Camera Calibration with Two Arbitrary Coplanar Circles

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Abstract. In this paper, we describe a novel camera calibration method to estimate the extrinsic parameters and the focal length of a camera by using only one single image of two coplanar circles with arbitrary radius.
We consider that a method of simple operation to estimate the *extrinsic parameters* and the *focal length* of a camera is very important because in many vision based applications, the position, the pose and the zooming factor of a camera is adjusted frequently.
An easy to use and convenient camera calibration method should have two characteristics: 1) the calibration object can be produced or prepared easily, and 2) the operation of a calibration job is simple and easy. Our new method satisfies this requirement, while most existing camera calibration methods do not because they need a specially designed calibration object, and require multi-view images. Because drawing beautiful circles with arbitrary radius is so easy that one can even draw it on the ground with only a rope and a stick, the calibration object used by our method can be prepared very easily. On the other hand, our method need only one image, and it allows that the centers of the circle and/or part of the circles to be occluded.
Another useful feature of our method is that it can estimate the focal length as well as the extrinsic parameters of a camera simultaneously. This is because zoom lenses are used so widely, and the zooming factor is adjusted as frequently as the camera setting, the estimation of the focal length is almost a must whenever the camera setting is changed. The extensive experiments over simulated images and real images demonstrate the robustness and the effectiveness of our method.

1 Introduction

Calibration of the extrinsic camera parameters is an indispensable preparation for computer vision tasks such as environment recognition, 3D shape acquirement and so on. In many real vision based applications, camera setting is adjusted frequently, and whenever the camera setting has been altered, the extrinsic camera parameters have to be estimated again. In recent years, zoom lenses have being widely used, and zooming factor is adjusted as frequently as other camera parameters. Thus when the position or pose of a camera has been adjusted, the focal length might also have been altered in most cases. Therefore, we consider that a method of simple operation to estimate the *extrinsic parameters* and the *focal length* of a camera simultaneously is highly desired. Such a

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method should have two characteristics: 1) the calibration object can be produced or prepared easily, and 2) the operation of a calibration job is simple and easy.

One of the conventional methods to calibrate the extrinsic parameters and the focal length of a camera is to use point correspondence data. In order to get precise results, point correspondence data spreading over an image plane is necessary. However, it is difficult to prepare a lot of points with known 3D coordinates and to find the correspondence between the 3D points and their projection in the image. Although specially designed calibration objects can ease this job, producing such an object itself and setting it properly for the calibration is still complicated and time consuming, and sometime becomes impossible or impractical, e.g. in the case of wide observing area, such as a baseball stadium or a football playground.

The usage of point corresponding data can be avoided by using geometrical patterns such as straight lines and circles instead. Several researches using circular patterns, or conic patterns [2]-[6] have been reported so far. These camera calibration methods are for estimating intrinsic camera parameters, and they all use some special patterns and multi-view images.

Meng *et al.*[2] proposed a method using a pattern that consists of a circle and straight lines passing through its center. It needs at least three different views. Kim *et al.*[4] proposed a method that makes use of planar con-centric circles. It requires two views. Yang *et al.*[5] proposed a similar method except that con-centric ellipses are used instead of con-centric circles.

Other methods are about motion analysis or 3D interpretation of conic[7]-[16]. Although some of them can be used as a calibration method, they have some or all of the following disadvantages, 1) multi-view images are required, 2) only part of the extrinsic camera parameters can be estimated, 3) the focal length can not be estimated, 4) a specially designed calibration object is required.

Long[7] proposed a method to find the correspondence between conics in two views, and to estimate the relative orientation of the optical axis of two views. Dhome *et al.*[8] proposed a method to estimate the attitude and the position of a circle from an image assuming known focal length and radius. Kanatani and Wu[15],[16] reported methods to extract 3D information from conics in images. The intrinsic and extrinsic camera parameters are supposed to be known.

In this paper, we describe a novel camera calibration method to estimate the extrinsic parameters and the focal length of a camera by using only one single image of two coplanar circles with arbitrary radius. Because drawing beautiful circles with arbitrary radius is so easy that one can even draw it on the ground with only a rope and a stick, the calibration object used by our method can be prepared very easily. On the other hand, our method need only one image, and it allows that the centers of the circle and/or part of the circles to be occluded. These features make the operation of camera calibration using our method becoming very simple and easy. Another useful feature of our method is that it can estimate the focal length as well as the extrinsic parameters of a camera simultaneously.

The extensive experiments over simulated images and real images demonstrate the robustness and the effectiveness of our method.

2 Elliptical Cone and Circular Cross Section

In this section we describe the problem of estimating the direction and the center of a circle from one perspective view. M.Dhome[8] addressed this problem in a research about the pose estimation of an object of revolution. We give a rigorous description here, which is then used in the estimation of the focal length and the extrinsic parameters of the camera in the succeeding section.

2.1 Ellipses and Conic Surfaces

If a circle is projected on to the image plane with perspective projection, it shows an ellipse in general case. Considering a camera coordinate system that the origin is the optical center and the Z-axis is the optical axis, then the ellipse in the image can be described by the following equation,

$$Ax_e^2 + 2Bx_e y_e + Cy_e^2 + 2Dx_e + 2Ey_e + F = 0, \tag{1}$$

or in quadratic form as following,

$$\begin{pmatrix} x_e & y_e & 1 \end{pmatrix} \begin{pmatrix} A & B & D \\ B & C & E \\ D & E & F \end{pmatrix} \begin{pmatrix} x_e \\ y_e \\ 1 \end{pmatrix} = 0. \tag{2}$$

A bundle of straight lines passing through the optical center and the ellipse defines an oblique elliptical cone. Assuming that the focal length of the camera is f, the image plane can be expressed by $z = -f$. Then the oblique elliptical cone can be described by,

$$\mathbf{P} = k \begin{pmatrix} x_e & y_e & -f \end{pmatrix}^T, \tag{3}$$

where k is a scale factor describing the distance from the origin to $\mathbf{P}$. From Eq.(2) and Eq.(3) the equation to describe the oblique elliptical cone is derived,

$$\mathbf{P}^T \mathbf{Q} \mathbf{P} = 0, \tag{4}$$

where

$$\mathbf{Q} = \begin{pmatrix} A & B & -\frac{D}{f} \\ B & C & -\frac{E}{f} \\ -\frac{D}{f} & -\frac{E}{f} & \frac{F}{f^2} \end{pmatrix}. \tag{5}$$

Considering a supporting plane coordinate system that the origin is also the optical center, but the Z-axis is defined by the unit normal vector of the supporting plane of the circle to be viewed. Let z_0 be the Z coordinate of points on the plane, the points on the circle can be described by the following expression,

$$\begin{cases} (x - x_0)^2 + (y - y_0)^2 = r^2 \\ z = z_0 \end{cases} \tag{6}$$

where (x_0, y_0, z_0) is the center and r is the radius of the circle. A bundle of straight lines passing through the optical center and the circle defines an oblique circular cone described by the following equation,

$$\mathbf{P_c}^T \mathbf{Q_c} \mathbf{P_c} = 0, \tag{7}$$

where

$$\mathbf{Q_c} = \begin{pmatrix} 1 & 0 & -\frac{x_0}{z_0} \\ 0 & 1 & -\frac{y_0}{z_0} \\ -\frac{x_0}{z_0} & -\frac{y_0}{z_0} & \frac{x_0^2+y_0^2-r^2}{z_0^2} \end{pmatrix}. \tag{8}$$

Since the camera coordinate system and the supporting plane coordinate system have a common origin at the optical center, the transform between the two coordinate systems is a rotation. Because the oblique circular cone and the oblique elliptical cone are the same cone surface, there exists a rotation matrix $\mathbf{R_c}$ that transforms $\mathbf{P_c}$ to $\mathbf{P}$ as following,

$$\mathbf{P} = \mathbf{R_c} \mathbf{P_c}. \tag{9}$$

Since $k\mathbf{Q_c}$ for any $k \neq 0$ describes the same cone as of $\mathbf{Q_c}$, from Eq.(9), Eq.(7) and Eq.(4) we have,

$$k\mathbf{R_c}^T \mathbf{Q} \mathbf{R_c} = \mathbf{Q_c}. \tag{10}$$

In order to determine $\mathbf{R_c}$ and $\mathbf{Q_c}$ so that the unit normal vector of the supporting plane and the center of the circle can be obtained, we want to convert $\mathbf{Q}$ to a diagonal matrix first.

Let λ_1, λ_2, λ_3 be the eigen-values, and $\mathbf{v_1} = (v1x\ v_{1y}\ v_{1z})^T$, $\mathbf{v_2} = (v_{2x}\ v_{2y}\ v_{2z})^T$, $\mathbf{v_3} = (v_{3x}\ v_{3y}\ v_{3z})^T$ be the normalized eigen-vecters of $\mathbf{Q}$ respectively, $\mathbf{Q}$ can be expressed by the following equation,

$$\mathbf{Q} = \mathbf{V} \mathbf{\Lambda} \mathbf{V}^T, \tag{11}$$

where

$$\begin{cases} \mathbf{\Lambda} = \mathrm{diag}\{\lambda_1, \lambda_2, \lambda_3\} \\ \mathbf{V} = \begin{pmatrix} \mathbf{v_1} & \mathbf{v_2} & \mathbf{v_3} \end{pmatrix} \end{cases}. \tag{12}$$

Substituting Eq.(11) for $\mathbf{Q}$ in Eq.(10), the following equation is obtained,

$$k\mathbf{R}^T \mathbf{\Lambda} \mathbf{R} = \mathbf{Q_c}, \tag{13}$$

where

$$\mathbf{R} = \begin{pmatrix} r_{1x} & r_{2x} & r_{3x} \\ r_{1y} & r_{2y} & r_{3y} \\ r_{1z} & r_{2z} & r_{3z} \end{pmatrix} = \mathbf{V}^T \mathbf{R_c}. \tag{14}$$

From Eq.(13) we obtain following equations,

$$\begin{cases} \lambda_1(r_{1x}^2 - r_{2x}^2) + \lambda_2(r_{1y}^2 - r_{2y}^2) + \lambda_3(r_{1z}^2 - r_{2z}^2) = 0 \\ \lambda_1 r_{1x} r_{2x} + \lambda_2 r_{1y} r_{2y} + \lambda_3 r_{1z} r_{2z} = 0 \end{cases}. \tag{15}$$

Without losing generality, we assume that

$$\begin{cases} \lambda_1 \lambda_2 > 0 \\ \lambda_1 \lambda_3 < 0 \\ |\lambda_1| \ \ \geq |\lambda_2| \end{cases}. \tag{16}$$

By simplifying Eq.(15) and $\mathbf{R}^T\mathbf{R} = \mathbf{I}$, we obtain,

$$\mathbf{R} = \begin{pmatrix} g\cos\alpha & \mathrm{S}_1 g\sin\alpha & \mathrm{S}_2 h \\ \sin\alpha & -\mathrm{S}_1\cos\alpha & 0 \\ \mathrm{S}_1\mathrm{S}_2 h\cos\alpha & \mathrm{S}_2 h\sin\alpha & -\mathrm{S}_1 g \end{pmatrix}, \tag{17}$$

where α is a free variable, S_1 and S_2 are undetermined signs, and

$$\begin{cases} g = \sqrt{\frac{\lambda_2-\lambda_3}{\lambda_1-\lambda_3}} \\ h = \sqrt{\frac{\lambda_1-\lambda_2}{\lambda_1-\lambda_3}} \end{cases}, \tag{18}$$

By substituting Eq.(17) for $\mathbf{R}$ in Eq.(13), k, x_0/z_0, y_0/z_0 and r/z_0 are determined,

$$\begin{cases} k \ = \lambda_2 \\ \frac{x_0}{z_0} = -\mathrm{S}_2 \frac{\sqrt{(\lambda_1-\lambda_2)(\lambda_2-\lambda_3)}\cos\alpha}{\lambda_2} \\ \frac{y_0}{z_0} = -\mathrm{S}_1\mathrm{S}_2 \frac{\sqrt{(\lambda_1-\lambda_2)(\lambda_2-\lambda_3)}\sin\alpha}{\lambda_2} \\ \frac{r^2}{z_0^2} = -\frac{\lambda_1\lambda_3}{\lambda_2^2} \end{cases} \tag{19}$$

Because the Z-axis of the supporting plane coordinate system is the unit normal vector of the plane (denoted by $\mathbf{N}$), from Eqs.(9), (14) and (19), $\mathbf{N}$ and the center of the circle (denoted by $\mathbf{C}$) described in the camera coordinate system can be computed by the following expression,

$$\begin{cases} z_0 = \mathrm{S}_3 \frac{\lambda_2 r}{\sqrt{-\lambda_1\lambda_3}} \\ \mathbf{C} = z_0\mathbf{V}\mathbf{R}\begin{pmatrix} x_0/z_0 \\ y_0/z_0 \\ 1 \end{pmatrix} = z_0\mathbf{V}\begin{pmatrix} \mathrm{S}_2\frac{\lambda_3}{\lambda_2}\sqrt{\frac{\lambda_1-\lambda_2}{\lambda_1-\lambda_3}} \\ 0 \\ -\mathrm{S}_1\frac{\lambda_1}{\lambda_2}\sqrt{\frac{\lambda_2-\lambda_3}{\lambda_1-\lambda_3}} \end{pmatrix} \\ \mathbf{N} = \mathbf{V}\mathbf{R}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad = \mathbf{V}\begin{pmatrix} \mathrm{S}_2\sqrt{\frac{\lambda_1-\lambda_2}{\lambda_1-\lambda_3}} \\ 0 \\ -\mathrm{S}_1\sqrt{\frac{\lambda_2-\lambda_3}{\lambda_1-\lambda_3}} \end{pmatrix} \end{cases}, \tag{20}$$

where S_3 is also an undetermined sign.

Since a plane has two side, we let $\mathbf{N}$ be the normal vector indicating the side faced to the camera. Also, since the center of the circle is in front of the camera, the following stands,

$$\begin{cases} \mathbf{N} \cdot \begin{pmatrix} 0 \ 0 \ 1 \end{pmatrix}^T > 0 \\ \mathbf{C} \cdot \begin{pmatrix} 0 \ 0 \ 1 \end{pmatrix}^T < 0 \end{cases}, \tag{21}$$

from which two of the three undetermined signs in Eq.(20) can be determined, and we have two sets of possible answers about $\mathbf{N}$ and $\mathbf{C}$. In the case of unknown radius, the r is left a scale factor in Eq.(20).

2.2 Estimating the Extrinsic Parameters and the Focal Length Simultaneously

In this section, we describe a method to estimate the extrinsic parameters and the focal length of a given camera by using two coplanar circles with arbitrary radius. As described in the section 2.1, the unit normal vector of the supporting plane and the center of the circle can be determined from one perspective image, if the focal length is known.

In the case of unknown focal length, the symbol f in Eq.(5) leaves a variable, and it will remain in all the answers.

In order to determine the focal length so that the unit normal vector of the supporting plane and the center of the circle can be determined, we let the camera to view a scene consists of two coplanar circles. In this case, two ellipses will be detected, and according to section 2.1, two oblique elliptical cones can be formed from the detected ellipses if we give a focal length. From each of them, the normal vector of the supporting plane can be estimated independently.

If we have given a wrong focal length, each of the formed cones will be deformed in different ways and will not be similar to the real cone surfaces. In this case, the estimated unit normal vectors of the supporting plane from each of the two cones will not only be different from the real one, but will not be parallel to each other too. Only if we give the correct focal length, the unit normal vectors estimated from each of the detected ellipses will be the same.

Let $\mathbf{N_1}(f)$ denote the normal vector estimated from one of the two ellipses and $\mathbf{N_2}(f)$ denote the normal vector from the other one. Because the two circles are coplanar, $\mathbf{N_1}(f)$ and $\mathbf{N_2}(f)$ should be same. This constraint can be expressed by the following equation,

$$\mathbf{N_1}(f) \cdot \mathbf{N_2}(f) = 1. \tag{22}$$

Then by minimizing the following expression, the focal length f and the unit normal vector $\mathbf{N}(= \mathbf{N_1} = \mathbf{N_2})$ can be determined, and the ambiguity cased by the undetermined signs remained in Eq.(20) can be eliminated,

$$\left(\mathbf{N_1}(f) \cdot \mathbf{N_2}(f) - 1\right)^2 \to \min . \tag{23}$$

The centers of the two circles can also determined with Eq.(20). If the radius of the circles are unknown, then z_0 in Eq.(20) leaves a variable. Let $\mathbf{C_1}$ and $\mathbf{C_2}$ denote the centers of the two circles respectively, from Eq.(20) they can be expressed by,

$$\begin{cases} \mathbf{C_1} = z_0 \mathbf{C_{10}} \\ \mathbf{C_2} = z_0 \mathbf{C_{20}} \end{cases}, \tag{24}$$

where $\mathbf{C_{10}}$ and $\mathbf{C_{20}}$ can be computed from the detected ellipses. The distance between the two circle centers (d_{12}) can be calculated by the following expression,

$$d_{12} = |\mathbf{C_1} - \mathbf{C_2}| = |z_0||\mathbf{C_{10}} - \mathbf{C_{20}}|. \tag{25}$$

A world coordinate system O-XYZ can be defined by using the two circle centers as reference points and the unit normal vector of the supporting plane as a reference direction as following. Let $\mathbf{C_1}$ be the origin, $\mathbf{N}$ define the Z axis and the vector $\mathbf{C_2} - \mathbf{C_1}$

define the direction of the X axis of O-XYZ respectively, the origin $\mathbf{O}$, the unit vectors of X, Y, Z axes $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ can be obtained by the following equation,

$$\begin{cases} \mathbf{O} = \mathbf{C_1} \\ \mathbf{i} \;= \frac{\mathbf{C_{20}}-\mathbf{C_{10}}}{|\mathbf{C_{20}}-\mathbf{C_{10}}|} \\ \mathbf{k} \;= \mathbf{N} \\ \mathbf{j} \;= \mathbf{k} \times \mathbf{i} \end{cases} . \tag{26}$$

If one of the radius of the two circles, the distance between the two circle center or the distance between the optical center and the supporting plane is known, or if we use one of them as the unit length of O-XYZ, then z_0 can be determined from Eq.(20) or Eq.(25), thus $\mathbf{O}$ and all other parameters related to length will be determined.

Then the optical center $\mathbf{O}'$, the unit vectors $\mathbf{i}'$, $\mathbf{j}'$, and $\mathbf{k}'$ that define the X, Y and Z axis of the camera coordinate system described in the world coordinate system O-XYZ can be computed by the following equation,

$$\begin{cases} \mathbf{O}' = \begin{pmatrix} \mathbf{i}^T \\ \mathbf{j}^T \\ \mathbf{k}^T \end{pmatrix} \left[\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} - \mathbf{O} \right] \\ \mathbf{i}' \;= \begin{pmatrix} \mathbf{i}^T \\ \mathbf{j}^T \\ \mathbf{k}^T \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \\ \mathbf{j}' \;= \begin{pmatrix} \mathbf{i}^T \\ \mathbf{j}^T \\ \mathbf{k}^T \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \\ \mathbf{k}' \;= \begin{pmatrix} \mathbf{i}^T \\ \mathbf{j}^T \\ \mathbf{k}^T \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \end{cases} . \tag{27}$$

Therefore, by taking an image of a scene of two coplanar circles, the unit normal vector of the supporting plane containing the circles and the focal length can be estimated. And if the centers of the circle can be used as two reference point, then the full translation and rotation of the camera relative to the world coordinate system defined by the two circle centers and the normal vector of the supporting plane can be also determined. If neither of the radiuses of the circles or the distance between the two circle centers is available, the rotation and the translation of the camera can also determined except that a scale factor remains undetermined. In both cases, the centers of the circles need not to be viewable in the image.

3 Experimental Results

In order to exam the usefulness and the effectiveness of the proposed algorithm; we first tested our method using some simulated images, then using some real images.

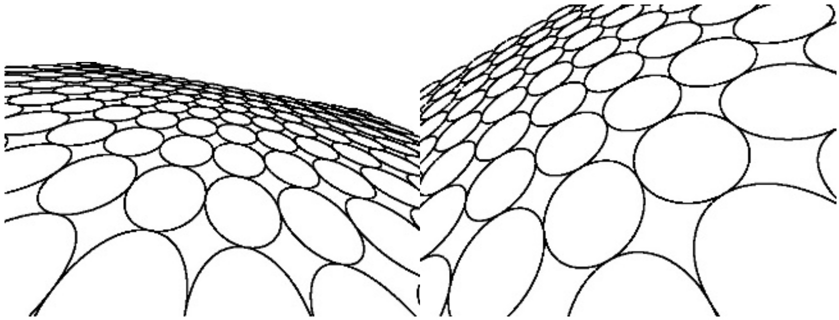

Fig. 1. Two sets of synthesized scenes of circles by CG: *case*-1 and *case*-2.

Table 1. Estimated camera parameters

case-1	RMS error	Standard deviation	*case*-2	RMS error	Stand deviation
f(pixel)	5.52	9.21	f(pixel)	7.19	11.89
β(degree)	0.36	0.47	β(degree)	0.11	0.15
θ(degree)	0.57	0.97	θ(degree)	0.51	0.85

3.1 Simulation Results

We used computer graphics to synthesize images of many different coplanar circle scenes. We first set the image resolution to 640×480 [pixel], the focal length to 200 [pixel], the tilt angle θ indicating the angle between the optical axis and the supporting plane to be 40 [degree], the roll angle β indicating the rotation about the optical axis to 10 [degree], and the distance between the optical center and the supporting plane to 3.0 [meter]. We called this camera setting as "*case*-1" hereafter. We use this camera setting to synthesize images of circles with a radius of 1.0 [meter].

Figure 1 shows an image containing all the circles used in the experiment of the "*case*-1" camera setting. We used 32 images containing two circles randomly selected from the ones shown in Figure 1.

We also have done a similar experiment using the camera setting called "*case*-2" of which the image resolution is same as the "*case*-1", the focal length is 300 [pixel], $\theta = 50$ [degree], $\beta = 30$ [degree], the distance between the optical center and the supporting plane and the radius of the circles are same as "*case*-1".

Figure 1 shows an image containing all the circles used in the experiment of the "*case*-2" camera setting. We used 17 images containing two circles randomly selected from the ones shown in Figure 1.

From each of the images, two ellipses were detected, which were used to estimate the unit normal vector of the supporting plane and the focal length. Then the estimated focal length and the tilt angle and the roll angle calculated from the estimated unit normal vector of the supporting plane were compared to the ground truth, which is the camera setting used to synthesize images with CG. The experimental results are summarized in Table 1 with suffixes 1 and 2.

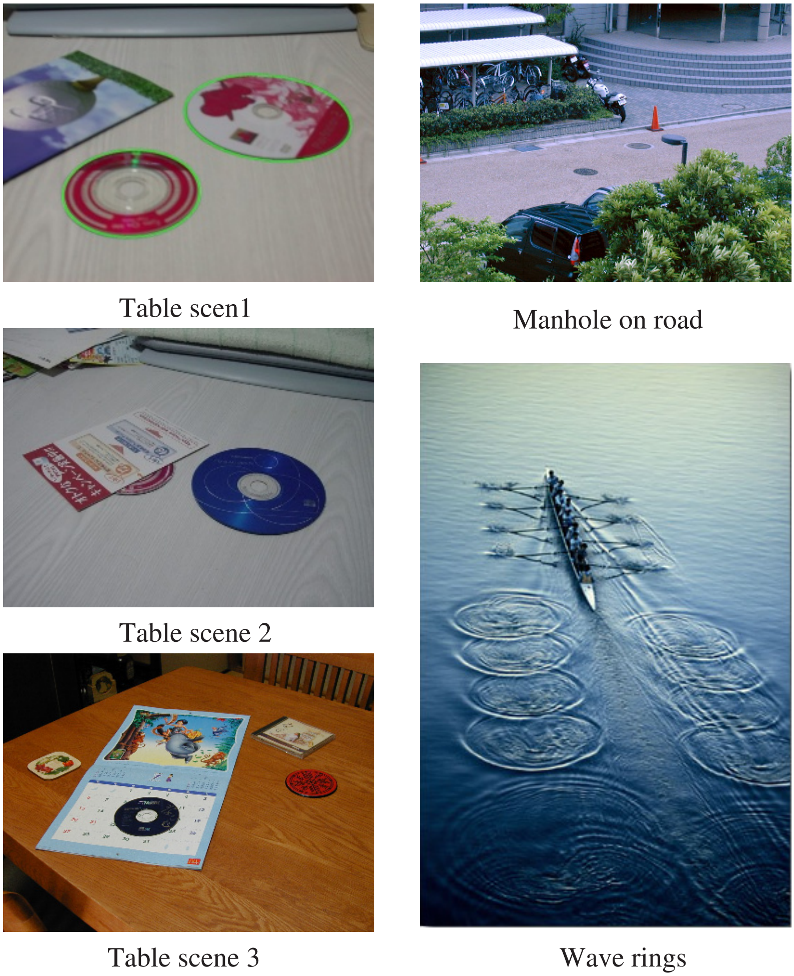

Fig. 2. Some images of real scene used the experiment.

Table 2. Experimental results estimated from images shown in Figure 2

Image name	Resolution[pixel]	Focal length[pixel]	Unit Normal vector
Table scene 1	640×480	901.0	(0.03, 0.83, 0.56)
Table scene 2	640×480	1140	(0.13, 0.84, 0.51)
Table scene 3	1600×1200	2209	(-0.04, 0.83, 0.56)
Manhole on road	1600×1200	4164	(0.05, 0.97, 0.26)
Wave rings	275×412	2615	(0.03, 0.92, 0.39)

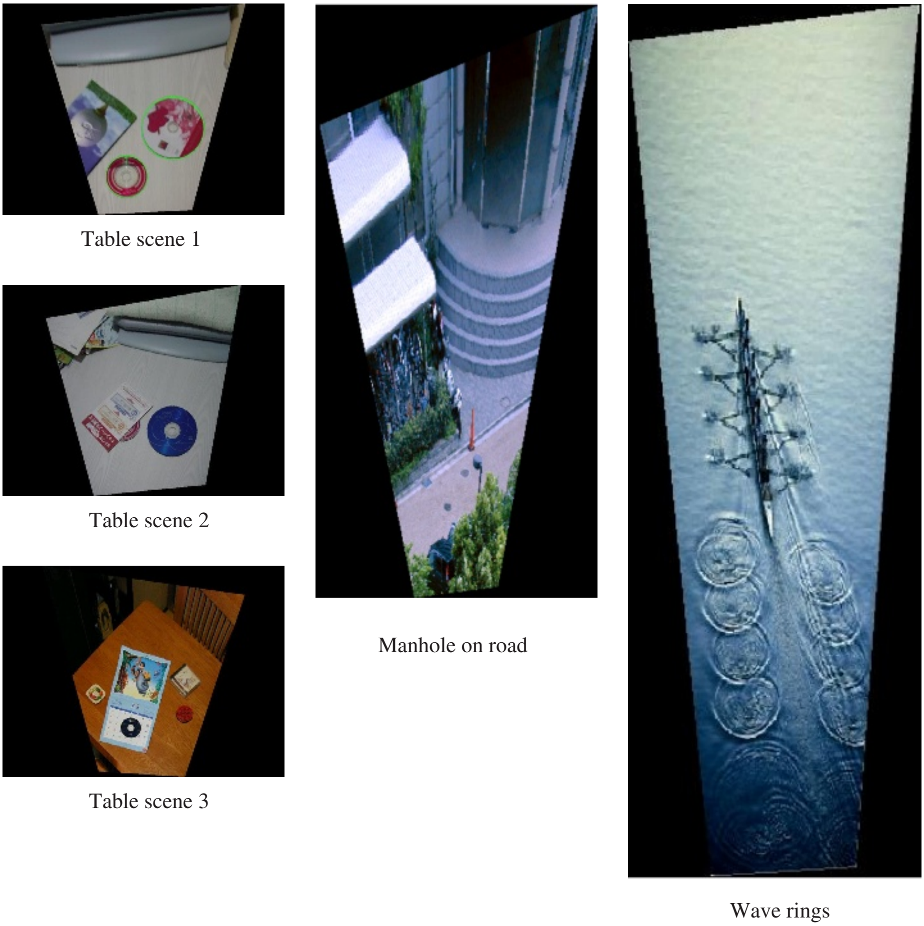

Fig. 3. Vertical views synthesized using estimated camera parameters.

3.2 Experiments with Real Images

We tested our method by applying it to many real images containing coplanar circle shape objects e.g., manholes on roads, CD discs on a table, widening rings on the water surface, and so on. Some of the images used in the experiment is shown in Figure 2).

The images were taken with two digital still cameras of different kind, and with a digital video camera. All of them are equipped with a zoom lens.

For each image, we detected the ellipses and used them to estimate the focal length and the unit normal vector of the supporting plane. The results are summarized in Table 1.

Since the ground truth of the camera setting including focal length, position and pose is not available, we used the estimated camera parameters to convert the image to a vertical view to the supporting plane by assuming a planar scene, and to see if it resembles the real scene. Figure 3.

In some of the converted images, the circular object doest not show a perfect circle. The considerable reasons are, 1) in our method, the intrinsic parameters except the focal length are assumed to be calibrated, but uncalibrated cameras were used in the experiment, 2) the radial distortion of the cameras are not compensated.

4 Conclusion

This paper has presented a new camera calibration method for estimating the focal length and the extrinsic camera parameters using circular patterns. This method allows us to estimate the extrinsic parameters and focal length simultaneously using one single view of two coplanar circles with arbitrary radius. Moreover, it does not require the whole circles or the centers to be viewable. These features make a very convenient calibration method because both the preparation of the calibration pattern and the operation of taking picture are quite easy.

Compared with existing method, our method can determine the focal length of camera as well as extrinsic camera parameters. Even in the case that the position and the size of the circles are not available, our method can still give the focal length and the normal vector of the supporting plane.

We will extend our method estimating image center and the radial distortion parameters in the future work.

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