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CAN ALL HADRONIC SYMMETRY BREAKING BE DUE TO WEAK INTERACTIONS?*

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ABSTRACT

We define, in very general terms, a class of theories in which it is possible for all symmetries such as SU(3) and SU(3) ⊗ SU(3) to be exact if one turns off all couplings of leptons and hadrons; but nevertheless, one can have ten to twenty percent violations of these symmetries coming about as a direct consequence of extremely weak lepton-hadron couplings. A framework is proposed for discussing these schemes which is independent of specific Lagrangian models and so leaves open the question of whether or not the Goldstone bosons appearing in these schemes are elementary or composite. For pedagogical purposes the last section of this paper is devoted to a brief discussion of a simple Lagrangian model having the various properties set out in our general scheme.

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INTRODUCTION

The interpretation of the successes of the current algebra plus PCAC hypotheses in terms of an approximate Goldstone symmetry[1,2] of the strong interactions is appealing if for no other reason than it systematizes approximations made in applying these ideas to discussions of physical processes. If, however, one is serious about the idea that the strong interactions are almost invariant with respect to the chiral $SU(3) \otimes SU(3)$ generated by the Gell-Mann algebra of vector and axial-vector charges, one cannot help wondering about the origins of the intimate relation that this scheme implies between the weak and electromagnetic interactions and strongly broken hadron symmetries.

To emphasize just how surprising this relationship is, even at the SU(3) level, let us contrast the way the CVC-hypothesis relates the approximate isotopic spin symmetry of the strong interactions to the algebra of isovector charges, and the analagous situation in which current algebra and the Cabbibo theory of weak interactions relate the approximate SU(3) symmetry of hadronic interactions to the SU(3) vector charge algebra.

In the first case, one argues that the relationship between the SU(2) of isovector charges and the SU(2) of strong interactions is not too surprising. After all, the SU(2) for strong interactions would be exact in the absence of weak and electromagnetic effects. Hence, if we assume that the entire non-invariant part of the total Hamiltonian, H_{tot}, is that describing the coupling of these would-be conserved currents to lepton currents, it is no surprise that commutation of H_{tot} with the isovector charges decomposes it into an SU(2) symmetrical piece — which includes the strong Hamiltonian — and a piece of order "α" and "G_W". In this picture it is the smallness of "α" and "G_W" which tells us that we may use perturbation theory to relate the real world to an approximately symmetrical one.

The second case is quite different. SU(3) is a symmetry which is apparently broken at the purely hadronic level by ten or twenty percent. Therefore, if one supposes the reason that the weak vector charges are the generators of an approximate SU(3) symmetry is that the only non-symmetrical piece of H_{tot} is that containing the corresponding currents; then one must face up to the fact that "α" and "G_W" are so small that it is difficult to see how a naive perturbative approach could lead to twenty percent violations of the observed symmetry, as opposed to one percent violations of SU(2). Clearly, the same problem faces us if we try to explain, in these terms, the interpretation of PCAC as an approximate Goldstone symmetry of the strong interactions.

Despite these problems, the idea that all hadronic symmetries are broken by very weak couplings of leptons and hadrons is so appealing that one hopes there might exist a scheme of this type amenable to perturbative treatment.

The purpose of this paper is to describe, in very general terms, how this is accomplished essentially automatically in a large class of Goldstone-Higgs type theories.[3] Physically, the essential feature of these schemes which allows a perturbation theory in small coupling constants to give rise to large hadronic symmetry violations is the existence of degenerate vacua. In general terms, in these theories — as in any problem involving degenerate perturbation theory — one must be careful to isolate potentially large effects intimately related to the degeneracy and its associated instabilities before proceeding with naive perturbation theoretic discussions.

Our general discussion will be based upon simple features abstracted from the various Weinberg-Higgs-Kibble type schemes so fashionable today.[4] We believe that this type of approach has several advantages.[5] First, it separates the details of constructing model Lagrangians and doing complicated calculations, from the important physical assumptions which govern the structure of low-energy theorems, etc. Second, we can expect that these general properties also describe worlds having composite Goldstone particles, as nothing we shall say requires the Goldstone particles involved to be elementary. Finally, we believe that discussion of the symmetry properties of these theories is simpler in this language.

The class of theories which can be constructed along the lines we shall describe is very large. Since the details of any approximately realistic scheme introduce complications which only obscure the important physics involved, we shall not — in this paper — engage in serious model building. Instead, for pedagogical reasons, we shall first discuss an unrealistic but easily understood $U(1) \otimes U(1)$ model which shows how small "lepton-hadron" couplings can lead to large violations of the "Goldberger-Treiman" relation. The general discussion of this model is given in Section 1 and we put off until Section 3 the exhibition of a Lagrangian model embodying these results. Section 2 is devoted to extending the discussion of Section 1 to a framework for building more realistic models which include the possibility of generating large SU(3) violation in the same way as violations of PCAC.

1. A SIMPLE MODEL

The model we shall discuss in this section describes the coupling of fictional "leptons" and "hadrons." It is contrived to mimic the pertinent features of a more realistic theory. In particular, the model has an exact "Goldberger-Treiman relation" in the limit in which all couplings to "leptons" are turned off. Moreover, as we shall show, very weak coupling of the "lepton world" to the "hadron world" will cause large violations of the corresponding relation for physically determinable coupling constants.

In order to keep our discussion as simple as possible we proceed in three stages. First we give a general characterization of the structure of the "lepton" and "hadron" worlds in the absence of any couplings between them. Next we discuss the important effects which occur when we couple these worlds without the introduction of vector mesons. Finally, we discuss the introduction of vector meson couplings and what happens when the Higgs phenomenon takes place.

A. Totally Symmetric Model

We begin by assuming that the uncoupled "lepton" and "hadron" worlds each separately possess a U(1) symmetry of Goldstone type. More precisely, we assume that in the absence of "lepton-hadron" couplings there exist two conserved commuting axial-vector currents $j_H^{5\mu}(x)$ and $j_L^{5\mu}(x)$. Furthermore, we assume that there exists a "hadronic" pseudoscalar Goldstone boson, denoted by $|\pi'\rangle$, and a "leptonic" Goldstone boson, $|\chi'\rangle$ such that

$$\langle \pi' | j_H^{5\mu}(0) | 0 \rangle = -iq^\mu f_{\pi'}^{(0)} \tag{1}$$

$$\langle \chi' | j_L^{5\mu}(0) | 0 \rangle = -iq^\mu f_{\chi'}^{(0)} \tag{2}$$

$$\langle \chi' | j_H^{5\mu}(0) | 0 \rangle = \langle \pi' | j_L^{5\mu}(0) | 0 \rangle = 0 . \tag{3}$$

The assumption that $\partial_\mu j_H^{5\mu} = \partial_\mu j_L^{5\mu} = 0$ combines with Eqs. (1) and (2) to force $m_{\pi'}^2 = m_{\chi'}^2 = 0$. Note, nothing keeps $|\pi'\rangle$ and $|\chi'\rangle$ from being composite particles (although in the model Lagrangian of Section 3 we shall use elementary π''s and χ''s).

If we now assume that the "hadron world" possesses "spin-$\frac{1}{2}$ nucleon" states, denoted by $|N\rangle$, and the "lepton world" has "spin-$\frac{1}{2}$ lepton" states, $|\ell\rangle$, then in the usual way derive exact Goldberger-Treiman like relations of the form

$$f_{\pi'}^{(0)} G_{\pi' NN}^{(0)} = m_N^{(0)} g_{ANN}^{(0)} \tag{4}$$

$$f_{\chi'}^{(0)} G_{\chi' \ell\ell}^{(0)} = m_\ell^{(0)} g_{A\ell\ell}^{(0)} , \tag{5}$$

$G_{\pi' NN}^{(0)}$ and $G_{\chi' \ell\ell}^{(0)}$, in Eqs. (4) and (5), denote the π' and χ' coupling constants to "nucleons" and "leptons" respectively, in the absence of any "lepton-hadron" coupling. Similarly, $g_{ANN}^{(0)}$ and $g_{A\ell\ell}^{(0)}$ are the axial vector form factors $g_{A\ell\ell}^{(0)}(q^2)$ and $g_{ANN}^{(0)}(q^2)$, defined in Eqs. (6) and (7), evaluated at $q^2 = 0$.

$$\langle N | j_H^{5\mu}(0) | N \rangle = \tfrac{1}{2} \bar{u}_N(p') \left\{ \gamma^5 \left(\gamma^\mu g_{ANN}^{(0)}(q^2) + q^\mu h_{ANN}(q^2) \right) \right\} u_N(p) \tag{6}$$

$$\langle \ell | j_\ell^{5\mu}(0) | \ell \rangle = \tfrac{1}{2} \bar{u}_\ell(p') \left\{ \gamma^2 \left(\gamma^\mu g_{A\ell\ell}^{(0)}(q^2) + q^\mu h_{A\ell\ell}(q^2) \right) \right\} u_\ell(p') \tag{7}$$

B. Coupling "Leptons" to "Hadrons"

Let us now consider what happens if we couple "hadrons" and "leptons" together by a set of small coupling constants $\{g_i\}$ in such a way that only the symmetry of "hadrons + leptons" under U(1) transformations is preserved.

First let us make a remark about the uncoupled case. If we define the states

$$| \chi > = + \cos\theta^{(0)} | \chi' > + \sin\theta^{(0)} | \pi' > \tag{8}$$

$$| \pi > = \cos\theta^{(0)} | \pi' > - \sin\theta^{(0)} | \chi' > \tag{9}$$

where $\cos\theta^{(0)} = f^{(0)}_{\chi'}/f^{(0)}$, $\sin\theta^{(0)} = f^{(0)}_{\pi'}/f^{(0)}$, and $f'^{(0)} = \sqrt{f^2_{\pi'} + f^2_{\chi'}}$, then Eqs. (1) and (2) tell us

$$< \chi | j^{5\mu}_H + j^{5\mu}_L | 0 > = -iq_\mu f^{(0)}, \quad < \pi | j^{5\mu}_H - j^{5\mu}_L | 0 > = 0 \tag{10}$$

$$< \pi | j^{5\mu}_H - j^{5\mu}_L | 0 > = -iq^\mu (2\cos\theta^{(0)}) f^{(0)}_\pi \tag{11}$$

$$< \chi | j^{5}_H - j^{5\mu}_L | 0 > = -iq^\mu \left(f^{(0)}_\chi \cos\theta^{(0)} - f^{(0)}_\pi \sin\theta^{(0)} \right) \tag{12}$$

Equations (10), (11), and (12) make it obvious that it is the invariance under combined U(1) rotations of "leptons" and "hadrons" which forces $| \chi >$ to remain massless; whereas, $| \pi >$ stays massless only if the difference current is conserved. It therefore follows that if we choose to lock the "lepton" and "hadron" worlds together so that the only conserved current is $j^{5\mu}_{H+L} \equiv j^{5\mu}_H + g^{5\mu}_L$, the world continues to possess one Goldstone boson $| \chi \{g_i\} >$ which is the limit $\{g_i\} \to 0$ goes smoothly over to $| \chi >$, and a massive particle

$|\,\pi(g_i)>$ which goes smoothly to $|\,\pi>$. Note, it is the relative size of $f_\pi^{(0)}$ and $f_\chi^{(0)}$ which essentially defines the correct states to use in any perturbation expansion and not the sizes of the $\{g_i\}$ or the details of the couplings involved.

Keeping this fact in mind, we also note that the Goldstone particle $|\,\chi>$ will satisfy a pair of exact Goldberger-Treiman like relations

$$f\,G_{\chi NN} = +m_N g_{ANN} \tag{13}$$

$$f\,G_{\chi \ell\ell} = +m_\ell g_{A\ell\ell}\,. \tag{14}$$

where $G_{\chi NN}$, g, g_{ANN}, etc., can be chosen to behave smoothly in the g_i's. In particular, we assume that as the g_i's tend to zero we have

$$f \to f^{(0)} + \theta(g_i) \tag{15}$$

$$g_{ANN} \to g^{(0)}_{ANN} + \theta(g_i) \tag{16}$$

$$g_{A\ell\ell} \to g^{(0)}_{A\ell\ell} + \theta(g_i) \tag{17}$$

and that there exists a g_i-dependent angle θ such that

$$G_{\chi NN} \to \cos\theta\, g^{(0)}_{\chi' NN} + \sin\theta\, G^{(0)}_{\pi'\ell\ell} + \theta\left\{g_i^2\right\} \tag{18}$$

$$G_{\chi\ell\ell} \to \cos\theta\, G^{(0)}_{\chi'\ell\ell} + \sin\theta\, g_{\pi'\ell\ell} + \theta\left\{g_i^2\right\}. \tag{19}$$

Note, that although we shall assume $\cos\theta \to f^{(0)}_{\chi'}/\sqrt{f^{(0)2}_{\chi'} + f^{(0)2}_{\pi'}}$ and $\sin\theta \to f^{(0)}_{\pi'}/\sqrt{f^{(0)2}_{\chi'} + f^{(0)2}_{\pi'}}$ it is entirely possible that in the case $f^{(0)}_{\chi'} >> f^{(0)}_{\pi'}$, $\sin\theta$ (which will be $<< 1$) can vary significantly over the desired range of

$\left\{g_i\text{'s}\right\}$ (say by a factor of 2). However, we shall see that if we take this into account by allowing θ to be a defining parameter of the coupled scheme then all important results become independent of this fact.

C. Adding Vector Mesons

What we have described to this point is the general structure of a theory of "hadrons" and "leptons" which is invariant under a simultaneous U(1) transformation. Due to this symmetry there is still one Goldstone boson, $|\chi\left\{g_i\right\}>$, and so we do not have a scheme which looks like a possible real world. This will now be taken care of by introducing a set of assumptions equivalent, for our purposes, to assuming that at the Lagrangian level we have extended the remaining U(1) symmetry to a gauge symmetry. The Higgs phenomenon in our general scheme will amount to the assumption that we have a massive vector meson, W^μ, coupled to "nucleons" and "leptons" by a universal coupling of the form $gW_\mu\, j^{5\mu}_{H+L}$. We assume that in general

$$m_W^2 \cong g^2 f^2 \tag{20}$$

and that "nucleon beta decay" goes primarily through W^μ exchange and so we can define a "weak coupling constant", G_W, associated with W^μ exchanges as

$$G_W \cong g^2/m_W^2 \cong 1/f^2\ . \tag{21}$$

We complete our definition of this scheme by assuming that the χ's Goldberger-Treiman like relations change smoothly in g and that Eqs. (15) through (19) are correct perturbation theoretic statements if we include g among the small constants $\left\{g_i\right\}$.

D. The "Real World's" Goldberger-Treiman Relation

Within the spirit of this approach we have

$$G_{\pi NN} \cong \cos\theta\, G^{(0)}_{\pi' NN} - \sin\theta\, G^{(0)}_{\chi' NN}\,. \tag{22}$$

Moreover, by definition $< \pi \mid j^{5\mu}_{H+L} \mid 0 > \; = \; 0$ implies that the process $\pi \rightarrow \ell + \ell$ cannot proceed by direct W^{μ} exchange of the form $\pi \rightarrow W^{\mu} \rightarrow \ell + \ell$. Hence, only direct π - lepton couplings and couplings induced by the fact that π is partially a χ' contribute to this process. Thus, we define

$$f_{\pi} \cong - \frac{1}{m_{\ell} G_W g_{A\ell\ell}} \left\{ \cos\theta\, g^{(0)}_{\pi'\ell\ell} - \sin\theta\, G^{(0)}_{\chi'\ell\ell} \right\}. \tag{23}$$

Equations (13), (14), (18) and (19) now yield

$$G^{(0)}_{\pi' NN} \cong (m_N g_{ANN} - f\cos\theta\, g_{\chi' NN})/f\sin\theta \tag{24}$$

$$G^{(0)}_{\chi'\ell\ell} \cong (m_{\ell} g^{(0)}_{A\ell\ell} - f^{(0)} \sin\theta\, g_{\pi'\ell\ell})/f\cos\theta \tag{25}$$

and so

$$G_{\pi NN} \cong \frac{\cos\theta}{f\sin\theta} \left[m_N g_{ANN} - \frac{f g_{\chi' NN}}{\cos\theta} \right] \tag{26}$$

and

$$f_{\pi} = \frac{(f\sin\theta)}{G_W f^2 \cos\theta} \left[1 - \frac{g_{\pi'\ell\ell}}{G_W m_{\ell} f\, g^{(0)}_{A\ell\ell} \sin\theta} \right] \tag{27}$$

Finally, using $f^2 G_W \cong 1$ we obtain

$$f_\pi G_{\pi NN} \cong \left[1 - \frac{g_{\pi' \ell\ell}}{G_W m_\ell f g_{A\ell\ell} \sin\theta}\right] \left[m_N g_{ANN} - \frac{g_{\chi' NN}}{(G_W)^{\frac{1}{2}} \cos\theta}\right] \tag{28}$$

Clearly the product of the leading terms in these expressions gives the usual PCAC result, $f_\pi G_{\pi NN} \cong m_N g_{ANN}$, and the remaining terms provide us with corrections to this relation. Assuming $G_W \approx 10^{-5}/m_N \approx 2 \times 10^{-7}/m_\pi^2$, $\sin\theta\, f \approx m_\pi$, $g_{A\ell\ell}^{(0)} \approx 1$ and $m_\ell \approx m_\pi/200$ we see that even if $|g_{\pi' NN}| \approx |g_{\chi' NN}| \approx 10^{-10}$ we still expect ten percent violations of the "Goldberger-Treiman" relation. (Note that in this case the $g_{\chi' NN}$ corrections, which correspond to nucleon mass shifts, will be negligible; but, there is no a priori reason to force this to be true.) Also note that our final result is independent of the fact that there can be significant fractional variations in $\sin\theta$ as we promised.

Summarizing, we note that the two important features of our model leading to the final result are (1) the way in which the large quantity $f = (G_W)^{-\frac{1}{2}}$ enters multiplying small quantities, and (2) the assumption that when the Higgs phenomenon takes place its only important effect is to remove the χ particle from on-shell states; it is important that it does not, except in a smooth way, affect relations following from the Goldstone nature of the coupled theory. All of these assumptions can be shown to hold in the correct renormalized perturbation theory of the model given in Section 3, as well as other models of this type.

2. SOME GENERAL REMARKS

We will now proceed to generalize our discussion and define a scheme having essentially the same structure. The defining assumptions of this scheme will be stated in terms of currents, their algebra and perturbation hypotheses; in the kind of situation we envisage these assumptions would replace the usual calculational techniques associated with the Gell-Mann current algebra plus a perturbation approach to PCAC.

To begin with, let us note that the simple $U(1) \otimes U(1)$ model already discussed suggests strongly that, for at least a large class of gauge theories, the important aspect of the theory will be the general way in which the separate "lepton-hadron" Goldstone symmetries coalesce to give a single combined symmetry scheme of Goldstone type. In these worlds, insofar as the symmetry properties of the theory are concerned, the only real importance of the Higgs phenomenon is that it provides us with a mechanism for eliminating whatever unwanted Goldstone bosons we happen to have. Of course, the resulting massive vector mesons must be considered when we try to make contact with usual theories of weak interactions, but that proceeds along the lines sketched in Section 1.

As we have already seen, at least some interesting results follow on the grounds of general hypotheses of this sort, without the necessity for believing specific Lagrangian models or engaging in complicated calculations. The same techniques readily extend to the general case and can be used to discuss violations of other PCAC identities such as the Adler self-consistency conditions, pion-nucleon scattering lengths, $\pi^0 \to 2\gamma$, etc. Details of how to do this and discussion of how one can use experimental information obtained from low energy hadronic processes to place limitations upon the possible structure of the lepton world will be discussed in another paper. In this paper we will be content to describe the general features which will be true for any scheme of this type.

Once again it is simplest to describe this scheme in two steps. First we shall describe the presumed structure of uncoupled worlds of hadrons and leptons, and then we shall discuss the general effects which occur due to coupling these worlds.

A. Uncoupled Worlds

The most general case we wish to consider is one in which the lepton and hadron worlds possess separate isomorphic chiral algebras of currents G_H and G_L. Although, we force the lepton and hadron current algebras to be the same we do allow the amount of spontaneous symmetry breaking in these worlds to be different. To be specific, we assume that there are subalgebras, $N_H \subset G_H$ and $N_L \subset G_L$, of currents whose charges annihilate the lepton and hadron vacua. [e.g. we might assume that $G_H \cong G_L \cong SU(3) \otimes SU(3)$ and that N_H is the SU(3) subalgebra defined by the hadronic vector currents and N_L is the one dimensional algebra consistent of the usual leptonic electromagnetic current.] The remaining conserved currents are assumed not to annihilate the vacuum.

This last hypothesis is precisely stated in the following way. Let us denote by $j^{\mu}_{H\alpha}$ ($\alpha = 1, \ldots, n$) a basis of G_H, the first m- of which are a basis for N_H; and let us denote by $j^{\mu}_{L\alpha}$ ($\alpha = 1, \ldots, n$) a basis for G_L, the first m' of which are a basis for N_L. We then suppose that in the hadronic world there are (n-m) hadronic Goldstone bosons, $|\pi'_\alpha>$ coupled by the currents $j^\mu_{H\alpha}$ $(\alpha = m+1, \ldots, n)$ to vacuum. In the same way, we assume there exist (n-m') Goldstone bosons in the lepton world, $|\chi'_\alpha>$, coupled by the currents $j^\mu_{L,\alpha}$ $(\alpha = m'+1, \ldots, n)$ to vacuum. In the totally symmetrical case we can classify the currents and the goldstone bosons into irreducible representations of N_H and N_L and if we let $|\pi^{(\sigma)}_\alpha>$ stand for a boson belonging to a representation of N_H of type 'σ' etc., we have in the totally symmetrical case

$$< \pi'^{(\sigma_1)}_\alpha | j^{(\sigma_2)\mu}_{H\beta} | 0 > = - iq^\mu \delta_{\sigma_1\sigma_2} \delta_{\alpha\beta} f^{(\sigma_1)}_{\pi'} \tag{29}$$

and

$$< \chi'^{(\sigma_1)}_\alpha | j^{(\sigma_2)\mu}_{L\beta} | 0 > = - iq^\mu \delta_{\sigma_1\sigma_2} \delta_{\alpha\beta} f^{(\sigma_1)}_{\chi'} \quad . \tag{30}$$

As in the U(1) × U(1) case, in the absence of hadron-lepton couplings the hadronic and leptonic Goldstone bosons satisfy exact Goldberger-Treiman like relations and exact low-energy theorems.

B. Coupling Leptons and Hadrons

If one turns on even extremely weak couplings between the hadron and lepton worlds — without introducing gauge mesons — there is a mixing of hadronic and leptonic Goldstone bosons $|\pi'_\alpha>$ and $|\chi'_\beta>$ the strength of which has nothing to do with the strength of the couplings between the worlds. Instead, the important quantities which determine the nature of this mixing are the constants $f^{(\sigma)}_{\pi'}$, $f^{(\sigma)}_{\chi'}$ and the way in which the G_{H+L} conserved currents go over to sums of $j^\mu_{H\alpha}$ and $j^\mu_{L\alpha}$ in the limit of vanishing lepton-hadron couplings. We can always choose the currents $j^\mu_{H+L,\alpha}$ so that they go over to definite hadronic currents, $j^{(\sigma)\mu}_{H\alpha}$, labeled by their N_H quantum numbers. In that event, the general situation we describe corresponds to the specific assumption

$$j^{\mu}_{H+L,\alpha} \rightarrow j^{\mu}_{H,\alpha} + X_{\alpha\beta} j^{\mu}_{L,\beta} \tag{31}$$

in the limit of vanishing lepton-hadron couplings. It is then clear that one can define a set of mixed Goldstone boson states $|X_\alpha>$ and $|\Pi_\alpha>$ such that the χ_α's stay massless as a consequence of the conservation of $j^{\mu}_{H+L,\alpha}$'s and the others require a mass as lepton-hadron couplings are turned on (this is not always a very small mass due to the large vacuum expectation values involved as is obvious in the Lagrangian discussion given in Section 3).

As in the $U(1)\otimes U(1)$ case, the particles which remain Goldstone bosons in the presence of couplings satisfy Goldberger-Treiman like relations, etc. which one can expect to behave smoothly as couplings to vector mesons are turned on.

Clearly, the number of Goldstone bosons in the coupled world will be at most n and at least max (m', m). Therefore, if we specify $X_{\alpha\beta}$, we have gone a long way towards constructing a general model.

If we now consider what happens when the Higgs phenomenon takes place we note that this amounts to introducing a set of gauge bosons W^{μ}_{α} coupled to the j^{μ}_{H+L} currents in the form $g W^{\mu}_{\alpha} j^{\alpha}_{H+L,\mu}$. The basic defining hypotheses which must be made are (1) that the W^{μ}_{α}'s have a mass matrix given by[6]

$$(M_W^2)_{\alpha\beta} = \sum_\delta g^2 < 0 | \tilde{j}_{H+L,\,\alpha} | \chi_\delta > < \chi_\delta | \tilde{j}_{H+L,\,\beta} | 0 > \tag{32}$$

where the states $| \chi_\delta >$ mean we sum over all remaining Goldstone boson states, and the $\sim$ indicates that we drop the factors of $(-iq^\mu)$ appearing in the formulae analogous to that given in Eq. (10).

This, of course, will mean that there will be as many massive vector bosons as there are Goldstone bosons $| \chi_\delta >$ and that they will fall into irreducible representations of whatever normal symmetries remain.

We would like to conclude this general description by pointing out that these hypotheses are in complete accord with the results of renormalizable Lagrangian field theories. Moreoever, once one departs from Abelian models (such as our $U(1) \otimes U(1)$ model) then the introduction of vector meson exchanges between the leptonic and hadronic worlds automatically implies — at the Lagrangian level — Goldstone boson exchanges between the two worlds. This will be true because in the non-Abelian case the vector mesons themselves transform under constant G_{H+L} transformations and so they automatically provide a mechanism for locking the lepton-hadron vacua together. (N. B. in our $U(1) \otimes U(1)$ case this was not true.) The terms in the Lagrangian corresponding to direct Goldstone exchange arise automatically as renormalization counterterms. Hence, one way of looking at the successive breaking scheme we have described is to say that insofar as the low energy structure of the theory is concerned it is the set of renormalization counterterms which play the key role. This is not a totally uninteresting remark, since it points up the fact that the very small couplings corresponding to the Goldstone boson exchange could be thought of as due entirely to second order weak effects.

3. A SIMPLE LAGRANGIAN MODEL

The purpose of this section is entirely pedagogical and it is unnecessary in order to understand the arguments given in the previous sections. Nevertheless, for those unfamiliar with the way the Higgs phenomenon takes place and/or the way the Goldberger-Treiman relation comes about in model Lagrangians, this section provides a brief discussion of all these points. The total Lagrangian we shall consider is

$$
\begin{aligned}
\mathscr{L} = \; & \overline{\psi}_N (i\not{\partial} + g\not{W})\psi_N - G_{\pi NN}\overline{\psi}_N(\sigma' + i\gamma_5\pi')\psi_N \\
& + \tfrac{1}{2}\left[\left((\partial^\mu + igW^\mu)(\sigma' - i\pi')\right)\left((\partial_\mu + igW_\mu)(\sigma' + i\pi')\right)\right] \\
& - B^2(\sigma'^2 + \pi'^2 - f_\pi^{(0)2}) \\
& + \overline{\psi}_\ell (i\not{\partial} + g\not{W})\psi_\ell - G_{\chi\ell\ell}\overline{\psi}_\ell(\phi' + i\gamma_5\chi')\psi_\ell \\
& + \tfrac{1}{2}\left[\left((\partial^\mu + igW^\mu)(\phi' - i\chi')\right)\left((\partial_\mu - igW_\mu)(\phi' + i\chi')\right)\right] \\
& - C^2(\phi'^2 + \chi'^2 - f_\chi^{(0)2}) - \tfrac{1}{4}(\partial_\mu W_\nu - \partial_\nu W_\mu)^2 - \frac{1}{Z\alpha}(\partial_\mu W^\mu)^2 \\
& - g_{\chi NN}\overline{\psi}_N(\phi' + i\gamma_5\chi')\psi_N - g_{\pi\ell\ell}\overline{\psi}_\ell(\sigma' + i\gamma_5\pi')\psi_\ell \\
& + \epsilon(\sigma'\phi' + \pi'\chi')^2 .
\end{aligned}
\tag{33}
$$

This Lagrangian is not the most general one we should write down if we were really going to pay attention to giving a correct prescription for renormalizing the theory defined by it; however, since most of the really important points to be made can be heuristically arrived at on the basis of semi-classical arguments, we shall ignore the niceties.

Clearly under these conditions even this simple Lagrangian seems formidable enough. In order to simplify our discussion let us approach it in three stages.

First, let us consider the case $g = \epsilon = g_{\chi NN} = g_{\pi \ell\ell} = 0$. In that event $\mathscr{L}_{tot}$ decomposes into

$$\begin{aligned} \mathscr{L}_H = & \bar{\psi}_N (i\not\partial)\psi_N - G_{\pi NN}\bar{\psi}_N(\sigma' + i\gamma_5\pi')\psi_N \\ & + \tfrac{1}{2}(\partial_\mu \sigma')^2 + \tfrac{1}{2}(\partial_\mu \pi')^2 - B^2(\sigma'^2 + \pi'^2 - f_\pi^{(0)2})^2 \end{aligned} \tag{34}$$

and

$$\begin{aligned} \mathscr{L}_L = & \bar{\psi}_L (i\not\partial)\psi_L - G_{\chi\ell\ell}\bar{\psi}_\ell(\phi' + i\gamma_5\chi')\psi_\ell \\ & + \tfrac{1}{2}(\partial_\mu \phi')^2 + \tfrac{1}{2}(\partial_\mu \chi')^2 - C^2(\phi'^2 + \chi'^2 - f_\chi^{(0)2})^2 . \end{aligned} \tag{35}$$

Following the usual semi-classical argument we observe that in order to define a perturbation theory using fields whose vacuum expectation value is zero, we should rewrite $\mathscr{L}_H$ so that the fields involved are the ones for which the potentials $B^2(\sigma^2 + \pi^2 - f_\pi^{(0)2})^2$ and $C^2(\phi^2 + \chi^2 - f_\chi^{(0)2})^2$ have minima for zero values of the fields. This is accomplished in the usual manner by defining

$$\sigma' = \sigma + f_\pi^{(0)} ; \quad \phi' = \phi + f_\chi^{(0)} \tag{36}$$

and rewriting Eqs. (34) and (35) as

$$\mathscr{L}_H = \overline{\psi}_N (i\not\partial - G^{(0)}_{\pi NN} f^{(0)}_\pi)\psi_N - G^{(0)}_{\pi NN}\overline{\psi}_N(\sigma + i\gamma_5\pi')\psi_N$$
$$+ \tfrac{1}{2}\left[(\partial_\mu\sigma)^2 - 8B^2 f_\pi^{(0)2}\sigma^2\right] + \tfrac{1}{2}(\partial_\mu\pi')^2 - B^2(\sigma^2 + \pi'^2) - 4B^2 f_\pi^{(0)}\sigma(\sigma^2 + \pi'^2) \tag{37}$$

and

$$\mathscr{L}_L = \overline{\psi}_L (i\not\partial - G_{\chi\ell\ell} f^{(0)}_\chi)\psi_\ell - G_{\chi\ell\ell}\overline{\psi}_\ell(\phi + i\gamma_5\chi')\psi_\ell$$
$$+ \tfrac{1}{2}\left[(\partial_\mu\phi)^2 - 8C^2 f_\chi^{(0)}\phi^2\right] + \tfrac{1}{2}(\partial_\mu\chi)^2 - C^2(\phi^2 + \chi'^2) - 4C^2 f_\chi^{(0)}\phi(\phi^2 + \chi'^2) \tag{38}$$

Quantizing these Lagrangians leads to a hadron world with a massless π and a massive σ and a Goldberger-Treiman relation $m_N^{(0)} = G^{(0)}_{\pi NN} f_\pi^{(0)}$. Similarly for the lepton world.

If we now consider what happens if we still let $g = 0$ but allow all other mixed coupling constants to be finite, we see that the proper potential to minimize is

$$B^2(\sigma'^2 + \pi^2 - f_\pi^{(0)2})^2 + C^2(\phi'^2 + \chi^2 - f_\chi^{(0)2}) - \epsilon\ (\sigma'\phi' + \pi\chi)^2 .$$

It is trivial to show that we can solve this by letting $\pi = \chi = 0$ and defining $\sigma' = \sigma + f_\pi$ and $\phi' = \phi + f_\chi$ so that at the minimum $\sigma = \phi = 0$. For the case $f_\chi^{(0)} >> f_\pi^{(0)}$ it is easy to show that f_χ is very nearly $f_\chi^{(0)}$ but that f_π can be appreciably different from $f_\pi^{(0)}$. Nevertheless, if we rewrite Eq. (33) with $g = 0$, σ and ϕ replacing σ' and ϕ' and defining

$$\pi = \cos\theta\, \pi' - \sin\theta\, \chi' \quad ; \qquad \chi = \cos\theta\, \chi' + \sin\theta\, \pi' \tag{39}$$

where $\cos\theta = f_\chi/\sqrt{f_\pi^2 + f_\chi^2}$ and $\sin\theta = f_\pi/\sqrt{f_\chi^2 + f_\pi^2}$, we find that we have a quantizable Lagrangian for which χ is a massless field, π describes a field whose mass is approximately $m_\pi^2 \approx 2(\epsilon f_\chi^2) \approx \frac{2\epsilon}{G_W} \approx 1.2\epsilon \times 10^7 m_\pi^2$. Hence, as before $\epsilon \approx 10^{-7}$ gives reasonable values for the π field mass. Furthermore, we see that

$$m_N = f_\pi G_{\pi NN} + f_\chi g_{\chi NN} = f(\sin\theta\, G_{\pi NN} + \cos\theta\, g_{\chi NN}) \tag{40}$$

and

$$m_\ell = f_\chi G_{\chi\ell\ell} + f_\pi g_{\pi\ell\ell} = f(\cos\theta\, G_{\chi\ell\ell} + \sin\theta\, g_{\pi\ell\ell}) , \tag{41}$$

which is what we obtained from general arguments. (Recall the f_π in Eq. (39) corresponds to $f\sin\theta$ in our general discussion and not the experimentally determined π-decay constant.)

It is a simple exercise to convince oneself that letting $g \neq 0$ just gives a W meson of mass $m_W^2 = g^2 f^2$ and since the minimization problem is unchanged (except to higher orders in g) one obtains essentially the same Goldberger-Treiman relation given in Eq. (40).

CONCLUSION

At this point we would like to add a few remarks concerning our general discussion and point out directions in which this work might be extended. As we have already stated, there appear to be a large class of theories for which all hadronic symmetry breaking can be reasonably assumed to be due to weak couplings of hadrons to leptons. Moreover, we have conjectured that as a general feature of such theories it is the Goldstone nature of the decoupled leptonic and hadronic worlds which govern these effects and not the details of the coupling scheme involved. We believe one of the important advantages of proceeding to explore these questions in the general manner we have outlined is that one does not have to worry whether or not the Goldstone bosons involved are to be thought of as elementary or composite. Hopefully, even if one could construct a solvable model of quarks binding to form a set of hadronic Goldstone particles, the general features of the physics involved in coupling these to leptons would be the same. Another point worth mentioning is that identities based upon the low energy theorems for the Goldstone bosons of the coupled lepton-hadron worlds would have to be free of anomalies. At the Lagrangian level this is essential in order to be able to carry through the renormalization program in gauge theories. For models this can be accomplished either by cancelling lepton anomalies against hadron anomalies or by basing models upon anomaly free symmetry schemes. Still one more point that is worth speculating upon is that if one assumes in the lepton world all but a simple U(1) subsymmetry is of the Goldstone type, one will automatically generate an isospin violating piece of the strong interactions. Presumably, the largest effect of these terms will be seen in the meson mass spectrum and it might provide us with a completely self consistent way around the failure of Dashen's sum rule for the electromagnetic mass differences of the pseudoscalar mesons, as well as a different way of treating processes such as $\pi^0 \rightarrow 2\gamma$ and $\eta \rightarrow 3\pi$.

FOOTNOTES

1. The original suggestion that PCAC is related to a slightly broken chiral symmetry is due to Nambu and his collaborators [see Y. Nambu and D. Lurie, Phys. Rev. 125, 1429 (1962) and earlier papers cited therein]. The first paper suggesting the relation between modern work on current algebra and a chiral symmetry scheme seems to be S. Weinberg, Phys. Rev. Letters 16, 163 (1966). An extensive list of later references on SU(3) × SU(3) is contained in Weinberg's report in Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968, edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968), p. 253.

2. More detailed discussion of how conventional ideas of PCAC are related to a perturbative approach about a Goldstone symmetry limit can be found in R. Dashen, Phys. Rev. 183, 1245 (1969); R. Dashen and M. Weinstein, Phys. Rev. 183, 1261 (1969); R. Dashen and M. Weinstein, Phys. Rev. 188, 2331 (1969).

3. The original discussion of the mechanism of eliminating unwanted Goldstone bosons from a theory by adding vector mesons is P. W. Higgs, Phys. Rev. 145, 1156 (1966) and the extension of these ideas to the non-Abelian case was discussed by T. W. B. Kibble, Phys. Rev. 155, 1554 (1967). The extension of these ideas to a discussion of weak interactions of leptons is discussed in S. Weinberg, Phys. Rev. Letters 19, 1264 (1967).

4. An extensive list of papers realted to such theories can be found in J. D. Bjorken et al., SLAC-PUB-1107, September 1972 (submitted to Phys. Rev.) and T. Hagiwara and B. W. Lee, Stoney Brook preprint October 1972.

5. Some of the general points made in this paper have been independently discussed in the context of a specific model in the paper by T. Hagiwara and B. W. Lee in reference 4. Also, some features of this approach apply to the scheme given by I. Bars, M. B. Halpern and M. Yoshimura, Phys. Ref. D6, 696 (1972) and in Department of Physics and Lawrence Berkeley Laboratory preprint (October 13, 1972).

6. This way of defining a generalized Higgs mechanism, although correct, is — to our mind — lacking in elegance. It would be much nicer if one could give a different, more directly intuitive set of assumptions from which Eq. (32) follows directly. One way of deriving Eq. (32) based upon a set of assumptions which are not more economical than the ones we make in Section 3 is the following. Define $j^{\mu}_{H+L+W,\alpha} \cong j^{\mu}_{H+L,\alpha} + \frac{(m^2_W)_{\alpha\beta}}{g} W^{\mu}_{\beta}$ (where $(m^2_W)_{\alpha\beta}$ is the W-meson (mass)^2 matrix. Assume

$$\lim_{g\to 0} (m^2_W)_{\alpha\beta}/g \to 0 \tag{1}$$

$$< \chi_\alpha | j^{\nu}_{H+L+W,\beta} | 0 > \;=\; -iq^{\nu} < X_\alpha | \tilde{j}^{\nu}_{H+L,\beta} | 0 > + \,\theta(g) \tag{2}$$

$$< W^{\mu}_{\alpha} | j^{\nu}_{H+L+W} | 0 > \;=\; \frac{-i}{g} \left\{ (m^2_W)_{\alpha\beta}\, g^{\mu\nu} - k^{\mu}k^{\nu} \right\} . \tag{3}$$

The desired result follows from

$$< 0 | T\left(j^{\mu}_{H+L+W,\alpha}(k)\, j^{\nu}_{H+L+W,\beta}(-k)\right) | 0 > \;=$$

$$= iq^{\mu}q^{\nu} \left(\sum_{\delta} \frac{< 0 | \tilde{j}_{H+L,\alpha} | X_\delta > < X_\delta | \tilde{j}_{H+L,\beta} | 0 >}{q^2} \right)$$

$$+ \frac{1}{g^2} \left((m_W)^2_{\alpha_\sigma}\, g^{\mu\lambda} - k^{\mu}k^{\lambda} \right) \frac{i}{(k^2 - m^2_{W\,\sigma\rho})} \left((m_W)^2_{\rho\beta}\, g^{\lambda\nu} - k^{\lambda}k^{\nu} \right)$$

$$+ \,\theta(g)$$

N. B. the first two terms for the right hand side of this are of order unity if $m_W^2 \approx g^2$.

Taking the divergence of both sides and <u>assuming</u> $\partial_\mu j^\mu_{H+L+W}$ is of order "g" we obtain

$$\theta_1^\mu(g) = iq^\nu \left[\sum_\delta < 0 \,|\, \tilde{j}_{H+L+W,\alpha} \,|\, X_\delta > < X_\delta \,|\, \tilde{j}_{H+L+W,\beta} \,|\, 0 > - \frac{1}{g^2} (m_W^2)_{\alpha\beta} \right] + \theta_2^\mu(g) \, .$$

Cancelling terms of order unity gives the desired relation.