossover or being reproduced in the mutation.

In summary, this study proposes a standardized-value-based SA_GA method, which primarily adopts the multinomial trial process to select the chromosome and associated genes used in the crossover and mutation. In detail, as two genes are identified for the crossover, the corresponding standardized values are exchanged. Similarly, in the case of the gene being extracted for the mutation, the corresponding standardized value is generated using the multivariate Monte Carlo simulation method. Eventually, the resulting standardized values of model parameters are transformed into the real values and are imported into the rainfall–runoff model. A graphical illustration of the above-mentioned crossover and mutation process in the proposed SA_GA method is shown in Figure 1.

Model framework

Based on the concept of the proposed SA_GA method above, the procedure of calibrating model parameters using SA_GA is introduced below:

Step[1]: Ensure the statistics of sensitive model parameters, i.e. mean and standard deviation.

Step[2]: Identify the sensitive model parameters using the standardized regression equation.

Step[3]: Reproduce initial k sets of standardized values of sensitive model parameters z_{θ_i} and transform them into the real values $\theta_i = z_{\theta_i} \times s_{\theta_i} + \bar{\theta}_i$.

Step[4]: Import the generated sensitive model parameters into the rainfall–runoff model to calculate objective function values using Equation (3).

Step[5]: Check to see if the minimum of the objective function is less than the criterion. If so, the optimal model
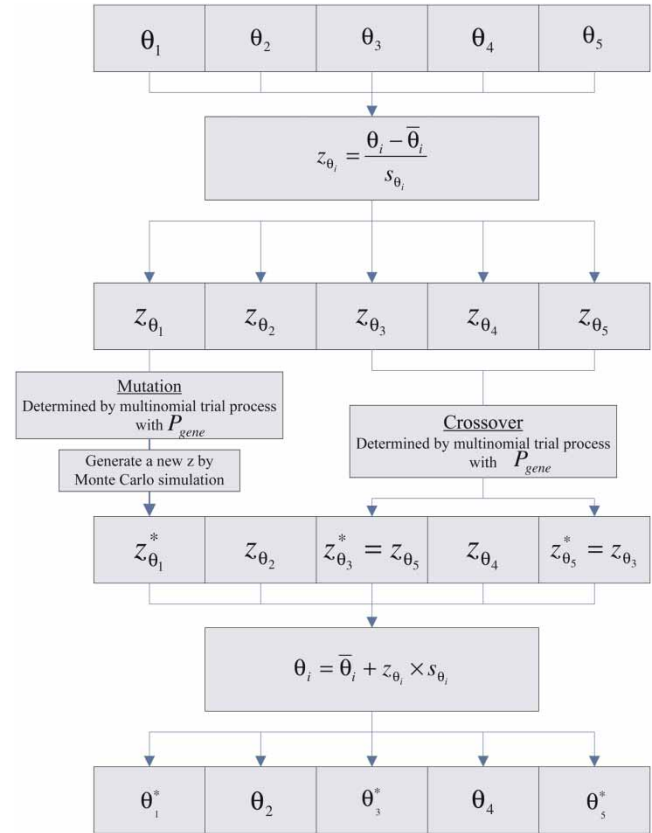
Figure 1 │ The graphical illustration of the crossover and mutation in the case of population size being 5.

parameters can be obtained. Otherwise, continue with the following steps.

Step[6]: Calculate the probability of selecting chromosomes P_{chrom} using Equation (5) to select the chromosomes, in which associated genes would be used in the crossover and mutation through the multinomial trial process.

Step[7]: Calculate the probability of extracting genes P_{gene} using Equation (6) to identify genes in the chromosomes selected at Step [6] for the crossover and mutation through the multinomial trial process.

Step[8]: Perform the crossover and mutation for genes extracted at Step [7] to yield new chromosomes. Then, the resulting standardized values $\hat{\theta}_i = \hat{z}_{\theta_i} \times s_{\theta_i} + \bar{\theta}_i$ are transformed into the real values $\hat{z}_{\theta_i}$. Return to Step [4].

Note that in GA, the criteria should be given in advance to stop the GA operators and find the optimal solution.

Generally speaking, the criterion is defined as a specific value of the objective function, i.e. the minimum value. However, in the case of the criterion being extremely small, it is difficult to stop the GA operators and find the optimum. In addition to the criterion with respect to the objective function, this study also adopts a criterion for the number of generations of genes. That is, in cases where the objective function values are significantly greater than the criterion, the number of generating genes exceeds the limitation so that the genes with the smallest objective function value are regarded as the optimum in this study.

Sacramento soil moisture accounting (*SAC-SMA*) model

Description of *SAC-SMA* model

Recently, many rainfall–runoff models have been commonly used to estimate runoff, for example, unit hydrograph method, Sacramento Soil Moisture Accounting model, *TOP-MODEL*. The Sacramento Soil Moisture Accounting (*SAC-SMA*) model is a lumped conceptual watershed model used for operational river forecasting (Burnash 1995). It was originally derived from the Stanford Watershed model, and designed for the Sacramento River system, US. The *SAC-SMA* model describes the mathematical equation that accounts for each process with the transformation of rainfall into outflow towards a river and it has 17 parameters, which are defined in Table 1, and a runoff function. The runoff function is a distribution of runoff in time. The triangle runoff function, of which the peak occurs at the middle of the period, is adopted in this study. In summary, the runoff function has two parameters: duration and peak ratio. As a result, the *SAC-SMA* model belongs to a 19-parameter conceptual rainfall–runoff model.

Sensitivity analysis for *SAC-SMA* parameters

The sensitivity analysis for evaluating the effect of parameters of the *SAC-SMA* model on the estimation of runoff characteristics is carried out by using the normal regression analysis in this study. To derive the standardized regression equation of runoff characteristics resulting from the *SAC-SMA* parameters, the hyetograph of a hypothetical rainstorm event, the duration of which is 96 h, is applied

Table 1 | Description of parameters of *SAC-SMA* model (Ajami *et al.* 2004)

Parameters	Description
UZTWM	Upper zone tension-water capacity (mm)
UZFWM	Upper zone free-water capacity (mm)
UZK	Upper zone recession coefficient
PCTIM	Percent of impervious area
ADIMP	Percent of additional impervious area
SARVA	Fraction of segment covered by streams, lakes and riparian vegetation
ZPERC	Minimum percolation rate coefficient
REXP	Percolation equation exponent
LZTWM	Lower zone tension water capacity (mm)
LZFSM	Lower zone supplementary free-water capacity (mm)
LZFPM	Lower zone primary free-water capacity (mm)
LZSK	Lower zone supplementary recession coefficient (mm)
LZPK	Lower zone primary recession coefficient (mm)
PFREE	Percentage percolating directly to lower zone free water
SIDE	Ratio of deep recharge water going to channel base-flow
RESERV	Percentage of lower zone free water not transferable to lower zone tension water
SSOUT	Fixed rate of discharge lost from the total channel flow (mm/Δt)
DF_L	Period of runoff distribution function
DF_P	Maximum ratio of runoff distribution function
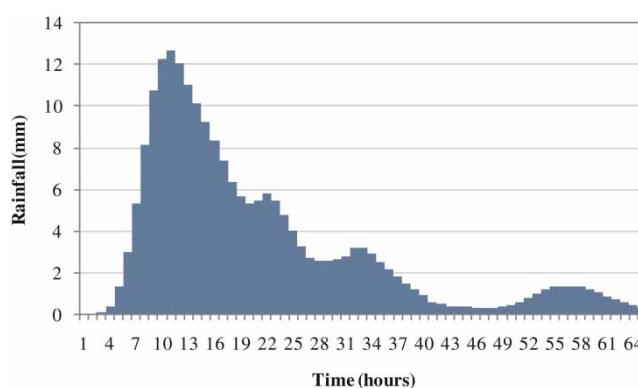
Figure 2 | Hypothetical hyetograph for the sensitivity analysis for *SAC-SMA* model parameters.

(see Figure 2), and 250 simulation sets of *SAC-SMA* parameters are obtained by the Monte Carlo simulation method with their assumed statistics. Following Tang *et al.* (2007), this study refers to the above-mentioned allowance
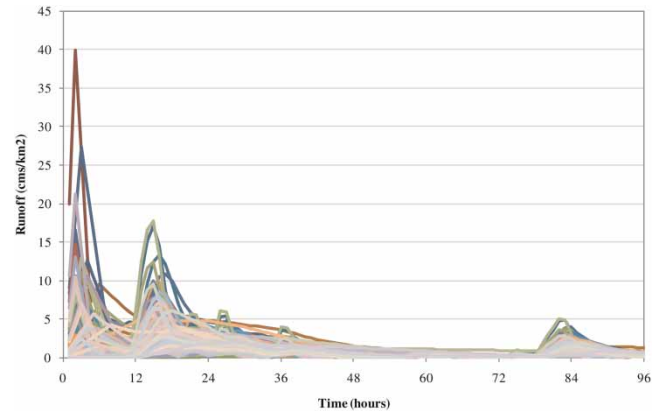
range to hypothesize the statistics (mean and standard deviation) of the parameters as shown in Table 2.

Figure 3 presents the resulting runoff hydrograph and the associated peak discharge that could be obtained for a watershed, in which the corresponding area is assumed as 1 km^2. Figure 4 shows the coefficients of the *SAC-SMA* parameters. As shown in Figure 4, ten parameters, PCITEM, ADIMP, UZTWM, UZFWM, UZK, ZPERC, REXP, LZTWM, DF_L and DF_P, show more influence on the runoff characteristics than the remaining parameters due to the significantly larger absolute coefficients. For the peak discharge, DF_L has the greatest absolute value of the coefficient (0.33) and the LZPK has the smallest absolute coefficient (0.00759). Additionally, the negative coefficients of UZTWM, LZTWM and DF_F imply that the peak discharge would decrease with an increase in the above parameters. In summary, ten parameters would significantly contribute to the uncertainty of estimated runoff characteristics, and they are defined as sensitive model parameters in this study. Therefore, in the *SAC-SMA* model, the

Figure 3 | Summary of estimated runoff by using *SAC-SMA* model with generated parameters.

above-mentioned ten sensitive parameters should be calibrated and the remaining parameters could be adopted as the mean values as shown in Table 2.

Note that Q_p denotes the peak discharge and T_p stands for the time to the peak. V, V_u and V_d are the entire runoff volume and runoff volume at the rising as well as the recession limbs.

MODEL VALIDATION

This study primarily develops a standardized-value-based GA (SA_GA) method to effectively calibrate the associated optimal sensitivity by integrating the genetic algorithm and sensitivity analysis for model parameters. As compared with SGA, in which the genes for the crossover and mutation are randomly selected, SA_GA extracts the genes by means of the multinomial trial process with a selection probability P_{gene} calculated by Equation (6). As the adaptive GA operators (the number of generation, population size, crossover and mutation ratios) are very useful for the parameter calibration (Siriwardene & Perera 2006), this study compares the proposed SA_GA method with SGA in the calculation of fitness values associated with optimal parameters of the *SAC-SMA* model mentioned above in the context of varying GA operators. Moreover, as the outcomes of hydrological models are sensitive to the number of rainstorm events used in parameter calibration (Peters-Lidard et al. 2008), this study also estimates runoff using the optimal *SAC-SMA* parameters calibrated by SA_GA and SGA, respectively.

Table 2 | Hypothetical statistics of *SAC-SMA* model parameters for the sensitivity analysis

Parameters	Allowable range (Tang et al. 2007)	Mean	Standard deviation
UZTWM	1.0–150	59.840	29.744
UZFWM	1.0–150	51.582	27.334
UZK	0.1–0.5	0.520	0.312
PCTIM	0.0–0.1	0.206	0.113
ADIMP	0.0–0.4	0.194	0.100
SARVA	***	0.010	0.005
ZPERC	1.0–250	45.987	26.496
REXP	0.0–5.0	1.490	0.798
LZTWM	1.0–500	237.242	111.061
LZFSM	1.0–1000	201.514	108.814
LZFPM	1.0–1000	58.964	32.799
LZSK	0.01–2.5	0.082	0.040
LZPK	0.0001–0.025	0.049	0.024
PFREE	0.0–0.6	0.197	0.105
SIDE	***	0.099	0.059
RESERV	***	0.304	0.157
SSOUT	***	0.001	0.000
DF_L	***	17.125	5.950
DF_P	***	0.932	0.890

***No well known allowable range.

Figure 4 | Summary of coefficients of normal regression for *SAC-SMA* model parameters.

	uztwm	uzfwm	uzk	pctim	adimp	riva(sarva)	zperc	rexp	lztwm	lzfsm	lzfpm	lzsk	lzpk	pfree	side	RESERV	SSOUT	DF_L	DF_P
Qp	-0.1442	0.01789	0.00806	0.24648	0.22594	0.02618	0.01707	0.16948	-0.26233	0.02962	0.00419	-0.01605	-0.00759	0.02628	0.01398	0.08011	0.02405	-0.33068	0.24494
Tp	0.19889	0.08789	0.1217	-0.00043	0.04125	0.00228	-0.03654	-0.01088	0.14133	-0.00691	0.00381	-0.04313	0.0183	0.02224	-0.04274	0.02946	-0.06505	0.47478	-0.20478
V	-0.24823	0.07568	0.14263	0.41149	0.46023	-0.00868	-0.05602	0.04903	-0.34079	-0.04345	-0.01394	-0.0239	-0.01172	0.00923	0.00767	0.00756	0.00186	-0.03149	0.10575
Vp	-0.18885	0.11225	0.12502	0.39472	0.33334	-0.03137	-0.08189	0.04558	-0.32071	-0.00778	0.0228	0.01174	0.01997	-0.00231	-0.0309	0.05632	-0.04746	0.06671	0.05526
Vd	-0.23639	0.04554	0.11857	0.35864	0.4396	0.00643	-0.03002	0.06269	-0.30765	-0.0444	-0.02558	-0.0356	-0.02339	0.0156	0.02402	-0.00355	0.02544	-0.11117	0.13662

This includes various rainstorm events and calculates the corresponding performance indices. By comparing performance indices, SA_GA and SGA are demonstrated in the accuracy and reliability of estimated runoff.

Performance indices

The comparison of estimated runoff is widely made using two methods; one is graphical comparison and the other is performance indices. The graphical comparison mainly evaluates varying trend of runoff with respect to time. The latter type, performance indices, assesses the difference between the estimated and observed runoff characteristics. This study primarily focuses on the accuracy of runoff in time and scale. This is such that three performance indices for the measurement of temporal varying trend and total volume of runoff as well as the peak discharge, which are commonly used in the evaluation of rainfall–runoff models, are summarized below:

1. Efficiency coefficient (CE) (Nash & Sutcliffe 1970)

$$CE = 1 - \frac{\sum (q_{obs}(t) - q_{est}(t))^2}{\sum (q_{obs}(t) - \bar{q}_{obs})^2} \qquad (7)$$

where $\bar{q}_{obs}$ represents the mean of observed discharge. If CE approaches one, this means that the estimated runoff model is a good fit with the observed value.

2. Error of peak discharge (EQP)

$$EQP = \frac{(q_{p,est} - q_{p,obs})}{q_{p,obs}} \times 100\% \qquad (8)$$

in which $q_{p,est}$ and $q_{p,obs}$ are the estimated and observed peak discharge, respectively. Note that EQP is probably a negative value, which means that the estimated value is less than the observed one.

3. Root mean square error (RMSE)

$$RMSE = \left[\frac{1}{n_f} \sum_{t=1}^{n_{flow}} (q_{obs}(t) - q_{est}(t))^2 \right]^{0.5} \qquad (9)$$

in which $q_{obs}(t)$ and $q_{est}(t)$ are estimated and observed discharge at time t, respectively.

Study area and data

In this study, the Chu-Kou sub-basin in Baj-Hang River watershed in Taiwan (see Figure 5) is applied to the

model development and validation. Table 3 lists the geo-metrical information of associated rainfall and water level gauge in the Baj-Hang River watershed. For the model vali-dation, observed hourly rainstorm events are selected according to two rules: one is that the runoff coefficient, which is the runoff volume divided by the rainfall amount, is less than or equal to one, and the other is

that the peak discharge is significantly greater than other events in the same year. Based on the above rule, Table 4 lists rainstorm events from the Chu-Kou sub-basin of the Baj-Hang River watershed, in which the selected rainstorm events are mostly typhoon events recorded from 2004 to 2008 and show the peak discharge between 200 and 1,300 m^3/s.
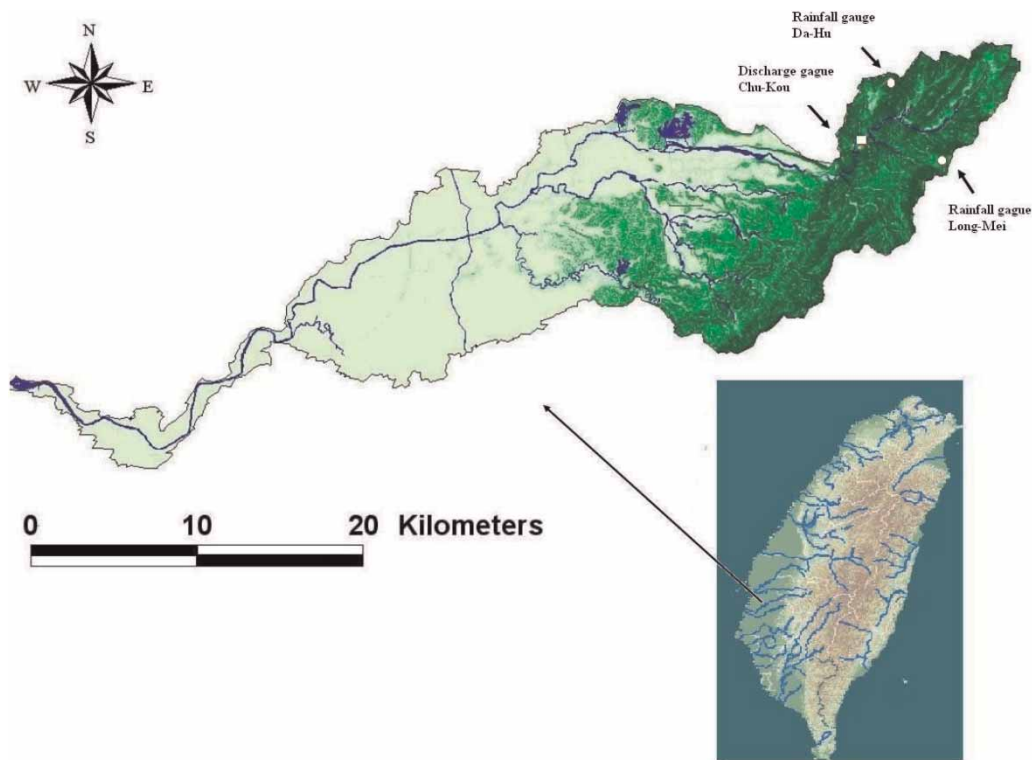
Figure 5 | Baj-Hang River watershed and associated discharge and rainfall gauges.

Table 3 | Discharge gauges and associated rainfall gauges in study basins

Watershed	Sub-basin	Gauges		
Baj-Hang	Chu-Kou	Discharge	Chu-Kou	$TM\_X = 208946.8$
				$TM\_Y = 2593107.0$
	Area $= 83.15$ km^2	Rainfall	Da-Hu	$TM\_X = 210373$
				$TM\_Y = 2597161$
			Long-Mei	$TM\_X = 213633$
				$TM\_Y = 2590031$

Table 4 | Rainstorm events recorded in study basins for the model application

Sub-basin	Events		Occurrence period	Peak discharge (m³/s)
Chu-Kou	EV1	SEPAT	20070818–20070821	643.6
	EV2	WIPHA	20071006–20070920	224.5
	EV3	KORSA	20071006–20071008	1276
	EV4	KAMLAEGI	20080717–20080720	830.84
	EV5	FONGWONG	20080728–20080730	410.6
	EV6	SINLAKU	20080914–20080915	359.34
	EV7	JANGMI	20080928–20080916	523.83

Sensitivity analysis for *SAC-SMA* parameters

Effect of GA operators

As the proposed SA_GA method calculates the probability of extracting genes used in the crossover and mutation related to the sensitivity of model parameters, which significantly differ from SGA, SA_GA and SGA should be compared according to the fitness values associated with the optimum under various GA operators, i.e. the generation number, population size, crossover and mutation ratios. In this study, the average fitness values for the ten best chromosomes (i.e. optimal parameters) are adopted in the model validation. Note the statistics (the mean and standard deviation) of *SAC-SMA* parameters as shown in Table 2 are adopted in the model validation.

Number of generation

This section evaluates the change in the fitness values with various numbers of gene-generation. The number of gene-generation is hypothesized as 50, 100, 150, 200, 250, 300, 350, 400, 450 and 500. Other operators are assumed to be a population size of 50 with crossover and mutation ratios of 0.5 and 0.01, respectively. Moreover, seven rainstorm events (see Table 4) recorded in the study area of the Baj-Hang River watershed are applied in the calibration of *SAC-SMA* parameters. Note that the criterion, the minimum objective function, is assigned as 0.1. Figure 6 shows the comparison of the calculated fitness values associated with the optimal parameters by SA_GA and SGA with
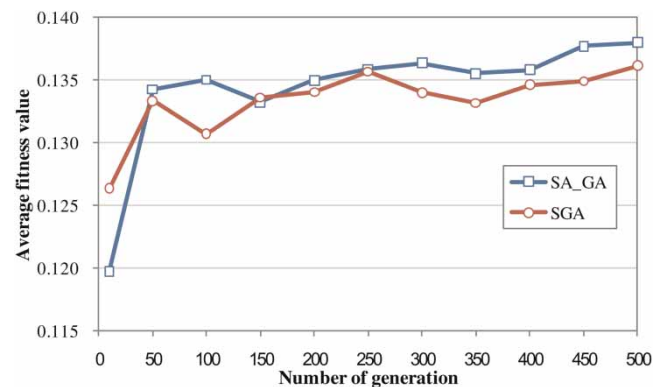
Figure 6 | Average fitness values corresponding to optimal parameters of *SAC-SMA* model calibrated by SA_GA and SGA with various number of generation.
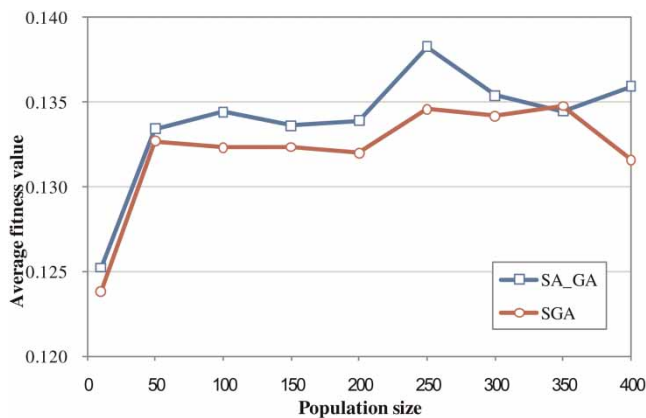
the number of gene-generation, i.e. 10, 50, 100, 150, 200, 250, 300, 350, 450 and 500. It is observed that the average fitness values increase from approximately 0.125 to 0.135 with the number of generation varying from 10 to 500. It also can be seen that the fitness values calculated by SA_GA are generally greater than those by SGA. On average, the calculated fitness value by SA_GA is approximately 0.133, whereas the fitness value by SGA is about 0.134. This implies that the proposed SA_GA method can provide the optimal model parameters in which the estimated runoff can fit better to the observed data than SGA under the generation number. Moreover, in view of Figure 6, as the number of generation increases from 10 to 50, the average fitness value quickly rises. However, for the number of generation exceeding 50, the average fitness value gradually increases. It can be said that a generation of 50 is suitable for the proposed SA_GA method, which is also regarded as the criterion for number of generating genes.

Population size

This section discusses the effect of population size on the fitness value resulting from SA_GA and SGA. Using the various population sizes, including 10, 50, 100, 150, 200, 250, 300, 350 and 400, the fitness values associated with optimal parameters are calculated by SA_GA and SGA with the generation number of 50, mutation ratio of 0.5 and crossover ratio of 0.05 as shown in Figure 7. Figure 7 shows that the average fitness values are positively related to the population size, in which the value approximately increases from 0.125 to 0.135. It also can be seen that as the population size exceeds 50, the average fitness value approaches a constant 0.35 (SA_GA) and 0.32 (SGA). Moreover, it is observed that the fitness values from SA_GA are greater than ones from SGA using various population sizes. Similar to the generation number, the optimal parameters calibrated by SA_GA can produce reliable runoff in which the corresponding difference from the observed data is less than those with the optimal parameters determined by SGA. In summary, a population size of 50 could provide a stable and reliable result for the parameter calibration in the proposed SA_GA model. As a result, a population size of 50 would be adopted in the proposed SA_GA method.

Crossover ratio

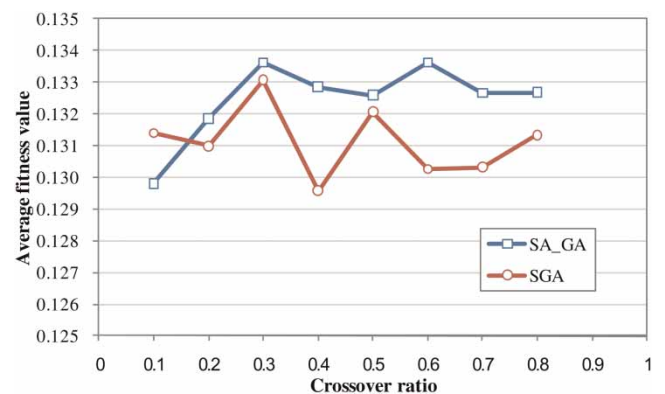Similar to the number of generation and population size, the average fitness value would be calculated by SA_GA and GA with varying crossover ratios, i.e. 0.1, 0.2,…, 0.8. The remaining GA operators are hypothesized as the generation number of 50, population size of 50 and mutation ratio of 0.5. Figure 8 shows the comparison of the average fitness values calculated by SA_GA and SGA with various crossover ratios. It is observed that the change in the average fitness value is dissimilar to the results from the number of generation and population size. That is, the average fitness value oscillates between 0.13 and 0.135 (on average 0.132) under the consideration of varying crossover ratios. As the average fitness values calculated by SA_GA are generally greater than ones by SGA, this indicates that the difference of estimated runoff from the observed data with the calibrated parameters by SA_GA is significantly less than those by SGA. In addition, referring to Figure 8, in the case of the crossover ratio being 0.5, the average fitness value is approximately 0.1325, which approaches the mean. Therefore, this study adopts the crossover ratio of 0.5 in the proposed SA_GA method.

Mutation ratio

As mentioned previously, this section evaluates the effect of mutation ratio on the fitness value with a generation number of 50, population size of 50 and crossover ratio of 0.05. Figure 9 shows the change in the average fitness values calculated by SA_GA and SGA under various mutation ratios, including 0.01, 0.02,…, 0.1. In Figure 9, similar to the results from the crossover ratio mentioned above, the average fitness also oscillates between 0.13 and 0.135 (on average

Figure 7 │ Average fitness values corresponding to optimal parameters of *SAC-SMA* model calibrated by SA_GA and SGA with various population sizes.

Figure 8 │ Average fitness values corresponding to optimal parameters of *SAC-SMA* model calibrated by SA_GA and SGA with various crossover ratios.
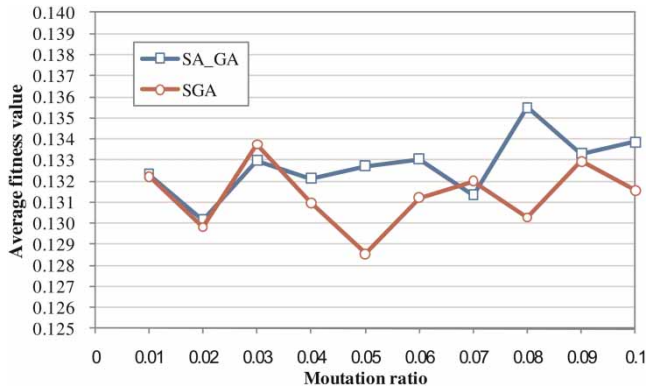
Figure 9 | Average fitness values corresponding to optimal parameters of *SAC-SMA* model calibrated by SA_GA and SGA with various mutation ratios.

0.132), and the average fitness values calculated by SA_GA are generally greater than ones by SGA. This provides evidence that SA_GA can calibrate the optimal parameters in which the corresponding difference of estimated runoff to the observed data is less than those for SGA for various mutation ratios. Moreover, the proposed SA_GA adopts a mutation ratio of 0.05, in which the associated fitness value approaches the mean value 0.132.

Specifically, the proposed SA_GA method can provide the optimal parameters that can produce runoff associated with a better fitness to the observed data than those by SGA under various GA operators. Also, the GA operators, i.e. a generation number of 50, population size of 50, and crossover and mutation ratios of 0.5 and 0.05, are suggested to be adopted in the proposed SA_GA model.

Effect of number of rainstorm events calibrated

Basic concept

The results mentioned previously focus on comparison between SA_GA and SGA in the parameter calibration of *SAC-SMA* model in which the fitness values associated with optimal parameters are calculated for varying GA operators. This section primarily compares SA_GA and SGA in the estimation of runoff using the optimal *SAC-SMA* parameters calibrated using different numbers of rainstorm events, which are defined as calibration events. In detail, this study estimates runoff using the optimal *SAC-SMA*

parameters calibrated by SA_GA and SGA, respectively. This is performed with various numbers of calibration events and calculates the corresponding performance indices for the comparison of SA_GA and SGA in the estimation of runoff. As seven rainstorm events recorded in the study area of the Baj-Hang River (see Table 4) are adopted, the number of calibration events considered is 1, 2, 3, 4, 5 and 6 events. In other words, the corresponding 6, 5, 4, 3, 2 and 1 rainstorm events, which are defined as the validation events, should be used in the estimation of runoff and calculation of performance indices.

Comparison of performance indices

Figure 10 shows the average performance indices for estimated runoff of validation events using the *SAC-SMA* model with optimal parameters calibrated by SA_GA and SGA using various number of calibration events. As shown in Figure 10(a), it is observed that the efficiency of coefficient (CE) increases from approximately 0.5 to 0.6 in relation to the number of calibration events. As CE measures the fitness of estimated runoff to the observed data, the calibrated optimal parameters using more calibration events can produce runoff, which better fits to the varying trend of runoff in time. In addition, the average CE of estimated runoff with optimal parameters calibrated by SA_GA is greater than SGA with different numbers of calibration events (except for 1 and 2 calibration events). It can be said that the proposed SA_GA method can provide the optimal parameters that have a higher likelihood of capturing the temporal behaviour of runoff.

The error of peak discharge (EQP) of estimated runoff is shown in Figure 10(b). The average EQP is negatively proportional to the number of calibration events. Furthermore, the average EQP for SA_GA decreases from about 18 to 5% with the increase of calibration events, whereas the average EQP for SGA decreases from approximately 18 to 10%. That is, the EQP values of estimated runoff with the calibrated optimal parameters by SA_GA are less than those by SGA. This implies that the calibrated optimal parameters by SA_GA can estimate the peak discharge in such a way that the corresponding accuracy and reliability is influenced less by the number of calibration events with SGA.
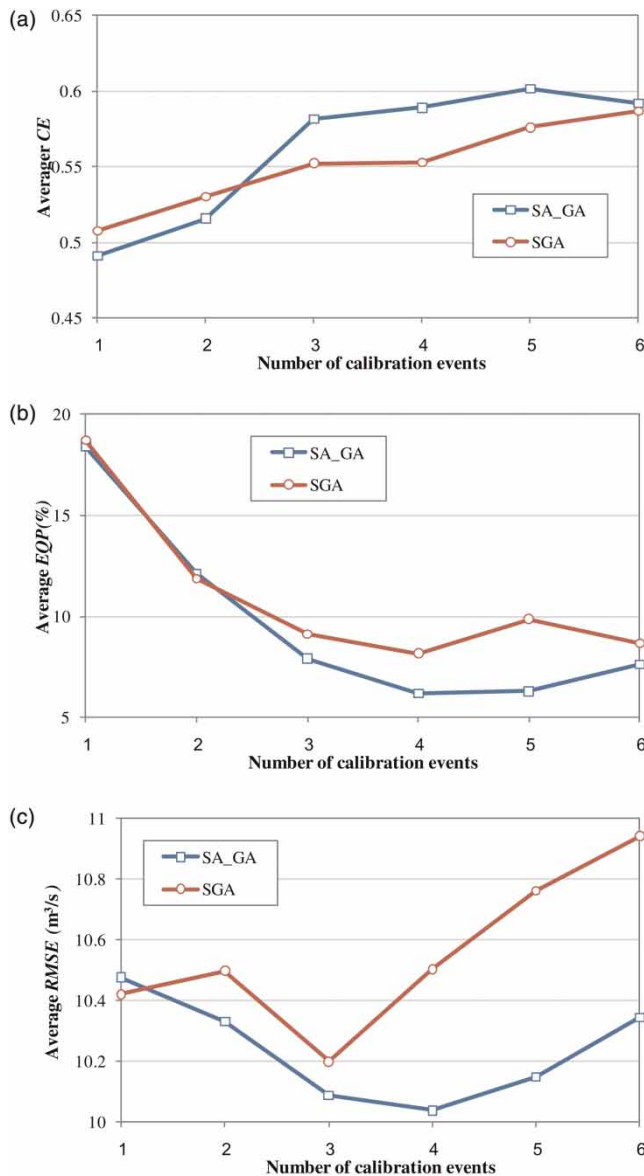
Figure 10 | Average performance indices of estimated runoff by *SAC-SMA* model with optimal parameters calibrated by SA_GA and SGA. (a) Coefficient of efficiency CE, (b) Error of peak discharge EQP, (c) Root mean square of error RMSE.

In addition to the performance indices CE and EQP, this study compares SA_GA with SGA using the RMSE of estimated runoff. Figure 10(c) presents the comparison of the average RMSE of estimated runoff by *SAC-SMA* model with the calibrated optimal parameters by SA_GA and SGA under various numbers of calibration events. The average RMSE decreases with the number of calibration events in cases where the number of calibration events is less than 3. However, when using 4, 5 and 6 calibration

events, the average RMSE rises. As such, there is an increased ratio of RMSE for SGA from 10.2 to 10.85 m³/s that is greater than one for SA_GA from 10 m³/s to 10.3 mm m³/s. Notably, the average values of RMSE for SA_GA are significantly less than those for SGA. In summary, SA_GA can obtain the optimal parameters that produce more accurate and reliable runoff volume than SGA.

Specifically, the proposed SA_GA method can be applied in the estimation of runoff by means of providing the optimal parameters of the conceptual rainfall–runoff model. This system can capture the change in runoff with respect to time and more accurately estimate runoff, especially for the peak discharge.

CONCLUSIONS

This study proposes a parameter-calibration method (SA_GA) for the conceptual rainfall–runoff model based on the GA by taking into account runoff estimation sensitivity to model parameters carried out by the standardized regression equation. As compared with the simple genetic algorithm (SGA), the proposed SA_GA method adopts the multinomial trial process with a probability of gene calculated from the sensitivity to model parameters (see Equation (6)) to identify the genes that are exchanged in the crossover and are reproduced in the mutation.

To compare the proposed SA_GA method with SGA in the parameter calibration of the conceptual rainfall model, a 19-parameter Sacramento Soil Moisture Accounting (*SAC-SMA*) model is adopted. Additionally, seven rainstorm events recorded in the Baj-Hang River watershed of South Taiwan are used in the model validation. According to the results from an evaluation of effect of GA operators, i.e. the number of generation, population size and crossover and mutation ratios, the proposed SA_GA method can provide the optimal parameters that are associated with better fitness to the observed data than SGA. Moreover, as compared with SGA in the estimation of runoff with optimal parameters using various numbers of calibration events, the proposed SA_GA method can provide the optimal parameters that can capture the change in runoff, especially for the peak discharge.

In addition to the *SAC-SMA* model, the proposed SA_GA method could be applied in other hydrological models associated with large numbers of parameters. Moreover, in SA_GA, the standardized values of model parameters are hypothesized as the non-correlated variables. However, some correlation should exist among standardized values of model parameters. Therefore, SA_GA could be improved by taking into account the correlation between model parameters. Also, as the single objective function Equation (3) adopted in this study focuses on the difference between estimated and observed runoff, especially for high observed runoff, the corresponding optimal parameters may ignore the effect of low runoff, such as base-flow. To remedy this omission, the multi-objective function (e.g. Madsen 2000; Seibert 2000) would be adopted in the proposed SA_GA method.

REFERENCES

Abdulla, F. & AL-Badranih, L. 2000 Application of a rainfall–runoff model to three catchments in Iraq. *Hydrolog. Sci.* **45** (1), 13–25.

Agrawal, R. K. & Singh, J. K. 2003 Application of a genetic algorithm in the development and optimisation of a non-linear dynamic runoff model. *Biosystems Eng.* **86** (1), 87–95.

Ajami, N. K., Gupta, H., Wagener, T. & Sorooshian, S. 2004 Calibration of a semi-distributed hydrologic model for streamflow estimation along a river system. *J. Hydrol.* **298**, 112–135.

Avila, S. L., Carpes, W. P., Vasconcelos, J. R. & Vasconcelos, J. A. 2004 Optimization of an offset reflector antenna using genetic algorithms. *IEEE Trans. Magn.* **40** (2), 1256–1259.

Back, T., Hammel, U. & Schwefel, H. P. 1997 Evolutionary computation: comments on the history and current stat. *IEEE Trans. Evol. Comput.* **1**, 3–17.

Bessaou, M. & Siarry, P. 2001 A genetic algorithm with real-value coding to optimize multimodal continuous function. *Struct. Multidisciplinary Optimization* **23** (1), 63–74.

Burnash, R. J. C. 1995 The NWS (National Weather Service) river forecast system-catchment model. In: *Computer Models of Watershed Hydrology* (V. P. Singh, ed.). Water Resour. Publ., Highlands Ranch, CO.

Castaings, W., Dartus, D., Le Dimet, F. X. & Saulnier, G. M. 2007 Sensitivity analysis and parameter estimation for the distributed modeling of infiltration excess overland flow. *Hydrol. Earth Syst. Sci. Discuss.* **4**, 363–405.

Chaubey, I., Haan, C. T., Grunwald, S. & Salisbury, J. M. 1999 Uncertainty in the model parameters due to spatial variability of rainfall. *J. Hydrol.* **220**, 48–61.

Cullmann, J., Mishra, V. & Peters, R. 2006 Flow analysis with WaSiM-ETH-model parameter sensitivity at different scales. *ADGEO* **9**, 73–77.

Fang, T. & Ball, J. E. 2007 Evaluation of spatially variable control parameters in a complex catchment modeling system: a genetic algorithm application. *J. Hydroinformatics* **9** (3), 163–173.

Franchini, M. 1996 Use of a genetic algorithm combined with a local search method for the automatic calibration of conceptual rainfall–runoff models. *Hydrolog. Sci.* **41** (1), 21–39.

Franchini, M. & Galeati, G. 1997 Comparing several genetic algorithm schemes for the calibration of conceptual rainfall–runoff models. *Hydrolog. Sci.* **42** (3), 357–379.

Franchini, M., Galeati, G. & Berra, S. 1998 Global optimization techniques for the calibration of conceptual rainfall–runoff model. *Hydrolog. Sci.* **43** (3), 443–458.

Goldberg, D. E. 1989 *Genetic Algorithms in Search Optimization and Machine Learning*. Addison-Wesley Publishing Company, Reading, Massachusetts, USA.

Goldberg, D. E. & Deb, K. 1991 A comparative analysis of selection schemes used in genetic algorithms. In: *Foundation of Genetic Algorithms* (G. J. E. Rawlins, ed.). Morgan Kaufmann, San Mateo, Cambridge, pp. 69–93.

Holland, J. 1975 *Adaptation in Natural and Artificial System*. The University of Michigan Press, Arbor, Michigan.

Hossain, F., Anagnostou, E. N., Dinku, T. & Borga, M. 2004 Hydrological model sensitivity to parameter and radar rainfall estimation uncertainty. *Hydrolog. Process.* **18**, 327–3201.

Jain, A. & Srinivasulu, S. 2004 Development of effective and efficient rainfall–runoff using integration of deterministic, real-coded genetic algorithms and artificial neural network techniques. *Water Resour. Res.* **40** (W04302), 1–12.

Janikow, C. Z. & Michalewicz, Z. 1991 An experimental comparison of binary and floating point representations in genetic algorithm. In: *Proceedings of the Fourth International Conference on Genetic Algorithm* (R. K. Belew & L. B. Booker, eds.). Morgan Kaufmann Publishers, Inc., San Mateo, CA, USA, pp. 31–36.

Krishnakumar, K. 1989 Micro-genetic algorithm for stationary and non-stationary function optimization. *SPIE Intelligent Control and Adaptive System* **1196**, 289–296.

Madsen, H. 2000 Automatic calibration of a conceptual rainfall–runoff model using multiple objectives. *J. Hydrol.* **235**, 276–288.

Madsen, H., Wilson, G. & Ammentorp, H. C. 2002 Comparison of different automated strategies for calibration of rainfall–runoff models. *J. Hydrol.* **261**, 48–59.

Mayer, D. G., Belward, J. A., Widell, H. & Burrage, K. 1999 Survival of the fittest – genetic algorithm versus evaluation strategies in the optimization of systems models. *Agr. Syst.* **60**, 113–122.

Milivojevic, N., Simic, Z., Orlic, A., Milivojevic, B. & Stojanovic, B. 2009 Parameter estimation and validation of the proposed SWAT based rainfall–runoff model- method and outcomes. *J. Serbian Soc. Comput. Mech.* **3** (1), 86–100.

Nash, J. E. & Sutcliffe, J. V. 1970 River flow forecasting through conceptual models: Part I – a discussion of principles. *J. Hydrol.* **10**, 282–290.

Ndiritu, J. G. & Daniell, T. M. 2001 An improved genetic algorithm for rainfall–runoff runoff model calibration and function optimization. *Math. Comput. Model.* **33**, 695–706.

Peters-Lidard, C. D., Mocko, D. M., Garcia, M., Santanellno Jr., J. A. , Tischler, M. A., Moran, M. S. & Wu, Y. 2008 Role of precipitation uncertainty in the estimation of hydrologic soil properties using remotely sensed soil moisture in a semiarid environment. *Water Resour. Res.* **44**, W05S18.

Salomon, R. 1996 The influence of different coding schemes on the computation complexity of genetic algorithms in function optimization. *Lecture Notes Comput. Sci.* **1141**, 227–235.

Seibert, J. 2000 Multi-criteria calibration of a conceptual runoff model using a genetic algorithm. *Hydrol. Earth Syst. Sci.* **4** (2), 215–224.

Siriwardene, N. R. & Perera, B. J. C. 2006 Selection of genetic algorithm operators for urban drainage model parameter optimisation. *Math. Comput. Model.* **44**, 415–429.

Speed, T. P. & Yu, B. 1993 Model selection and prediction: normal regression. *Ann. Inst. Statist. Math.* **45** (1), 35–54.

Subret, B. 2008 Global sensitivity analysis using polynomial chaos expansions. *Reliab. Eng. Syst. Saf.* **93** (7), 964–979.

Tang, Y., Reed, P., Wagener, T. & Werkhoven, K. V. 2007 Comparing sensitivity analysis method to advance lumped watershed model identification and evaluation. *Hydrol. Earth Syst. Sci.* **11**, 793–817.

Wang, Q. J. 1991 The genetic algorithm and its application to calibration conceptual rainfall–runoff models. *Water Resour. Res.* **27** (9), 2467–2471.

Wang, Q. J. 1997 Using genetic algorithm to optimize model parameters. *Environ. Model. Software* **12** (1), 27–34.

Wardlaw, R. & Sharif, M. 1999 Evaluation of genetic algorithm for optimal reservoir operation. *J. Water Resour. Plann. Manag.* **125** (1), 25–33.

Wu, S. J., Tung, Y. K. & Yang, J. C. 2006 Stochastic generation of hourly rainstorm events. *Stoch. Environ. Res. Risk Assess.* **21** (2), 195–212.

Yu, P. S., Yang, T. C. & Chen, S. J. 2001 Comparison of uncertainty analysis methods for a distributed rainfall–runoff model. *J. Hydrol.* **244**, 43–59.